PREDICTION OF THE DURATION OF AN EVENT

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Abstract. Given an event that has lasted an amount of time $x > 0$, we give an estimate (in the absence of more information) of the possible duration of the event. The estimated duration is $e \cdot x$.

The purpose of this note is to give an estimation of the duration of an event. Suppose we are asked "How long their marriage is going to last?" Who knows, and of course this depends on the couple, their particular situation and their personalities. But if we are not given more information about them? Then the first reaction of a mathematician is to make statistics: take all couples and compute the mean of their duration. If we are told that they have been a couple for some time $x > 0$ until now, how longer it will take the partnership? Then we are into a conditional probabilitiy: taken all couples that have been dating x, what is the average of the duration of their relation? But what if we do not know in which moment of the history is this couple living, or in which region of the world. And what if we are not allowed (or we do not have the chance) to recopilate dates for our statistics? In the absence of all this information we are starting to guess. Sure, there are useful facts: couples do not last more than a lifetime, which has an upper bound of 100 years.

Let us give a further twist. Suppose that we want to buy a criptocurrency. We are afraid that this event (with no similar example in the past) will not last much. In which criptocurrency we could invest? Suppose there are two, like Bitcoin and another less common one like Dogecoin. Which one will last longer? Common sense tells you that the first one. Bitcoin has been here for $x_1 = 14$ years and Dogecoin for $x_2 = 10$ years. Our answer is based on this little input . . .

Beautiful examples are the duration of a religion, or the time existence of a species. For the former, a prophet may pray in front of a crowd of people, and they may pay little attention. This religion can last as long as the speech, or the speaker can make a group of people and they can be preaching their faith for several years. Cases like the Christian religion of the Muslim religion show that it can last for centuries and more. You never know. What you know is that if a religion has been here for a lot of years, it will likely be alive for as many. Similar consideration applies to political ideologies (think of Marxim) or social systems (think of Medieval society), though probably there is not much difference to a religion. The case of how long a species will be on Earth is more illuminating. A species can be around for some thousand years, or for some hundred million years. But it can last for a day or some seconds. Just imagine that one individual has the mutation to be a new species, or several procreate but are soon killed. We cannot detect them because they are not around long enough in such case. What we know (better said, predict) is that a species that has been here for long time (think of an ant) will be for longer. The most that we expect is that the duration will not be more than thousand million years, just because it is the lifetime of the Earth itself. From seconds to thousand million years, the interval is too wide to be helpful.

And . . . what about the duration of a Physics particle? It can be from nanoseconds to the duration of the universe!

These examples show that in the absense of information, we are a bit lost just guessing. The question is: given an event that has already lasted an amount of time $x > 0$ (and not even telling which type of event in particular), what is a reasonable estimate for the duration of the event? In this note we propose a short computation (a model) to give a resonable answer, or in more colloquial words, an educated guess.

Main result: *If an event has lasted* $x > 0$*, and we do not have more information, we expect that it will last* $e \cdot x$*.*

This phenomenom is already mentioned by several authors under the name *Lindy effect*, for instance the newspaper articles [1] and [3], and in Nassim Taleb's famous books "Antifragile" [4] and "The black Swan" [5]. It is supported on the common sense idea that things that have survived long are more durable, and hence will likely live longer. However the Lindy effect was not supported yet by a mathematical computation (though simple it might be), and what it is striking (or not!) is the appearance of the famous mathematical number $e = 2.71812818286\ldots$

Another important caveat is given in the clean statement of the hypothesis of the theorem: "under no extra information". One is tempted to use this kind of results as a hammer applying it everywhere once that one has become aware of it. But in general, in real life, all concrete events come to us with extra information that gives a precisation beyond the blind guess $e \cdot x$.

Now you know how long your marriage is going to last! Unless you have the extra information that you have already decided to ask for divorce next week!

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1. The model and the computation

We start with an event of some duration in time, say $\ell > 0$, that is unknown to us, and we know that the event has lasted a given time $x > 0$.

Let us introduce the random variable X with support in $[0, \ell]$ that tells us that the event is active at a time $y \in [0, \ell]$. With no more information, our best model is in principle that X is an uniform distribution. It makes no sense to write $P(X = x)$ since this probability is zero, because we have a continuous random variable. In reality, we have a small interval around x, of the form $[x, x + h]$ and we deal with the probability $P(X \in [x, x + h])$. Moreover, the time-scale is understood in terms of x. This means that when we say that something has lasted 70 days, it means something like $[70, 71]$ days, and for 70 seconds, it means $[70, 71]$ seconds. In plain terms, we have $h = \varepsilon\, x$ in fact. Thus we can write

$$P(X \in [x, x + \varepsilon\, x]) = \frac{\varepsilon\, x}{\ell}\,.$$

Denote by L the random variable parametrizing the possible values of ℓ. The previous information can be translated to the fact that the probability is

$$P(L \in [\ell, \ell + \varepsilon\, \ell]) = \frac{\varepsilon\, x}{\ell}\,.$$

We again have the uncertainty small interval designed for L, as before. It is in fact a conditional probability (with the condition that the value x is given), but we shall not make this explicit.

What we have to compute is the expentancy value $E[L]$. To make the computation, we introduce the (more natural) random variable $T = \ln(L)$ (here we could use decimal logarithm $T = \log(L) =$

$(\log e)\ln(L)$, but this is just a linear rescalling). Phylosophically, we move the question to a logarithmic scale, which seems natural since the time-units of ℓ are somehow expected to be of the same range as that of x, as the choice of the moment x in the interval $[0,\ell]$ is chosen ramdonly.

We set as well $t=\ln(\ell)$, and we have

$$P(T\in[t,t+\varepsilon])=\varepsilon\, x\, e^{-t}\,,$$

using that $\ln(1+\varepsilon)\approx\varepsilon$ and $\ell^{-1}=e^{-t}$. Now let $F(t)$ the distribution function of T. We have got

$$F(t)=x\,e^{-t}\,,\qquad t\in[\ln(x),\infty).$$

Note that

$$\int_{\ln(x)}^{\infty} x\,e^{-t}\,\mathrm{d}t=\Big[-x\,e^{-t}\Big]_{\ln(x)}^{\infty}=x\,e^{-\ln(x)}=1,$$

as it should be for a distribution function of a probability.

The expectancy value is now

$$E[T]=\int_{\ln(x)}^{\infty} t\,x\,e^{-t}\,\mathrm{d}t=\Big[-x(t+1)e^{-t}\Big]_{\ln(x)}^{\infty}=x(\ln(x)+1)e^{-\ln(x)}=\ln(x)+1.$$

Unravelling, for $L=e^{T}$, we have the expectancy value

$$e^{\ln(x)+1}=e^{\ln(x)}\cdot e=e\cdot x.$$

Remark. It is natural to have a linear relation from x to ℓ. The double is x, the double should be ℓ. What is not tantamount is that the linear factor should be the number e.

References

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