Exotic Quarkonia in Effective field theory

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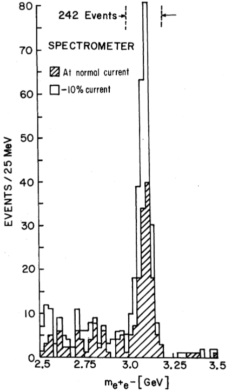
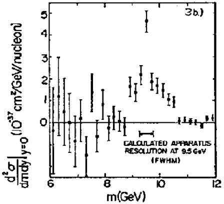
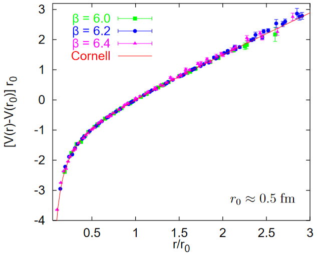




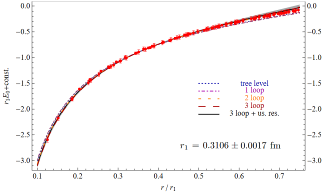
- Quarkonia are bound states of a heavy quark and a heavy anti-quark.
- J/ψ was discovered in Brookhaven and SLAC in 1974 and it was identified as $c\bar{c}$.
- Three years later the Υ was discovered and interpreted as as $b\bar{b}$.

- They were recognized early on as nonrelativistic bound states.
- Can be classified with principal and angular quantum numbers.
- Spectrum can be obtained from the Schrödinger eq. with a suitable potential.
- QCD analog of the Hydrogen atom.

Static potential



Bali Phys.Rept. 343 (2001)

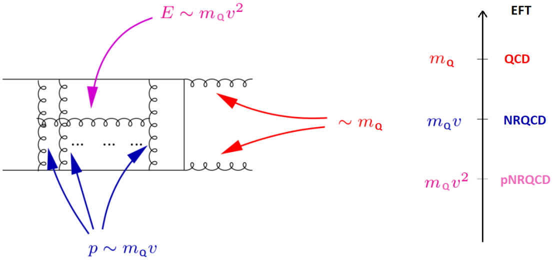


Bazavov et al Phys.Rev.D90 (2014)

- The quark-antiquark potential was computed in perturbation theory.
- Leading order: one gluon exchange. Coulombic potential.
- However, the spectrum is not Coulombic.

- Phenomenological potentials were developed.
- Cornell potential: $a/r + \sigma r + \Lambda$.
- Non perturbative expressions of the potential as a Wilson loop.
- Eventually the potential was obtained from lattice QCD.

How to connect the phenomenological picture to QCD?



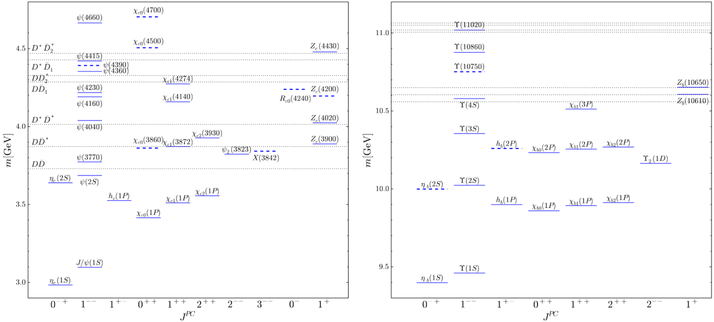
$\Rightarrow$ Build EFTs exploiting $m_Q \gg m_Q v \gg m_Q v^2$ ($v \ll 1$ heavy quark relative velocity)

- NRQCD, integrate out $m_Q \gg \Lambda_{\rm QCD}$.
  - Still two dynamical scales.
- potential NRQCD (pNRQCD), integrate out $mv \sim 1/r$.
  - Schrödinger eq. as eqs. of motion at LO.
  - **Weak coupling**: $m_Q v \gg \Lambda_{QCD} \Leftrightarrow r \ll 1/\Lambda_{QCD}$ (short distance).

    Pineda, Soto Nucl.Phys.Proc.Suppl.64 (1998); Brambilla, Pineda, Soto, Vairo Nucl.Phys.B566 (2000)
  - **Strong coupling**: integrate out $m_Q v$ and Λ_{QCD} simultaneously (any distance).

    Brambilla, Pineda, Soto, Vairo Phys.Rev.D 63 (2001); Pineda, Vairo Phys.Rev.D 63 (2001)

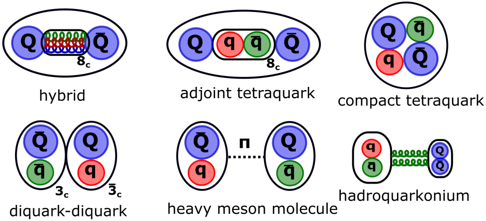
- This picture described the quarkonium spectrum very well and even predicted new states.
- This changed with the discovery of $X(3872)$ in 2003.



- Dozens more unexpected states discovered close and above open flavor thresholds.
- Properties unlike standard quarkonium and/or exotic quantum numbers.

Exotic Quarkonium: Models

- Must contain a heavy quark-antiquark pair and light degrees of freedom (d.o.f).
- Many models have been proposed...but no coherent, comprehensive framework yet.



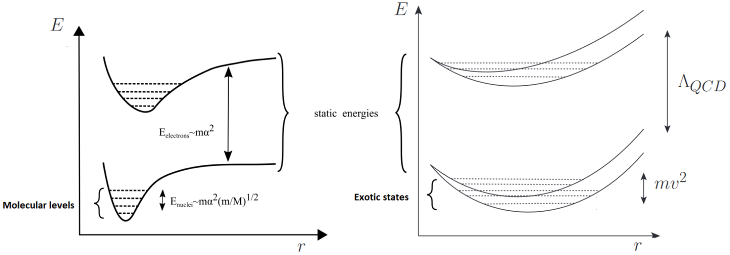
- Major challenge to our understanding of QCD spectrum!

Exotic Quarkonium: Born-Oppenheimer approximation

Adiabatic expansion: $\Lambda_{\rm QCD} \gg m_Q v^2$

- Heavy quarks are nonrelativistic and almost static.
- Light d.o.f adapt almost instantaneously to the heavy quarks.
- Exotic quarkonium are QCD analogs to diatomic molecules.

Brambilla, Krein, JT, Vairo Phys.Rev.D 97 (2018)



Static energies: Energy levels of the light d.o.f for static heavy quarks.

Static energies in NRQCD

- The static energies of a $Q\bar{Q}$ system are defined as the energies of the eigenstates of NRQCD in the static limit.

$$\mathcal{O}^{\dagger}_{\kappa^p \Lambda^\sigma_\eta}(\boldsymbol{r},\, \boldsymbol{R})|0\rangle \sim |\Lambda^\sigma_\eta\rangle^{(0)} + \dots$$

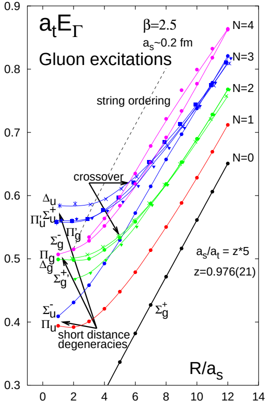
$$E_{\Lambda^\sigma_\eta}(\boldsymbol{r}) = \lim_{t\to\infty} \frac{i}{t} \ln\langle 0|\mathrm{Tr}\left[\mathcal{O}_{\kappa^p \Lambda^\sigma_\eta}(t/2,\, \boldsymbol{r},\, \boldsymbol{R})\mathcal{O}^{\dagger}_{\kappa^p \Lambda^\sigma_\eta}(-t/2,\, \boldsymbol{r},\, \boldsymbol{R})\right]|0\rangle$$

- Nonperturbative quantity.

Static eigenstates quantum numbers:

- $Q\bar{Q}$ distance $\boldsymbol{r}$.
- Light d.o.f quantum numbers: spin κ, parity p, charge conjugation, flavor...
- Representation $\boldsymbol{\Lambda^\sigma_\eta}$ of $D_{\infty\, h}$.
  - Λ rotational quantum number, eigenvalue of $|\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_\kappa|$.
  - η eigenvalue of CP: $g \hat{=} +1$, $u \hat{=} -1$
  - σ eigenvalue of reflections (only $\Lambda = 0$ states)

- **Quenched** lattice NRQCD.

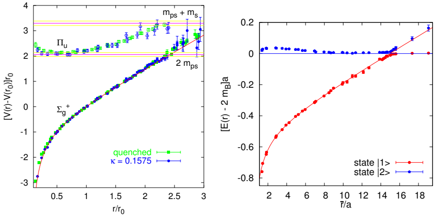


Juge, Kuti, Morningstar Phys.Rev.Lett.90 (2003)

Recent precision computation:

Capitani et al Phys.Rev.D 99 (2019)

SESAM/TCL Col. Phys.Rev.D62 (2000), Phys.Rev.D71 (2005)

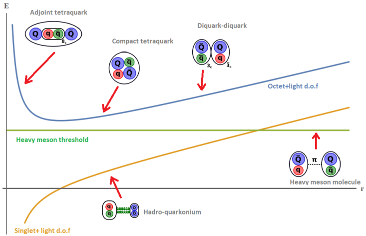


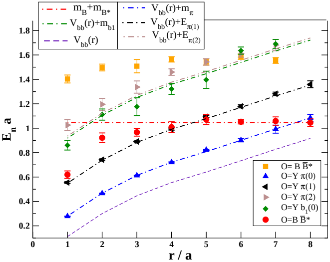
- Short distances: $D_{\infty h} \to O(3) \times C$.
- Long distances: linear (string-like).
- No significant difference between quenched and unquenched results.
- However with dynamic light quarks new states appear such as thresholds.

Static energies for tetraquarks?

Brambilla, Vairo, Polosa, Soto Nucl.Phys.Proc.Suppl.185 (2008); Braaten, Langmack, Smith Phys.Rev.D90 (2014)

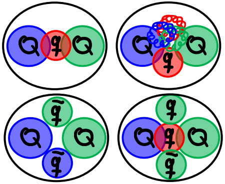
Consider light d.o.f with light-quark flavor.



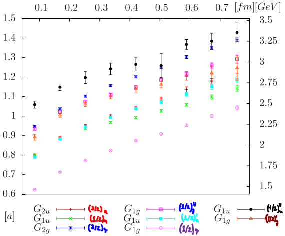


Prelovsek et al Phys.Lett.B 805 (2020)

- However: No lattice data available yet.
- Each model can be though as a different region of the wave function?



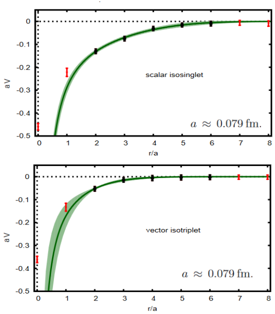
- The same picture also applies to other doubly heavy hadrons: Soto, JT Phys.Rev.D 102 (2020) 1, 014012 *and* 014013
  - Double heavy baryons QQq.
  - Hybrids baryons $QQgq$.
  - Tetraquaks $QQ\bar{q}\bar{q}$.
  - Pentaquaks $QQqq\bar{q}$.
  - ...



QQ static energy for one light quark.

Najjar, Bali PoS LAT2009 (2009)

$N_f = 2$, $a = 0.084$ fm, $L \simeq 1.3$ fm, $m_\pi \simeq 783$ MeV.



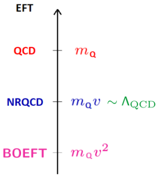
$\bar{Q}\bar{Q}$ static energy for two light quarks.

Bicudo, Wagner Phys.Rev.D87 (2013)

Born-Oppenheimer EFT

- We aim to build an EFT for the heavy quark-quark or heavy quark-antiquark bound states in the minima of the static energies.

- Energy gaps:
  - Heavy quarks are non relativistic $m_Q \gg \Lambda_{\rm QCD}$.
  - Adiabatic expansion $\Lambda_{\rm QCD} \gg m_Q v^2$.



- No assumption on the scale of the interquark distance $r \sim 1/(m_Q v)$ respect to $\Lambda_{\rm QCD}$ as in strongly coupled pNRQCD.

1. Identify the relevant κ^p light d.o.f states for a particular problem.
2. One $\Psi_{\kappa^p} = \Psi^{\alpha\, i}_{\kappa^p}(t, \boldsymbol{r}, \boldsymbol{R})$ field for each light d.o.f state:
   - α light d.o.f spin index $\alpha = -\kappa, .., 0, \ldots\kappa$.
   - i heavy quark spin index (corresponding to a singlet or a triplet).
   - $\boldsymbol{r}$ heavy quark distance.
   - $\boldsymbol{R}$ heavy quark center of mass.
3. Nonrelativistic Lagrangian

$$\mathcal{L} = \sum_{\kappa^p} \Psi^{\dagger}_{\kappa^p} \left[i\partial_t - h_{\kappa^p}\right] \Psi_{\kappa^p}$$

4. The Hamiltonian density is organized as an expansion in $1/m_Q$

$$h_{\kappa^p} = \frac{\boldsymbol{p}^2}{m_Q} + \frac{\boldsymbol{P}^2}{4m_Q} + V^{(0)}_{\kappa^p}(\boldsymbol{r}) + \frac{1}{m_Q} V^{(1)}_{\kappa^p}(\boldsymbol{r},\, \boldsymbol{p}) + \mathcal{O}(1/m_Q^2)$$

5. The potential terms are matrices in the heavy and light d.o.f spin spaces.

How to build the most general set of operators?

1. Building blocks
   - $1/m_Q$ suppressed: $\boldsymbol{S}_1$, $\boldsymbol{S}_2$, $\boldsymbol{L}_{QQ}$, $\boldsymbol{p} = -i\boldsymbol{\nabla}_r$, ($\boldsymbol{P} = -i\boldsymbol{\nabla}_R$)
   - No suppression: $\hat{\boldsymbol{r}}$, $\boldsymbol{S}_\kappa$, ($\hat{\boldsymbol{R}}$)
2. The amount of $1/m_Q$-suppressed terms is limited to the overall order.
3. Build all operators invariant under:
   - $O(3)$, parity, charge conjugation...
   - And $D_{\infty h}$.

Problems:

- How to build operators belonging to irreducible representations $D_{\infty h}$?
- Factors of $(\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_\kappa)^{2n}$ are free...

Irreducible representations of $D_{\infty h}$:

- Project $O(3)$ representations (κ^p) into $D_{\infty h}$ ones (Λ_η).
- The eigenvectors of the projection of the light d.o.f spin into the heavy quark axis are

$$(\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_\kappa)\, P_{\kappa\lambda} = \lambda\, P_{\kappa\lambda} \quad \lambda = -\kappa, .., 0, ..., \kappa$$

then the projectors are defined

$$\mathcal{P}_{\kappa\Lambda} = \sum_{\lambda=\pm\Lambda} P_{\kappa\lambda} P^\dagger_{\kappa\lambda}$$

- All the scalar operators can be decomposed into projectors since

$$(\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_\kappa) = \sum_\lambda \lambda\, P_{\kappa\lambda} P^\dagger_{\kappa\lambda} \Rightarrow (\hat{\boldsymbol{r}} \cdot \boldsymbol{S}_\kappa)^{2n} = \sum_\Lambda \Lambda^{2n} \mathcal{P}_{\kappa\Lambda} \quad (\Lambda = |\lambda|)$$

- LO (static potential)

$$V^{(0)}_{\kappa^p}(\boldsymbol{r}) = \sum_{\Lambda} V^{(0)}_{\kappa^p\Lambda}(r)\mathcal{P}_{\kappa\Lambda}$$

- NLO ($\mathcal{O}(1/m_Q)$)

$$\begin{aligned}
V^{(1)}_{\kappa^p}(\boldsymbol{r}) =& V^{(1)}_{\kappa^p\mathrm{SI}}(\boldsymbol{r}) + V^{(1)}_{\kappa^p\mathrm{SD}}(\boldsymbol{r})\\
V^{(1)}_{\kappa^p\mathrm{SD}}(\boldsymbol{r}) =& \sum_{\Lambda\Lambda'}\mathcal{P}_{\kappa\Lambda}\left[V^{sa}_{\kappa^p\Lambda\Lambda'}(r)\boldsymbol{S}_{QQ}\cdot(\mathcal{P}^{\mathrm{c.r.}}_{10}\cdot\boldsymbol{S}_{\kappa}) + V^{sb}_{\kappa^p\Lambda\Lambda'}(r)\boldsymbol{S}_{QQ}\cdot(\mathcal{P}^{\mathrm{c.r.}}_{11}\cdot\boldsymbol{S}_{\kappa})\right.\\
&\left.+V^{l}_{\kappa^p\Lambda\Lambda'}(r)\left(\boldsymbol{L}_{QQ}\cdot\boldsymbol{S}_{\kappa}\right)\right]\mathcal{P}_{\kappa\Lambda'}
\end{aligned}$$

- $V^{(1)}_{\kappa^p\mathrm{SI}}$ decomposes as the static potential.
- $2\boldsymbol{S}_{QQ} = \boldsymbol{\sigma}_{QQ} = \boldsymbol{\sigma}_{Q_1}\mathbb{1}_{2\,Q_2} + \mathbb{1}_{2\,Q_1}\boldsymbol{\sigma}_{Q_2}$, $(\mathcal{P}^{\mathrm{c.r.}}_{10})^{ij} = \hat{\boldsymbol{r}}^i\hat{\boldsymbol{r}}^j$, $(\mathcal{P}^{\mathrm{c.r.}}_{11})^{ij} = \delta^{ij} - \hat{\boldsymbol{r}}^i\hat{\boldsymbol{r}}^j$.

Matching to NRQCD

Soto, JT Phys.Rev.D 102 (2020)

- Matching condition

$$\mathcal{O}_{\kappa^p}(t) = \chi_c^\top(t,\, \boldsymbol{x}_2)\phi(t,\, \boldsymbol{x}_2,\, \boldsymbol{R})\mathcal{Q}_{\kappa^p}(t,\, \boldsymbol{R})\phi(t,\, \boldsymbol{R},\, \boldsymbol{x}_1)\psi(t,\, \boldsymbol{x}_1) = \sqrt{Z_{\kappa^p}}\Psi_{\kappa^p}(t,\, \boldsymbol{r},\, \boldsymbol{R})$$

  - $\mathcal{Q}_{\kappa^p}$ light d.o.f operator.
  - ψ, χ heavy quark fields in NRQCD.
  - $\phi(t,\, \boldsymbol{R},\, \boldsymbol{x}_1)$ Wilson line.

- Match the NRQCD and BOEFT correlators and expand both sides in $1/m_Q$

$$\langle 0|\,T\{\mathcal{O}_{\kappa^p}(t/2)\mathcal{O}^\dagger_{\kappa^p}(-t/2)\}|0\rangle = \sqrt{Z_{\kappa^p}}\langle 0|\,T\{\Psi_{\kappa^p}(t/2)\Psi^\dagger_{\kappa^p}(-t/2)\}|0\rangle\sqrt{Z^\dagger_{\kappa^p}}$$

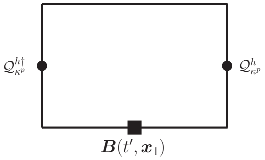
- At leading order

$$V^{(0)}_{\kappa^p\Lambda}(\boldsymbol{r}) = E_{\kappa^p\Lambda_\eta}(\boldsymbol{r}) = \lim_{t\to\infty}\frac{i}{t}\ln$$

- At NLO insertions of $\boldsymbol{D}^2(t', \boldsymbol{x}_1)/(2m_Q)$ or $c_F g \boldsymbol{\sigma}_1 \cdot \boldsymbol{B}(t', \boldsymbol{x}_1)/m_Q$ are needed.
- For instance:

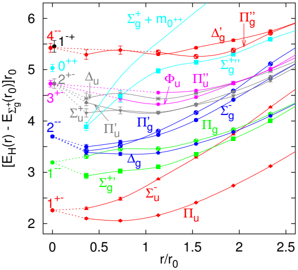
$$V^{sa}_{\kappa^p \Lambda\Lambda'} = -\, c_F \lim_{t\to\infty} \frac{\delta_{\Lambda\Lambda'}}{t} \frac{\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\right]}{\mathrm{Tr}\left[\mathcal{P}_{\kappa\Lambda}\langle 1\rangle^{\kappa^p}_{\square}\right]} \int_{-t/2}^{t/2} dt' \frac{\mathrm{Tr}\left[\left(\boldsymbol{S}_\kappa \cdot \mathcal{P}^{\mathrm{c.r.}}_{10}\right)\cdot\left(\mathcal{P}_{\kappa\Lambda}\langle g\boldsymbol{B}(t',\boldsymbol{x}_1)\rangle^{\kappa^p}_{\square}\mathcal{P}_{\kappa\Lambda}\right)\right]}{\mathrm{Tr}\left[\left(\boldsymbol{S}_\kappa \cdot \mathcal{P}^{\mathrm{c.r.}}_{10}\right)\cdot\left(\mathcal{P}_{\kappa\Lambda}\boldsymbol{S}_\kappa\mathcal{P}_{\kappa\Lambda}\right)\right]}$$

$$\langle \boldsymbol{B}(t',\boldsymbol{x}_1)\rangle^{h\kappa^p}_{\square} =$$



- Similar expressions (but more involved) for V^{sb} and V^l in Soto, JTC, Phys.Rev.D 102 (2020)

Quarkonium hybrids: heavy-quark antiquark pair + a nontrivial gluonic state.



Juge, Kuti, Morningstar Phys.Rev.Lett.90 (2003)

$O(3)$	$D_{\infty h}$
1^{+-}	Σ_u^-, Π_u
1^{--}	$\Sigma_g^{+\prime}$, Π_g
2^{+-}	Σ_u^+, $\Pi_u^\prime$, Δ_u
2^{--}	Σ_g^-, $\Pi_g^\prime$, Δ_g

$$\mathcal{Q}^{\alpha}_{1+-}(t,\boldsymbol{x}) = \left(\boldsymbol{e}^{\dagger}_{\alpha}\cdot\boldsymbol{B}(t,\boldsymbol{x})\right) ,$$

$$\mathcal{Q}^{\alpha}_{1--}(t,\boldsymbol{x}) = \left(\boldsymbol{e}^{\dagger}_{\alpha}\cdot\boldsymbol{E}(t,\boldsymbol{x})\right) ,$$

$$\mathcal{Q}^{\alpha}_{2+-}(t,\boldsymbol{x}) = \mathcal{C}^{2\,\alpha}_{1\,m_1\,1\,m_2}\left(\boldsymbol{e}^{\dagger}_{m_1}\cdot\boldsymbol{D}(t,\boldsymbol{x})\right)\left(\boldsymbol{e}^{\dagger}_{m_2}\cdot\boldsymbol{E}(t,\boldsymbol{x})\right) ,$$

$$\mathcal{Q}^{\alpha}_{2--}(t,\boldsymbol{x}) = \mathcal{C}^{2\,\alpha}_{1\,m_1\,1\,m_2}\left(\boldsymbol{e}^{\dagger}_{m_1}\cdot\boldsymbol{D}(t,\boldsymbol{x})\right)\left(\boldsymbol{e}^{\dagger}_{m_2}\cdot\boldsymbol{B}(t,\boldsymbol{x})\right) ,$$

$$\ldots$$

$$\mathcal{P}_{10} = \mathbb{1}_3 - (\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_1)^2 ,$$

$$\mathcal{P}_{11} = (\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_1)^2 ,$$

$$\mathcal{P}_{20} = \mathbb{1}_5 - \frac{5}{4}(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_2)^2 + \frac{1}{4}(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_2)^4 ,$$

$$\mathcal{P}_{21} = \frac{4}{3}(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_2)^2 - \frac{1}{3}(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_2)^4 ,$$

$$\mathcal{P}_{22} = -\frac{1}{12}(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_2)^2 + \frac{1}{12}(\hat{\boldsymbol{r}}\cdot\boldsymbol{S}_2)^4 .$$

- At LO we have the Shrodinger eq.

$$i\partial_t \Psi_{1^{+-}} = \left(-\frac{\boldsymbol{\nabla}_r^2}{m_Q} + V_{\Sigma_u^-}\mathcal{P}_{10} + V_{\Pi_u}\mathcal{P}_{11} \right) \Psi_{1^{+-}}$$

  We can diagonalize the potential to recover the traditional Born-Oppenheimer formulation

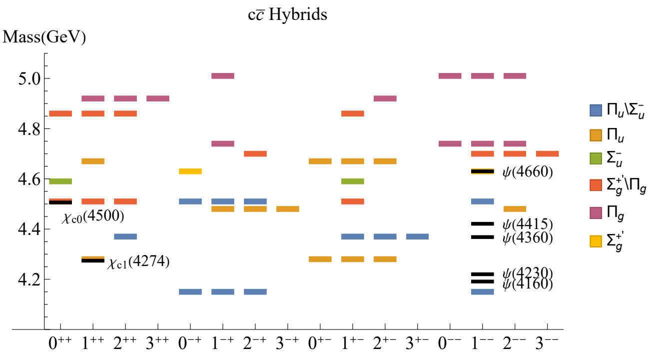
$$i\partial_t \left(P_{1\lambda} \cdot \Psi_{1^{+-}}\right) = \left\{ \left(-\frac{\boldsymbol{\nabla}_r^2}{m_Q} + V_{|\lambda|} \right) \delta_{\lambda\lambda'} + \underbrace{P_{1\lambda} \cdot \left[\frac{\boldsymbol{\nabla}_r^2}{m_Q}, P_{1\lambda'} \right]}_{\text{Nonadiabatic coupling}} \right\} \left(P_{1\lambda'} \cdot \Psi_{1^{+-}}\right)$$

  $\lambda, \lambda' = -1, 0, 1$

- Non adiabatic coupling: couples different static states.
  - Subleading in diatomic molecules
  - Should be kept in hybrids due to the short distance degeneracy of Σ_u^- and Π_u.
  - Angular wave functions are eigenfunctions of $(\boldsymbol{L}_{Q\bar{Q}} + \boldsymbol{S}_1)^2$ with eigenvalue $\ell(\ell+1)$.

Heavy Hybrids: Charmonium Spectrum at leading order

Berwein, Brambilla, JT, Vairo Phys.Rev.D92 (2015); Pineda, JT Phys.Rev.D 100 (2019)

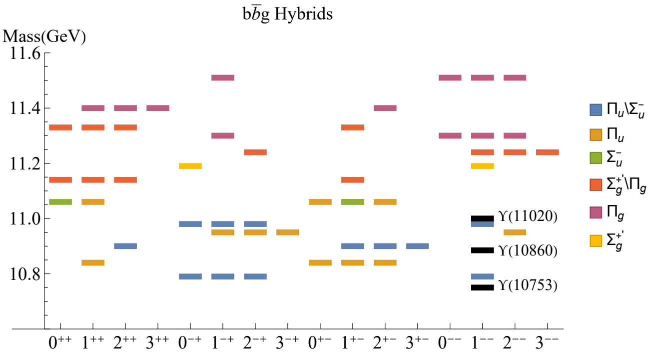


For a state associated to a 1^{pc} gluon operator and a solution of the Schrödinger eq. with ℓ angular quantum number we have

Λ	Angular mom. $Q\bar{Q}$	$j(s_{Q\bar{Q}}=0, s_{Q\bar{Q}}=1)$	Parity	C-parity
Σ	0	$\{0,1\}$	p	$-(-1)^s c$
Π	ℓ	$\{\ell,\{\ell-1,\ell,\ell+1\}\}$	$-(-1)^\ell p$	$(-1)^{\ell+s} c$
$\Sigma\backslash\Pi$	$(\ell-1)\backslash(\ell+1)$	$\{\ell,\{\ell-1,\ell,\ell+1\}\}$	$(-1)^\ell p$	$-(-1)^{\ell+s} c$

Heavy Hybrids: Bottomonium Spectrum at leading order

Berwein, Brambilla, JT, Vairo Phys.Rev.D92 (2015); Pineda, JT Phys.Rev.D 100 (2019)



There are only 3 neutral exotic bottomonium states, all 1^{--}:

- $\Upsilon(10753)$ mass within 40 MeV of ground state hybrid.
- $\Upsilon(10860)$ lays very close to B_s^* pair threshold, likely a molecular state.
- $\Upsilon(11020)$ mass within 20 MeV of first excited hybrid.

Hyperfine splittings in quarkonium hybrids

Brambilla, Lai, Segovia, JT, Vairo Phys.Rev. D99 (2019); Brambilla, Lai, Segovia, JT Phys.Rev.D 101 (2020); Soto, JT Phys.Rev.D 102 (2020)

The degeneracy of the heavy quark spin multiplets is broken at $\mathcal{O}(1/m_Q)$.

- For $\kappa = 1$ with the representation $\left(\boldsymbol{S}_1^i\right)^{jk} = -i\epsilon_{ijk}$:

$$\mathcal{L}^{hf} = \frac{1}{m_Q}\Psi_1^{\dagger\,i}\left[V^{sa}(r)\boldsymbol{S}^k_{QQ}(\boldsymbol{S}^k_1)_{ij} + \left(V^{sb}(r) - V^{sa}(r)\right)\boldsymbol{S}^k_{QQ}\left(\hat{\boldsymbol{r}}^i\hat{\boldsymbol{r}}^l(\boldsymbol{S}^k_1)_{lj} - \hat{\boldsymbol{r}}^l(\boldsymbol{S}^k_1)_{li}\hat{\boldsymbol{r}}^j\right)\right]\Psi_1^j$$

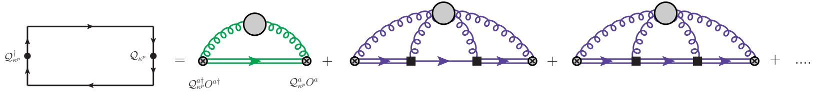
- Hyperfine contributions to the mass can be computed with standard perturbation theory.
- Unfortunately no lattice determinations of the spin-dependent potentials are available.

⇒ We need extra approximations or models.

Short-distance expansion

Short distance: $r \ll 1/\Lambda_{\rm QCD}$

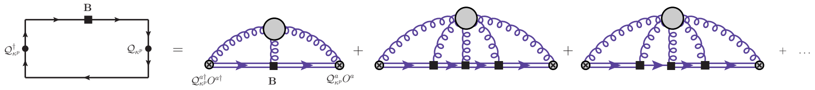
- Instead of integrating out $m_Q v$ and $\Lambda_{\rm QCD}$ at the same time we can do one scale at a time.
- Integrating out $m_Q v$ is equivalent to the multipole expansion.
- The heavy quark pair can be decomposed into a color singlet or octet (O^a) field.
- Hybrid quarkonium field $\Psi_{1^{+-}} = \sqrt{Z_B}\boldsymbol{B}^a O^a$



$\Rightarrow$ Potentials: non analytic term from integrating out $m_Q v$ perturbatively plus and expansion in r^2 with unknown nonperturbative coefficients

$$E_{1^{+-}\Lambda_\eta}(\boldsymbol{r}) = V_o^{(0)}(r) + \Lambda_{1^{+-}} + b_{1^{+-}\Lambda} r^2 + \cdots = V^{(0)}_{1^{+-}\Lambda}(\boldsymbol{r})$$

Short-distance expansion

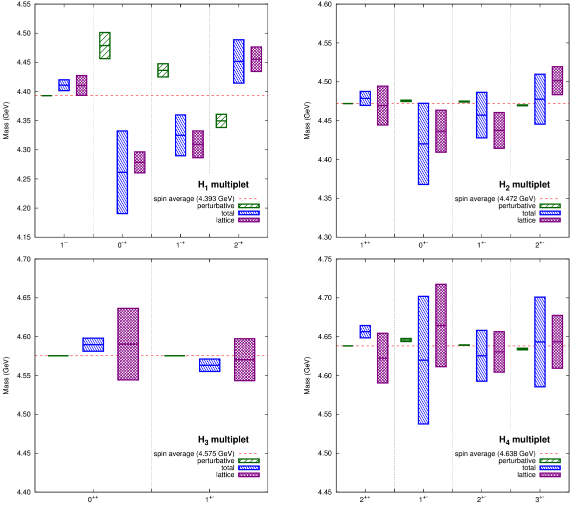


$$V^{sa}_{1^+11}(\boldsymbol{r}) = \Delta^{sa\,(0)}_{1^+11} + \Delta^{sa\,(1)}_{1^+11} r^2 + \dots$$
$$V^{sb}_{1^+10}(\boldsymbol{r}) = \Delta^{sb\,(0)}_{1^+11} + \Delta^{sb\,(1)}_{1^+11} r^2 + \dots$$

- Obtain the nonperturbative coefficients by fitting the hyperfine splittings of charmonium hybrids lattice data.
- Use the same values of the coefficients to obtain predictions for the bottomonium hybrid hyperfine splittings .

Lattice data: Hadron Spectrum col. JHEP 1612 (2016)

$cg\bar{c}$ (fitted)

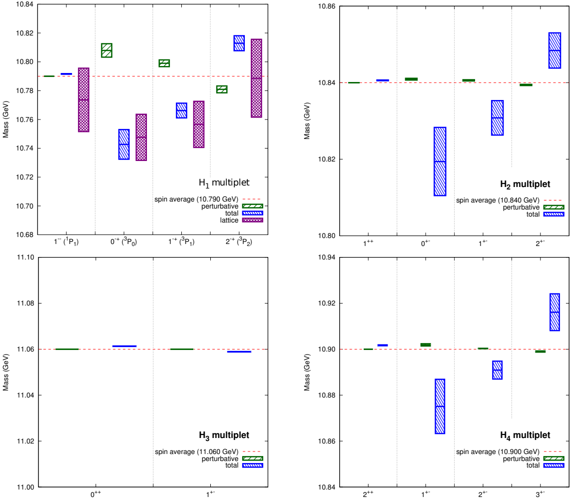


Bottomonium Hybrids Spin splittings

Brambilla, Lai, Segovia, JT, Vairo Phys.Rev. D99 (2019); Brambilla, Lai, Segovia, JT Phys.Rev.D 101 (2020)

Lattice data: Hadron Spectrum col. JHEP 02 (2021)

$bg\bar{b}$ (prediction)



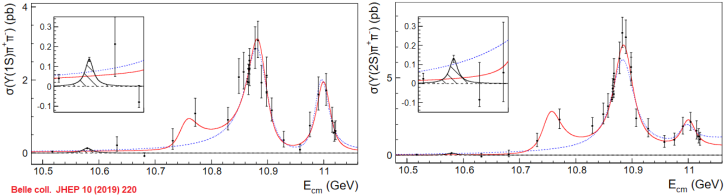
Hybrid to standard bottomonium transitions

JT, Passemar Phys.Rev.D 104 (2021)

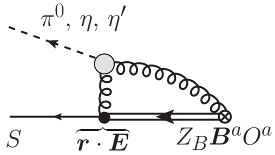
- Not many J^{PC} are easily accessible experimentally:
  - It is difficult to use spectrum predictions to validate different approaches.
  - Cannot test heavy quark spin symmetry multiplets in different approaches.

$\Rightarrow$ The nature of many exotic quarkonium states is still not settled.

- On the other hand information on decay channels is always available.
- Many exotic states discovered in channels with standard quarkonium and light-quark mesons.



- We have studied these transitions in our EFT formalism in the short-distance approximation.



Transitions from the LO singlet-octet operator:

$$\langle S_m \mathcal{O}_\pi | g\mathrm{Tr}\left[\mathrm{S}^\dagger \boldsymbol{r}\cdot\boldsymbol{E}\,\mathrm{O}\right]|H_n\rangle \sim Z_B^{-1/2}\langle \mathcal{O}_\pi | g^2 \boldsymbol{E}\cdot\boldsymbol{B}|0\rangle\langle\phi^{(m)}|\boldsymbol{r}\cdot\hat{\boldsymbol{r}}_\lambda|\psi_\lambda^{(n)}\rangle$$

- Z_B can be related to the gluon condensate.
- Heavy quark matrix element:
  - Selection rules for final quarkonium states: $\Delta s = 0$, $\ell = l$.
  - $\Upsilon(10753)$ and $\Upsilon(11020)$ decay into $h_b(m^1P_1)$ and light quark mesons.
- Light-quark meson production:
  - Allowed final light-quark states: 0^{-+}, $I = 0$ such as π^0, η, η', η-like resonances or odd numbers of pseudoscalar mesons.
  - Matrix elements for production of π^0, η, η' can be determined from $U(1)_A$ anomaly and a mixing scheme. Feldmann, Kroll, Stech, Phys.Rev.D58 (1998); Kroll, Mod. Phys. Lett. A20 (2005)

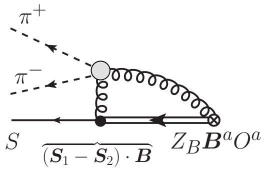
LO Transitions

JT, Passemar, accepted to Phys.Rev.D arXiv:2104.03975

- LO Transition widths:

$$\Gamma_{\Upsilon(10753)\to h_b(1P)\pi^0} = 2.57(\pm 1.03)_{\rm m.e.}(\pm 0.14)_{Z_B}(\pm 0.16)_{\omega_{\pi^0}}\ {\rm keV}$$
$$\Gamma_{\Upsilon(10753)\to h_b(1P)\eta} = 2.29(\pm 0.92)_{\rm m.e.}(\pm 0.13)_{Z_B}(\pm 0.08)_{\omega_{\eta}}\ {\rm MeV}$$
$$\Gamma_{\Upsilon(10753)\to h_b(2P)\pi^0} = 0.168(\pm 0.067)_{\rm m.e.}(\pm 0.009)_{Z_B}(\pm 0.010)_{\omega_{\pi^0}}\ {\rm keV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(1P)\pi^0} = 2.04(\pm 0.82)_{\rm m.e.}(\pm 0.11)_{Z_B}(\pm 0.13)_{\omega_{\pi^0}}\ {\rm keV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(1P)\eta} = 2.04(\pm 0.81)_{\rm m.e.}(\pm 0.11)_{Z_B}(\pm 0.07)_{\omega_{\eta}}\ {\rm MeV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(1P)\eta'} = 9.23(\pm 3.69)_{\rm m.e.}(\pm 0.51)_{Z_B}(\pm 0.39)_{\omega_{\eta'}}\ {\rm MeV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(2P)\pi^0} = 0.104(\pm 0.042)_{\rm m.e.}(\pm 0.006)_{Z_B}(\pm 0.006)_{\omega_{\pi^0}}\ {\rm keV}$$
$$\Gamma_{\Upsilon(11020)\to h_b(2P)\eta} = 81.8(\pm 32.7)_{\rm m.e.}(\pm 4.6)_{Z_B}(\pm 2.7)_{\omega_{\eta}}\ {\rm keV}$$

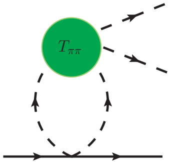
- Uncertainties labeled by the origin m.e.=multipole expansion, ω=Production matrix element.



Transitions from the NLO singlet-octet operator:

$$\langle S_m \mathcal{O}_{\pi\pi}| \frac{g c_F}{m_Q} \mathrm{Tr}\left[\mathrm{S}^\dagger (\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot \boldsymbol{B}\, \mathrm{O}\right] |H_n\rangle \sim \frac{c_F}{m_Q} Z_B^{-1/2} \langle \mathcal{O}_{\pi\pi}|\boldsymbol{B}^2|0\rangle \langle \phi^{(m)}|(\boldsymbol{S}_1 - \boldsymbol{S}_2)\cdot \hat{\boldsymbol{r}}_\lambda |\psi_\lambda^{(n)}\rangle$$

- Heavy quark matrix element:
  - Selection rules: $\Delta s = 1$, $l = \ell \pm 1$.
  - $\Upsilon(10753)$ and $\Upsilon(11020)$ decay into $\Upsilon(m^3S_1)$ or $\Upsilon(m^3D_1)$ and light quark mesons.
- Light-quark meson production:
  - Allowed final light-quark states: 0^{++} and $I = 0$ such as $\pi^+\pi^-$, K^+K^-, pairs of π^0 or η as well as f_0 resonances.
  - We use a dispersive representation for the matrix elements for the production $\pi^+\pi^-$, K^+K^-. Donoghue, Gasser, Leutwyler, Nucl.Phys.B343 (1990); Moussallam, Eur.Phys.J.C 14 (2000)



- Decompose $\langle \mathcal{O}_{\pi\pi} | \boldsymbol{B}^2 | 0 \rangle$ into S ($F^{(0)}$) and D ($F^{(2)}$) wave pieces.
- From Watson's Theorem we obtain the imaginary part of $F^{(I)}$ corresponding to two-pion and two-kaon rescattering.

Muskhelishvili-Omnès problem:

Muskhelishvili, *Singular integral equations*; Omnes, Nuovo Cim.8 (1958)

- Imaginary part is known.
- Analytic in the complex s-plane, except on the cuts.
- Real in the real s axis below the cuts.

$\Rightarrow$ A general form of the form factors is

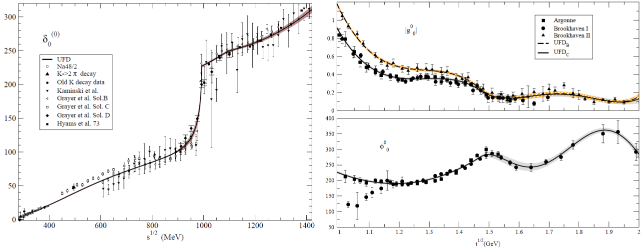
$$n_P F_P^{(I)}(s) = \Omega_{PP'}^{(I)}(s) Q_{P'}^{(I)}(s)\,, \qquad P, P' = \pi,\, K$$

Dispersive representation: Omnès Functions

- Ω-matrix satisfies the singular integral equations

$$\boldsymbol{\Omega}(s) = \frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \frac{ds'}{s'-s}\left(\boldsymbol{T}_l^0(s')\right)^* \boldsymbol{\Sigma}(s')\boldsymbol{\Omega}(s')$$

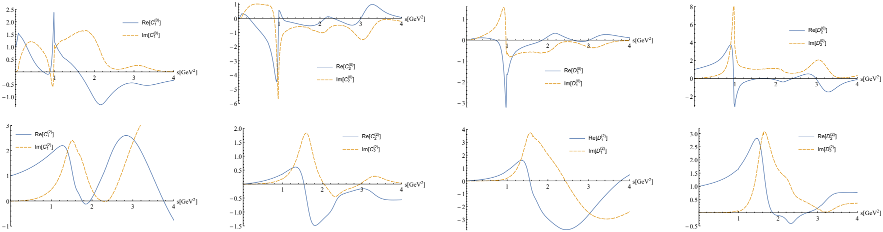
- Input needed: $\pi\pi \to \pi\pi$ and $\pi\pi \to K\bar{K}$ T-matrix for each partial wave.



- Parametrizations (and figure) from Garcia-Martin et al Phys.Rev.D83 (2011); Pelaez, Rodas Eur.Phys.J. C78 (2018)

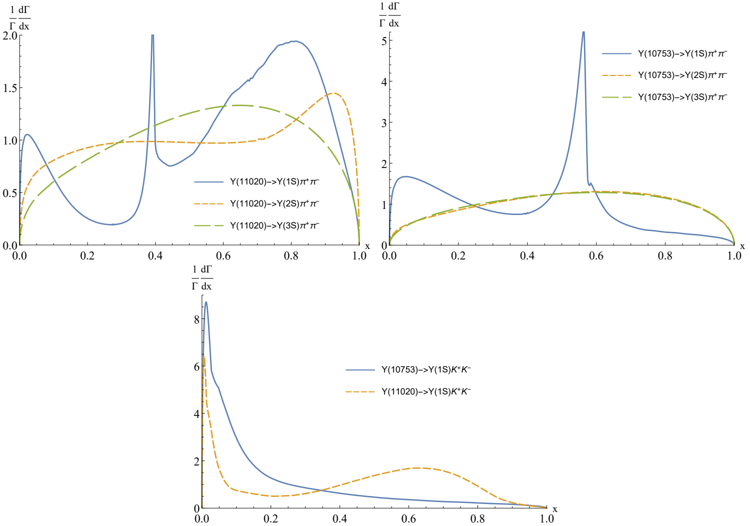
Dispersive representation

- (Numerical) $\Omega^{(I)}$ solutions Moussallam, Eur.Phys.J.C 14 (2000); Descotes-Genon Ph.D. thesis (2000)



- $\boldsymbol{Q}^{(I)}(s) = (Q_1^{(I)},\, Q_2^{(I)})$ are subtraction polynomials obtained by matching to a chiral representation.
- Chiral low-energy constants determined using the Scale anomaly, Feynman-Hellmann theorem, and one free parameter from quarkonium transitions.

  Voloshin, Zakharov, Phys.Rev.Lett.45 (1980); Novikov, Shifman, Z.Phys.C8 (1981); Chivukula et al, Annals Phys. 192 (1989); Pineda, JTC, Phys.Rev.D100 (2019)



$x = (s - 4m^2_{GB})/(m_{\bar{Q}Qg} - m_{\bar{Q}Q} - 4m^2_{GB})$

NLO Transitions widths

JT, Passemar, accepted to Phys.Rev.D arXiv:2104.03975

- NLO transition widths:

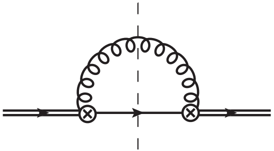
$$\Gamma_{\Upsilon(10753)\to\Upsilon(1S)\pi^+\pi^-} = 43.4(\pm 17.3)_{\rm m.e.}(\pm 2.4)_{Z_B}(\pm 8.6)_{\alpha_s}(^{+0.5}_{-0.0})_{\kappa}\ \text{keV}$$
$$\Gamma_{\Upsilon(10753)\to\Upsilon(2S)\pi^+\pi^-} = 2.75(\pm 1.10)_{\rm m.e.}(\pm 0.15)_{Z_B}(\pm 0.55)_{\alpha_s}(^{+0.13}_{-0.12})_{\kappa}\ \text{keV}$$
$$\Gamma_{\Upsilon(10753)\to\Upsilon(3S)\pi^+\pi^-} = 0.98(\pm 0.39)_{\rm m.e.}(\pm 0.05)_{Z_B}(\pm 0.19)_{\alpha_s}(\pm 0.03)_{\kappa}\ \text{eV}$$
$$\Gamma_{\Upsilon(11020)\to\Upsilon(1S)\pi^+\pi^-} = 99.1(\pm 39.6)_{\rm m.e.}(\pm 5.5)_{Z_B}(\pm 19.7)_{\alpha_s}(^{+26.3}_{-21.8})_{\kappa}\ \text{keV}$$
$$\Gamma_{\Upsilon(11020)\to\Upsilon(2S)\pi^+\pi^-} = 3.96(\pm 1.58)_{\rm m.e.}(\pm 0.22)_{Z_B}(\pm 0.70)_{\alpha_s}(^{-0.16}_{+0.17})_{\kappa}\ \text{keV}$$
$$\Gamma_{\Upsilon(11020)\to\Upsilon(3S)\pi^+\pi^-} = 1.33(\pm 0.53)_{\rm m.e.}(\pm 0.07)_{Z_B}(\pm 0.27)_{\alpha_s}(\pm 0.02)_{\kappa}\ \text{keV}$$
$$\Gamma_{\Upsilon(10753)\to\Upsilon(1S)K^+K^-} = 3.98(\pm 1.59)_{\rm m.e.}(\pm 0.22)_{Z_B}(\pm 0.79)_{\alpha_s}(^{-0.50}_{+0.67})_{\kappa}\ \text{keV}$$
$$\Gamma_{\Upsilon(11020)\to\Upsilon(1S)K^+K^-} = 5.93(\pm 2.37)_{\rm m.e.}(\pm 0.33)_{Z_B}(\pm 1.18)_{\alpha_s}(^{+1.75}_{-1.18})_{\kappa}\ \text{keV}$$

- Experimental **ranges** for the widths Belle col. JHEP 10, 220 (2019)

$$\Gamma^{\rm exp}_{\Upsilon(11020)\to\Upsilon(1S)\pi^+\pi^-} = 49 - 118\ \text{keV} \Leftarrow \textbf{Promising agreement!}$$
$$\Gamma^{\rm exp}_{\Upsilon(11020)\to\Upsilon(2S)\pi^+\pi^-} = 13 - 225\ \text{keV}$$
$$\Gamma^{\rm exp}_{\Upsilon(11020)\to\Upsilon(3S)\pi^+\pi^-} = 13 - 98\ \text{keV}$$

Semi-inclusive widths

- If the energy gap between a hybrid and a standard quarkonium state is large semi-inclusive transition widths can be computed. Oncala, Soto Phys.Rev.D96 (2017)

$\Gamma_{\text{s.i.}} = \langle H_n | \text{Im}[$  $] | H_n \rangle$

$$\Gamma^{\text{LO}}_{\Upsilon(11020)\to h_b(1P)} = 20(\pm 9)_{\alpha_s}\ \text{MeV}$$

$$\Gamma^{\text{NLO}}_{\Upsilon(10753)\to \Upsilon(1S)} = 9.7(\pm 3.8)_{\alpha_s}\ \text{MeV}$$

$$\Gamma^{\text{NLO}}_{\Upsilon(11020)\to \Upsilon(1S)} = 7.3(\pm 2.5)_{\alpha_s}\ \text{MeV}$$

$$\Gamma^{\text{NLO}}_{\Upsilon(11020)\to \Upsilon(2S)} = 1.1(\pm 0.5)_{\alpha_s}\ \text{MeV}$$

- The sum of semi-inclusive widths for $\Gamma^{\text{LO+NLO}}_{\Upsilon(11020)} = 28.4 \pm 9.4$ MeV is compatible with the experimental value of the total width $\Gamma^{\text{exp}}_{\Upsilon(11020)} = 24^{+8}_{-6}$ MeV.

Summary and Outlook

Summary:

- We have proposed an EFT framework to describe doubly heavy hadrons.
- Quarkonium hybrids:
  - Spectrum including non-adiabatic corrections and hyperfine splittings.
  - Bottomonium transitions in the short distance approximation.
- It has also been applied to doubly heavy baryons.

  Soto, JT, Phys.Rev.D 102 (2020), Phys.Rev.D 104 (2021)
- More input from lattice on static energies needed!

Outlook

- String models to evaluate Wilson loops.
- Incorporate mixing with thresholds.
- T_{cc}^{++} and other tetraquarks.

Thank you for your attention