Optimal polynomial approximants. An electrostatic approach.

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The concept of optimal polynomial approximants arises from the study of the so-called cyclic functions. In particular, in this talk we focus on cyclic functions in the well-known Hardy space D_0 of analytic functions $f(z) = \sum_{k=0}^{\infty} a_k z^K$ on the unit disk $\mathbb{D}$, equipped with the norm

$$\|f\|_0^2 := \sum_{k=0}^{\infty} |a_k|^2 < \infty.$$

A function $f \in D_0$ is said to be cyclic if the subspace $f\,\mathbb{P}$ is dense in D_0 ($\mathbb{P}$ denotes the space of all polynomials). In this context, a polynomial $p_n \in \mathbb{P}_n$ is called an optimal polynomial approximant (abbreviately, an OPA) to $1/f$ if

$$\|1 - p_n f\|_0 \leq \|1 - qf\|_0,$$

for any $q \in \mathbb{P}_n$.

In this talk we are interested in the connection of these OPA with the theory of orthogonal polynomials in the Unit Circle (OPUC), and the consequent asymptotic properties and, especially, electrostatic interpretation of their zeros.

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