

Orthonet meeting

Madrid, 19 y 20 de diciembre de 2024

## LADDER OPERATORS FOR GENERALIZED ZERNIKE OR DISK POLYNOMIALS

MISAEEL E. MARRIAGA

### Abstract

The aim of this work is to report on several ladder operators for generalized Zernike polynomials which are orthogonal polynomials on the unit disk  $\mathbf{D} = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$  with respect to the weight function  $W_\mu(x, y) = (1 - x^2 - y^2)^\mu$  where  $\mu > -1$ . These polynomials can be expressed in terms of the univariate Jacobi polynomials and, thus, we start by deducing several ladder operators for the Jacobi polynomials. Due to the symmetry of the disk and the weight function  $W_\mu$ , it turns out that it is more convenient to use complex variables  $z = x + iy$  and  $\bar{z} = x - iy$ . Indeed, this allows us to systematically use the univariate ladder operators to deduce analogous ones for the complex generalized Zernike polynomials. Some of these univariate and bivariate ladder operators already appear in the literature. However, to the best of our knowledge, the proofs presented here are new. Lastly, we illustrate the use of ladder operators in the study of the orthogonal structure of some Sobolev spaces.

**Key words:** Zernike polynomials, ladder operators.

M. E. Marriaga,

Departamento de Matemática Aplicada,

Ciencia e Ingeniería de Materiales y Tecnología Electrónica, Univ. Rey Juan Carlos (Spain)

`misael.marriaga@urjc.es`

---

MEM has been supported by the research project [PID2021- 122154NB-I00], *Ortogonalidad y Aproximación con Aplicaciones en Machine Learning y Teoría de la Probabilidad* funded by MICIU/AEI/10.13039/501100011033 and by “ERDF A Way of making Europe”. The author has also been supported by the Comunidad de Madrid multiannual agreement with the Universidad Rey Juan Carlos under the grant Proyectos I+D para Jóvenes Doctores, Ref. M2731, project NETA-MM..