

# Zeros of Sobolev polynomials: bounded point evaluations and matrix inequalities

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## Abstract

The main aim of this work is to apply the matrix approach to orthogonal polynomials associated with infinite Hermitian positive definite matrices developed in our previous works ([2],[3],[4]), to address a significant problem concerning the location of zeros of orthogonal polynomials in the context of Sobolev inner products. These inner products are associated with a set of measures supported in the complex plane or, more generally, with matrix Sobolev inner products as introduced in [5]. It is well established that, within this general framework, the boundedness of the multiplication operator by  $z$  with respect to such inner products provides a sufficient condition to guarantee the boundedness of the zeros of the associated Sobolev orthogonal polynomials. To establish conditions that ensure the boundedness of this operator, we use the concept of bounded point evaluations for a measure and its extension within the framework of infinite Hermitian positive-definite matrices.

Furthermore, we provide examples and applications to illustrate these results. Additionally, we introduce certain matrix inequalities that relate the norm of polynomials to the norm of their derivatives. These inequalities are used to present new examples of Sobolev orthogonal polynomials with bounded sets of zeros.

## References

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