

Asymptotic and numerical approximations to the zeros of parabolic cylinder functions

Diego Ruiz-Antolín

Dpto. de Matemática Aplicada y Ciencias de la Computación, Universidad de Cantabria
(Spain)

e-mail: `diego.ruizantolin@unican.es`

The zeros of parabolic cylinder functions have numerous applications in science and engineering. They are used, for example, in the analysis of fluid flow in cylindrical channels and in the scattering of electromagnetic waves at parabolic boundaries.

In this talk, we present uniform asymptotic approximations for the real and complex zeros of the parabolic cylinder function $U(a, z)$, involving certain combinations of the zeros of Airy functions [1]. The expansions are valid for a large in absolute value (whether positive or negative), and uniformly for unbounded z (real or complex).

The accuracy of the approximations for the complex zeros is tested using a method (implemented in Maple) for finding the complex zeros of solutions of second-order ODEs [2]. A fixed-precision implementation of this method for parabolic cylinder functions will also be discussed in the talk. The numerical algorithm incorporates, among other techniques, Taylor series and Liouville-Green expansions of $U(a, z)$ and its derivative in the region where the zeros are located [3].

Work in collaboration with **T.M. Dunster** (SDSU, USA), **A. Gil** (UC, Spain) and **J. Segura** (UC, Spain).

References

- [1] T.M. Dunster, A. Gil, D. Ruiz-Antolin, J. Segura. Uniform asymptotic expansions for the zeros of parabolic cylinder functions. Submitted.
- [2] J. Segura. Computing the complex zeros of special functions. *Numer. Math.* 124 (4) (2013) 723-752.
- [3] T.M. Dunster, A. Gil, D. Ruiz-Antolin, J. Segura. A numerical algorithm for computing the zeros of parabolic cylinder functions in the complex plane. In progress.