Programme:

- **Thursday, December 19:**
  - 3:30 PM – 5:00 PM: **IPs meeting**
  - 5:00 PM – 6:00 PM: **Opening plenary lecture "*Luis Vigil*" by Andrei Martínez-Finkelshtein, *Flow of the zeros of polynomials under iterated differentiation***
  - *6:00 PM – 6:30 PM: Coffee break*
  - 6:30 PM – 7:10 PM: Diego Ruíz Antolín, *Asymptotic and numerical approximations to the zeros of parabolic cylinder functions and numerical approximations to the zeros of parabolic cylinder functions*
  - 7:15 PM – 7:55 PM: Iván Area, *Digital-twin approach for mathematical modelling*
- **Friday, December 20:**
  - 9:00 AM – 10:00 AM: **Plenary lecture by Alberto Enciso, *Spectral geometry and Bessel functions***
  - 10:00 AM – 10:45 AM: María José Cantero, *Khrushchev's formula on the real line*
  - 10:45 AM – 11:30 AM: Ramón Orive, *Optimal polynomial approximants. An electrostatic approach.*
  - *11:30 AM – 12:00 AM: Coffee break | **POSTER SESSION***
  - 12:00 AM – 12:45 PM: Junior talk. Juan E F Díaz, *Multiple type I polynomials in the Askey scheme*
  - 12:45 PM – 1:30 PM: Junior talk. Juan Antonio Villegas, *Extension of multiple orthogonality to the bivariate case*
  - *1:30 PM – 3:30 PM: Lunch*
  - 3:30 PM – 4:10 PM: Antonio J. Durán, *Brenke polynomials with real zeros and the Riemann Hypothesis*
  - 4:15 PM – 4:55 PM: Juan Luis Varona, *Machin's type formulas for π*
  - *5:00 PM – 5:30 PM: Coffee break*
  - 5:30 PM – 6:30 PM: **Closing plenary lecture "*Orthogonality and Applications*" by Walter Van Assche, *Rational approximations for Catalan's constant***
  - 6:30 PM – 8:00 PM: **ORTHONET researchers' assembly**

Digital-twin approach for mathematical modelling

Iván Area
Departamento de Matemática Aplicada II
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The work we present revolves around the theory of Stieltjes derivatives applied to ODE as well as to EDP models, a field which has started expanding recently with important findings and a wide range of applications. On one hand, the ideas are applied to infectious diseases by considering ODEs involving Stieltjes derivatives. Moreover, the importance of these derivatives are shown by considering a mathematical model to analyze the population dynamics of Vespa Velutina.

Khrushchev's formula on the real line

M.J. Cantero, L. Moral, L. Velázquez

Abstract

Khrushchev's formula was introduced in 2001 by Sergei Khrushchev to take advantage of continued fraction methods to study Orthogonal Polynomials on the Unit Circle (OPUC).

He surprised OPUC community with new and deep results in OPUC theory which revolutionized this theory. Key for these results is the so-called Khrushchev's formula that identifies the Schur function of the orthogonality measure modified by the corresponding OPUC.

Curiously, the analogue of Khrushchev's formula for Orthogonal Polynomials on the Real Line (OPRL) was not known until 2018. It was uncovered by Grünbaum and Velázquez, who obtained the OPRL version via operator theory. This result had the drawback of being only valid for the determinate case. The new approach to OPRL Khrushchev's formula presented in this talk shows its validity for any measure. This allows us to use such a tool to obtain information about convergence properties of OPRL even in the indeterminate case.

We also present a simple diagrammatic proof of OPRL Khrushchev's formula which sheds light on its graph theoretical meaning.

MULTIPLE TYPE I POLYNOMIALS IN THE ASKEY SCHEME

AMÍLCAR BRANQUINHO[1], JUAN DÍAZ[2], ANA FOULQUIÉ-MORENO[2], MANUEL MAÑAS[3] & THOMAS WOLFS[4]

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[2] *UNIVERSIDADE DE AVEIRO*, PORTUGAL
[3] *UNIVERSIDAD COMPLUTENSE DE MADRID*, SPAIN
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Abstract

Multiple orthogonal polynomials are a generalization of standard orthogonal polynomials' theory that arise from considering orthogonality conditions respect to, not one, but an arbitrary number of weight functions. This leads, on one hand, to the type II polynomials, that have been widely studied and many families are already known. On the other hand; there are the multiple type I polynomials, that have not been studied as extensively. Recently; an important step in this direction has been given with the finding of explicit expressions for the type I polynomials corresponding to some well known families such as Hahn, Meixner, Kravchuk, Charlier, Jacobi–Piñeiro, Laguerre or Hermite; see [1], [2], [3] or [4]. These ones are expressed through special functions like the generalized hypergeometric or Kampé de Fériet series.

References

[1] A. Branquinho, J. E. F. Díaz, A. Foulquié-Moreno & M. Mañas, *Hypergeometric Expressions for Type I Jacobi–Piñeiro Orthogonal Polynomials with Arbitrary Number of Weights*, Proceedings of the American Mathematical Society Series B **11** (2024), https://doi.org/10.1090/bproc/225

[2] A. Branquinho, J. E. F. Díaz, A. Foulquié-Moreno & M. Mañas, *Classical multiple orthogonal polynomials for arbitrary number of weights and their explicit representation*, https://arxiv.org/abs/2404.13958

[3] A. Branquinho, J. E. F. Díaz, A. Foulquié-Moreno, M. Mañas & T. Wolfs, *Integral and hypergeometric representations for multiple orthogonal polynomials*, https://arxiv.org/abs/2407.15001

[4] A. Branquinho, J. E. F. Díaz, A. Foulquié-Moreno, M. Mañas & T. Wolfs, *Classical discrete multiple orthogonal polynomials: hypergeometric and integral representations*, https://arxiv.org/abs/2409.16254

Spectral geometry and Bessel functions

Alberto Enciso
Instituto de Ciencias Matemáticas

Bessel functions play a central role in the study of the eigenvalues of the Laplacian, both in domains in Euclidean space and in more general contexts. In this talk, we will review some open problems and recent results where Bessel functions are prominently featured.

Flow of the zeros of polynomials under iterated differentiation

Andrei Martínez-Finkelshtein

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Assume we have a sequence of polynomials whose asymptotic zero distribution is known. What can be said about the zeros of their derivatives? Especially if we differentiate each polynomial several times, proportional to their degree? This simple-to-formulate problem has recently attracted the attention of several researchers. The problem and its solution methods have exciting connections with free probability, random matrices, approximation theory on the complex plane, and nonlinear PDE, such as the inviscid Burgers equation.

Optimal polynomial approximants. An electrostatic approach.

Ramón Orive
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ORTHONET Meeting, December 19–20, 2024.

The concept of optimal polynomial approximants arises from the study of the so-called cyclic functions. In particular, in this talk we focus on cyclic functions in the well-known Hardy space D_0 of analytic functions $f(z) = \sum_{k=0}^{\infty} a_k z^K$ on the unit disk $\mathbb{D}$, equipped with the norm

$$\|f\|_0^2 := \sum_{k=0}^{\infty} |a_k|^2 < \infty.$$

A function $f \in D_0$ is said to be cyclic if the subspace $f\,\mathbb{P}$ is dense in D_0 ($\mathbb{P}$ denotes the space of all polynomials). In this context, a polynomial $p_n \in \mathbb{P}_n$ is called an optimal polynomial approximant (abbreviately, an OPA) to $1/f$ if

$$\|1 - p_n f\|_0 \leq \|1 - qf\|_0,$$

for any $q \in \mathbb{P}_n$.

In this talk we are interested in the connection of these OPA with the theory of orthogonal polynomials in the Unit Circle (OPUC), and the consequent asymptotic properties and, especially, electrostatic interpretation of their zeros.

This is a joint work with J. Sánchez Lara (Univ. Granada) and D. Seco Forsnacke (ULL).

Asymptotic and numerical approximations to the zeros of parabolic cylinder functions

Diego Ruiz-Antolín

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The zeros of parabolic cylinder functions have numerous applications in science and engineering. They are used, for example, in the analysis of fluid flow in cylindrical channels and in the scattering of electromagnetic waves at parabolic boundaries.
In this talk, we present uniform asymptotic approximations for the real and complex zeros of the parabolic cylinder function $U(a, z)$, involving certain combinations of the zeros of Airy functions [1]. The expansions are valid for a large in absolute value (whether positive or negative), and uniformly for unbounded z (real or complex).
The accuracy of the approximations for the complex zeros is tested using a method (implemented in Maple) for finding the complex zeros of solutions of second-order ODEs [2]. A fixed-precision implementation of this method for parabolic cylinder functions will also be discussed in the talk. The numerical algorithm incorporates, among other techniques, Taylor series and Liouville-Green expansions of U(a,z) and its derivative in the region where the zeros are located [3].

Work in collaboration with **T.M. Dunster** (SDSU, USA), **A. Gil** (UC, Spain) and **J. Segura** (UC, Spain).

References

[1] T.M. Dunster, A. Gil, D. Ruiz-Antolin, J. Segura. Uniform asymptotic expansions for the zeros of parabolic cylinder functions. Submitted.

[2] J. Segura. Computing the complex zeros of special functions. Numer. Math. 124 (4) (2013) 723-752.

[3] T.M. Dunster, A. Gil, D. Ruiz-Antolin, J. Segura. A numerical algorithm for computing the zeros of parabolic cylinder functions in the complex plane. In progress.

Rational approximations for Catalan's constant

Walter Van Assche

Department of Mathematics
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Catalan's constant $G = \sum_{k=0}^{\infty} (-1)^k/(2k+1)^2$ is closely related to π^2 but so far it is not known whether or not this is an irrational number. One possible way to prove irrationality is to construct rational approximants that are better than possible for a rational number. We will give a few constructions for rational approximants but so far none of them is good enough to prove irrationality.s equation.

Machin's type formulas for π

Juan Luis Varona

Departamento de Matemáticas
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In 1706, John Machin used a formula expressing π as a combination of two arctangents and, with it, calculated 100 decimal places of π. Quite a few similar formulas are known today, which are called Machin-type formulas.

In the talk we will discuss some issues related to these formulas, and we will see a procedure for finding infinitely many such formulas, with any number of summands. In particular, this allows us to show that there are Machin-type formulas with Lehmer measure as small as one wishes.

The content of the talk comes from this article: A. Gasull, F. Luca and J. L. Varona, *Three essays on Machin's type formulas*, Indag. Math. (N.S.) 34 (2023), 1373-1396.

EXTENSION OF MULTIPLE ORTHOGONALITY TO THE BIVARIATE CASE

JUAN ANTONIO VILLEGAS, LIDIA FERNÁNDEZ

Abstract

Multiple Orthogonal Polynomials (MOPs) in a single variable generalize the standard theory by satisfying orthogonality conditions with respect to several measures, and they hold significant importance in various applications, including Hermite-Padé rational approximation, random matrix theory, and integrable systems.

However, multiple orthogonality has been explored mainly in the univariate case. In this talk, I will start by presenting some preliminaries on multiple orthogonality in the univariate case, followed by some definitions of the two main types of multiple orthogonality, examples and extended results, closing with a discussion on the relation between the univariate and bivariate cases.

Keywords: Orthogonal Polynomials, Approximation Theory, Applications, Multiple orthogonality.
AMS Classification: 33C45, 33C50, 42C05.

Bibliography

[1] C. F. Dunkl and Y. Xu *Orthogonal Polynomials of Several Variables*, Cambridge University Press (2014).
[2] M. E. H. Ismail, *Classical and quantum orthogonal polynomials in one variable*, Encyclopedia of mathematics and its applications, Cambridge University Press (2005).
[3] T. Koornwinder, Two-Variable Analogues of the Classical Orthogonal Polynomials. *Elsevier eBooks*, (1975), 435–495.
[4] A. Martínez–Finkelshtein and W. Van Assche, WHAT IS...A multiple orthogonal polynomial? *Notices of the American Mathematical Society*, 63 (2016), 1029–1031.
[5] W. Van Assche *Orthogonal and multiple orthogonal polynomials, random matrices, and Painlevé equations*, "Orthogonal Polynomials" (M. Foupouagnigni, W. Koepf, eds), Tutorials, Schools and Workshops in the Mathematical Sciences, Springer Nature Switzerland (2020) 629–683.

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Universidad de Granada; IMAG-María de Maeztu, grant "IMAG CEX2020–001105–M"; GOYA: "Grupo de Ortogonalidad Y Aplicaciones" and "Departamento de Matemática Aplicada".