

Modeling and simulation of a gas distribution pipeline network

A. Herrán-González*, J.M. De La Cruz, B. De Andrés-Toro, J.L. Risco-Martín

Department of Computer Architecture and Automatic Control, Complutense University, 28040 Madrid, Spain

Received 1 September 2005; received in revised form 1 February 2008; accepted 12 February 2008

Available online 29 February 2008

Abstract

This research study focuses on the modeling and simulation of a gas distribution pipeline network with a special emphasis on gas ducts. Gas ducts are the most important components of such kind of systems since they define the major dynamic characteristics. Isothermal, unidirectional flow is usually assumed when modeling the gas flow through a gas duct. This paper presents two simplified models derived from the set of partial differential equations governing the dynamics of the process. These models include the inclination term, neglected in most related papers. Moreover, two numerical schemes are presented for the integration of such models. Also, it is shown how the pressure drop along the pipe has a strong dependency with the inclination term. To solve the system dynamics through the proposed numerical schemes a based MATLAB-Simulink library was built. With this library it is possible to simulate the behavior of a gas distribution network from the individual simulation of each component. Finally, the library is tested through three application examples, and results are compared with the existing ones in the literature.

© 2008 Elsevier Inc. All rights reserved.

Keywords: Gas distribution pipeline networks; Gas ducts; Mathematical model; Inclination term; Partial differential equations (PDE's); Numerical scheme

1. Introduction

Gas distribution pipeline networks are systems with hundreds or thousands of kilometers of pipes and production, storage and distribution centers, compression stations, and many other devices like valves and regulators. These types of systems work at high pressures and use compression stations to supply to the gas enough energy to be moved along long distances. When gas flows through the network, it suffers energy and pressure losses due to the friction between the gas and the inner walls of gas ducts but also due to the heat transfer between gas and the environment. If demanded gas has to be supplied to delivery points with a specified pressure, the undesired pressure drops along the network must be periodically restored. This task is performed by compression stations installed on the network, but these usually consume over 3% or 5% of total gas transported, [1]. On the other hand, if an increase of gas pressure takes place, certain safety limits

* Corresponding author.

E-mail address: aherrang@fis.ucm.es (A. Herrán-González).

Nomenclature

c	isothermal speed of sound (m s^{-1})
D	gas duct diameter (m)
e	specific internal energy (J kg^{-1})
f	friction factor
h	specific enthalpy (J kg^{-1})
I	No. of points of spatial mesh
L	gas duct length (m)
M	molecular mass of gas (kg mol^{-1})
p	gas pressure (bar)
q	mass flow rate (kg s^{-1})
Q	volumetric flow rate ($\text{m}^3 \text{s}^{-1}$)
Q_n	volumetric flow rate in normal conditions ($\text{m}^3 \text{s}^{-1}$)
R_u	universal gas constant ($\text{J mol}^{-1} \text{K}^{-1}$)
R_g	gas constant ($\text{J kg}^{-1} \text{K}^{-1}$)
S	gas duct cross-sectional area (m^2)
t	temporal coordinate (s)
T	temperature of gas (K)
v	velocity (m/s)
x	axial coordinate (m)
Z	compressibility factor
Δt	time step (s)
Δx	mesh spatial size (m)
Ω	heat flow per unit length ($\text{J m}^{-1} \text{s}^{-1}$)
ρ	gas density (kg m^{-3})
ρ_n	gas density in normal conditions (kg m^{-3})
θ	inclination angle ($^\circ$)
τ	tangential stress (N)

could be exceeded. In these cases it is necessary to activate an emergency mechanism to avoid such contingency. To cope with it the network has pressure regulators able to reduce the pressure until reaching values which are within these limits. As in the compressor stations case, these devices consume a fraction of the total gas transported by the network.

In literature, there are different PDE models to describe the behavior of gas dynamics inside a gas duct with constant circular section. However, in most cases they make reference to a horizontal gas duct. Ke and Ti, [2], begin with the continuity and conservation of momentum equations and, after rejecting dispersive and inclination terms, make an electrical analogy of the system in terms of R , L and C in function of gas duct characteristics. Finally, they extend this formulation to networks and propose an algorithm together with some application examples. Osiadacz, [3], proposes different models of the system with their corresponding numerical methods for a horizontal gas duct but without making any reference to the computational time used in relation to the accuracy of the results and the step length employed in the integration of these models. Martko et al. [4], is one of the few authors who keeps the inclination term in the equations; nevertheless, he does not develop an efficient numerical method to solve them since he makes use of the program PIPESIM.

The objective of this research study is the implementation of a gas distribution pipeline network simulator to evaluate different operation scenes, taking into account the previously mentioned features for such kind of systems. Having an efficient simulator is very useful to acquire a deeper knowledge of the system but also to test possible future control strategies. Here, two numerical schemes were developed to integrate the simplified models derived from the original one on the basis of the usual operation conditions. These algorithms converge to a satisfactory solution even in the case of maintaining the inclination term in the equations. In addi-

tion, they are easily implemented using the MATLAB-Simulink tool. The paper continues in Section 2 with a brief review of the set of PDE’s governing the dynamics of the process. Also, two simplified models are derived from the original one. Section 3 shows a brief description of the numerical schemes proposed to integrate these models, together with the mesh to be employed for a computer implementation in each case. Section 4 describes the MATLAB-Simulink library developed to simulate such kind of systems makes. It also shows the necessary initial and boundary conditions for the problem to be well posed, together with some preliminary examples showing the pressure profiles calculated from both models. Section 5 shows the results obtained by the proposed numerical schemes in three application examples: a straight gas duct, a gas duct under three typical pressure ranges and a gas pipeline network, comparing in all cases these results with data taken from literature. Finally, the conclusions of the study are presented in Section 6.

2. Modeling of the system

2.1. Gas duct modeling

It is known that one of the most interesting issues in a gas distribution system is the search of the pressure that should be applied to the gas from the compression stations to reach the consumption points with the required conditions by the client. The gas flow inside a gas duct is governed by some dynamic laws, constituting a “hydrodynamic” system, [5]. The set of PDE’s describing the one-dimensional gas flow dynamic through a gas duct is obtained by applying the conservation of mass, moment and energy to the infinitesimal control volume shown in Fig. 1, [6].

This equation for the space and time dependent density $\rho(x, t)$, pressure $p(x, t)$, speed $v(x, t)$ and temperature $T(x, t)$ are shown in Eq. (1), where S is the cross-sectional area of gas duct, D their diameter, T the temperature of the gas, e the specific internal energy, h the specific enthalpy, $h = e + p/\rho$, τ the tangential stress between gas and the inner wall of gas duct and Ω the heat transfer between the gas and its surroundings per unit length. Since the system has three equations and four unknown variables, it is necessary an additional equation to have a complete system. This is indeed the gas state equation shown in Eq. (2) where M is the molecular mass of gas, Z is a variable compressibility and $R_g = R_u/M$.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0,$$

$$\frac{\partial}{\partial t}(\rho v S) + \frac{\partial}{\partial x}(p S + \rho v^2 S) + |\tau| \pi D + \rho S g \sin \theta = 0,$$

$$\frac{\partial}{\partial t} \left[\left(e + \frac{1}{2} v^2 \right) \rho S \right] + \frac{\partial}{\partial x} \left[\left(h + \frac{1}{2} v^2 \right) \rho v S \right] - \Omega + \rho S g v \sin \theta = 0,$$

$$p = \rho \cdot \frac{Z R_u T}{M} = \rho \cdot Z R_g T.$$

In order to solve Eqs. (1) and (2) it is necessary to know the value of the thermal flow term, Ω . There are two especially important cases: (a) isothermal flow ($T = \text{constant}$), where the energy equation becomes redundant except for calculating the value of Ω ; and (b) adiabatic flow ($\Omega = 0$), that includes the particular isentro-

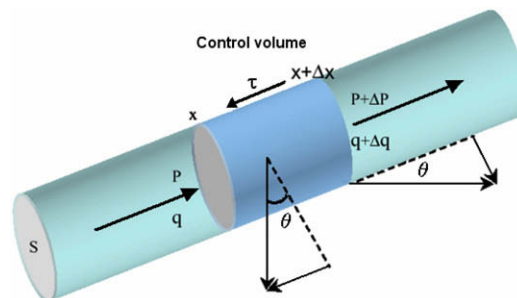


Fig. 1. Control volume in a gas duct.

pic flow case. These two cases can be considered as two extremes. Isothermal flow corresponds to slow dynamic changes in which temperature changes within the gas are sufficiently slow so as to be cancelled out by heat conduction between the gas duct and its surroundings. Adiabatic flow corresponds to fast dynamic changes in the gas where the slow effects of heat conduction can be ignored. Isentropic flow is only possible when friction effects are negligible, and it is not normally valid in gas transmission problems. In the general case, where $\Omega \neq 0$ and there is no thermal equilibrium between gas duct and the ground, we will need more equations to model the heat conduction process.

As consumer demand is usually expressed in terms of mass flow, it is advisable to replace the speed v in Eq. (1) by flow q defined by Eq. (3). On the other hand, the friction can be represented by means of the Fanning factor f using Eq. (4). Then, expressing Eqs. (1) and (2) in function of friction factor and mass flow, and assuming isothermal flow since it is the commonest approach, the one-dimensional gas flow dynamics inside a gas duct is described by the set of PDE's shown in Eq. (5). As it is seen, the friction term in Eq. (5) contains the expression $q|q|$ instead of q^2 , which ensures that the friction force is always against the gas movement. If there is no reverse flow on the network, then q^2 can be kept

$$q = \rho v S = \rho Q, \tag{3}$$

$$f = \frac{|\tau|}{\frac{1}{2}\rho v^2}, \tag{4}$$

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{1}{S} \frac{\partial q}{\partial x} &= 0, \\ \frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left(S p + \frac{q^2}{S \rho} \right) + \frac{2 f q |q|}{D S \rho} + \rho S g \sin \theta &= 0. \end{aligned} \tag{5}$$

2.2. Simplified models

It has been already introduced an important simplification in Eq. (1) for slow dynamic effects, the assumption of isothermal flow. Even so, the nonlinear PDE's shown in Eq. (5) must be solved. Neglecting the friction term, the equations of the non viscous gas classic dynamics in which pressure waves propagate through the gas at the speed of sound without any damping are obtained. Neglecting the inertia terms it results in a creeping motion, described by parabolic PDE's related to heat conduction and diffusion, and the undulatory propagation phenomenon disappears. The flow in real gas ducts is a mixture of these two extremes, but it is possible to neglect some terms to make the computation simpler, [7]. In order to decide what terms to neglect it is needed to make an estimation of the magnitude of each term for typical values of the variables involved in the process. As an example, it is considered a gas duct for high pressure gas transmission, where the dynamic variations are stimulated by demand fluctuations, and it takes hours to complete a significant change. The values given for the variables at these conditions, together with the estimated magnitude for the terms in the moment equation are the following:

$$\begin{aligned} x = 10^5 \text{ m} & \quad \frac{\partial q}{\partial t} \approx 0.1 \\ D = 1 \text{ m} & \quad \frac{\partial}{\partial x} (S p) \approx 50 \\ t = 3600 \text{ s} & \\ f = 0.002 & \quad \Rightarrow \frac{\partial}{\partial x} \left(\frac{q^2}{S \rho} \right) \approx 0.04 \\ p = 70 \text{ bar} & \\ \rho = 50 \text{ kg/m}^3 & \quad \frac{2 f q |q|}{D S \rho} \approx 15 \\ v = 10 \text{ m/s} & \quad \rho S g \sin \theta \approx 385 \sin \theta \end{aligned}$$

A first simplification of the equations can be made neglecting the third term in Eq. (5). Then, the gas flow dynamics through a gas duct can be represented by the system of PDE's shown in Eq. (6) where it has been used the relation $p = c^2 \rho$ valid for an isothermal process and the relation $q = \rho v S = \text{constant} = \rho Q = \rho_n Q_n$ to express the model in function of flow rate in normal conditions, $Q_n(x, t)$, and pressure $p(x, t)$

$$\text{Model A} \begin{cases} \frac{\partial p}{\partial t} = -\frac{c^2 \rho_n}{S} \frac{\partial Q_n}{\partial x} \\ \frac{\partial Q_n}{\partial t} = -\frac{S}{\rho_n} \frac{\partial p}{\partial x} - \frac{2fc^2 \rho_n}{DS} \frac{Q_n^2}{p} - \frac{Sg \sin \theta}{\rho_n c^2} p. \end{cases} \quad (6)$$

Another simplified model can be derived from the hypothesis that the boundary conditions do not change quickly and that the capacity of gas duct is relatively large. In this case, the first term can be also eliminated from the model. Then, expressing again the model in function of flow rate in normal conditions, $Q_n(x, t)$, and pressure, $p(x, t)$, it results the set of PDE's shown in Eq. (7)

$$\begin{cases} \frac{\partial p}{\partial t} = -\frac{c^2 \rho_n}{S} \frac{\partial Q_n}{\partial x} \\ \frac{\partial p}{\partial x} = -\frac{2fc^2 \rho_n^2 Q_n^2}{DS^2 p} - \frac{g \sin \theta}{c^2} p, \end{cases} \quad (7)$$

Now, using $\partial_x(p^2) = 2p\partial_x(p)$, this system of equations becomes the system shown in Eq. (8). This system was used by Larson and Wismer in [8] for unsteady flow simulation

$$\begin{cases} \frac{\partial p}{\partial t} = -\frac{c^2 \rho_n}{S} \frac{\partial Q_n}{\partial x} \\ \frac{\partial p^2}{\partial x} = -\frac{4fc^2 \rho_n^2 Q_n^2}{DS^2} - \frac{2g \sin \theta}{c^2} p^2. \end{cases} \quad (8)$$

Differentiating with respect to x the second equation in Eq. (8) and using the first one to eliminate the term $\partial_x(Q_n)$, the model can be written by Eq. (9)

$$\frac{\partial^2 p^2}{\partial x^2} = \frac{8fc^2 \rho_n^2 Q_n}{DS^2} \frac{S}{c^2 \rho_n} \frac{\partial p}{\partial t} - \frac{2g \sin \theta}{c^2} \frac{\partial p^2}{\partial x}. \quad (9)$$

Finally, using again the relation $\rho Q = \rho_n Q_n$, and taking into account that $\partial_t(p^2) = 2p\partial_t(p)$ and $S = \pi(D/2)^2$, the second simplified model can be written as shown in Eq. (10). This is a nonlinear parabolic model in which: (a) assuming that α is constant (true for the case in which the flow variations through gas duct are slow), it becomes a linear model with respect to p^2 ; and (b) assuming that $Q(x, t)$ is an average over all the length of the gas duct in each time interval Δt , it becomes a second order linear parabolic PDE with respect to p^2 in each Δt

$$\text{Model B} \begin{cases} \frac{\partial^2 p^2}{\partial x^2} + \beta \frac{\partial p^2}{\partial x} = \alpha \frac{\partial p^2}{\partial t} \\ \text{where } \alpha = \frac{16fQ}{D^3 c^2 \pi} \text{ and } \beta = \frac{2g \sin \theta}{c^2}. \end{cases} \quad (10)$$

3. Numerical resolution

Analytical solution of PDE's implies compact expressions which provide the continuous variation of the dependent variables through the region in which they are defined. In contrast, numerical solution can only provide an answer in a set of usually uniformly spaced discrete points of this region, called grid points. The accuracy in numerical resolution grows by increasing the number of points on the grid but it increases the computational requirements. In this section a pair of numerical schemes for numerical resolution of gas models is proposed, [9]. Firstly, the method of characteristics is applied to solve model A. Secondly, the Crank–Nicolson method is applied to solve model B.

3.1. Method of characteristics

Defining the coefficients shown in Eq. (11), model A shown in Eq. (6) can be rewritten as shown in Eq. (13). The characteristic equations for this system are the ones shown in Eq. (13), [9]. This set of characteristics is similar to the shown in Osiadacz [3], but considering the inclination term

$$\alpha_1 = \frac{c^2 \rho_n}{S}; \quad \alpha_2 = \frac{S}{\rho_n}; \quad \alpha_3 = \frac{2fc^2 \rho_n}{DS}; \quad \alpha_4 = \frac{Sg \sin \theta}{\rho_n c^2}, \tag{11}$$

$$\frac{\partial p}{\partial t} + \alpha_1 \frac{\partial Q_n}{\partial x} = 0 \tag{12}$$

$$\frac{\partial Q_n}{\partial t} + \alpha_2 \frac{\partial p}{\partial x} + \alpha_3 \frac{Q_n^2}{p} + \alpha_4 p = 0 \tag{13}$$

$$\left. \begin{aligned} dx - c dt &= 0 \\ -\alpha_2 dp - \alpha_1 \alpha_2 \frac{1}{c} dQ_n - \left(\alpha_3 \frac{Q_n^2}{p} + \alpha_4 p \right) dx &= 0 \end{aligned} \right\} C_+$$

$$\left. \begin{aligned} dx + c dt &= 0 \\ -\alpha_2 dp + \alpha_1 \alpha_2 \frac{1}{c} dQ_n - \left(\alpha_3 \frac{Q_n^2}{p} + \alpha_4 p \right) dx &= 0 \end{aligned} \right\} C_-$$

The system shown in Eq. (13) can be integrated over a grid of characteristics like those shown in Fig. 2 in the rank $0 \leq t \leq T, 0 \leq x \leq L$. Assuming a linear approximation, the slopes of the lines satisfy the condition shown in Eq. (14). In order to get a consistent numerical scheme, it is necessary that Δx and Δt to be linked by means of Eq. (14) so that the numerical integration of the equations was carried out over the characteristic curves. In such a case the system shown in Eq. (13) becomes the one shown in Eq. (15).

$$\pm c \frac{\Delta t}{\Delta x} \leq 1, \tag{14}$$

$$C_- : dp - \frac{c \rho_n}{S} dQ_n + \left(\frac{2f}{D} \left(\frac{c \rho_n}{S} \right)^2 \frac{Q_n^2}{p} + \frac{g \sin \theta}{c^2} \right) dx = 0, \tag{15}$$

$$C_+ : dp + \frac{c \rho_n}{S} dQ_n + \left(\frac{2f}{D} \left(\frac{c \rho_n}{S} \right)^2 \frac{Q_n^2}{p} + \frac{g \sin \theta}{c^2} \right) dx = 0.$$

Finally, using the coefficients shown in Eq. (16) and denoting by p_A the pressure in the layer A, p_B the pressure in layer B, Q_{nA} the flow rate in layer A and Q_{nB} the flow rate in layer B, the system shown in Eq. (15) can be approximated by Eq. (17) for $i = 0, 1, \dots, I - 1$.

$$\alpha_5 = \frac{c \rho_n}{S}, \quad \alpha_6 = \frac{2f}{D} \left(\frac{c \rho_n}{S} \right)^2 dx, \quad \alpha_7 = \frac{g \sin \theta}{c^2} dx \quad \text{with} \quad dx = \frac{\Delta x}{2}, \tag{16}$$

$$C_- : p_{i,B} - p_{i+1,A} - \alpha_5 (Q_{ni,B} - Q_{ni+1,A}) - 0.5 \alpha_6 \left(\frac{Q_{ni,B}^2}{p_{i,B}} + \frac{Q_{ni+1,A}^2}{p_{i+1,A}} \right) - 0.5 \alpha_7 (p_{i,B} + p_{i+1,A}) = 0, \tag{17}$$

$$C_+ : p_{i,B} - p_{i,A} + \alpha_5 (Q_{ni,B} - Q_{ni,A}) + 0.5 \alpha_6 \left(\frac{Q_{ni,B}^2}{p_{i,B}} + \frac{Q_{ni,A}^2}{p_{i,A}} \right) + 0.5 \alpha_7 (p_{i,B} + p_{i,A}) = 0.$$

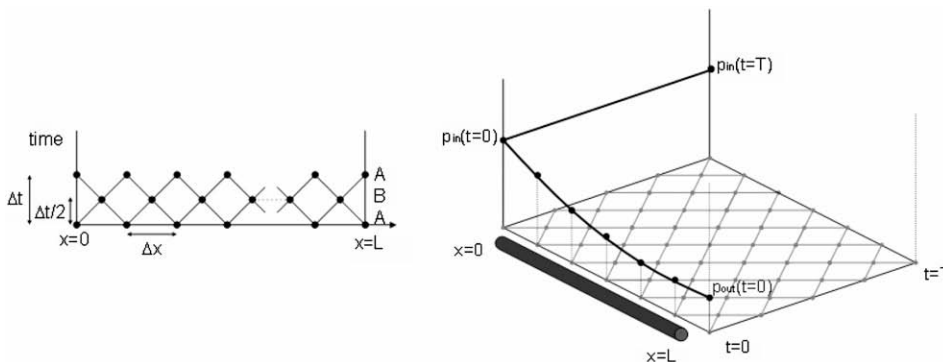


Fig. 2. Numerical scheme for the method of characteristics.

3.2. The Crank–Nicolson method

With the variable change $g = p^2$, model B shown in Eq. (10) can be written as shown in Eq. (18). In this equation, each term can be approximated by its finite differences scheme, [9]. A forward difference approximation for the temporal derivative at point (i, n) is the one shown in Eq. (19). For the spatial derivative, it can be taken into account that a forward difference at point (i, n) can be seen as a central difference at point $(i, n + 1/2)$. Moreover, this central difference can be seen as an average of the central differences at points $(i, n + 1)$ and (i, n) as it is shown in Fig. 3. Applying this idea to the first and second order spatial derivatives it gives Eqs. (20) and (21), respectively

$$\frac{\partial^2 g}{\partial x^2} + \beta \frac{\partial g}{\partial x} = \alpha \frac{\partial g}{\partial t}, \tag{18}$$

$$\left(\frac{\partial g}{\partial t}\right)_{i,n} = \frac{g_{i,n+1} - g_{i,n}}{\Delta t}, \tag{19}$$

$$\left(\frac{\partial g}{\partial x}\right)_{i,n} = \frac{1}{2} \left(\frac{g_{i+1,n} - g_{i-1,n}}{2\Delta x} + \frac{g_{i+1,n+1} - g_{i-1,n+1}}{2\Delta x} \right), \tag{20}$$

$$\left(\frac{\partial^2 g}{\partial x^2}\right)_{i,n} = \frac{1}{2} \left(\frac{g_{i-1,n} - 2g_{i,n} + g_{i+1,n}}{\Delta x^2} + \frac{g_{i-1,n+1} - 2g_{i,n+1} + g_{i+1,n+1}}{\Delta x^2} \right). \tag{21}$$

Using these approximations over the model shown in Eq. (18), it gives Eq. (22) as the equation of the proposed numerical scheme for $i = 2, 3, \dots, I - 1$, where coefficients a and b are given in Eq. (23). This equation corresponds to an implicit method. The most important characteristic of Crank–Nicolson method is that it is consistent and stable independently of the step length used. Then, the only aspect to be considered for the choice of one or another step length is the desired accuracy of the results

$$(a - b) \cdot g_{i-1,n+1} + (-2a - 2) \cdot g_{i,n+1} + (a + b) \cdot g_{i+1,n+1} = (-a + b) \cdot g_{i-1,n} + (2a - 2) \cdot g_{i,n} + (-a - b) \cdot g_{i+1,n} \tag{22}$$

$$a = \frac{\Delta t}{\alpha \Delta x^2}; \quad b = \frac{2\Delta t \beta}{\alpha \Delta x} = \frac{a\beta \Delta x}{2}. \tag{23}$$

4. Computer implementation

Dynamical behavior of each component of the network can be simulated on a computer using some modeling language. For this purpose, MATLAB-Simulink tool was used through the called S-functions, [10]. An S-function is a computer language description of a Simulink block. S-functions allow adding own blocks to

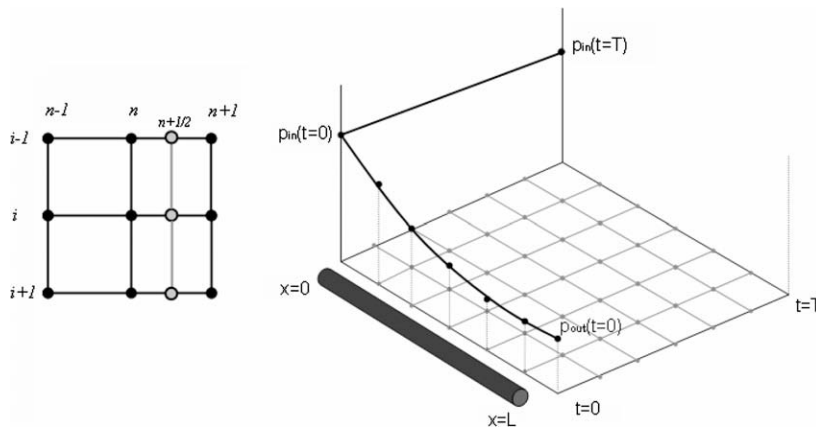


Fig. 3. Numerical scheme for the Crank–Nicolson method.

Simulink models. With them it is possible to create own blocks in MATLAB, C, C++, Fortran or Ada. By following a set of simple rules, it is very easy to implement own algorithms in an S-function. Once the S-function is written and its name placed in an S-Function block (available in the User-Defined Functions block library), it is also possible to customize the user interface by using masking. The most common use of S-functions is to create custom Simulink blocks, but they can be used for a variety of applications. An advantage of using S-functions is that it is possible to build a general purpose block that can be used many times in a model, varying parameters with each instance of the block.

4.1. MATLAB-Simulink library

Based on a S-functions philosophy a MATLAB-Simulink component library was built to simulate gas distribution pipeline networks. As it is shown in Fig. 4, the library is composed by several elements as gas ducts, compressor station, regulators, valves, etc. Since this paper focus on the gas ducts modeling and simulation, only this component will be explained in detail.

As it can be seen in Fig. 4 the block implementation of a gas duct has two inputs and two outputs. Fig. 5 shows the internal implementation of such a block. Input ports specify the gas flow demand at the outlet and the gas pressure at the inlet of the gas duct (these are the necessary boundary conditions to integrate both simplified models). After integration the block gives through output ports the gas pressure at the outlet and gas flow at the input of the gas duct. Both input and output signals pass through a zero-order-hold to simulate an external data sampling each T seconds needed for future discrete-time control applications.

Fig. 5 also shows how the internal implementation of a gas duct has been masked to customize the user interface. Clicking twice over the block a window appears to specify all the needed parameters to simulate the gas duct behavior. This window let the user specify the physical characteristic of a gas duct but also the desired parameters for the numerical schema to be used to solve the model. Then clicking the box “gas duct” it is possible to specify the length, diameter, friction factor, density of gas, etc, and the initial conditions from which to start the simulation. Similarly, clicking the box “simulator” it is possible to select between the characteristic method or Crank–Nicolson method to solve model A or B respectively, and the step lengths to be used for time and spatial discretization. These steps can be linked through Eq. (14) or selected independently by activating the box “fixed temporal step”.

4.2. Initial and boundary conditions

As an application example of the algorithms developed in the previous section, the calculation of the temporal variation of pressure and flow rate in discrete points of a gas duct is carried out. The initial conditions were determined on the basis of the gas duct state at the start time of the simulation. Suppose that the initial state is at $t = 0$. Thus, $Q_n(x, 0) = f_Q(x)$ is the initial flow rate and $p(x, 0) = f_p(x)$ the initial pressure through the gas duct. In addition, it is assumed here a steady initial state. In these conditions, flow and pressure gas profiles can be calculated forcing conditions shown in Eq. (24) over model A. Solving its results in a constant gas flow

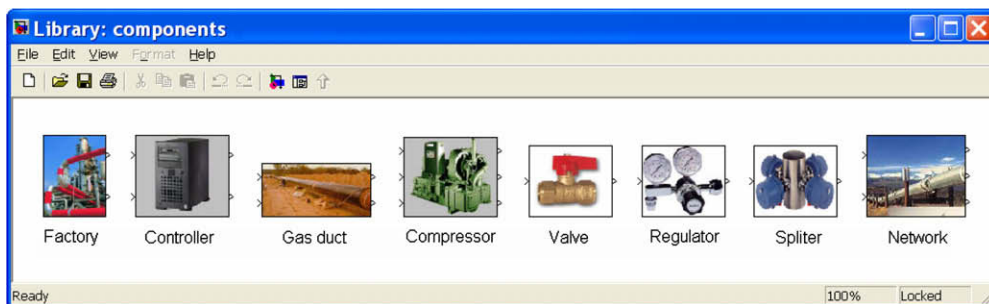


Fig. 4. Developed MATLAB-Simulink library.

over all the gas duct $Q_n(x,0) = Q_{n0}$, and the pressure profile given by Eq. (25) for the case of an inclined gas duct, and by Eq. (26) for an horizontal one, where coefficients σ and ξ are shown in Eq. (27)

$$\frac{\partial p}{\partial t} = 0, \quad \frac{\partial Q_n}{\partial t} = 0, \tag{24}$$

$$p(x) = \sqrt{\left(p(0)^2 - \frac{\xi}{\sigma}(e^{\xi x} - 1)\right)e^{-\xi x}}, \tag{25}$$

$$p(x) = \sqrt{p(0)^2 - \xi x}, \tag{26}$$

$$\sigma = \frac{2g \sin \theta}{c^2}, \quad \xi = \frac{f}{D} \left(\frac{2\rho_n c Q_{n0}}{S}\right)^2. \tag{27}$$

Taking into account that $c^2 = ZRT$, coefficients σ and ξ have some terms, such as T , Z , R and f , that are difficult to know accurately. For this reason, pressure drop could be far to which would be obtained with a good

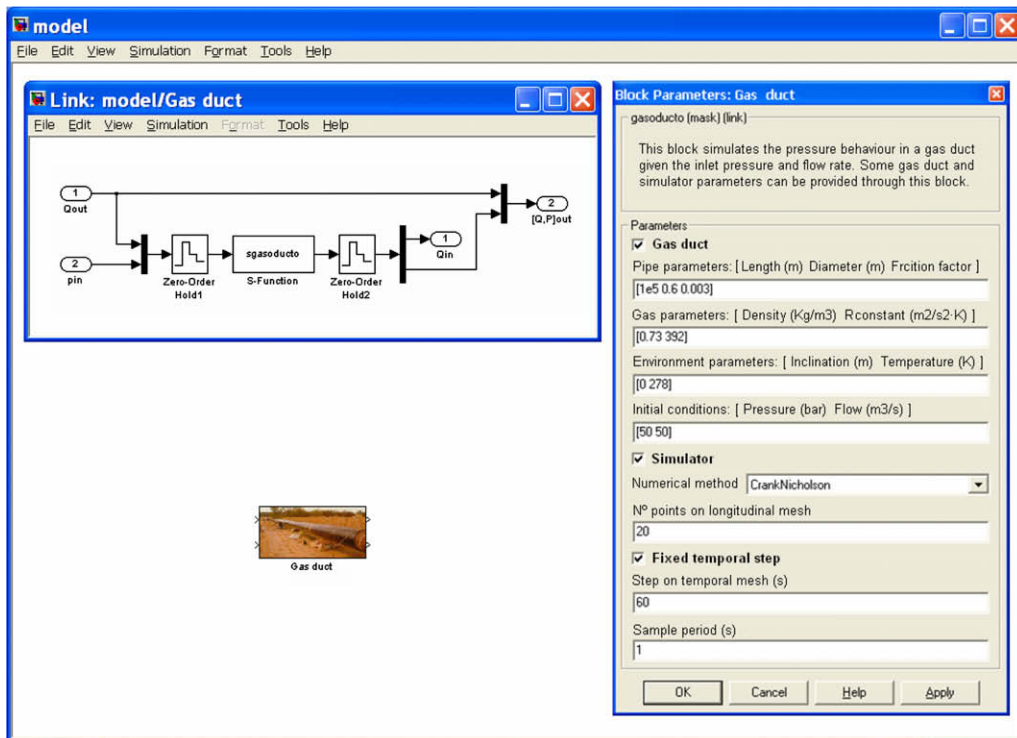


Fig. 5. Gas duct block implementation.

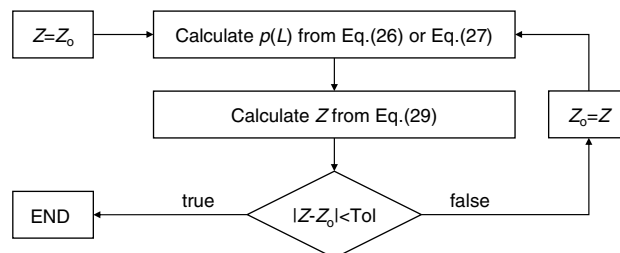


Fig. 6. Algorithm used to calculate Z and p_L .

knowledge of such terms. Among these factors, approximate average values for T and R are considered. The value of Z can be approximated from an initial value by the algorithm shown in Fig. 6. This algorithm makes use of Eq. (28) to estimate the value of Z in each step. Eq. (28) is an empirical equation proposed by [11]. Once Z and p_L are calculated, the pressure profile through the gas duct in steady state conditions is determined by Eq. (25) or Eq. (26).

$$Z = 1 - \frac{p_{\text{ave}}}{390} \quad \text{where} \quad p_{\text{ave}} = \frac{1}{2}(p(0) + p(L)). \tag{28}$$

Boundary conditions are dependent on the form in which gas duct is fed. It is assumed that the gas duct under study takes the gas from a compression station at $x = 0$ and supplies the gas to a consumer located at $x = L$ according to a time variable curve demand $Q_n(L, t)$. This demand is usually a periodic function as shown in Fig. 7. In addition it is assumed that introducing appropriate changes in the capacity of the compression station the pressure at the beginning of the gas duct can be kept constant. Suppose the following values for the parameters of the problem. With these data, the next two sections try to find the temporal evolution of the pressure along the gas duct by means of both numerical schemes developed in the previous section

$$\begin{aligned} L &= 10^5 \text{ m} & \rho_n &= 0.73 \text{ kg/m}^3 & p(0, t) &= 50 \text{ bar} & f &= 0.003 \\ D &= 0.6 \text{ m} & R &= 392 \text{ m}^2/(\text{s}^2 \text{ K}) & Q_n(x, 0) &= 50 \text{ m}^3/\text{s} \\ H &= 100 \text{ m} & T &= 278 \text{ K} & t_{\text{max}} &= 2 \text{ days.} \end{aligned}$$

4.3. Integration of models A and B

As it was said in Section 3.1, model A can be integrated through a grid of characteristic curves. Fig. 8 shows the pressure profile through the gas duct obtained when $I = 20$ points are used to perform the integration over the characteristic curves with spatial and temporal step sizes linked by Eq. (14). As it can be seen, inlet pressure is kept constant due to the imposed boundary condition, and decreases from inlet to outlet along the gas duct during the operating period in response to the cyclical behavior of the gas flow demanded at the outlet. Such response decreases monotonically from outlet to inlet along the gas duct, showing that transient propagates from outlet to inlet in this case.

Section 3.2 showed how model B can be solved using the Crank–Nicolson method. In this case it is needed to know the value of pressure boundary condition at the outlet of the gas duct in each time instant. This value can be calculated from Eq. (25) for the general case of an inclined gas duct. To do this, it has been assumed a steady flow throughout the last discrete element of the gas duct, that is, between x_{I-1} and x_I . Then evaluating

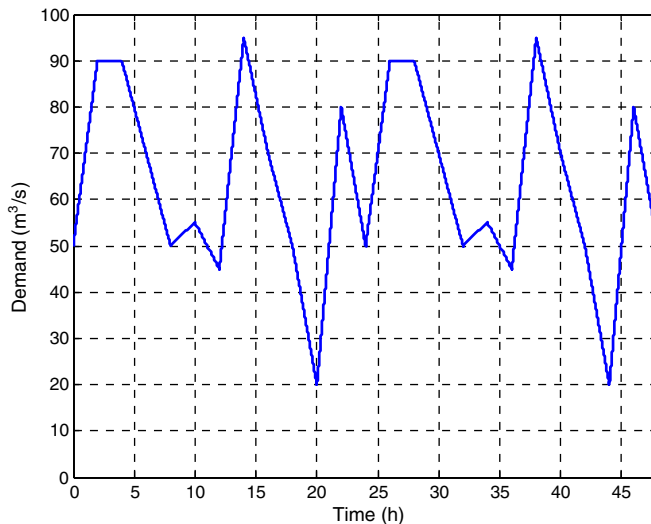


Fig. 7. Typical gas flow demand (2 days).

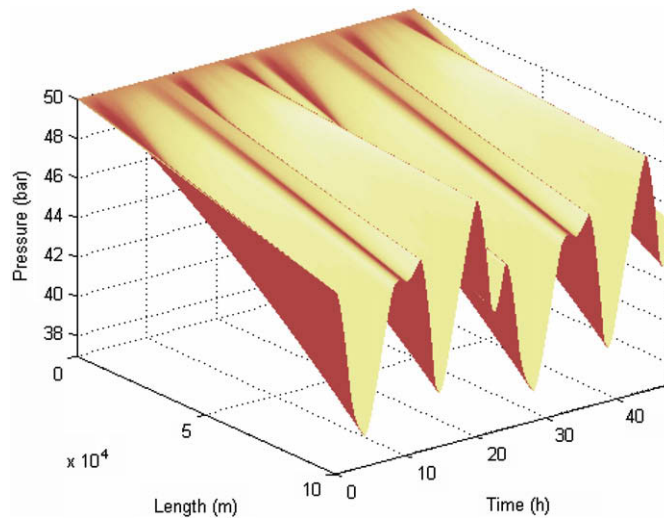


Fig. 8. Pressure profile calculated solving model A with the method of characteristics.

Eq. (25) for the last portion of the gas duct (between x_{I-1} and x_I) at time t_{j+1} , it gives Eq. (29), where $p_{I-1,j}$ has been used instead of $p_{I-1,j+1}$ since its value is unknown till next time instant, and $Q_{n0} = Q_{nI,j+1}$ is given by the boundary condition

$$p_{I,j+1} = \sqrt{\left(p_{I-1,j}^2 - \frac{\gamma}{\sigma} (e^{\xi \Delta x} - 1) \right) e^{-\xi \Delta x}} \tag{29}$$

Once the algorithm is applied, the pressure profile through the gas duct has a similar behavior to the one shown in Fig. 8 for the method of characteristics. As it was said before, this method is unconditionally stable and consistent and then the selection of the step lengths will only depend on the desired accuracy of the results. In contrast, it has an increase in the computational time for small values of Δx and Δt . So, the usual situation is to reach a compromise between the computational time and the precision of the results. Fig. 9 shows the pressure profile at the outlet of the gas duct calculated by both models for the same example.

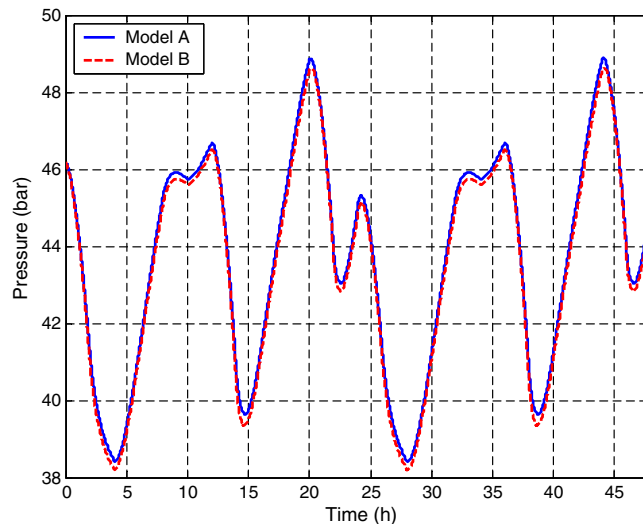


Fig. 9. Pressure profile at the outlet calculated by solving models A and B.

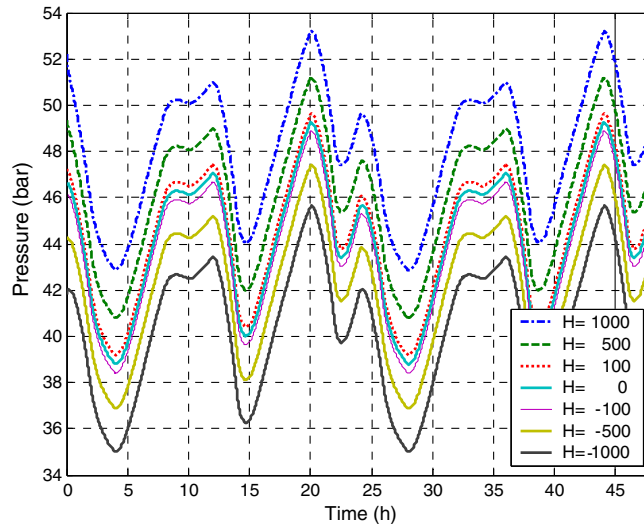


Fig. 10. Outlet pressure for different values of the inclination.

Finally, to see the effect of the inclination term over the results, the gas duct has been simulated for several values of the level difference between the inlet and the outlet. To do that, model A was used. Results are presented in Fig. 10, showing how the pressure profile at the outlet changes largely from the previously obtained results. Therefore, the inclination term must be taken into account. In addition, it is shown how for ascending gas ducts ($H > 0$) there is an increase of the outlet pressure as the gas is compressed over gas flow direction. The opposite happens for descending gas ducts ($H < 0$).

5. Application examples

To show the utility of the developed MATLAB-Simulink library three application examples were solved. First and second examples only involved a straight gas duct while the third one is a gas pipeline network. All the examples were taken from literature to compare results with the existing ones.

5.1. Straight gas duct

First of all, the results obtained by the numerical schemes proposed in this paper are compared with those obtained by the method proposed in Osiaacz [3] and Ke and Ti [2]. To do that, it is assumed an horizontal gas duct with constant circular section of 0.6 m diameter and 100 km length. The inlet pressure is held to 50 bar and the flow demand changes according to the curve shown in Fig. 11. The operation temperature is 278 K, the fluid density is 0.73 kg/m^3 , the specific gravity is 0.6 and the friction factor is 0.003. Running the developed numerical schemes, the obtained results are shown in Fig. 13. The simulations carried out by the numerical schemes presented here were made with a value of $I = 20$ and keeping the condition imposed by Eq. (14). Other values of these parameters produce a variation in computational time without modifying excessively the results. To solve this example with the developed library it is just to create the Simulink model shown in Fig. 12. Once the model is created, the user interface shown in Fig. 5 leads the user to specify all the parameters for the problem to be solved. Fig. 13 shows the results for this example showing how both models produce similar results to Osiaacz [3].

5.2. A gas duct in three typical pressure ranges

The second application example is a network that was used by the London research service (LRS) to demonstrate the versatility of their program PAN, [2]. Initially, the gas duct is in a steady state and then the

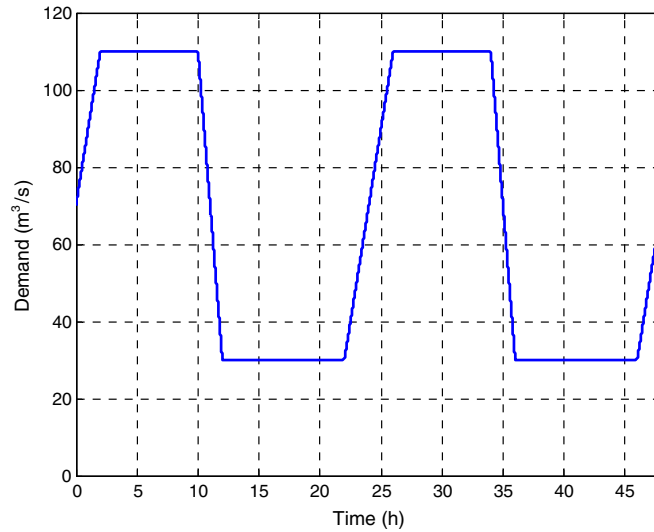


Fig. 11. Gas flow demand for the first application example.

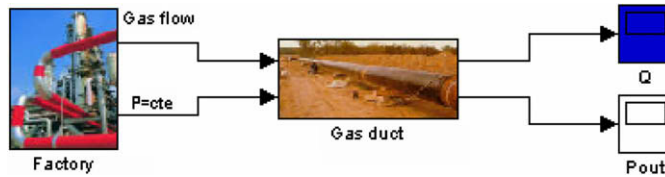


Fig. 12. Outlet pressure for the first application example.

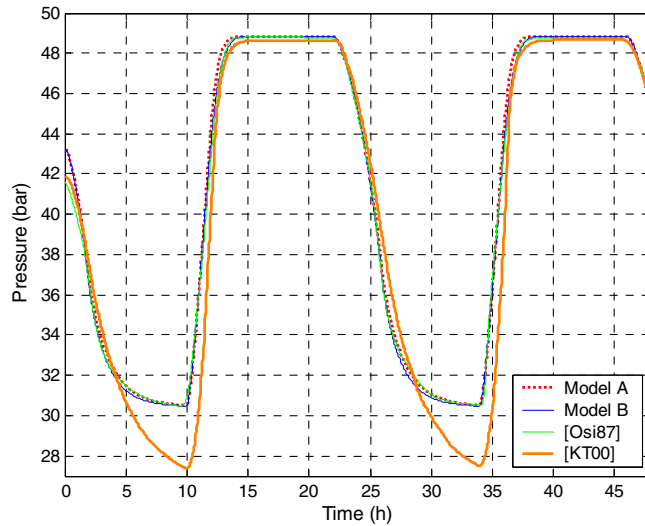


Fig. 13. Outlet pressure for the first application example.

demand is increased in a 50% step change remaining constant at this value during the rest of the simulation. In these conditions, the inlet pressure remains constant, while the pressure at the outlet fell to a new steady state. The analysis was carried out for pressure conditions shown in Table 1 which correspond to three typical operation pressure ranges in gas distribution networks, low, medium and high pressure ranges. Fig. 14 shows

Table 1
Data for the second application example

Solution	L (mile)	D (in.)	Q (MSCFH)	P (psig)
LRS 1	80	18	1500	350
LRS 2	20	12	800	180
LRS 3	6	14	300	25
LRS 4	3	14	80	2

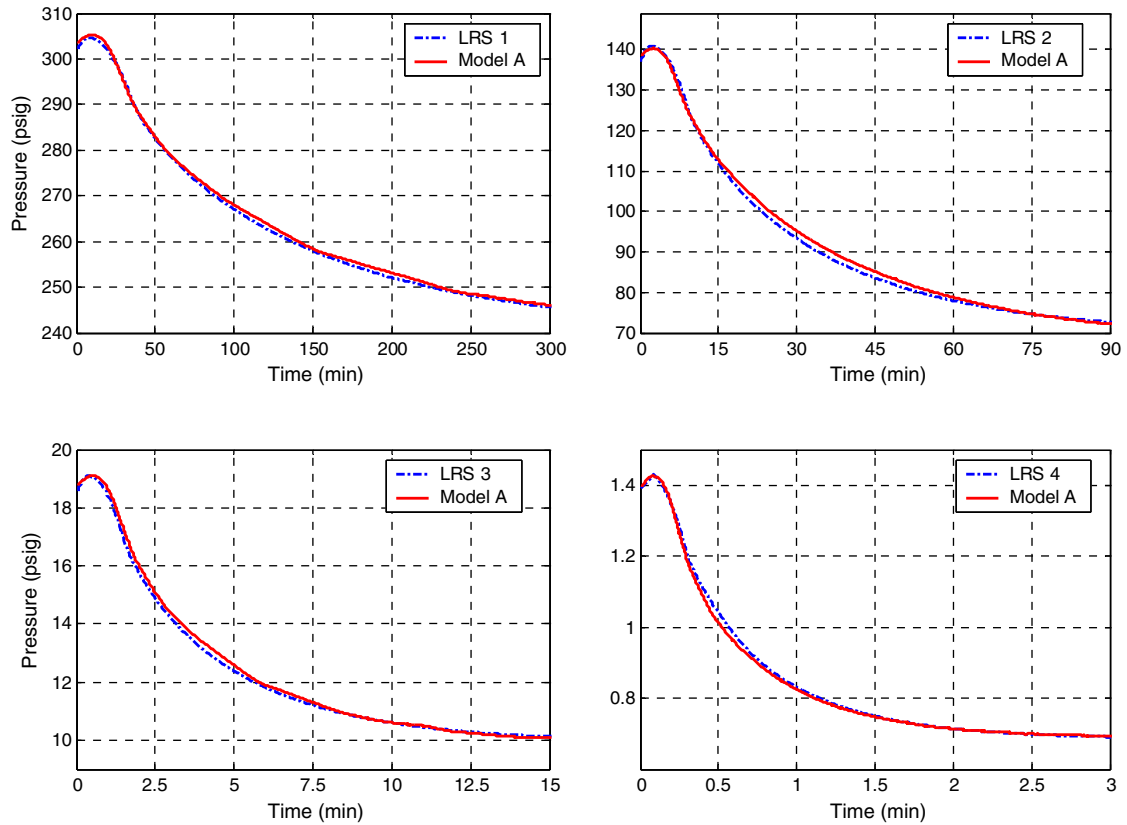


Fig. 14. Outlet pressure for the LRS simulated cases.

results after the simulation is performed. As it can be seen from these figures, similar response to literature is found for all the proposed pressure ranges. Then, the developed simulator can satisfactorily be used for the simulation of gas distribution networks in all these ranges.

5.3. A gas pipeline network

Finally, the developed library is used to simulate the pressure behavior in a pipeline network. In this case, the network is composed by three gas ducts connected according to the scheme shown in Fig. 15. Data for three gas ducts of the network are shown in Table 2. Node 1 is the pressure source with a constant pressure of 50 bar. The loads at node 2 and 3 are known functions of time as shown in Fig. 16. The operational temperature is 278 K, the fluid density is 0.7165 kg/m³, the specific gravity is 0.6 and the friction factor is 0.003.

To simulate this application example, the gas pipeline network shown in Fig. 15 can be easily built on Simulink connecting three instances of a gas duct block. Fig. 17 shows the resulting Simulink model for the network shown in Fig. 15. In such a way it is very easy to simulate more complex networks with the developed library. Once all the physical parameters of each gas ducts are fixed and a numerical scheme is selected

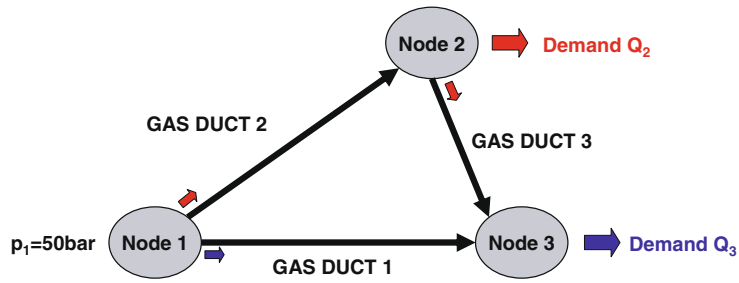


Fig. 15. Gas pipeline network of the third application example.

Table 2
Data for the third application example

Gas duct	Start node	End node	Diameter (m)	Length (m)
1	1	3	0.6	80,000
2	1	2	0.6	90,000
3	2	3	0.6	100,000

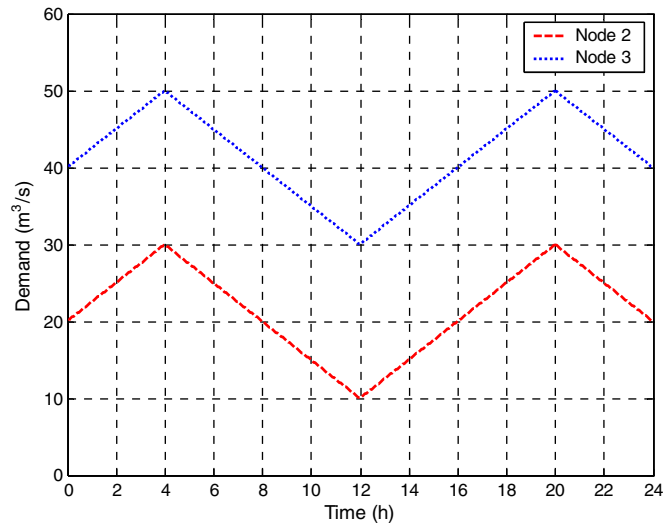


Fig. 16. Flow demand at nodes 2 and 3.

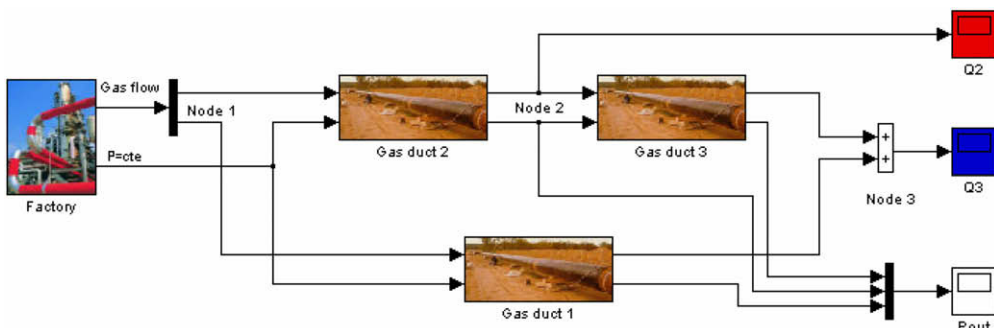


Fig. 17. Simulink model for the gas pipeline network shown in Fig. 15.

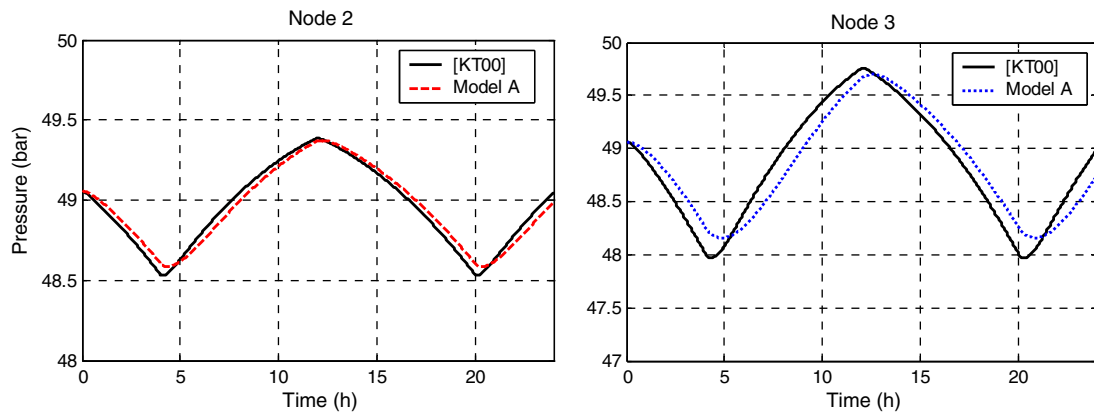


Fig. 18. Pressure at nodes 2 and 3.

the simulation can be performed just by clicking play button on Simulink project. Fig. 18 shows the temporal variation for pressure at nodes 2 and 3 together with the results presented in [2] for the same example. As it can be seen a similar response is calculated in both cases. There is only a little difference as a consequence of different models employed by both papers.

6. Conclusions

This research study focuses on the dynamic modeling and simulation of a gas distribution pipeline network. Gas ducts are the main component in such kind of networks, so they have been described in more detail. It has been given the set of PDE governing the pressure and flow dynamics and two simplified models have been derived. These simplified models keep the inclination term, avoided in most of existing literature on the matter. Two numerical schemes for the integration of these models were proposed. To implement the proposed numerical schemes MATLAB-Simulink tool was used. A library was built with all basic components in a gas network. Only gas duct block implementation was explained in detail since this was the objective of the paper. With this library, the behavior of pipeline networks can be simulated from the individual simulation of interconnected instances of a gas duct block performing the network. Examples solved in this paper show how the inclination effect cannot be neglected from the gas duct model equations, since there is a significant outlet pressure difference when different values of the inclination term are used. Finally, three application examples taken from literature were solved showing the flexibility and easy implementation with the developed MATLAB-Simulink library. Also, it was shown how results for these application examples match with the existing ones on literature for the same examples.

As a future study, the use of the developed library to simulate different operating conditions is suggested with the purpose of acquiring deeper knowledge of such kind of systems. Gas duct blocks can also be interconnected with other blocks like compressor station to form more complex networks. Finally, easy implementation of such gas pipeline networks is a very powerful tool to test different control strategies proposed for the system. For example, an optimal control could be implemented with the aim of reducing the cost of the distribution process in the network under certain operation conditions taking the discharge pressure at compressors and the flow distribution in bifurcations as the manipulated variables.

References

- [1] S. Wu, R.Z. Ríos-Mercado, E.A. Boyd, L.R. Scott, Model relaxations for the fuel cost minimization of steady-state gas pipelines networks, *Math. Comput. Model.* 31 (2000) 197–220.
- [2] S.L. Ke, H.C. Ti, Transient analysis of isothermal gas flow in pipeline network, *Chem. Eng. J.* 76 (2000) 169–177.
- [3] A.J. Osiadacz, *Simulation and Analysis of Gas Pipeline Networks*, E.& F.N. Spon, London, 1987.
- [4] D. Matko, G. Geiger, W. Gregoritz, Pipeline simulation techniques, *Math. Comput. Simul.* 52 (2000) 211–230.
- [5] J.F. Wendt, *Computational Fluids Dynamics*, second ed., Springer, 1995.

- [6] J. Pamponet, P. Neto, Modelado dinámico en redes de transporte de flujo compresible para aplicar en la detección de pérdidas en tiempo real, Bol. Téc. PETROBRAS, Rio de Janeiro, 45(2): abr./jun., 2002.
- [7] M.H. Goldwater, A.E. Fincham, Modeling of gas supply systems, in: G.A. Montgomerie, H. Nicolson (Eds.), Modeling of Dynamical Systems, vol. 2, IEE Cont. Eng. Series 13, 1981, pp. 150–177.
- [8] R.E. Larson, D.A. Wismer, Hierarchical control of transient flow in natural gas pipeline networks, in: IFAC Symposium on Distributed Parameter Systems, Banff, Alberta, Canada, 1971.
- [9] A.R. Mitchell, D.F. Griffiths, The Finite Difference Method in Partial Differential Equations, John Wiley and Sons, Chichester, 1997.
- [10] Simulink, Model-Based and System-Based design. Writing S-Functions, Version 5. The Math Works, Inc., 2003.
- [11] C.G. Segeler, M.D. Ringler, E.M. Kafka, Gas Engineers' Handbook, AGA, NY, 1969.