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Loop quantum cosmology and ultraviolet physics

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Standard cosmology and inflationary theory explain most of the evolution of the Universe and the Cosmic Microwave Background primordial fluctuations, but they do not take into account preinflationary dynamics. In this work, we study the scalar modes that, being observable today, were trans-Planckian before inflation, within the context of Loop Quantum Cosmology (LQC). We analyze the dynamics of these highly ultraviolet modes by introducing modified dispersion relations to their equation of motion. More precisely, we consider Ashtekar's LQC model with 141 e-folds, which is compatible with observations for the standard linear dispersion relation. When trans-Planckian effects are considered in this model, the power spectrum of primordial fluctuations is different from the standard result. In other words, this model of LQC suffers from a trans-Planckian problem.

I. INTRODUCTION

The classical cosmological description of the evolution of the Universe in terms of the old Big Bang model starts to fail as one goes back in time. This is reflected in the existence of different problems that arise in such a framework, namely, the flatness, horizon, and monopole problems [1]. The solution to them comes from the introduction of an early stage of the Universe with accelerated expansion, known as inflation [2, 3]. In fact, inflationary theories are not only good models because they can solve these problems. The main success of inflation is that it describes and explains the origin of the cosmological perturbations, measured in the Cosmic Microwave Background (CMB), through quantum fluctuations of the scalar and gravitational fields. The amplitude of these perturbations is determined by the power spectrum of primordial fluctuations, which turns out to be nearly scale invariant [4].

Although the inflationary theory succeeds in explaining the issues mentioned above, it also presents different problems. One of them, which will be the focus of this work, is the so-called trans-Planckian problem of inflationary cosmology [5]. In most current models, inflation involves a huge expansion of the Universe in order to solve the classical problems, which means that physical wavelengths that correspond to large-scale structures we observe today in the CMB were much smaller than the Planck length at the onset of inflation. This questions the validity of the standard results concerning the power spectrum and force to consider trans-Planckian effects.

This problem is analogous to the trans-Planckian problem of black hole physics [6]. In this context, it was shown that the thermal Hawking spectrum of black holes is robust against modifications of physics in the high energy sector, which were represented by modified dispersion relations that deviate from the standard one above some ultraviolet scale [7, 8].

In inflationary cosmology, the problem has been analyzed following the same approach [5, 9–15]. The equations of motion of perturbations were changed by introducing modified dispersion relations. The robustness of the predictions of classical cosmology was then analyzed by means of the so-called adiabaticity coefficient [10, 14], which accounts for the adiabatic evolution of perturbations modes while they are trans-Planckian. The result is that as long as there is scale separation between the ultraviolet scale and the rate of expansion of the Universe, the imprint in the power spectrum will be negligible, provided that the modified dispersion relation is monotonic.

However, all these analyses ignore the preinflationary evolution of the Universe. Naturally, if trans-Planckian physics is important in inflationary theories, so it will in a preinflationary stage, since the Universe is smaller and hence physical wavelengths too. There are different theories that attempt to introduce a preinflationary description of the Universe. Among them, Loop Quantum Cosmology (LQC) [16] has become in the recent years an important tool to do that, given its predictive power and the capacity to compute power spectra within this framework [17]. The main characteristic of LQC is that it removes the initial singularity of classical cosmology by means of a quantum bounce, due to quantum geometry effects. This leads to modified Einstein equations for the background and provides a well-defined evolution.

In this work, we study the trans-Planckian problem considering a preinflationary scenario described by LQC, following the same steps that have been taken in the context of inflation. In particular, we have considered the background evolution introduced in [18], for which there are 141 e-folds from the LQC bounce until today, and that is compatible with current observations for the standard linear dispersion relation [19]. This will allow us to determine whether the predictions of the primordial power spectrum are robust when this scenario is taken into consideration. With this analysis we also pretend to enhance our knowledge of the very early Universe and the physics beyond the Planck scale.

Explicitly, our objective is to confirm that trans-Planckian physics must be considered during the evolution of the Universe and, specifically, to show that these effects are important in a pre-inflationary scenario as the one we have considered. Moreover, we make a preliminary analysis which reveals that, in these concrete LQC model in which we have focused, the observable power spectrum is sensitive to modifications above an ultraviolet scale and, therefore, the trans-Planckian problem exists. This opens the question of whether this problem appears for every conceivable model within the context of LQC that is compatible with observations when possible trans-Planckian effects are ignored. It also motivates the study of the trans-Planckian problem from deep, fully-geometrical arguments, rather than by introducing modifications to the theory by hand, in order to delve into this subject from a theoretical point of view.

The remainder of this work is structured as follows. In Sec. II we briefly review the inflationary theory and the origin of primordial fluctuations that can be measured in the CMB, whose amplitude is fixed by the power spectrum. In Sec. III we give an overview of the trans-Planckian problem of inflationary cosmology and the methodology used to study it. In Sec. IV we introduce the main ideas of LQC both for the background and the perturbations, and proceed as in the previous section to analyze the trans-Planckian problem within LQC. Sec. V is devoted entirely to the numerical results and their discussion for the particular LQC model under consideration. Finally, in Sec. VI we summarize the main results of this work, its limitations and future research paths in this area. The convention used in this work is $\hbar = c = 1$ and $m_{\rm Pl}^2 = 1/G$. We also use Planckian units: $\ell_{\rm Pl} = t_{\rm Pl} = m_{\rm Pl}^{-1}$.

II. INFLATION AND GENERATION OF FLUCTUATIONS

A. Inflation and slow-roll regime

The main idea of inflation [2, 3] is that the scale factor evolves nearly exponentially, and the Universe is suffering an accelerated expansion. This can be achieved in many ways, but the usual and simpler approach is based on the existence of a scalar field, known as inflaton [20]. An inflaton $\phi(t)$ can be described as a perfect fluid, so that its energy density and pressure are:

$$\rho = \dot{\phi}^2/2 + V(\phi), \qquad p = \dot{\phi}^2/2 - V(\phi), \tag{1}$$

where $V(\phi)$ is the potential of the inflaton and the dot means derivative with respect to cosmological time t. Hence, if one assumes that the Universe content is that of an inflaton during inflation, the Einstein equations for a Friedmann-Lemaître-Robertson-Walker (FLRW) Universe can be written as:

$$H^2 = \frac{8\pi G}{3}\rho = \frac{8\pi G}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \tag{2a}$$

$$\dot{H} = -4\pi G\left(\rho + p\right) = -4\pi G\dot{\phi}^2, \tag{2b}$$

where $H(t) = \dot{a}(t)/a(t)$ is the Hubble parameter and $a(t)$ is the scale factor. From here it follows the equation of motion:

$$\ddot{\phi} + 3H(t)\dot{\phi} + V'(\phi) = 0. \tag{3}$$

Thus, in order to have a nearly constant Hubble parameter during inflation which leads to a nearly exponential expansion, one must impose $p \approx -\rho$. In virtue of (1), this results in the condition $\dot{\phi}^2 \ll V(\phi)$ for the inflaton during a sufficiently long period of time. This can be achieved if the friction term dominates in (3), so that $\ddot{\phi}$ may be neglected. This is known as the slow-roll regime, which is characterized by the slow-roll conditions

$$\epsilon_H = -\dot{H}/H^2 \ll 1\,, \qquad \eta_H = -\ddot{\phi}/(\dot{\phi}H) + \epsilon_H \ll 1. \tag{4}$$

The scale factor increases exponentially, as long as $V(\phi)$ is positive and decreases over time, when the slow-roll conditions are met. The so-called number of e-folds $N = \log[a(t_f)/a(t_i)]$ quantifies this exponential increase, where t_i and t_f are the times when inflation starts and ends. It is necessary that $N \gtrsim 65$ for inflation to solve the problems of classical cosmology. This allows to interpret inflation as a period of the evolution where the Universe is filled with the inflaton, which is slowly rolling on the potential $V(\phi)$, thus producing a nearly constant Hubble parameter. When the conditions (4) cease to be valid inflation ends, the inflaton begins to oscillate, and the Universe starts decelerating.

B. Primordial fluctuations

Now let us briefly review the theory of cosmological fluctuations (see [4] for an exhaustive review) which explain the origin of primordial perturbations measured in the CMB by means of the power spectrum. We consider linear cosmological perturbations around an homogeneous background.

There exist different sorts of metric perturbations, namely scalar, vector, and tensor perturbations, that can be treated independently at a linear level due to the symmetry properties of the background. In this work, we will focus on scalar perturbations, as they leave an observable imprint in the CMB. Tensor perturbations admit a similar analysis but are related to primordial gravitational waves, which have not been observed yet, while vector perturbations are diluted in cosmological evolution [21].

Scalar perturbations are described by five scalar functions, four related to the metric and the fifth denoting the perturbation of the scalar field, $\delta\phi$. However, due to the gauge freedom and the connection between metric and matter perturbations through the Einstein equations, there is only a degree of freedom, that we will take as the comoving curvature perturbation, defined as [1]:

$$\mathcal{R} = \Psi + H\delta\phi/\phi, \tag{5}$$

where Ψ is the invariant Bardeen potential which accounts for the metric perturbations and ϕ is the background solution of the scalar field. This quantity is gauge invariant. Its dynamics can be conveniently expressed in terms of the so-called Mukhanov-Sasaki variable $v = z\mathcal{R}$, where $z = a\dot{\phi}/H$. Up to linear order in perturbations and decomposing v in Fourier modes one gets the Mukhanov-Sasaki equation [4]:

$$v_k'' + \omega_k^2(\eta) v_k = 0, \qquad \omega_k^2(\eta) = k^2 - z''/z, \tag{6}$$

where $k = |\vec{k}|$ is the comoving wavenumber of the mode v_k and the prime denotes derivative with respect to conformal time η defined via $d\eta = a\,dt$. In other words, scalar perturbations can be fully studied through a scalar field v whose modes v_k satisfy the equation of a harmonic oscillator with a time dependent frequency ω_k which is determined by the background geometry[1]. We see that z''/z introduces a scale in the dynamics: in the sub-Hubble limit $k^2 \gg z''/z$ the modes oscillate with constant amplitude k (they do not feel the curvature of spacetime), while in the super-Hubble limit $k^2 \ll z''/z$ the modes behave as $v_k \sim z$ (they do feel the curvature), implying that $\mathcal{R}$ is constant for those modes. Thus, $\mathcal{R}$ is the appropriate variable to analyze through evolution, since it freezes outside the horizon, hence yielding initial conditions for perturbations when they re-enter the horizon.

Notice that, since the modes v_k have a time dependent frequency, the associated Hamiltonian depends explicitly on time and hence the choice of the vacuum of the theory cannot be done in a time-independent way. Thus, one has to pick an initial time η_0 and define there the vacuum as the lowest energy state. In the limit where $\eta_0 \to -\infty$ this state is called the Bunch-Davies vacuum [22] and corresponds to the recovery of plane wave solutions in the asymptotic past for sub-Hubble modes (with $k \gg aH$).

Equation (6) can be solved analytically in the slow-roll regime. Up to first order in the slow-roll parameters (where ϵ_H and η_H are constant), the resulting solution, in the super-Hubble limit $k^2 \ll z''/z$ and considering the Bunch-Davies vacuum, is:

$$|v_k(\eta)| \sim \frac{1}{\sqrt{2k}} \left(\frac{k}{aH}\right)^{\frac{1}{2}-\nu}, \tag{7}$$

where $\nu = 3/2 + 3\epsilon_H - \eta_H$.

The power spectrum is just the Fourier transform of the spacetime two-point correlation function of the curvature perturbation:

$$\mathcal{P}_\mathcal{R} = \frac{k^3}{2\pi^2} \left|\frac{v_k}{z}\right|^2. \tag{8}$$

It is a measurement of the contribution to the variance of $\mathcal{R}$ of modes with comoving wavenumber k, that is, of quantum zero-point fluctuations. This power spectrum may be evaluated at super-horizon scales, due to constancy of $\mathcal{R}$ for those modes. Substituting $z = a\dot{\phi}/H$ and (7) in (8), the power spectrum can be written as:

$$\mathcal{P}_\mathcal{R}(k) = A_S \big(k/k_*\big)^{n_S - 1}, \tag{9}$$

where n_S is the scalar spectral index (or tilt), which in terms of the slow-roll parameters is given by

$$n_S = 1 - 6\epsilon_H + 2\eta_H, \tag{10}$$

and A_S is the scalar amplitude specified by

$$A_S = \frac{1}{\pi\epsilon_H}\left(\frac{H}{m_{\rm Pl}}\right)^2\Big|_{k_*=aH}, \tag{11}$$

where k_* is commonly known as the pivot scale from which the power spectrum is measured. The most recent measurements [23] provide the following values for the pivot scale $k_* = 0.05\,\mathrm{Mpc}^{-1}$:

$$A_S = (2.092 \pm 0.034)\cdot 10^{-9}, \ n_S = 0.9626 \pm 0.0057. \tag{12}$$

This means that the power spectrum of fluctuations is nearly scale invariant ($n_S \approx 1$) and so the slow-roll parameters are very small.

III. TRANS-PLANCKIAN PROBLEM IN INFLATION

Despite the great success of the inflationary theory, which we have summarized in Sec. II, this theory also faces several problems. One of them, which is the one that concerns this work, is commonly referred to as the trans-Planckian problem of inflationary cosmology [5].

In most inflationary models based on an inflaton, the inflationary stage lasts a very large number of e-folds. As a consequence, due to the fact that physical wavenumbers $\kappa = k/a$ at different times are related through $\kappa(t_1)a(t_1) = \kappa(t_2)a(t_2)$, one gets that $\kappa(t_i) = e^N \kappa(t_f)$. Hence, physical wavenumbers and energies corresponding nowadays to large-scale structures that can be measured in CMB were larger (indeed, much larger) than the Planck mass at the beginning of the inflationary stage.

This is clearly a severe issue, since it implies that the power spectrum of cosmological fluctuations (which is calculated on pure classical gravity, low energy grounds) depends as well on high energy physics. Moreover, the power spectrum we observe today may be altered by any slight modification of physics above the Planck scale. Therefore, to compute it, it is necessary to be aware of trans-Planckian effects through the evolution of perturbations. However, these effects of trans-Planckian physics are yet unknown, and thus the only way to proceed is by introducing reasonable modifications to the theory that try to simulate those effects.

The usual approach to do so is by introducing modified dispersion relations in the equation of motion (6), as was

[1] For tensor perturbations one has the same structure, but replacing z by the scale factor a.

first done in [5] following the steps of the analog problem in black holes physics [6]. In this case, due to the spacetime expansion, the analysis is not just an extension of what was done with black holes.

In the standard cosmology frame, as previously stated, fluctuations can be studied via a scalar field v with time-dependent frequency whose modes obey the Mukhanov-Sasaki equation (6). When considering possible trans-Planckian effects, a way of implementing them is by modifying the standard frequency to:

$$\omega_F^2(\eta) = \left[a(\eta)F(\kappa)\right]^2 - z''/z, \tag{13}$$

where $F(\kappa) = F(k/a)$ is some nonlinear function that deviates from the standard linear dispersion relation for physical wavenumbers $\kappa \gg \kappa_c$ and recovers the linear behaviour for $\kappa \ll \kappa_c$, where κ_c is some ultraviolet scale (expected to be of the order of the Planck mass). When doing so, a non-Lorentz invariant dispersion relation results, so one must stipulate the reference frame where the dispersion relation is defined.

Since now the dispersion relation is nonlinear, the vacuum cannot be defined as the Bunch-Davies state. Here, we will take the adiabatic approach [24], where the adiabatic vacuum emanates from minimizing the energy of the field (equivalent to say that modes with $\kappa \gg H$ are in their ground state [25]), as was shown in [5]. This vacuum is the positive frequency WKB solution of the Mukhanov-Sasaki equation with modified dispersion relation, appropriately normalized.

Different modified dispersion relations have been considered until now, some of them shown in Figure 1. This includes the so-called Unruh dispersion relation F_{U} [7] or the generalized Corley-Jacobson dispersion relation F_{CJ} (introduced in [5] based on the one used in [8]):

$$F_{\mathrm{U}}(\kappa) = \kappa_c \tanh\left(\kappa/\kappa_c\right), \tag{14}$$

$$F_{\mathrm{CJ}}(\kappa) = \kappa\sqrt{1 + b_m(\kappa/\kappa_c)^{2m}}, \tag{15}$$

where b_m reflects the subluminal ($b_m < 0$) or superluminal ($b_m > 0$) character of F_{CJ}. In this last family of modified dispersion relations, the nonlinear term must be understood as the first term of a power expansion of a generic dispersion relation; otherwise, one would get (for the subluminal case) pathological behaviour for physical wavenumbers $\kappa > \kappa_c|b_m|^{2m}$, where $F_{\mathrm{CJ}}(\kappa)$ becomes complex. Furthermore, in this case the energy may not be bounded from below and the definition of vacuum would not be clear. For these reasons, we will not consider this modified dispersion relation and we will focus only on monotonic dispersion relations. Other modified dispersion relations have been studied in different works [26].

In most works, the robustness of the predictions of inflationary cosmology against trans-Planckian physics is studied in terms of the adiabaticity coefficient, defined as [14] (for an alternative, but equivalent definition see [5]):

$$\varepsilon(\eta, k) = \left|\omega'/\omega^2\right|. \tag{16}$$

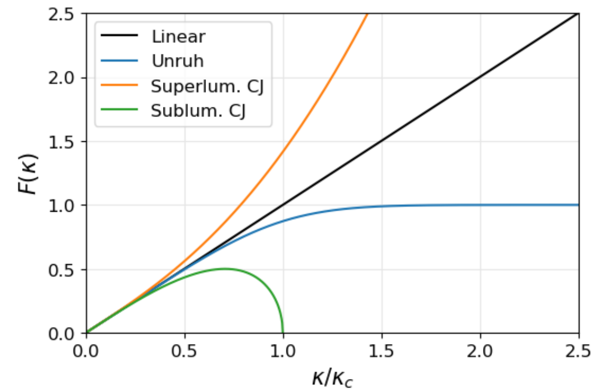


Figure 1: Sketch of the different dispersion relations considered in this work. For the Corley-Jacobson dispersion relation we have considered $m = 1$ and $b_m = \pm 1$.

This expression is only valid when the modes are inside the horizon, that is, when $\omega^2 > 0$. In the case $\omega^2 < 0$, the modes fail to be adiabatic as they cross the horizon and instead of oscillating they suffer an exponential amplification. Nevertheless, observable modes are already sub-Planckian when they cross the horizon during slow-roll and thus ω^2 becomes negative when the dispersion relation is the standard one. In particular, we are interested in the value of $\varepsilon(\eta, k)$ during inflation, when a concrete mode is trans-Planckian. Hence, in its evaluation, the second term in (13) can be safely neglected, since observable modes are inside the horizon while being trans-Planckian and their dispersion relation rules over spacetime expansion. Under this approximation, the adiabaticity coefficient can be readily computed:

$$\varepsilon(\eta, k) \approx \left|\frac{H}{F} - \frac{H\kappa}{F^2}\frac{dF}{d\kappa}\right| = \frac{H}{\kappa_c}\left|\frac{d}{d\kappa}\left(\frac{\kappa\kappa_c}{F}\right)\right|. \tag{17}$$

It is easy to see that ε is bounded by H/κ_c for every monotonic dispersion relation. Therefore, one can conclude that as long as the scale separation condition

$$H/\kappa_c \ll 1 \tag{18}$$

is met, the adiabaticity coefficient is $\varepsilon(\eta, k) \ll 1$ for all monotonic modified dispersion relation [14], and modifying the standard dispersion relation above the scale κ_c will not have an imprint on the power spectrum [12]. This is in good agreement with the analytical results obtained in [5, 10]. Physically, this can be seen as if the modes with $\kappa > \kappa_c$ (which are affected by modified dispersion relations) have enough time to adapt themselves to the standard vacuum solution provided that their evolution is adiabatic, since $F \geq H$ for those κ values that satisfy the adiabaticity condition. Moreover, according to (11) and (12), scale separation (18) is satisfied during slow-roll, as long as $\kappa_c \sim m_{\mathrm{Pl}}$, and we have $H/\kappa_c \lesssim 10^{-5}$.

IV. LOOP QUANTUM COSMOLOGY AND THE TRANS-PLANCKIAN PROBLEM

A. Loop Quantum Cosmology: Background

Let us first summarize the evolution of the homogeneous background obtained in LQC (for a complete review of this theory and its derivation, consult [16]).

The quantum geometry effects that this theory introduces allow to remove the classical Big Bang singularity of FLRW models, replacing it by a quantum bounce, hence giving rise to a well-defined background evolution of the Universe. In fact, LQC leads to a family of semi-classical states that follow well defined trajectories. These trajectories correspond to an effective dynamics encoded in the following modified Einstein equations with quantum corrections [16]:

$$H_{\rm LQC}^2 = \frac{8\pi G}{3}\rho\big(1-\rho/\rho_*\big), \tag{19a}$$

$$\dot{H}_{\rm LQC} = -4\pi G\,(\rho+p)\,\big(1-2\rho/\rho_*\big), \tag{19b}$$

where ρ_* is a critical density of the order of the Planck density (usually taken to be $\rho_* \approx 0.41\rho_{\rm Pl}$ due to geometrical arguments). These equations lead to:

$$\dot{\rho} + 3H_{\rm LQC}\,(\rho+p) = 0. \tag{20}$$

A few remarks about these equations are in order. As stated before, equation (19a) leads to a bounce, where the Hubble parameter vanishes, when $\rho = \rho_*$. This is a feature of LQC that is not present in GR and guarantees that physical quantities (such as the energy density or the Ricci scalar) that diverge in GR are bounded in LQC. Moreover, the term ρ/ρ_* is negligible a few Planck times after the bounce, so one can use GR soon after it. One may notice as well that (20) has the same structure as in GR, despite the fact that Einstein equations are modified.

The scalar field in LQC can also be described as a perfect fluid obeying (1) and thus reducing (20) to (3), but now with the Hubble parameter given by (19a). This set of equations is analytically intractable except for the free scalar field case (and a few others). In this case where $V(\phi) = 0$ the analytical solution in terms of cosmological time t is (from now on, the subscript LQC will be omitted unless necessary for comparison with GR):

$$a(t) = \left[(t/t_*)^2+1\right]^{1/6}, \quad \rho(t) = \rho_* a^{-6}(t), \tag{21}$$
$$\phi(t) = \phi(0) + \frac{1}{\sqrt{12\pi G}}\,\mathrm{arcsinh}\,\big(t/t_*\big),\ H(t) = \frac{t}{3t_*^2 a^6(t)}\,,$$

where we have chosen as normalization the scale factor at the bounce $a(0)=1$, and $t_* = (24\pi G\rho_*)^{-1/2}$ is the time when the Hubble parameter achieves its maximum value $H(t_*) = \sqrt{2\pi G\rho_*/3}$. For $\rho_* = 0.41\rho_{\rm Pl}$ this maximum is $H(t_*) \approx 0.93\,m_{\rm Pl}$. The Hubble parameter is depicted in Figure 2, along with the classical GR Hubble parameter. Similar plots can be portrayed for the other background variables, leading to the conclusion that LQC cures the problems that standard cosmology have for a free scalar field and enables to define a preinflationary dynamics.

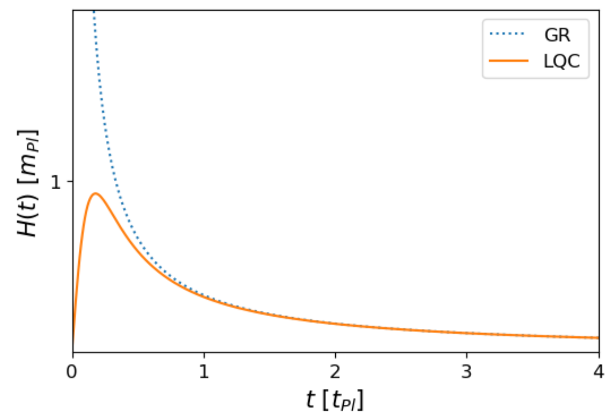


Figure 2: Hubble parameter for a free scalar field in GR and LQC (with $\rho_* = 0.41\rho_{\rm Pl}$). $H_{\rm LQC}$ is always bounded and the GR behaviour is restored soon after the bounce. For larger (smaller) values of ρ_*, GR is recovered sooner (later) and the maximum of $H_{\rm LQC}$ is different.

In addition to solving the singularity problem, LQC also provides natural initial conditions that ensure the inflationary paradigm with the required e-folds for suitable potentials [27, 28], after a very short phase of superinflation takes place. To make the inflationary stage happen we need a non-vanishing potential, which requires numerical treatment. As a toy model, we will take the quadratic potential

$$V(\phi) = m^2\phi^2/2. \tag{22}$$

In Sec. V we will see that, as long as $m\phi(0) \ll \dot{\phi}(0)$, the background evolution is that of the free field case from the bounce to a short time after the maximum of H (which corresponds to kinetic dominance) and, after some time, the Hubble parameter goes to a non-vanishing constant, thus causing inflation (which implies a potential dominance). Moreover, numerical simulations show that under this assumption the background evolution for non-vanishing potentials does not depend strongly on the choice of initial conditions or on the concrete shape of the potential, and can be well-described by the zero potential solution at early times; thus, the maximum of H and the time when it is reached depend only on the value of ρ_*.

B. Loop Quantum Cosmology: Perturbations

We next consider cosmological perturbation theory using LQC, in order to account for Planck scale physics. This would enhance our knowledge of the Universe evolution and enable to connect Planck era with observable quantities. In this case, in addition to possible effects of trans-Planckian physics in ultraviolet modes, modifications on infrared modes could arise as well due to a different evolution of the background: since these modes feel the curvature at the bounce (their wavelength is bigger than the curvature radius of the Universe at that time) they can evolve to an excited state. Indeed, observable modes could in principle exit and re-enter the curvature radius before inflation takes place, thus reaching the onset of inflation in a non Bunch-Davies vacuum.

Different strategies have been followed to accomplish this task (see [17] for a review), namely, the so-called dressed metric approach [29–32], the deformed algebra approach [33], the hybrid quantization [34, 35], or the separate universe loop quantization [36]). For our purposes we will consider hybrid LQC. This is because the separate universe approach is only valid for infrared modes, whereas the deformed algebra approach is discarded as it is incompatible with observations [37]. On the other hand, hybrid and dressed metric approaches are based mainly on the same grounds, but their equations of motion for perturbations are not the same, due to the different ways of quantizing. We choose hybrid LQC because, unlike the dressed metric approach, it leads to hyperbolic equations of motion at the bounce, where we will set initial conditions for perturbations [38].

In hybrid LQC, when neglecting the back-reaction of the perturbation modes, the equation for scalar perturbation modes has the same structure as (6) but with a time-dependent frequency [38]

$$\omega_k^2(\eta) = k^2 + s(\eta), \quad s = -\frac{4\pi G}{3} a^2\left(\rho - 3p\right) + a^2 u, \tag{23}$$

where u is an effective potential given by

$$u = V''(\phi) + 48\pi G V(\phi)\left(1 - \frac{V(\phi)}{\rho}\right) + 6\frac{a'\phi'}{a^3\rho}V'(\phi). \tag{24}$$

In the classical limit where the effective dynamics reduces to GR, that is, when $\rho_* \to \infty$, we recover the classical Mukhanov-Sasaki equation (6).

C. Trans-Planckian problem in Loop Quantum Cosmology

We may now wonder how and why LQC and Planck scale physics may affect the power spectrum, in analogy with Sec. III that focused solely on inflation. An extra drawback appears in the context of LQC, as was mentioned before, namely, the relevance of the background in modes which are sensitive to the curvature. Moreover, LQC introduces a scale as it already happened in inflation. In this case, this scale results from the competition between the physical wavenumber of the modes and the Ricci scalar in the Mukhanov-Sasaki equation (equivalently, between the physical wavelength and the curvature radius). The discrimination between which modes feel the curvature and which do not is precisely the value of the Ricci scalar at the bounce, where it is maximum, and constitutes the characteristic energy scale of LQC:

$$\kappa_{\rm LQC} = \sqrt{R(0)/6} = (\sqrt{3}t_*)^{-1} \approx 3.21\, m_{\rm Pl}. \tag{25}$$

Modes with $\kappa \lesssim \kappa_{\rm LQC}$ will be the ones which feel the curvature since they have a wavelength longer than the LQC wavelength. Consequently, they exit and enter the curvature radius in the bouncing stage, before they exit again in inflation, and can have an imprint on the power spectrum due to background effects.

Another central issue is the choice of initial conditions for the perturbations. There are different alternatives, namely, setting them far away before the bounce (in the contracting branch) or at the bounce itself. The former allows to set initial conditions at some point where effects of modified dispersion relations disappear, but requires fixing them at the concrete points where $s(\eta) = 0$ [39]. The latter, although facing the problem that the bounce is the most affected by LQC, at least guarantees that the equation of motion for perturbations is hyperbolic there and thus initial conditions can be set up in an intuitive way. However, it is only the infrared and intermediate parts of the spectrum which are sensitive to the choice of initial conditions. For a review of this topic and the computation of the power spectrum for several monomial potentials with the standard dispersion relation, see [39].

Despite that, the methodology is the same as before. One includes in (23) modified dispersion relations (replacing k^2 by a^2F^2) and analyzes how fair the assumption of WKB solution is by means of the pertinent adiabaticity coefficient when modes are trans-Planckian.

To our knowledge, not much work has been done in the trans-Planckian problem in LQC. In particular, only in [40] modified dispersion relations were considered within the framework of LQC. In that work, the emphasis was placed in computing the power spectrum in different approaches and comparing with the standard scenario, rather than understanding the possible modification and its origin qualitatively. The main result was that the power spectrum is modified when considering modified dispersion relations and that this change depends on the concrete value of the ultraviolet scale κ_c, possibly leading to a power spectrum with oscillations or enhancement in its ultraviolet sector, thus making relevant the trans-Planckian effects. This opens the question of whether such effects affect or not the observable window.

The calculation of the adiabaticity coefficient can be carried out easily for the kinetic dominated regime close to the bounce, as it is well approximated by the free scalar field case, where the explicit background solution is known and $u = 0$, yielding $s_0(\eta) = 8\pi G\rho a^2/3$. In this case, we have:

$$\varepsilon_0(\eta, k) = \frac{HF^3}{\kappa_c \mathcal{W}_0^3}\left|\frac{d}{d\kappa}\left(\frac{\kappa\kappa_c}{F}\right) - \frac{16\pi G}{3}\frac{\rho\kappa_c}{F^3}\right|, \tag{26}$$

where

$$\mathcal{W}_0 = \frac{\omega}{a} = \sqrt{F^2 + \frac{8\pi G}{3}\rho}. \tag{27}$$

Let us qualitatively analyze this adiabaticity coefficient.

First, notice that it is proportional to H, so that the adiabaticity condition $\varepsilon_0 \ll 1$ for trans-Planckian modes again follows from the condition $H/\kappa_c \ll 1$. Moreover, according to (19a), we have that $\varepsilon_0(\eta, k) \propto (1-\rho/\rho_*)^{1/2}$. Hence, as long as F is not too steep, all the modes satisfy the adiabaticity condition very close to the bounce (where $\rho \approx \rho_*$). Moreover, $\varepsilon_0(\eta, k)$ is exactly zero at the bounce. Therefore, all the modes can be set up at the bounce in their adiabatic vacuum, regardless of the specific modified dispersion relation and the physical wavenumber of the modes.

Second, when the evolution of perturbations is fully determined by its dispersion relation (that is to say, when $\mathcal{W}_0 \approx F$), we recover the inflationary result (17).

A similar result to (26) can be derived when the potential cannot be ignored, with an extra term that depends only on the specific potential and on the background, but not on the modified dispersion relation. In this case, we do not have an analytical solution for the background evolution, so we will need to compute $\varepsilon(\eta, k)$ numerically.

V. NUMERICAL RESULTS

A. Background dynamics

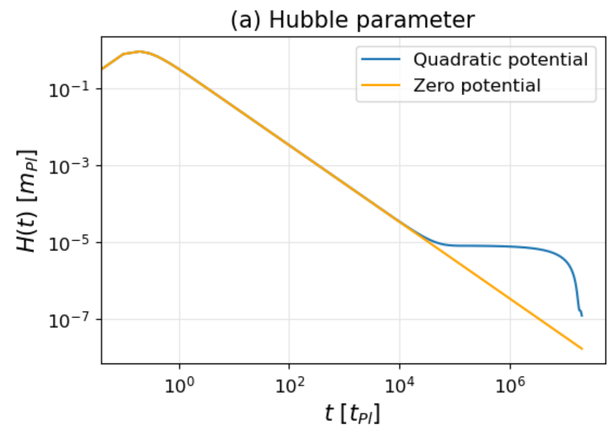
We have considered the quadratic potential (22) with $m = 1.2 \cdot 10^{-6}\, m_{\mathrm{Pl}}$ and the initial condition at the bounce $\phi(0) = 1.033\, m_{\mathrm{Pl}}$, following [18]; the remaining initial condition $\dot{\phi}(0)$ is fixed by $\rho(0) = \rho_* = 0.41\rho_{\mathrm{Pl}}$. In this way, we are considering a model in LQC that, while being compatible with observations for the standard dispersion relation, it also introduces some LQC imprints in the CMB, as we will see later.

The simulation has been done in cosmological time t, and run up to $2 \cdot 10^7\, t_{\mathrm{Pl}}$ in order to reach inflation. The background evolution is depicted in Figure 3. As can be seen in Figure 3a, the Hubble parameter evolves near the bounce as if the scalar field was free and, around $t = 10^4\, t_{\mathrm{Pl}}$, quadratic and zero-potential lines begin to differentiate. From $t \sim 10^5\, t_{\mathrm{Pl}}$ to $t \sim 10^7\, t_{\mathrm{Pl}}$, $H(t)$ is roughly constant, which means that the scale factor there grows exponentially and inflation is taking place. To get further insight, we have also plotted in Figure 3b the parameter of state, defined as

$$w(\phi) = \frac{p}{\rho} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)}. \tag{28}$$

Near the bounce there is kinetic dominance and hence $w \approx 1$, whilst during slow-roll inflation $w \approx -1$, thus confirming potential dominance. After inflation, since the scalar field begins to oscillate, so does w.

We have also calculated the number of e-folds, resulting in 4.57 e-folds from the bounce to the onset of inflation, and 67.78 e-folds during the inflationary period (that is, around 72 e-folds for the whole simulation).



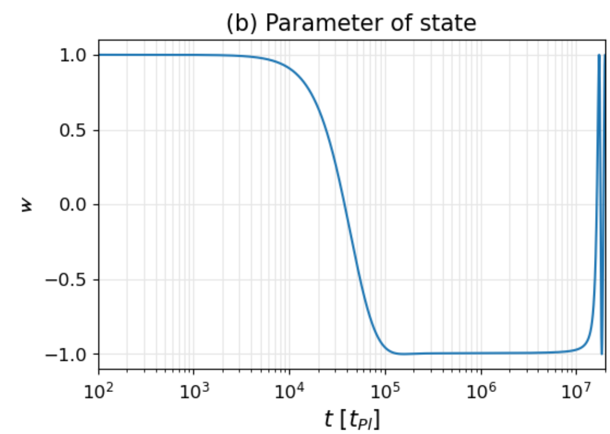


Figure 3: Background evolution. (a) Hubble parameter under the quadratic potential, compared to the zero-potential case. (b) Parameter of state of the inflaton.

B. Perturbations

Now we compute the adiabaticity coefficient of the different modes while being trans-Planckian. At this point, we have to choose a specific value for the ultraviolet scale κ_c. We have chosen $\kappa_c = \kappa_{\mathrm{LQC}}$, since it seems natural that the ultraviolet modifications arise when the energy scale of LQC is dominant, although other choices are also possible. We also need the observable window measured in the CMB. For our model we have 141 e-folds from the bounce until today [18], which means that the observable window today, $\kappa_{\mathrm{today}} \in [10^{-4}\,\mathrm{Mpc}^{-1}, 0.5\,\mathrm{Mpc}^{-1}]$, is at the bounce $\kappa(0) \in [0.9\, m_{\mathrm{Pl}}, 4504\, m_{\mathrm{Pl}}]$. We have evolved this window from the bounce to the end of the simulation along with the curvature radius. The result is shown in Figure 4 in terms of physical wavelengths, where the ultraviolet scale $\lambda_c = 1/\kappa_c \approx 0.31\ell_{\mathrm{Pl}}$ is also depicted.

We see that the observable window presents the two kinds of modes that we have been discussing. On the one hand, it contains modes whose physical wavelength is below λ_c, not only at the bounce, but also at the onset

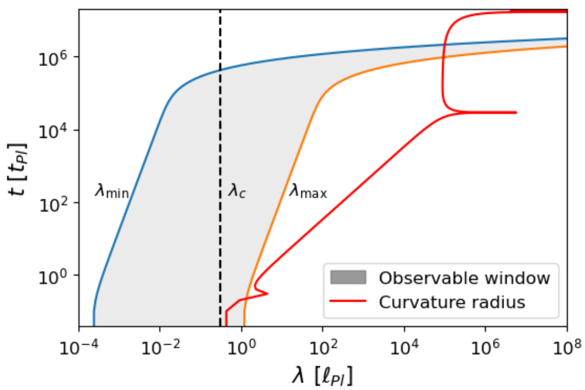


Figure 4: Evolution of the observable window (shaded region) from the bounce to inflation. The curvature radius (red line) is plotted to see when do the modes of wavelength λ feel the curvature. The dashed vertical line represent the ultraviolet scale λ_c.

of inflation, which means they are trans-Planckian. On the other hand, some modes are outside the curvature radius at the bouncing epoch and thus feeling the curvature of spacetime (carrying the effects of LQC). These most infrared modes of the observable window will then have an imprint on the power spectrum due to LQC effects, reflected in the loss of the nearly scale invariance in that sector [17]. Notice as well that all these modes cross the horizon during inflation and when doing so they are no longer trans-Planckian, that is to say, their physical wavelength is well above the ultraviolet scale λ_c.

It is now time to compute the adiabaticity coefficient for the different modified dispersion relations that affect the ultraviolet sector while the modes are trans-Planckian. This will allow us to determine if the ultraviolet part of the observed power spectrum is robust against trans-Planckian effects in this model within LQC.

We have calculated the adiabaticity coefficient $\varepsilon(\eta, k)$ both for the Unruh and superluminal Corley-Jacobson modified dispersion relations, as well as for the linear one. The results are depicted in Figure 5. The curves in orange and blue correspond to the endpoints of the observable window and the ultraviolet scale λ_c is the dashed black line. Since these three functions are monotonic, the calculation of this parameter for the limits of the window is enough to enclose all the observable modes.

We now summarize the main results that follow from the analysis of these graphics. The first thing to notice is that the observable mode with the longest physical wavelength is not affected by the concrete dispersion relation we are using. This is natural, since modifications to the dispersion only affect those modes whose wavelength is below or comparable to the ultraviolet scale λ_c, and this concrete mode is sufficiently above it. Another thing to remark is that the adiabaticity coefficient increases dramatically around $10^6\, t_{\rm Pl}$, due to the fact that modes are crossing the horizon, but there they are already sub-Planckian, as seen before.

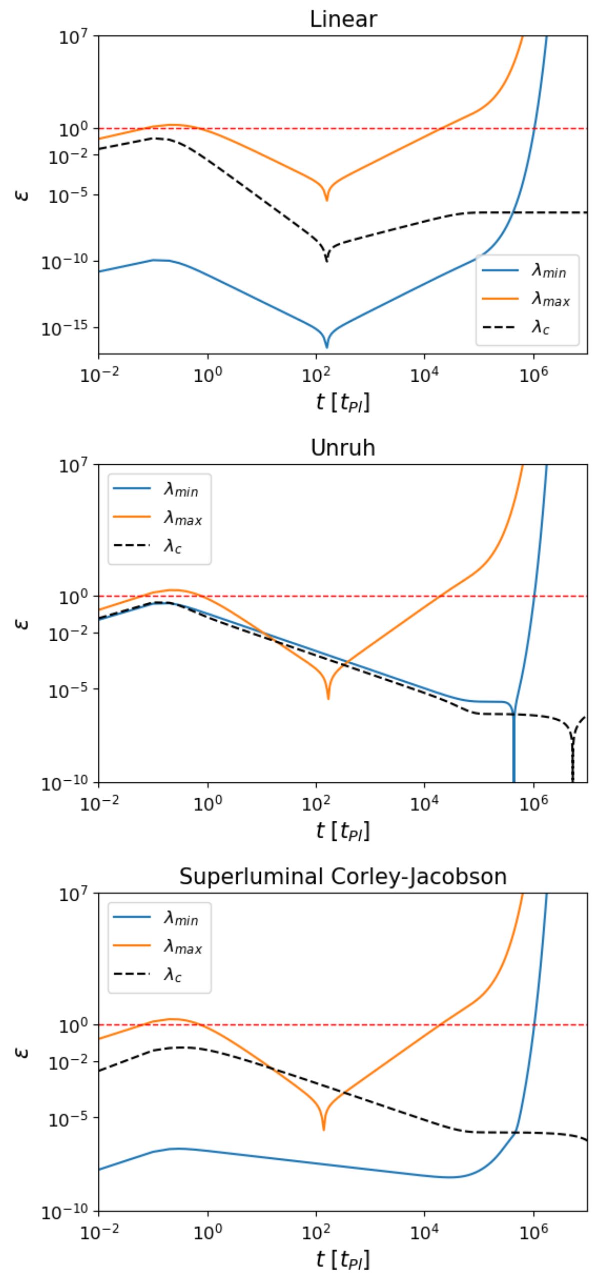


Figure 5: Adiabaticity coefficient for the different dispersion relations considered in this work. Upper: linear. Middle: Unruh. Lower: superluminal Corley-Jacobson. The line in red is to visualize the value of one. All the lines depicted tend to zero at the bounce ($t = 0$).

Focusing first in the linear dispersion relation (upper graph), we see that both observable modes depicted present the same behaviour for the adiabaticity coefficient. In particular, for the shortest physical wavelength mode we observe today, it remains far below one at every instant while being trans-Planckian. On the other hand, for the longest physical wavelength mode, it reaches values of order one during the bouncing phase, at the times when $H(t)$ is maximum. This is understandable, since this mode has a physical wavelength bigger than the curvature radius at this early stage, thus being affected by the curvature of spacetime. However, after this bouncing stage, its adiabaticity coefficient is small (of order 10^{-2}), but the effects of LQC have already been captured and they will be reflected in the power spectrum. We have also plotted the corresponding adiabaticity coefficient for the ultraviolet scale λ_c, just for comparison with the other cases, although in this case there is no such a ultraviolet scale at all.

We continue the analysis with the Unruh dispersion relation (middle graph). We see big differences with respect to the linear one. The main one is that both lines are much closer during the bouncing phase. Moreover, the most ultraviolet mode has an adiabaticity coefficient that is almost equal to that of the ultraviolet scale λ_c during the whole evolution until it exits the horizon. We can view this as if the Unruh dispersion relation makes this mode less adiabatic; this is related with the particular form of (14), which makes modes with $\kappa \gg \kappa_c$ saturate to the value of κ_c. Due to this, the whole observable window is non-adiabatic right after the bounce, being most of the modes trans-Planckian. We therefore conclude that the Unruh dispersion relation will leave an imprint in the power spectrum, possibly breaking the nearly scale-invariance obtained with the standard dispersion relation, specially in the most ultraviolet part of the observable power spectrum.

Finally, we focus on the superluminal Corley-Jacobson dispersion relation (lower graph). We see that, despite the fact that the ultraviolet modes become less adiabatic than in the linear case, they are always far below one during the bouncing phase and while being trans-Planckian. In particular, the adiabaticity coefficient of the ultraviolet scale λ_c is always less than one, but not by much, and this means that we can expect slight modifications to the power spectrum, less important than in the Unruh case.

This analysis shows that in this model (which is compatible with observations for the linear dispersion relation [19]) the trans-Planckian problem is present and thus these effects must be considered in the computation of the power spectrum. Moreover, it would be desirable to know the precise way in which these effects arise in the quantum theory to confirm that the same results we have obtained in this work apply there. At this point, it is important to mention that we have made this analysis considering the value $\lambda_c = 0.31\ell_{\rm Pl}$ for the ultraviolet scale. In principle, one could imagine any other reasonable value and the results could be very different. We have to recall as well that the presence of a trans-Planckian problem depends strongly on the number of e-folds from the bounce until today and, therefore, these results should not be considered as general, but only as an example of the existence of the trans-Planckian problem within the context of LQC.

VI. CONCLUSIONS

In this work, we have dealt with the trans-Planckian problem from inflationary and LQC perspectives. In order to describe trans-Planckian effects, we have considered two different modified dispersion relations above some ultraviolet scale κ_c, namely, the ones introduced by Unruh and by Corley and Jacobson. We have argued that the imprints that these modifications may leave in the power spectrum can be studied in terms of the adiabaticity coefficient of the modes while they were trans-Planckian. In particular, we have found that this parameter is of order H/κ_c, so that when this quantity is very small we do not expect any change. More precisely, these dispersion relations present this characteristic during inflation, but the same does not apply to the preinflationary scenario introduced in [18] that we have considered. In this last case we have seen that modified dispersion relations may have an imprint on the power spectrum, thus confirming the existence of a trans-Planckian problem in this model within the context of LQC.

This work complements the one carried out in [40], where the power spectra with modified dispersion relations were computed in different LQC approaches to perturbations, but no insight was placed in understanding the origin of the modifications to those power spectra.

This solves our initial question of whether the trans-Planckian problem is present or not in a preinflationary Universe described by LQC. These results are of particular interest, since the prediction of modifications to the power spectrum and their possible observational measurement could tell us about the history of the Universe before inflation took place and if the observable modes behaved always following a linear dispersion relation.

Nevertheless, this analysis presents some limitations that are important to mention. These are mainly three. The first one is that we have restricted ourselves to two modified dispersion relations to account for trans-Planckian effects, but we do not know if these are the ones that best fit the possible deviations of the linear regime for the modes beyond Planck scale. It would then be interesting, if possible, to find the exact form that the dispersion relation of the modes has following geometrical arguments of LQC. In addition to this, we have chosen a specific ultraviolet scale, namely, the one that provides LQC related to the curvature radius at the bounce. This choice above others seems to be justified, but there is no argument that confirms that we have to take $\kappa_c = \kappa_{\rm LQC}$. Lastly, in this work we have only carried out a qualitative analysis of the trans-Planckian problem, for a particular model with a specific number of e-folds and for a concrete potential, focusing on the evolution of the adiabaticity coefficient for trans-Planckian modes, rather than computing the power spectrum for completeness. We leave these calculations, as well as the consideration of other suitable potentials and models with different number of e-folds, for future research. However, the study carried out here anticipates what the results will be.

To conclude, we affirm that the trans-Planckian problem exists in the model we have considered within LQC, and that this effect must be taken into account in the evaluation and computation of observable quantities, such as the power spectrum of primordial fluctuations measured in the CMB.

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