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Física trans-planckiana en el universo primitivo

Trans-Planckian physics in the early Universe

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Física trans-planckiana en el universo primitivo
Trans-Planckian physics in the early Universe

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The trans-Planckian problem in inflationary cosmology highlights that modes observable today may have originated at wavelengths smaller than the Planck length during an extended inflationary phase. To investigate potential ultraviolet corrections, we study two paradigmatic modified dispersion relations applied to a massive scalar field in a cosmological background. Particle production rates, the two-point correlation function, and the energy-momentum tensor are rendered finite via adiabatic regularization. By deriving necessary and sufficient criteria for the WKB expansion's validity, we show that, under broad adiabaticity conditions, standard inflationary predictions remain stable against such high-energy modifications.

CONTENTS

I. INTRODUCTION

The theory of cosmological inflation has become the prevailing paradigm in early universe cosmology. Initially proposed to resolve the flatness, horizon, and monopole problems of the standard Big Bang model [1], inflation posits a brief epoch of accelerated expansion that drives the spatial curvature to near zero and stretches microscopic regions to cosmological scales. Beyond solving these problems, inflation provides a mechanism for the generation of primordial density perturbations: quantum fluctuations of the inflaton field are redshifted beyond the Hubble radius and later re-enter as classical seeds for large scale structure [2]. Observations of the Cosmic Microwave Background (CMB) anisotropies have confirmed the resulting nearly scale-invariant spectrum with remarkable precision [3].

Notwithstanding these triumphs, a conceptual subtlety arises when one traces observable perturbation modes back to the onset of inflation. If inflation endures for more than the minimal sixty e-foldings or so, then the comoving wavelengths corresponding to CMB scales would have been shorter than the Planck length at the very beginning of the inflationary phase. At such trans-Planckian scales, the classical concept of spacetime and the standard methods of quantum field theory in curved backgrounds might lose their validity. This is the so-called trans-Planckian problem of inflationary cosmology [4, 5]. One must confront the possibility that predictions based on extrapolating low-energy effective field theory may receive significant corrections from unknown ultraviolet (UV) physics, potentially altering the power spectrum.

Most trans-Planckian studies in cosmology assume that at the onset of inflation perturbations were in an adiabatic vacuum state, neglecting any preceding dynamics. Yet if new physics is operative at the Planck scale, its imprint may be even stronger during a pre-inflationary epoch [6]. This consideration opens a novel window: rather than undermining the predictive power of inflation, the trans-Planckian problem may provide an observational handle on quantum gravity phenomena through subtle imprints in the primordial spectrum and higher-order correlators [7].

A closely related issue was identified in black hole physics. Hawking radiation is derived by tracing outgoing modes backward in time to arbitrarily short distances near the event horizon, where quantum gravitational effects are expected to dominate. Remarkably, analyses incorporating modified dispersion relations (MDRs)—in which the usual linear relation between frequency and wavenumber is deformed at high energies—have shown that the thermal character of Hawking radiation is robust over a wide range of UV deformations [8, 9]. This suggests that MDRs can serve as useful phenomenological proxies for the unknown Planck-scale theory, encoding potential quantum gravity corrections in a simple, controllable manner.

In the inflationary context, MDRs play an analogous role. By substituting the standard dispersion relation with one that deviates at a characteristic UV scale, one aims to effectively model the influence of quantum gravity on the evolution of perturbation modes [10, 11]. Two archetypal forms are often considered: the Corley-Jacobson (CJ) relation, which introduces a smooth high-momentum correction preserving monotonicity, and the Unruh relation, which imposes a frequency cutoff reflecting a maximal propagation speed in the deep UV. These deformations capture qualitatively different behaviors—smooth adiabatic bending versus abrupt high-frequency suppression—and thus enable us to assess how sensitive inflationary observables are to Planck-scale physics and to characterize potential observational signatures in both the scalar and tensor sectors.

A crucial element in any such analysis is the consistent removal of UV divergences from physical observables. In particular, the energy-momentum tensor must be renormalized so that its expectation value remains finite and continues to serve as a reliable source in the semiclassical Einstein equations. Without proper subtraction of divergent parts, one risks introducing spurious anomalies and violating energy conservation in the backreaction of quantum fields on the background geometry. However, the introduction of MDRs brings its own challenges, as the high-momentum deformations alter the UV behavior and can generate new divergent structures. In doing so, one must ensure that the extended subtraction preserves locality and energy-momentum conservation, thereby maintaining the consistency of the semiclassical dynamics under Planck-scale modifications. In cosmological scenarios, one typically adopts adiabatic regularization [12–14] to systematically remove UV divergences from both the two-point correlation function and the trace of the energy-momentum tensor. This procedure can be implemented for the standard linear dispersion relation as well as for monotonic superluminal modifications. However, the Unruh dispersion relation exhibits additional UV structure that complicates the usual subtraction algorithm. In this work, we present the explicit implementation of adiabatic regularization for the two-point function and the energy-momentum tensor in the cases of standard and CJ dispersions, and we introduce a novel subtraction scheme specifically tailored to handle the Unruh dispersion relation.

This thesis is organized as follows. In Sec. II, we present the key concepts of the canonical quantization of the field under consideration and formalize the notion of modified dispersion relations. We then review in Sec. III the concept of adiabaticity. We also derive conditions for the validity of the adiabatic WKB expansion, ensuring that the vacuum state remains well defined and that particle interpretation is meaningful even in the trans-Planckian regime. In Sec. IV, we apply the adiabatic regularization framework to renormalize the two-point correlation function and the trace of the energy-momentum tensor in both the standard and CJ dispersion cases, and introduce a novel subtraction scheme for the Unruh dispersion relation. We conclude in Sec. V with a discussion of the robustness of inflationary predictions and the prospects for probing trans-Planckian physics in forthcoming observations.

Notation: For the spacetime metric we use the signature $(-,+,+,+)$. We employ units such as $c=\hbar=1$. Furthermore, m_{P} and ℓ_{P} denote the Planck mass and the Planck length, respectively.

II. PRELIMINARIES

A. Canonical quantization

Consider a field $\phi(x)$ propagating in spacetime. The metric $g_{\mu\nu}$ will be treated as a given unquantized external field. Now, let us introduce a real, massive scalar field minimally coupled to gravity. In the framework of classical field theory, we can compute the equations of evolution of the dynamical field ϕ by writing Euler-Lagrange equations. For a generic spacetime, these equations take the Klein-Gordon (KG) form:

$$(\Box - m^2)\phi(x) = 0, \tag{1}$$

where $\Box$ is the D'Alembertian operator and m is the mass of the field.

Given a globally hyperbolic spacetime, the KG inner product is defined as [15]:

$$(\phi_1,\phi_2)_{\text{KG}} = i\int_{\Sigma_t} d^3x\sqrt{\mathfrak{h}}n^\mu\left(\phi_1^*\partial_\mu\phi_2 - \phi_2\partial_\mu\phi_1^*\right), \tag{2}$$

where t is a global time function, Σ_t is a Cauchy hypersurface defined by $t=$ const., n^μ is a timelike vector orthonormal to the surface and future-oriented, $\mathfrak{h}$ is the absolute value of determinant of the induced metric on the hypersurface, and ϕ_1 and ϕ_2 belong to the space of complex solutions of Eq. (1). A key property of the KG inner product is that it is preserved under temporal evolution.

We shall consider a Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime with flat space-like sections, described in the coordinate chart $x=(t,\vec{x})$, where t denotes cosmic time and $\vec{x}$ the comoving spatial coordinates. Then, the line element takes the form: $ds^2 = -dt^2 + a^2(t)d\vec{x}^2$ with $a(t)$ being the scale factor.

Let $\{f_{\vec{k}}(x), f^*_{\vec{k}}(x)\}$ be a complete set of complex solutions to the KG equation, which we will refer to as mode functions, or simply modes. We require these modes to satisfy the following orthonormality relations:

$$(f_{\vec{k}}, f_{\vec{q}})_{\text{KG}} = \delta^{(3)}(\vec{k}-\vec{q}), \qquad (f_{\vec{k}}, f^*_{\vec{q}})_{\text{KG}} = 0, \tag{3}$$

from which we deduce that

$$(f^*_{\vec{k}}, f^*_{\vec{q}})_{\text{KG}} = -\delta^{(3)}(\vec{k}-\vec{q}). \tag{4}$$

The most general real solution to the KG equation can be written as a linear combination of $f_{\vec{k}}$ and $f^*_{\vec{k}}$,

$$\phi(t,\vec{x}) = \int d^3k \left(a_{\vec{k}} f_{\vec{k}}(t,\vec{x}) + a^*_{\vec{k}} f^*_{\vec{k}}(t,\vec{x}) \right), \tag{5}$$

where $a_{\vec{k}}$ and $a^*_{\vec{k}}$ are the so-called annihilation and creation coefficients, respectively.

Because of spatial homogeneity and isotropy, we can choose the mode functions $f_{\vec{k}}$ of the form:

$$f_{\vec{k}}(x) = \frac{1}{\sqrt{2(2\pi)^3 a^3(t)}} F_k(t) e^{i\vec{k}\cdot\vec{x}}, \tag{6}$$

with $k = |\vec{k}|$ and the functions F_k satisfy the equation:

$$\ddot{F}_k + \omega_k^2 F_k = 0, \tag{7}$$

where the dot is the derivative with respect to t and

$$\omega_k^2 \equiv \kappa^2 + m^2 + \sigma, \qquad \sigma \equiv -\frac{3}{2}\frac{\ddot{a}}{a} - \frac{3}{4}\frac{\dot{a}^2}{a^2}. \tag{8}$$

Here we denote $\kappa = k/a$ the physical (observed) wavenumber, whereas k represents the comoving one.

In canonical quantization [14], the classical field is promoted to an operator by elevating the creation and annihilation coefficients to operators that satisfy standard commutation relations:

$$\hat{\phi}(x) = \int d^3k \left(f^*_k(x)\hat{a}^\dagger_k + f_k(x)\hat{a}_k \right). \tag{9}$$

Note that the preceding quantization procedure could also be carried out using conformal time η, related to cosmological time t by $dt = a(\eta)\, d\eta$. We have verified that the two quantization schemes are formally equivalent and yield identical physical predictions. Hence, for convenience, we shall adopt the cosmological time quantization throughout this work.

B. Modified dispersion relations

In standard quantum field theory in curved spacetimes, the dispersion relation for a free scalar field is given by Eq. (8). However, in high-energy (or trans-Planckian) regimes—particularly when quantum-gravitational effects are expected to become significant—this relation may require modifications [16]. One convenient way to encode such trans-Planckian corrections is to replace the usual momentum term with a function $\mathcal{K}(\kappa)$, so that

$$\omega_k^2 = \mathcal{K}^2 + m^2 + \sigma\,. \tag{10}$$

To modify the UV behavior, $\mathcal{K}$ is endowed with a new characteristic momentum scale, κ_c. For physical wavenumbers well below this threshold ($\kappa \ll \kappa_c$), one recovers the standard linear relation $\omega_k \approx \kappa$, while for $\kappa \gtrsim \kappa_c$ the function $\mathcal{K}$ encodes the departure from relativistic behavior. Importantly, κ_c is introduced purely as a phenomenological scale and does not signal any deformation of spacetime symmetries; it merely marks the regime in which new high-energy physics effects become relevant.

Multiple MDRs, characterized by different functions $\mathcal{K}$, have been studied in the literature. In our work, we focus on two commonly encountered relations: superluminical CJ's and Unruh's.

To construct the function $\mathcal{K}^2$ within an effective field theory framework, we can expand this function in a power series of κ, suppressing higher-order contributions by powers of the scale κ_c. This expansion must be performed in even powers. Indeed, only local differential operators can appear in the effective action, and these depend analytically on the square of the physical momentum. Any odd power would require a nonlocal, fractional operator and is therefore excluded by the requirement of locality.

First, we consider the dispersion relations obtained by truncating the previous power-series expansion at fourth order [8]. There are two main branches of these relations: those in which the coefficient of the κ^4 term is positive—referred to as superluminal modifications—and those in which this coefficient is negative—referred to as subluminal modifications. However, we do not pursue subliminal CJ relations, since they fail to produce stable, cutoff-independent predictions for the primordial power spectrum (see, e.g., [16]), violate the WKB adiabaticity condition at high momenta, and lead to pathological behavior. Consequently, our analysis focuses exclusively on the superluminal CJ relation,

$$\mathcal{K}^2 = \kappa^2 + \frac{\kappa^4}{\kappa_c^2}, \tag{11}$$

where we recall $\kappa = k/a$.

Another significant example is provided by the Unruh dispersion relation [9]:

$$\mathcal{K} = \kappa_c \tanh\left(\kappa/\kappa_c\right). \tag{12}$$

This relation exhibits the interesting property that $\mathcal{K}$ saturates at the scale κ_c in the high-energy limit. Although a power-series expansion of $\mathcal{K}^2$ also produces a negative coefficient at order κ^4, the complete form remains strictly positive for all κ and thus avoids UV instabilities.

In the context of FLRW cosmology with MDRs, one may ask whether quantization in conformal time is physically distinct from quantization in cosmic time. By employing the same Klein–Gordon inner product (2), we found that—analogously to the standard dispersion case—this product remains conserved under the MDR evolution, ensuring its validity as a KG inner product. Consequently, the quantizations in cosmic and conformal time coincide even in the presence of MDRs. To demonstrate this, one introduces the associated KG inner product for the modified field equations. When generalizing the usual equations to include MDRs, the spatial Laplacian Δ is replaced by a self-adjoint, positive operator $D(\Delta, a)$. Under these circumstances, D and Δ share the same eigenfunctions although their eigenvalues are modified in general due to the dependence on the scale factor. As a result, the mode basis remains unchanged and the corresponding quantizations in cosmic and conformal time yield the same vacuum state and particle interpretation.

III. ADIABATICITY

A. Adiabatic Vacua

Defining a vacuum state in a time-dependent background is nontrivial due to the absence of a global timelike Killing vector. In a FLRW spacetime, one seeks an approximation to the true ground state of the field when the background evolves slowly. This motivates the notion of an adiabatic vacuum: a state constructed so that each mode experiences only gradual changes in its effective frequency, minimizing particle production.

To implement this idea, one uses the Wentzel-Kramers-Brillouin (WKB) approximation to construct mode solutions to the field equation. We first note that [14]:

$$F_k(t) \;=\; \frac{1}{\sqrt{W_k(t)}}\exp\Big(-i\int^t W_k(t')\,dt'\Big), \tag{13}$$

is a solution of the mode equation (7) provided that $W_k(t)$ satisfies

$$W_k^2(t) \;=\; \omega_k^2(t) \;+\; \left(\frac{3}{4}\frac{\dot W_k^2(t)}{W_k^2(t)} \;-\; \frac{1}{2}\frac{\ddot W_k(t)}{W_k(t)}\right). \tag{14}$$

To solve Eq. (14) systematically, $W_k(t)$ is expanded in an asymptotic series in inverse powers of a dimensionless parameter τ that measures the slowness of variation:

$$W_k(t) \;=\; \sum_{n=0}^{\infty}\frac{W_k^{(n)}(t)}{\tau^n}. \tag{15}$$

Substituting this series into Eq. (14) and matching order by order yields

$$W_k^{(0)}(t) = \omega_k(t), \tag{16}$$

$$W_k^{(2)}(t) = \frac{3}{4}\,\frac{\dot\omega_k^2(t)}{\omega_k^3(t)} \;-\; \frac{1}{2}\,\frac{\ddot\omega_k(t)}{\omega_k^2(t)}. \tag{17}$$

Higher-order terms can also be computed, but we omit their expression since they are not particularly illuminating; moreover, all odd-order terms vanish identically.

The adiabatic vacuum of order n at an initial time t_0 is defined by requiring that the exact mode functions and their first time derivatives coincide with their WKB approximations up to order n at $t = t_0$. Concretely, the zeroth-order adiabatic vacuum is specified by the initial conditions

$$F_k(t_0) = \frac{1}{\sqrt{2\,\omega_k(t_0)}},$$
$$\dot F_k(t_0) = \left(-i\,\omega_k(t_0) \;-\; \frac{\dot\omega_k(t_0)}{2\,\omega_k(t_0)}\right)\frac{1}{\sqrt{2\,\omega_k(t_0)}}. \tag{18}$$

Higher-order adiabatic vacua impose higher WKB order initial conditions for F_k.

Thus, while the adiabatic regime refers to the interval during which the frequency $\omega_k(t)$ varies slowly (justifying the WKB approximation), the adiabatic vacuum corresponds to the specific choice of initial conditions that align the mode functions with their WKB counterparts.

B. Validity conditions

Having introduced the notion of adiabatic vacua, we now turn to the question of when the adiabatic approximation is justified and what criteria govern its validity. If the background evolution significantly departs from adiabaticity, particle production can become substantial and may even dominate the energy density of the field. In the context of inflation, such backreaction could undermine the assumption that the inflaton potential is responsible for the accelerated expansion [17]. For these reasons, it is crucial to identify the conditions under which the WKB-based adiabatic approximation remains valid.

In the literature, a conflict has arisen when addressing the validity conditions of adiabaticity. On the one hand, there is the eikonal condition presented in reference [18]:

$$\epsilon \equiv \left|\left(\frac{\dot\omega_k}{\omega_k^2}\right)\right| \ll 1. \tag{19}$$

On the other hand, there is the following condition presented in reference [19]:

$$\mathcal{Q} = \left|\frac{3}{4}\left(\frac{\dot\omega_k(t)}{\omega_k(t)^2}\right)^2 - \frac{1}{2}\frac{\ddot\omega_k(t)}{\omega_k^3(t)}\right| \ll 1. \tag{20}$$

The condition for performing the adiabatic approximation and obtaining an adiabatic vacuum is that $W_k(t) \approx \omega_k(t)$. To ensure this, we have found that besides condition (20), there is actually one more, which is:

$$\mathcal{I} \equiv \left|\int^t \left(\frac{3}{8}\frac{(\dot\omega_k(t))^2}{\omega_k^3(t)} - \frac{1}{4}\frac{\ddot\omega_k(t)}{\omega_k^2(t)}\right)\,dt'\right| \ll 1. \tag{21}$$

This condition has gone unnoticed in the literature, and yet it is, in fact, at least as restrictive as the other.

Although in many cosmological scenarios the condition given by equation (19) is commonly considered sufficient to guarantee adiabaticity, we have found that this condition does not necessarily imply that conditions (20) or (21) are satisfied individually. Moreover, we have also established a mutual relationship between these conditions. Explicitly, we have seen that $\mathcal{Q} \ll 1$ if and only if

$\epsilon \ll 1$ and $\left|\ddot{\omega}_k/\omega_k^3\right| \ll 1$. This equivalence helps to explain the discrepancies observed in the numerical analysis of MDRs in de Sitter spacetime reported in [20].

These conditions arise from truncating the WKB series, although this is not strictly rigorous: as an asymptotic expansion, the WKB series guarantees a vanishing relative error under suitable assumptions but does not control the absolute error. Nevertheless, their use is fully justified here, since a more rigorous yet practical analysis is not feasible within this framework [21].

A subtle issue in adiabaticity arises because truncating the WKB series need not converge if higher-order terms diverge. In principle, one might have $\omega_k(t) = 0$ at some instant t, with its first and second derivatives also vanishing, potentially satisfying the above adiabatic conditions despite $\omega_k = 0$. To exclude this pathological case, we require that ω_k have no degenerate critical points coinciding with its zeros. In a slow-roll (nearly de Sitter) inflationary background, this condition forces $\mathcal{K}(\kappa)$ to be κ–independent at t, which contradicts the construction $\mathcal{K}(\kappa) \sim \kappa$ as $\kappa \to 0$. Hence, no modified dispersion relation can produce such degenerate zeros, and the previously derived conditions guarantee that no higher-order adiabatic term diverges.

A numerical example

Let us consider a model of cosmological inflation lasting approximately 60 e-folds for a scalar perturbation with a mass of $m = 1.21 \times 10^{-6}\, m_{\rm P}$ [22]. In this inflationary scenario, one can study the adiabaticity of the mode evolution by employing conditions (20) and (21) and comparing them with condition (19). The numerical results obtained are presented in Fig. 1.

We observe that, for all dispersion relations considered, each adiabatic condition remains satisfied throughout the entire evolution. Moreover, the numerical value of ϵ is consistently much smaller than that of $\mathcal{Q}$. As shown before, no direct implication links them; however, in typical slow-varying backgrounds one expects the second time derivative of the frequency to be much smaller than the first, so this hierarchy is natural. Consequently, ϵ alone cannot serve as a reliable estimator of adiabaticity.

IV. RENORMALIZATION

Many physically meaningful quantities, such as the action and the energy-momentum tensor, are quadratic in the fields and their derivatives, evaluated at a single spacetime point. This local structure leads to well-known divergences in their expectation values. In flat spacetime, such divergences can often be handled by normal ordering in the case of free fields. However, the situation becomes more subtle in curved backgrounds: even free fields interact with the underlying geometry, giving rise to new types of divergences. Additionally, vacuum energy must be addressed with particular care, as it can have real gravitational consequences through its role in the Einstein field equations. In this section, we will analyze the renormalization of the two-point correlation function for the standard, the superluminal CJ, and Unruh dispersion relations. Finally, we will study the renormalization of the trace of the energy-momentum tensor for the first two. The Unruh relation presents greater difficulties and remains a topic for future work.

One may determine all components of the expectation value of the energy-momentum tensor $\langle T_{\mu\nu}\rangle$ from its trace $\langle T\rangle$ by exploiting the homogeneity and isotropy of the background together with the conservation equation $\nabla_\mu \langle T^{\mu\nu}\rangle = 0$. Consequently, in this work we shall focus exclusively on the renormalization of the trace $\langle T\rangle$.

A. Two point correlation functions

The first physical observable we shall examine is the two-point correlation function. Its relevance extends beyond its role in the computation of the energy-momentum tensor; it also holds intrinsic interest as its Fourier transform defines the power spectrum.

Let us consider the two-point correlation function of a scalar field minimally coupled to gravity, as introduced in Sec. II. The UV divergence arises solely in the coincidence limit, that is, when the two spacetime points are brought together. In a homogeneous and isotropic background, this yields:

$$\langle \phi^2(t,\vec{x})\rangle = \frac{1}{4\pi^2 a^3(t)} \int_0^\infty \frac{dk\, k^2}{W_k(t)}. \tag{22}$$

This quantity generally presents divergences in the limit $k \to \infty$ which must be regularized and renormalized in order to obtain physically meaningful results. The method of adiabatic subtraction proves to be particularly effective [23]. This procedure relies on the adiabatic expansion of mode functions, which provides a controlled approximation valid in the high-momentum regime. A key feature of this expansion is that each successive adiabatic order introduces additional inverse powers of the momentum k, leading to stronger suppression in the UV limit. Consequently, for dispersion relations in which the frequency ω_k increases with k, higher-order terms yield integrands that decay more rapidly at large momenta. This ensures that, beyond a certain order, the corresponding contributions to physical observables become UV finite. However, this property no longer holds if ω_k ceases to grow monotonically with k, as is the case, for instance, with the Unruh dispersion relation. In such situations, the convergence of the adiabatic subtraction terms must be reassessed with care.

The scheme of adiabatic subtraction consists in removing, mode by mode, any adiabatic term that contains UV divergences for generic parameter values. Crucially, when a given adiabatic order contains any divergent contribution, the entire term must be subtracted—including its finite parts. By subtracting full adiabatic orders, one

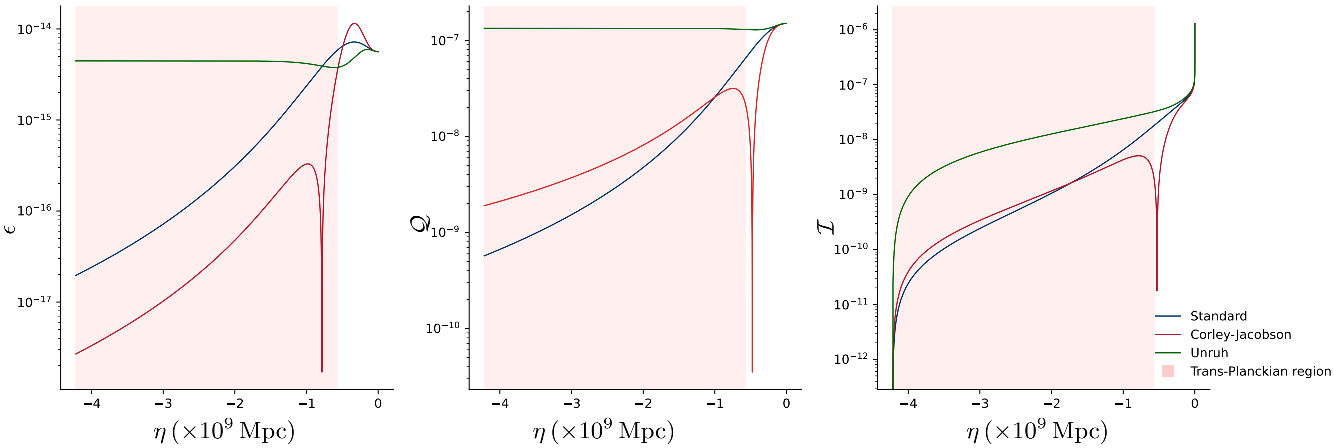

FIG. 1. Joint evolution of $\mathcal{Q}$, ϵ and $\mathcal{I}$ as functions of the conformal time η (in Mpc), chosen here for simplicity, for the comoving mode $k = 10^{-2}\,\mathrm{Mpc}^{-1}$, which is the most infrared mode that remained trans-Planckian at the onset of inflation. The red-shaded region indicates the interval during which this mode was trans-Planckian.

ensures the covariant conservation of the renormalized energy–momentum tensor $\nabla_\mu \langle T^{\mu\nu}\rangle_{\rm ren} = 0$.

When the divergent contributions depend on a parameter so that, for certain values, the integral becomes finite but, for others, it diverges, the subtraction must still be applied even if the raw integral converges. Enforcing this subtraction guarantees that the renormalized result remains a smooth, continuous function of the theory's parameters, preserving both mathematical consistency and physical continuity.

Let us now apply this renormalization scheme to Eq. (22). To this end, we expand the numerator in a series, formally obtaining

$$\frac{1}{W_k(t)} = \sum_n \frac{\left(W_k(t)^{-1}\right)^{(n)}}{\tau^n}. \tag{23}$$

In particular, by using the expressions given in (15), we obtain:

$$\begin{aligned}
\left(W_k(t)^{-1}\right)^{(0)} &= \omega_k^{-1}, \\
\left(W_k(t)^{-1}\right)^{(2)} &= \omega_k^{-2} W_k^{(2)}, \\
\left(W_k(t)^{-1}\right)^{(4)} &= \omega_k^{-3}\left(W_k^{(2)}\right)^2 - \omega_k^{-2} W_k^{(4)},
\end{aligned} \tag{24}$$

where all odd-order terms vanish. By substituting the results from Eq. (23)–(24) in Eq (22), one arrives at

$$\begin{aligned}
\langle \phi^2(t,\vec{x})\rangle = \frac{1}{4\pi^2 a^3(t)} \int_0^\infty dk\, k^2 \Big\{ \omega_k^{-1} + \omega_k^{-2} W_k^{(2)} \\
+ \omega_k^{-3}\left(W_k^{(2)}\right)^2 - \omega_k^{-2} W_k^{(4)} + \dots \Big\},
\end{aligned} \tag{25}$$

where the ellipsis denotes higher adiabatic orders not displayed. This expression holds in full generality, irrespective of the particular dispersion relation employed.

In this framework, the adiabatic expansion plays a dual role: it makes the UV structure of the theory manifest, and it provides a systematic prescription for subtracting divergences in a manner that is both minimal and consistent. In doing so, it ensures that expectation values of composite observables—such as correlation functions and components of the energy-momentum tensor—remain finite and physically meaningful in curved spacetime.

B. Standard Dispersion Relation

We begin by analyzing the standard dispersion relation, whose frequency is given by Eq. (8), with $\kappa = k/a$ denoting the physical wavenumber. According to the general renormalization procedure, one must identify and subtract the terms in the adiabatic expansion of Eq. (25) that lead to UV divergences.

In the high-momentum regime, the frequency asymptotically behaves as $\omega_k \to \kappa$, so that the leading term in the integrand scales as $k^2\,\omega_k^{-1} \sim k$. This contribution diverges quadratically and must therefore be removed. The next-to-leading-order term scales as $k^2 (W_k^{-1})^{(2)} \sim k^0$, leading to a logarithmic divergence, which must also be subtracted. In contrast, the fourth-order term behaves as $k^2 (W_k^{-1})^{(4)} \sim k^{-2}$, and is thus convergent in the UV regime.

Accordingly, the renormalized two-point function for the standard dispersion relation takes the form:

$$\begin{aligned}
\langle \phi^2(t,\vec{x})\rangle_{\rm ren} = \frac{1}{4\pi^2 a^3(t)} \int_0^\infty dk\, k^2 \\
\times \left(\frac{1}{W_k(t)} - \omega_k^{-1} - \left(W_k(t)^{-1}\right)^{(2)} \right).
\end{aligned} \tag{26}$$

We now turn to the vacuum expectation value of the trace of the renormalized energy-momentum tensor, $\langle T\rangle_{\rm ren}$. For a scalar field one might invoke the standard relation $\langle T\rangle = -m^2\,\langle\phi^2\rangle$, but the UV divergences in $\langle T\rangle$

exceed those in $\langle\phi^2\rangle$, so that $m^2\ \langle\phi^2\rangle_{\rm ren}$ alone cannot render the trace finite. Indeed, for instance, $\langle T^0{}_0\rangle$ contains a divergent contribution proportional to the integral of $\dot F_k(t)$ (and hence of $\dot W_k(t)$), which raises the adiabatic order to the next one. Consequently, UV divergences persist through fourth adiabatic order and must be removed by subtracting all lower orders. This defines the renormalized tensor $\langle T_{\mu\nu}\rangle_{\rm ren}$, whose trace is rendered finite:

$$\langle T\rangle_{\rm ren} = -\frac{m^2}{4\pi^2 a^3(t)}\int_0^\infty dk\, k^2\Bigg(\frac{1}{W_k(t)} - \omega_k^{-1} - \big(W_k^{-1}(t)\big)^{(2)} - \big(W_k^{-1}(t)\big)^{(4)}\Bigg). \tag{27}$$

After a lengthy calculation one finds

$$\langle T\rangle_{\rm ren} = -m^2\ \langle\phi(x)^2\rangle_{\rm ren} - \frac{1}{128\pi^2}\Bigg\{\frac{\dot a^4}{2a^4} + \frac{29\,\dot a^2\ddot a}{15a^3} + \frac{3\,\ddot a^2}{10a^2} - \frac{\ddddot a}{5a} - \frac{3\,\dot a\ \dddot a}{5a^2}\Bigg\}, \tag{28}$$

which is a well-known result [23].

C. Superluminal Dispersion Relation

To carry out the renormalization procedure for the superluminal CJ dispersion relation, we adopt the same formalism as before. We recall that the dispersion relation was defined in Sec. II by $\mathcal{K}^2 = \kappa^2 + \kappa^4/\kappa_c^2$ (cf. Eq. (10)). Substituting into the general adiabatic expansion given in Eq. (25), one observes that in the UV limit $\omega_k \sim k^2$, so that $k^2\,\omega_k^{-1} \sim k^0$ which produces a linear divergence. Consequently, one must subtract this term to render the two-point function finite. Although the explicit form of the subtraction term is lengthy and does not add conceptual insight, we proceed to compute the next adiabatic correction. One finds that, in the UV regime, $k^2\big(W_k(t)^{-1}\big)^{(2)} \sim k^{-4}$, confirming that this higher-order term vanishes fast enough to yield convergence. Therefore, the two-point correlation function takes the form:

$$\langle\phi^2(t,\vec x)\rangle_{\rm ren} = \frac{1}{4\pi^2 a^3(t)}\int_0^\infty dk\, k^2\Big(\frac{1}{W_k(t)} - \omega_k^{-1}\Big). \tag{29}$$

It is worth noting that the superluminal relation achieves stronger UV suppression than the standard case. Whereas the usual dispersion yields $\omega_k \sim k$ at large momentum (inducing quadratic divergences), the quartic correction enforces $\omega_k \sim k^2$, so that inverse-frequency integrands decay more rapidly. This built-in damping of high-frequency modes illustrates how MDRs can serve as natural regulators for trans-Planckian excitations, ensuring a well-controlled quantum field behavior at ultra-high energies.

Let us now turn to the computation of the trace of the energy-momentum tensor for this modified dispersion relation. First, we introduce the Lagrangian density that reproduces the equation of motion for the scalar field with frequency determined by Eq. (10), where $\mathcal{K}$ is specified in Eq. (11). In fact, working backwards from the dispersion relation, one finds that the appropriate Lagrangian is

Since no closed-form expression for the trace of the energy-momentum tensor is available in the literature for this type of Lagrangian, we proceed to compute $T_{\mu\nu}$ explicitly and then take its trace. We employ Hilbert's prescription [15]. A straightforward calculation shows that

$$T_{\mu\nu} = \nabla_\mu\phi\,\nabla_\nu\phi - \frac{1}{2}g_{\mu\nu}\,\mathcal{L}_{\rm CJ} + \frac{1}{\kappa_c^2}\,\delta^i_\mu\,\delta^j_\nu\Big(\nabla_i\nabla_j(\Delta\phi) + \Delta(\nabla_i\nabla_j\phi)\Big). \tag{30}$$

The trace follows immediately:

$$T = -m^2\,\phi^2 + \frac{1}{\kappa_c^2}\,\phi\,\Delta^2\phi. \tag{31}$$

It should be emphasized that the renormalized trace cannot be obtained simply by applying adiabatic renormalization directly to the formal expression in Eq. (31). Indeed, for the same reason encountered in the standard dispersion-relation scenario—namely that $\langle T\rangle$ exhibits divergences even more severe than those of $\langle\phi^2\rangle$—one must subtract an additional adiabatic order. Moreover, when computing its vacuum expectation value one introduces a factor of k^4 in the integrand, which renders the integral divergent. Consequently, it is necessary to subtract contributions up to the second adiabatic order.

Thus, in complete analogy with the standard dispersion relation case, the renormalized expectation value of the trace is given by

$$\langle T\rangle_{\rm ren} = \langle\frac{1}{\kappa_c^2}\,\phi\,\Delta^2\phi - m^2\,\phi^2\rangle + \frac{1}{4\pi^2 a^3}\int_0^\infty dk\,k^2\Big(m^2 - \frac{k^4}{a^4\,\kappa_c^2}\Big)\Big[\omega_k^{-1} + \big(W_k(t)^{-1}\big)^{(2)}\Big]. \tag{32}$$

There does not exist closed-form analytic solution for this integral in the general case, and hence it must be evaluated numerically in the regimes of physical interest. In spite of the absence of an analytic form, the result of the numerical evaluation is finite.

D. Unruh Dispersion Relation

While the superluminal dispersion relation ameliorates the UV behavior by accelerating the growth of the frequency with respect to momentum, the Unruh dispersion follows a distinct behavior: it causes the frequency to saturate at large momenta. This saturation introduces new obstacles in the renormalization procedure, as we now discuss.

To examine this dispersion relation, recall the explicit form of Unruh's frequency,

$$\omega_k(t) = \sqrt{\kappa_c^2 \tanh^2\left(\frac{k}{a\kappa_c}\right) + m_{\rm eff}^2(t)}, \tag{33}$$

where, by virtue of Eq. (8), we define $m_{\rm eff}^2(t) \equiv m^2 + \sigma(t)$. One might then attempt to employ the adiabatic renormalization technique—used successfully in the previous two cases—to regularize the divergence of the two-point correlation function given by Eq. (25). However, a critical complication arises: upon performing the adiabatic expansion of the integrand, every term diverges. Subtracting all of these divergent contributions would inevitably drive the two-point correlation function to zero.

Likewise, extending the same procedure to the computation of the energy-momentum tensor also yields a trivial result. Thus, although this renormalization scheme is formally permissible, it fails to capture any nontrivial physical content for the problem at hand.

Furthermore, standard alternatives—such as dimensional regularization or heat kernel renormalization—cannot be applied in a straightforward manner to isolate and remove the divergence. In light of these challenges, we now propose an alternative renormalization strategy specifically designed to accommodate the unusual UV behavior inherent to the Unruh dispersion relation.

The first step is to introduce an arbitrary (but finite) scale μ, so that the two-point correlation function can be written as:

$$\langle \phi^2(t,\vec{x})\rangle = \frac{1}{4\pi^2 a^3(t)} \int_0^{\mu} \frac{dk\, k^2}{W_k(t)} + I_\mu(t), \tag{34}$$

where we have defined:

$$I_\mu(t) \equiv \frac{1}{4\pi^2 a^3(t)} \int_\mu^{\infty} \frac{dk\, k^2}{W_k(t)}. \tag{35}$$

This integral admits the following adiabatic expansion:

$$I_\mu(t) = \frac{1}{4\pi^2 a^3(t)} \int_\mu^{\infty} dk\, k^2 \Big\{ \omega_k^{-1} + \left(W_k(t)^{-1}\right)^{(2)} + \left(W_k(t)^{-1}\right)^{(4)} + \left(W_k(t)^{-1}\right)^{(6)} + \dots \Big\}. \tag{36}$$

This integral diverges, as every term in the integrand yields a divergent contribution, so the integral as a whole is unbounded. To analyze the leading term divergence, we perform a change of variables from k to ω_k, whose inverse Jacobian yields the density of states:

$$\rho(\omega_k) = \frac{a\kappa_c^2 \omega_k}{\sqrt{\omega_k^2 - m_{\rm eff}^2}} \left(\kappa_c^2 - (\omega_k^2 - m_{\rm eff}^2)\right)^{-1}. \tag{37}$$

Hence, the leading term becomes

$$I_\mu^{(0)} \equiv \frac{\kappa_c^2}{4\pi^2} \int_{\omega_\mu}^{\omega_m} d\omega \,\left(\kappa_c^2 - (\omega^2 + m_{\rm eff}^2)\right) \times \left[\tanh^{-1}\left(\frac{\sqrt{\omega^2 - m_{\rm eff}^2}}{\kappa_c}\right)\right]^2 \frac{\rho(\omega)}{[\omega_m^2 - \omega^2]}, \tag{38}$$

with $\omega_m^2 = \kappa_c^2 + m_{\rm eff}^2$ and ω_μ denoting the lower bound. This integral diverges as $\omega \to \omega_m$, since the denominator vanishes and the density of states simultaneously becomes singular. In contrast to earlier cases—where, the density of states did not introduce any new UV divergences—in the present scenario it instead aggravates the singularity.

To isolate this divergence, we set $x^2 \equiv \omega_m^2 - \omega^2$, yielding:

$$I_\mu^{(0)}(t) = \frac{\kappa_c^4}{4\pi^2} \int_0^{x_\mu} \frac{dx}{\sqrt{(\kappa_c^2 - x^2)\,(\omega_m^2 - x^2)}} \times \left[\tanh^{-1}\left(\sqrt{1 - \frac{x^2}{\kappa_c^2}}\right)\right]^2 x^{-1}. \tag{39}$$

Observe that, under this change of variables, the integral diverges as $x \to 0$, coming from the last factor of the integrand. This divergence is identical to the one previously expressed in other coordinates. To regularize it, we expand the integrand in a series around $x = 0$ and retain only the divergent contribution. By doing so, we obtain:

$$I_{\mu,\infty}^{(0)}(t) = \frac{\kappa_c}{4\pi^2 \omega_m} \int_0^{x_\mu} \frac{dx}{x} \left(\left[\ln\left(\frac{x}{\kappa_c}\right)\right]^2 - \ln(4)\ln\left(\frac{x}{\kappa_c}\right) + (\ln 2)^2 \right). \tag{40}$$

Note that this expression contains no finite terms in its expansion. Finally, we rewrite the integral in the original variables:

$$I_{\mu,\infty}^{(0)}(t) = \frac{\kappa_c}{4\pi^2 a\, \omega_m} \int_\mu^{\infty} dk \sqrt{\omega_k^2 - m_{\rm eff}^2} \left\{ \left[\ln\left(\frac{\sqrt{\omega_m^2 - \omega_k^2}}{\kappa_c}\right)\right]^2 - \ln(4)\ln\left(\frac{\sqrt{\omega_m^2 - \omega_k^2}}{\kappa_c}\right) + (\ln 2)^2 \right\}. \tag{41}$$

This integral isolates the divergent part associated with the first term of the adiabatic expansion. To characterize the UV divergence, we first make the change of variable from k to the dimensionless variable κ/κ_c and then replace the upper integration bound by $1/\varepsilon$, so that the previous expression is recovered in the limit $\varepsilon \to 0^+$. In this way, we obtain:

$$I_{\mu,\infty}^{(0),\varepsilon} = \frac{\kappa_c^3}{4\pi^2 \omega_m} \left\{ \frac{1}{3\varepsilon^3} + \left(\frac{\ln 4}{2} - \ln 2\right)\frac{1}{\varepsilon^2} + \left(\ln^2 2 - \frac{\ln^3 2}{3} - \ln 4 \ln^2 2\right)\frac{1}{\varepsilon} \right\} + \text{finite terms}, \tag{42}$$

such that $I^{(0),\varepsilon}_{\mu,\infty} \xrightarrow{\varepsilon\to 0^+} I^{(0)}_{\mu,\infty}$, and the remaining finite terms are not shown, as their explicit form adds no additional insight.

We now analyze the divergences of the remaining infinite terms in (36). Consider the generic form of any term in the expansion (23), which takes the form

$$\big(W_k(t)^{-1}\big)^{(2n)} = \sum_{m=1}^{N_{2n}} c_m^{(2n)}\,\omega_k^{-a_m} \prod_{l=1}^{M_m} \big[\omega_k^{(b_l)}\big]^{d_l}, \tag{43}$$

with real coefficients $c_m^{(2n)}$ and positive integers a_m, b_l, d_l, N_{2n} M_m. Each inverse adiabatic term is thus a linear combination of powers of ω_k and its derivatives. The integral corresponding to such a term, denoted $I_\mu^{(2n)}(t)$, is again divergent.

In the UV limit, the n-th derivative of ω_k becomes independent of k. Indeed, from Eq. (33) we have $\tanh\big(k/(a\kappa_c)\big) \to 1$ as $k \to \infty$, so all k-dependence in those derivatives vanishes. Consequently, in the limit $k \to \infty$ Eq. (43) can be written as

$$\big(W_k(t)^{-1}\big)^{(2n)} \sim \sum_{m=1}^{N_{2n}} c_m^{(2n)}\,\omega_m^{-a_m-\sum_{l=1}^{M_m} d_l} \prod_{l=1}^{M_m} \Big[\frac{d^{\,b_l} m_{\rm eff}^2}{dt^{\,b_l}}\Big]^{d_l}, \tag{44}$$

which shows that no new UV divergences arise from these terms. Therefore, the divergence can be isolated exactly as in (41). We denote the resulting divergent contribution by $I^{(2n)}_{\mu,\infty}(t)$, yielding:

$$I^{(2n)}_{\mu,\infty}(t) = \sum_{q=1}^{N_n} \frac{c_q^{(2n)}}{2\,\omega_m^{a_q-1+\sum_{l=1}^{M_q} d_l}} \prod_{l=1}^{M_m} \Big[\frac{d^{\,b_l} m_{\rm eff}^2}{dt^{\,b_l}}\Big]^{d_l}\; I^{(0)}_{\mu,\infty}(t)\,. \tag{45}$$

Thus $I^{(0)}_{\mu,\infty}(t)$ represents a universal divergence in the expansion. This expression holds for all $2n > 0$, covering every term in (36).

The absence of new divergences in this method is no coincidence—it reflects the underlying physics: effective frequencies that saturate in the UV naturally constrain the adiabatic expansion, ensuring that renormalization proceeds in harmony with the physical structure of the theory.

After the previous discussion, we find that the set of infinite divergences results in

$$\langle \phi^2(t,\vec{x})\rangle = \frac{1}{4\pi^2 a^3(t)} \int_0^\mu \frac{dk\,k^2}{W_k(t)} + \sum_{n=0}^{\infty} I^{(2n)}_{\mu,\infty}(t) + \mathcal{C}_\mu(t), \tag{46}$$

where the entire divergent part is contained in the term with the summation and $\mathcal{C}_\mu(t)$ contains the finite remanent left when we have isolated the divergence in (39):

$$\mathcal{C}_\mu(t) = I_\mu(t) - \sum_{n=0}^{\infty} I^{(2n)}_{\mu,\infty}(t), \tag{47}$$

which is a finite function of cosmological time. Therefore, we will say that the renormalized two-point correlation function is the finite function obtained by subtracting the divergent part of the previous expression, yielding:

$$\langle \phi^2(t,\vec{x})\rangle_{\rm ren} = \frac{1}{4\pi^2 a^3(t)} \int_0^\mu \frac{dk\,k^2}{W_k(t)} + c_\mu(t), \tag{48}$$

where now $c_\mu(t)$ is a finite function of t, but otherwise arbitrary for now. This finite remainder arises because, in subtracting the divergent contributions, one may always add any finite local contribution without spoiling the cancellation of infinities. Consequently, all those finite remnants accumulated throughout the subtraction procedure are absorbed into the function $c_\mu(t)$.

It remains to establish a scheme compatible with this renormalization method that enables the determination of this function while preserving the desired symmetries of the theory—most notably, the conservation of the energy-momentum tensor, which will be renormalized in future works using the same renormalization method that we have presented here. We will also try to set a meaningful relationship between the renormalization scale μ and the deformation scale κ_c, so that no new scale appears after renormalization.

In summary, extracting each UV pole and its coefficient explicitly ensures the subtraction of all divergences and demonstrates that the Unruh dispersion relation is renormalizable adiabatic order by adiabatic order. Moreover, having each divergence fully characterized clarifies the structure of the finite remainder, simplifies comparison between different regularization schemes, and makes it straightforward to track any residual scheme dependence.

As future work, we will introduce explicit counterterms to remove these divergences, thereby opening the door to the running of parameters and a transparent determination of renormalization-group behavior. Even in the absence of loop diagrams, we can implement an adiabatic expansion of the prefactor multiplying the leading divergent term, which allows a systematic, order-by-order adiabatic renormalization. Because all UV poles are already localized, this scheme not only streamlines the renormalization procedure but also highlights the fact that regularization reflects the physical taming of trans-Planckian modes by modified dispersion relations. Consequently, the divergences that appear are not pathological artifacts but indicators of the breakdown of the high-energy effective description—precisely the limitation that our renormalization corrects.

V. CONCLUSIONS

In this work, we have explored the influence of MDRs on quantum field dynamics during the inflationary period, motivated by the trans-Planckian problem. We focused on two representative MDRs—the superluminal Corley-Jacobson and Unruh dispersion relations—and analyzed their implications for particle creation and quantum fluctuations in an FLRW background.

Our analysis shows that the modification of the UV behavior of the dispersion relation affects the adiabatic expansion used in defining physically meaningful vacuum states. We have discussed and clarified the necessary and sufficient conditions under which the WKB approximation—and thus the zeroth-order adiabatic vacuum—is valid. These conditions are essential for interpreting the creation of particles and for ensuring the consistency of the quantum field description.

A major part of the study was devoted to the renormalization of divergent quantities, particularly the two-point correlation function $\langle \phi^2 \rangle$ and the trace of the energy-momentum tensor $\langle T \rangle$. We implemented adiabatic regularization techniques tailored to each dispersion relation. While the superluminal dispersion naturally regulates high-energy divergences due to its rapidly growing frequency, the Unruh relation requires a more refined approach, due to the saturation of the frequency at high momenta.

To address this, we have proposed a new renormalization scheme that isolates and consistently subtracts the divergent contributions associated with the Unruh dispersion relation. This scheme ensures the extraction of finite, physically meaningful quantities, and provides a viable route for handling trans-Planckian effects in scenarios where standard techniques fail for the Unruh dispersion relation.

These results emphasize that MDRs, though phenomenological, provide a viable framework to probe Planck-scale effects during inflation. However, their implementation requires careful attention to the structure of the vacuum, the validity of approximations, and the renormalization scheme. Future work will extend this analysis to the renormalization of the energy-momentum tensor and to scenarios beyond de Sitter or slow-roll inflation, possibly within a full effective field theory approach.

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[1] E. W. Kolb and M. S. Turner, *The Early Universe*. Addison-Wesley, 1990.

[2] V. Mukhanov, *Physical Foundations of Cosmology*. Cambridge University Press, 2005.

[3] G. Hinshaw et al., *Nine-year wilkinson microwave anisotropy probe (wmap) observations: Cosmological parameter results*, *Astrophys. J. Suppl.* **208** (2013) 19.

[4] J. Martin and R. H. Brandenberger, *The trans-Planckian problem of inflationary cosmology*, *Phys. Rev. D* **63** (2001) 123501.

[5] R. H. Brandenberger and J. Martin, *The robustness of inflation to changes in super-Planck-scale physics*, *Mod. Phys. Lett. A* **16** (2001) 999.

[6] J. Martin and R. H. Brandenberger, *The trans-Planckian problem of inflationary cosmology: A status report*, *Phys. Rev. D* **67** (2003) 083515.

[7] R. H. Brandenberger, *Trans-Planckian Censorship and Inflation*, *Class. Quant. Grav.* **39** (2022) 105003.

[8] S. Corley and T. Jacobson, *Black hole lasers*, *Phys. Rev. D* **54** (1996) 1568.

[9] W. G. Unruh, *Sonic analogue of black holes and the effects of high frequencies on black hole evaporation*, *Phys. Rev. D* **51** (1995) 2827.

[10] J. C. Niemeyer, *Inflation with a high frequency cutoff*, *Phys. Rev. D* **59** (1999) 123502.

[11] R. H. Brandenberger and J. Martin, *Trans-Planckian issues for inflationary cosmology*, *Int. J. Mod. Phys. A* **20** (2005) 5671.

[12] L. Parker and S. A. Fulling, *Adiabatic regularization of the energy-momentum tensor of a quantized field in homogeneous spaces*, *Phys. Rev. D* **9** (1974) 341.

[13] S. A. Fulling and L. Parker, *Renormalization in the theory of a quantized scalar field interacting with a robertson–walker spacetime*, *Ann. Phys.* **87** (1974) 176.

[14] N. D. Birrell and P. C. W. Davies, *Quantum Fields in Curved Space*. Cambridge University Press, 1982.

[15] R. M. Wald, *General Relativity*. University of Chicago Press, 1st ed., 1984.

[16] J. Martin and R. H. Brandenberger, *A Cosmological Window on Trans-Planckian Physics*, *astro-ph/0012031* (2001).

[17] T. Tanaka, *A comment on trans-Planckian physics in inflationary universe*, astro-ph/0012431.

[18] J. C. Niemeyer and R. Parentani, *Trans-Planckian dispersion and scale invariance of inflationary perturbations*, *Physical Review D* **64** (2001).

[19] R. H. Brandenberger and J. Martin, *On signatures of short distance physics in the cosmic microwave background*, *International Journal of Modern Physics A* **17** (2002) 3663.

[20] J. Macher and R. Parentani, *Signatures of trans-Planckian dispersion in inflationary spectra*, *Phys. Rev. D* **78** (2008) 043522.

[21] N. Fröman and P. O. Fröman, *Phase-Integral Method*. Springer-Verlag, 1996.

[22] L. J. Garay, M. L. González, M. Martín-Benito, and R. B. Neves, *Adiabatic approach to the trans-Planckian problem in loop quantum cosmology*, *Phys. Rev. D* **109** (2024) 123534.

[23] L. E. Parker and D. J. Toms, *Quantum Field Theory in Curved Spacetime: Quantized Fields and Gravity*. Cambridge University Press, 2009.