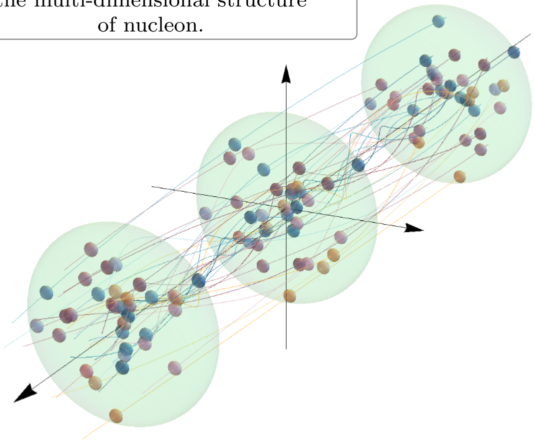
Nucleon tomography in momentum space

Alexey Vladimirov

IPARCOS meeting **16 June 2022**

Hadron is a 3D object

Nucleon tomography aims to explore the multi-dimensional structure of nucleon.



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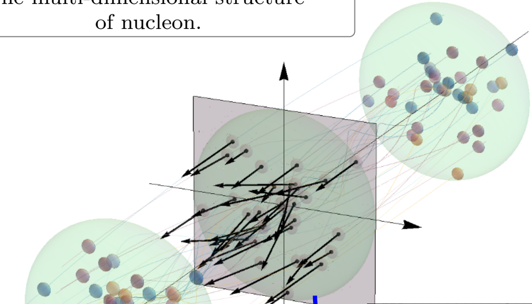
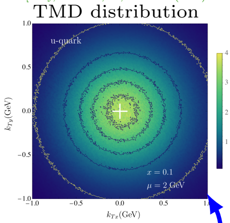
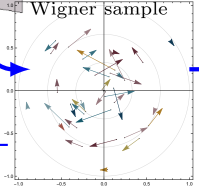
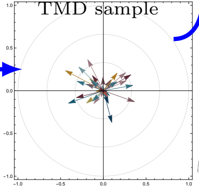
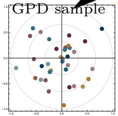


[Bury,Prokudin,AV, PRL 126 (2021)]

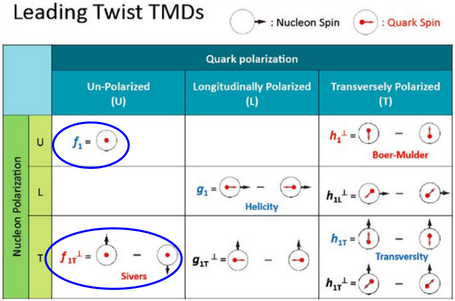








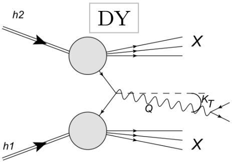
There are many TMD distributions
They describe different correlations between
quark's and hadron's spin and orbital momenta



$$F(x, k_T) = f_1(x, k_T^2) - i\frac{(\mathbf{s} \times \mathbf{k}_T)}{M} f_{1T}^{\perp}(x, k_T^2)$$

- Some TMDs (e.g. Sivers function) do not have analogy in naive/collinear physics

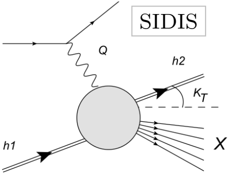
How to extract TMD distributions from the data?



PDF × PDF

LHC
Tevatron
Fermilab
RHIC

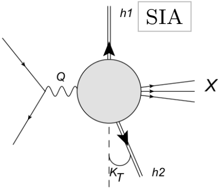
Best studied



PDF × FF

COMPASS
HERMES
JLab

Some studies



FF × FF

Belle
BES

Just started

At $Q^2 \gg q_T^2$ structure functions can be expressed via TMD parton distributions
$\Rightarrow$ **TMD factorization theorem**

$$F_{UU} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bp_T)} C\left(\frac{Q}{\mu}\right) F(x_1, b; \mu, \zeta) F(x_2, b; \mu, \bar{\zeta})$$

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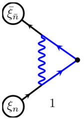
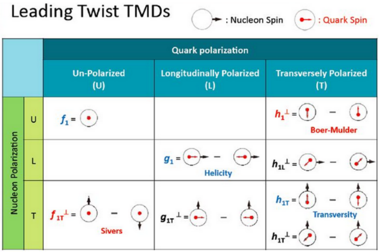
Perturbative
N^{4}LO

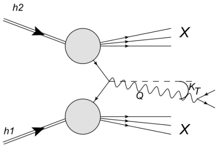
$$1 + a_s C_F \left(-\mathbf{L}_Q^2 + 3\mathbf{L}_Q - 8 + \frac{\pi^2}{6}\right) + O(a_s)$$



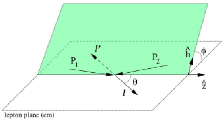
Nonperturbative & Universal
TMD distributions
(μ, ζ) -dependence is given by evolution



Note: TMDs are defined in b-space (not k_T-space)



Large amount of structure functions!



unpolarized

Sivers

$$
\begin{aligned}
\frac{d\sigma}{d^4q\,d\Omega} = \frac{\alpha_{em}^2}{F\,q^2}\times
&\Big\{\Big((1+\cos^2\theta)\,F_{UU}^1 + (1-\cos^2\theta)\,F_{UU}^2 + \sin 2\theta\cos\phi\,F_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UU}^{\cos 2\phi}\Big)\\
&+ S_{aL}\Big(\sin 2\theta\sin\phi\,F_{LU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LU}^{\sin 2\phi}\Big)\\
&+ S_{bL}\Big(\sin 2\theta\sin\phi\,F_{UL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UL}^{\sin 2\phi}\Big)\\
&+ |\vec{S}_{aT}|\Big[\sin\phi_a\Big((1+\cos^2\theta)\,F_{TU}^1 + (1-\cos^2\theta)\,F_{TU}^2 + \sin 2\theta\cos\phi\,F_{TU}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TU}^{\cos 2\phi}\Big)\\
&\qquad + \cos\phi_a\Big(\sin 2\theta\sin\phi\,F_{TU}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TU}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{bT}|\Big[\sin\phi_b\Big((1+\cos^2\theta)\,F_{UT}^1 + (1-\cos^2\theta)\,F_{UT}^2 + \sin 2\theta\cos\phi\,F_{UT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{UT}^{\cos 2\phi}\Big)\\
&\qquad + \cos\phi_b\Big(\sin 2\theta\sin\phi\,F_{UT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{UT}^{\sin 2\phi}\Big)\Big]\\
&+ S_{aL}\,S_{bL}\Big((1+\cos^2\theta)\,F_{LL}^1 + (1-\cos^2\theta)\,F_{LL}^2 + \sin 2\theta\cos\phi\,F_{LL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LL}^{\cos 2\phi}\Big)\\
&+ S_{aL}\,|\vec{S}_{bT}|\Big[\cos\phi_b\Big((1+\cos^2\theta)\,F_{LT}^1 + (1-\cos^2\theta)\,F_{LT}^2 + \sin 2\theta\cos\phi\,F_{LT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{LT}^{\cos 2\phi}\Big)\\
&\qquad + \sin\phi_b\Big(\sin 2\theta\sin\phi\,F_{LT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{LT}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{aT}|\,S_{bL}\Big[\cos\phi_a\Big((1+\cos^2\theta)\,F_{TL}^1 + (1-\cos^2\theta)\,F_{TL}^2 + \sin 2\theta\cos\phi\,F_{TL}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TL}^{\cos 2\phi}\Big)\\
&\qquad + \sin\phi_a\Big(\sin 2\theta\sin\phi\,F_{TL}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TL}^{\sin 2\phi}\Big)\Big]\\
&+ |\vec{S}_{aT}|\,|\vec{S}_{bT}|\Big[\cos(\phi_a+\phi_b)\Big((1+\cos^2\theta)\,F_{TT}^1 + (1-\cos^2\theta)\,F_{TT}^2 + \sin 2\theta\cos\phi\,F_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,F_{TT}^{\cos 2\phi}\Big)\\
&\qquad + \cos(\phi_a-\phi_b)\Big((1+\cos^2\theta)\,\bar{F}_{TT}^1 + (1-\cos^2\theta)\,\bar{F}_{TT}^2 + \sin 2\theta\cos\phi\,\bar{F}_{TT}^{\cos\phi} + \sin^2\theta\cos 2\phi\,\bar{F}_{TT}^{\cos 2\phi}\Big)\\
&\qquad + \sin(\phi_a+\phi_b)\Big(\sin 2\theta\sin\phi\,F_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,F_{TT}^{\sin 2\phi}\Big)\\
&\qquad + \sin(\phi_a-\phi_b)\Big(\sin 2\theta\sin\phi\,\bar{F}_{TT}^{\sin\phi} + \sin^2\theta\sin 2\phi\,\bar{F}_{TT}^{\sin 2\phi}\Big)\Big]\Big\}.
\end{aligned} \tag{57}
$$

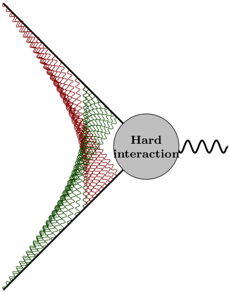
Feature of TMD distributions :
double-scale evolution & **non-peturbative evolution kernel**

$$\mu^2 \frac{d}{d\mu^2} F(x,b;\mu,\zeta) = \gamma_F(\mu,\zeta) F(x,b;\mu,\zeta) \quad \longleftarrow \text{UV}$$

$$\zeta \frac{d}{d\zeta} F(x,b;\mu,\zeta) = -\mathcal{D}(b,\mu) F(x,b;\mu,\zeta) \quad \longleftarrow \text{Rapidity}$$

CS kernel

This structure comes from the involved form of TMD operator (infinite staple gauge link)



Collins-Soper (CS) kernel
describes long-range forces of QCD.
It is one of the most fundamental observable
which provides us information about QCD vacuum.

[AV, PRL,125 (2020) 19]

Each experimental point is a convolution of
two TMDs and the **CS kernel**

$$\frac{d\sigma}{dX} = \sigma_0 \int \frac{d^2b}{(2\pi)^2} e^{i(bp_T)} C\left(\frac{Q}{\mu}\right) R[\mathcal{D}, \mu, Q] F(x_1, b) F(x_2, b)$$

q_T-dependence, common

Q-dependence
CS kernel

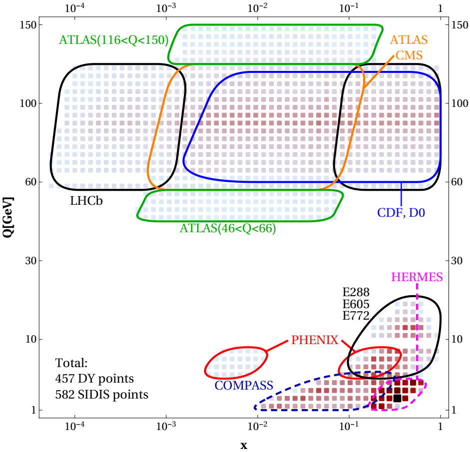
x_1, x_2-dependence
TMDs

To disentangle these functions one needs

- Multi-differential cross-section
- Large coverage

Requires combining together several experiments

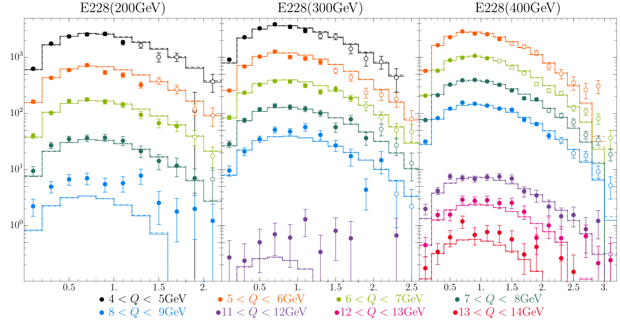
Current state-of-the-art is SV19 [I.Scimemi, AV, JHEP 06 (2020) 137]



SV19 extraction

- Data
  - Large energy span: $2 < Q < 150$GeV
  - SIDIS+DY
  - $q_T < 0.25Q$
- Theory
  - N^{3}LO evolution [AV, PRL 118 (2017)]
  - NNLO collinear matching [Echevarria, Scimemi, AV, JHEP 09 (20
  - ζ-prescription [Scimemi, AV, JHEP 08 (2018)]
- **artemide** github.com/VladimirovAlexey/artemide-public

Example of data description
Mid/Low-energy Drell-Yan



Example of data description
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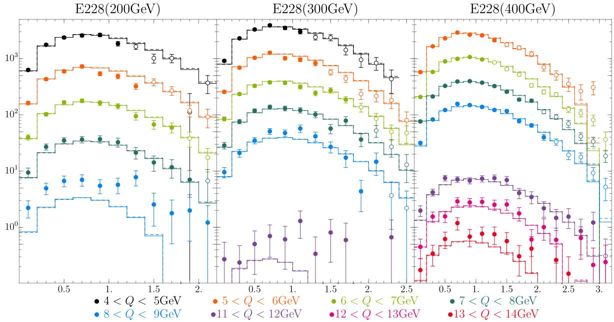
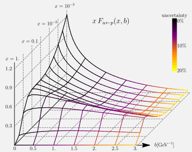
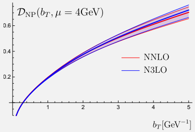




Using DY data we extract CS-kernel and unpolarized TMDPDF

This is only the first step!



Universality & the chain of extractions

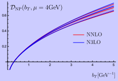
Low-energy
E228, E605,...
PHENIX

→ $R \times F_1 \times F_1$ ←

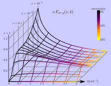
High-energy
ATLAS, LHCb,...
CDF, D0

Unpolarized DY

CS kernel



uTMDPDF



[V.Bertone,I.Scimemi,AV;1902.08474]

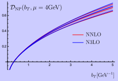
Unpolarized DY

Low-energy
E228, E605,...
PHENIX

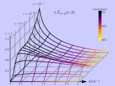
→ $R \times F_1 \times F_1$ ←

High-energy
ATLAS, LHCb,...
CDF, D0

CS kernel



uTMDPDF



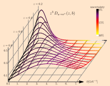
[V.Bertone,I.Scimemi,AV;1902.08474]

Unpolarized SIDIS

HERMES
COMPASS

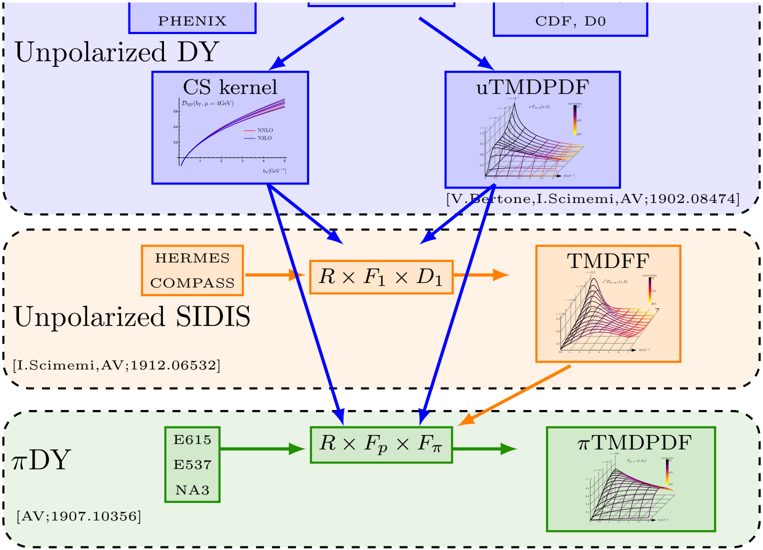
→ $R \times F_1 \times D_1$ →

TMDFF

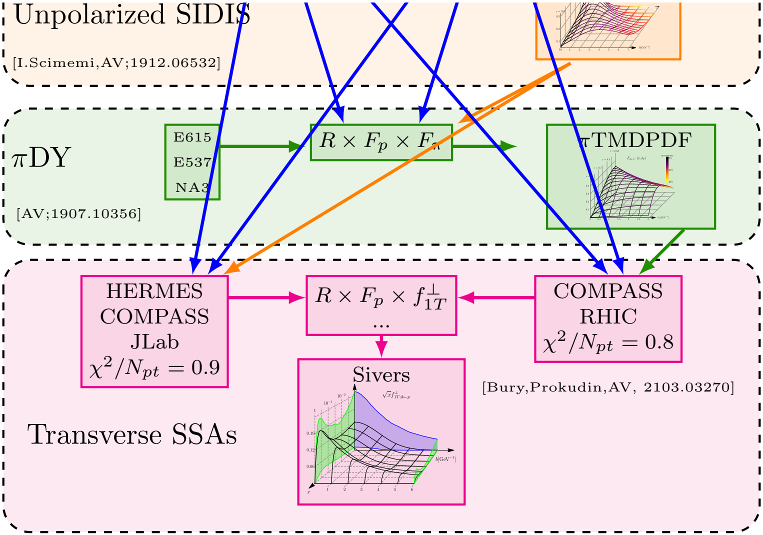


[I.Scimemi,AV;1912.06532]

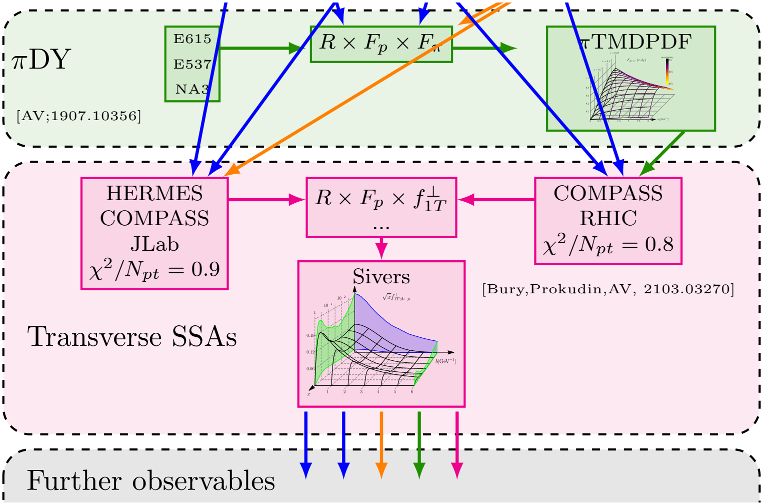
Universality & the chain of extractions



Universality & the chain of extractions



Universality & the chain of extractions

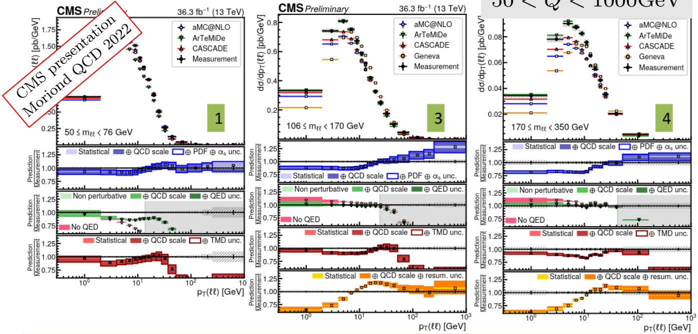


Why is it useful/interesting?

[in two words]

artemide is used by many experiments for generation of predictions

CMS measurements
$50 < Q < 1000$GeV



→ amc@NLO+ Pythia8 gives overall good description

→ Failing to describe the low p_T, failure increasing for higher m(ll)

→ ArTeMiDe gives perfect description in its validity region **!!!**

→ The QED FSR, added to the prediction from amc@NLO+ Pythia8 sample, shows the effect of migrations to low mass from the Z peak

unpol. DY:
CMS
LHCb
RHIC

πDY:
COMPASS

DY-SSA:
RHIC

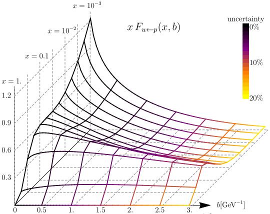
unpol.SIDIS:
COMPASS
JLab

SIDIS-SSA:
JLab

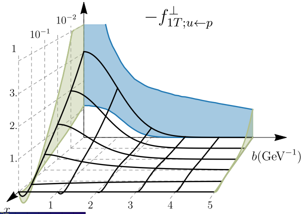
Tomographic picture of nucleon
Momentum density of an **unpolarized** quark in a **polarized** hadron

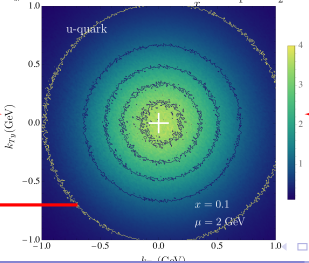
$$\rho_{1;q\leftarrow h^{\uparrow}}(x,\boldsymbol{k}_T,\boldsymbol{S}_T,\mu)=f_{1;q\leftarrow h}(x,k_T;\mu,\mu^2)-\frac{k_{Tx}}{M}f_{1T;q\leftarrow h}^{\perp}(x,k_T;\mu,\mu^2)$$

unpolarized TMDPDF



Sivers TMDPDF



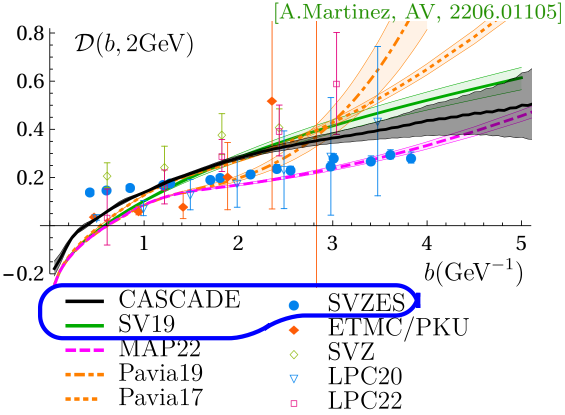


[PDG,2022]

Collins-Soper kernel

Exploring QCD vacuum structure with collider data

[AV, PRL,125 (2020) 19]



Lattice calculations

From the ratios of "quasi-TMD" distributions

[AV, Schäfer, 2002.07527]

Directly

From the "proper" ratios of cross-sections

[A.Martinez, AV, 2206.01105]

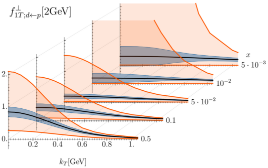
Each approach has its own systematic uncertanties

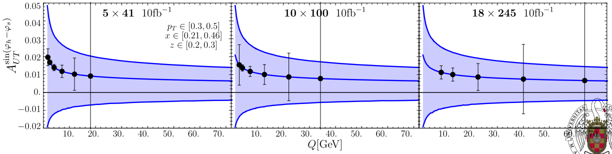
These are only a few examples

- Collinear distributions
- Spin and orbital momentum structure
- Sum-rules [spin, mass, tensor-charge]
- Beyond SM studies [W-mass]
- ...

At present **nucleon tomography** has detailed theory which is able to describe the modern generation of experiments.

But in the (nearest) future, the next generation of experiments will require a better/more accurate theory/techniques.





Backup slides

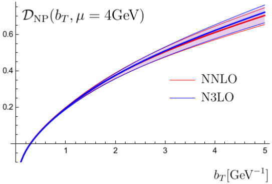
Collins-Soper kernel

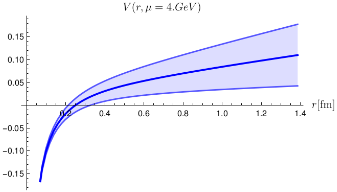
- CS-kernel was introduced long ago [Collins, Soper, Sterman,1982-1985]
- The fundamental importance for QCD has been recognized recently
  - Decorrelation from the TMD distributions [I.Scimemi, AV, JHEP 08 (2018)]
  - Duality to jet-production [AV, Phys.Rev.Lett. 118 (2017)]

$$\mathcal{D}(b,\epsilon^*) = 2\gamma_{\text{jet}}(\theta(b))$$

  - Definition and interpretation [AV, Phys.Rev.Lett. 125 (2020)]

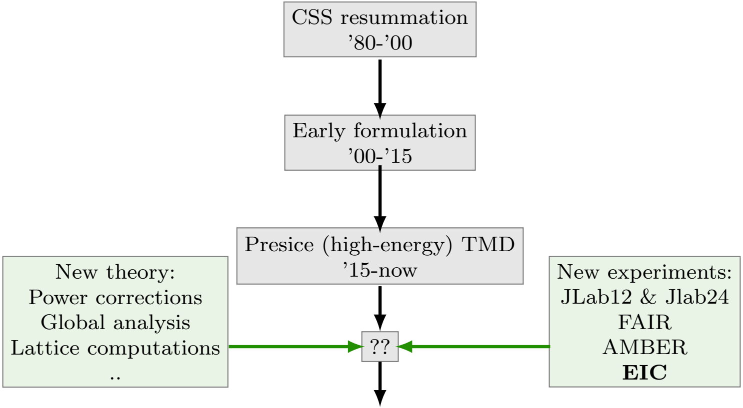
$$\mathcal{D}(b) \simeq \int_0^1 d\beta \langle 0|b^\nu F_{\nu+}(\beta b)[\text{staple contour}]|0\rangle$$



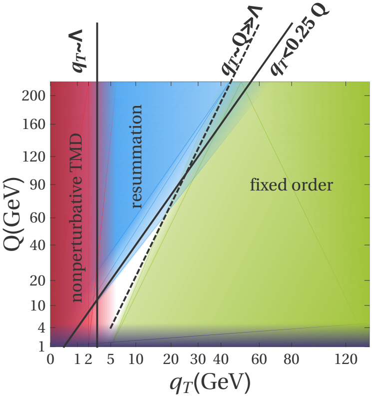


$$V(r) = r\frac{\pi}{4}\mathcal{D}''(0) + \frac{\mathcal{D}'(0)}{2} + \frac{r^2}{2}\int_r^\infty \frac{dx}{x^2}\frac{\mathcal{D}'(x)}{\sqrt{x^2-r^2}} + ...$$

What is next?


CSS resummation
'80-'00
Early formulation
'00-'15
Presice (high-energy) TMD
'15-now
New theory:
Power corrections
Global analysis
Lattice computations
..
??
New experiments:
JLab12 & Jlab24
FAIR
AMBER
EIC

TMD factorization is a very developed science
In fact, it is as good (and even better) than the collinear factorization
But **there are important limitations**



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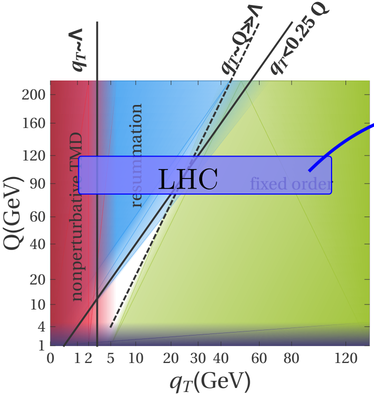
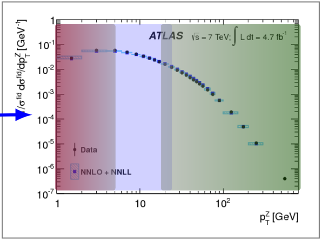
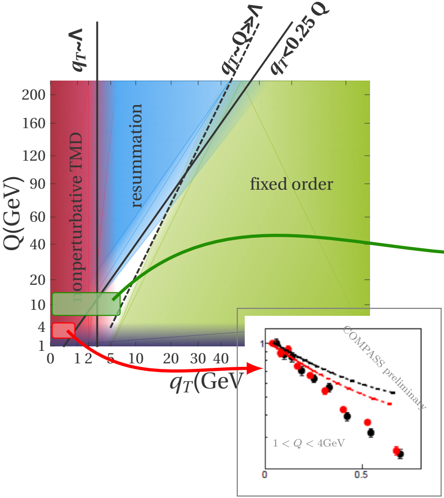




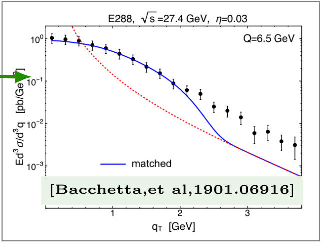
At high-energy (such as LHC)
different factorization regions overlap
therefore, the interpolation makes a good job.

TMD factorization is a very developed science
In fact, it is as good (and even better) than the collinear factorization
But **there are important limitations**



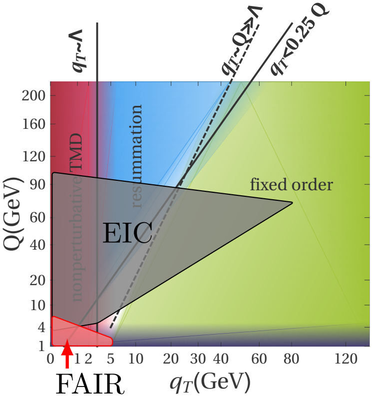
LP TMD factorization has limited region of application.

For SIDIS it cuts **the most part** of the data



[Bacchetta,et al,1901.06916]

TMD factorization is a very developed science
In fact, it is as good (and even better) than the collinear factorization
But **there are important limitations**



Phase space of all future experiments are centered directly in the transition region

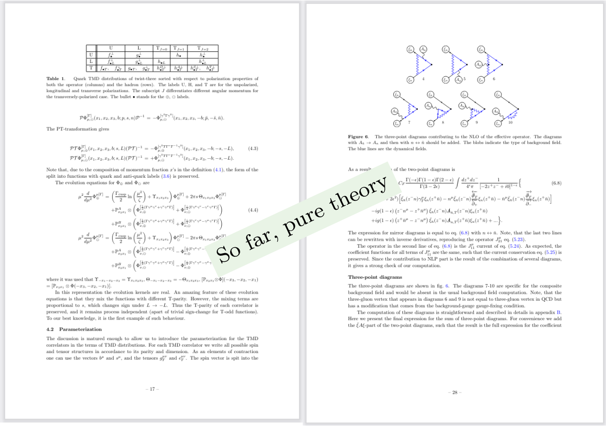
COMPASS, JLab have large contribution of power corrections

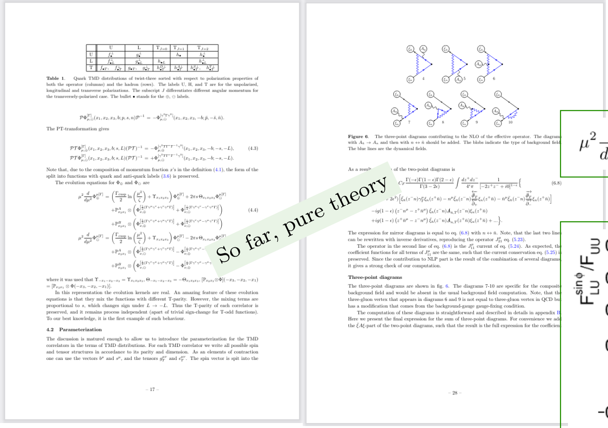
Important!
Similar problem is foreseen for GPD physics

The key to the problem is power corrections!
However, this is a very difficult problem of QCD (in general), and especially for multi-dimensional measurements.

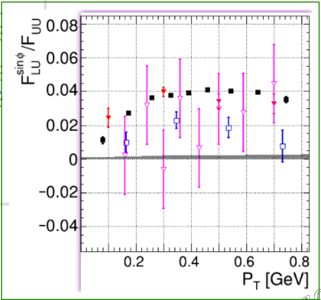
For the last ~ 2 years I concentrate on power corrections

- Power corrections to lattice observables [V.Braun,AV,18-21]
- Target mass corrections to TMDs (all powers!) [V.Moos,AV,JHEP 12 (2020)]
- **TMD operator expansion** [AV,V.Moos,I.Scimemi,JHEP 01 (2022)]
  - Novel method for TMD factorization
  - Next-to-leading power expression at NLO
  - Operator-level formalism
- TMDs of twist-3 (the first systematic study) [S.Rodini,AV, 2204.03856]
- Factorization for lattice-TMD operator at NLP [S.Rodini,AV, in prep.]


So far, pure theory



$$\mu^2 \frac{d}{d\mu^2} e = \left(\frac{\Gamma_{\text{cusp}}}{2} \ln \left(\frac{\mu^2}{\zeta} \right) + a_s C_F \right) e$$

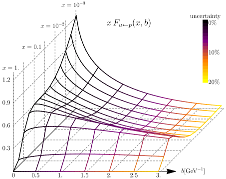


3D ⟶ 1D

$$\lim_{b\to 0} F(x,b) \simeq C(x,\ln(b^2)) \otimes f(x) + \textbf{power corrections}$$

[Echevarria,Scimemi,AV, 1604.07869]
[Gutierrez-Reyes, et al, AV, 1907.03780]
[Scimemi, Tarasov, AV, 1901.04519]
[Moos, AV, 2008.01744]
...

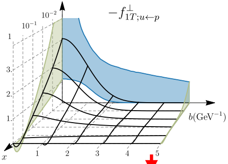
unpolarized TMDPDF



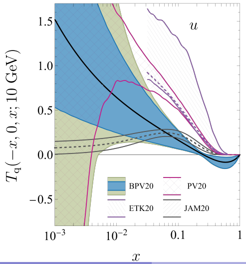
PDF are used as input
PDF - bias
[AV, et al, 2201.07114]

Joined analysis of
TMD + collinear data

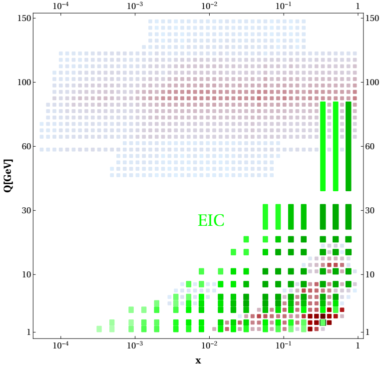
Sivers TMDPDF



Determining twist-3 PDF
$T_q \sim \langle h|\bar{q}\ F\ q|h\rangle$



Next generation of experiments
EIC, FAIR, AMBER, JLab24, EICC
Each of them has a research program dedicated to nucleon tomography



Complimentary

Altogether EIC, FAIR, AMBER, JLab24 will be able to measure relevant cross-section with **unprecedented precision** in **a wide kinematic range**

- from low energy [FAIR]
- till high energy [EIC]

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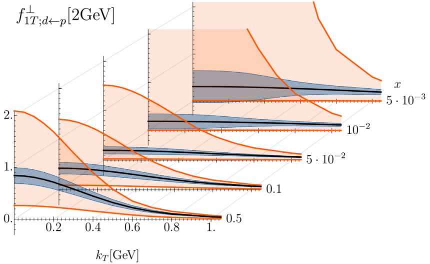

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