Control of electron and electron–hole pair dynamics on nonlinear lattice bilayers by strong solitons

Cite as: Chaos **31**, 083123 (2021); doi: 10.1063/5.0057084
Submitted: 17 May 2021 · Accepted: 15 July 2021 ·
Published Online: 23 August 2021

A. P. Chetverikov,[1,a)] W. Ebeling,[2] E. Schöll,[3,4,5] and M. G. Velarde[6,7]

AFFILIATIONS

[1]Institute of Physics, Saratov State University, Saratov, Saratov 410012, Russia
[2]Physics Department, Humboldt University, 12489 Berlin, Germany
[3]Institut für Theoretische Physik, Technische Universität Berlin, 10623 Berlin, Germany
[4]Bernstein Center for Computational Neuroscience Berlin, Humboldt-Universität, 10115 Berlin, Germany
[5]Potsdam Institute for Climate Impact Research, Telegrafenberg A 31, 14473 Potsdam, Germany
[6]Instituto Pluridisciplinar, Universidad Complutense, Paseo Juan XXIII, 1, Madrid 28040, Spain
[7]School of Architecture, Engineering and Design, Universidad Europea, Villaviciosa de Odón 28670, Spain

Note: This paper is part of the Focus Issue, In Memory of Vadim S. Anishchenko: Statistical Physics and Nonlinear Dynamics of Complex Systems.

[a)]**Author to whom correspondence should be addressed:** chetverikovap@info.sgu.ru

ABSTRACT

We consider the dynamics of electrons and holes moving in two-dimensional lattice layers and bilayers. As an example, we study triangular lattices with units interacting via anharmonic Morse potentials and investigate the dynamics of excess electrons and electron–hole pairs according to the Schrödinger equation in the tight binding approximation. We show that when single-site lattice solitons or M-solitons are excited in one of the layers, those lattice deformations are capable of trapping excess electrons or electron–hole pairs, thus forming quasiparticle compounds moving approximately with the velocity of the solitons. We study the temporal and spatial nonlinear dynamical evolution of localized excitations on coupled triangular double layers. Furthermore, we find that the motion of electrons or electron–hole pairs on a bilayer is slaved by solitons. By case studies of the dynamics of charges bound to solitons, we demonstrate that the slaving effect may be exploited for controlling the motion of the electrons and holes in lattice layers, including also bosonic electron–hole–soliton compounds in lattice bilayers, which represent a novel form of quasiparticles.

A focus of the school of Vadim Anishchenko in Saratov, in close collaboration with the school of Lutz Schimansky-Geier in Berlin, was the control of the dynamics of nonlinear systems.[1] One of the central aims of the studies in Saratov and in Berlin has been synchronization of two nonlinear dynamical systems, which is the strongest form of control between two systems. Here, we address the more specialized problem of how to synchronize the individual motion of two different particles, namely, microscopic charged (single or compound) particles and a lattice soliton. This is part of the important issue of influencing the motion of physical particles by tools of nonlinear lattice theory. In particular, we consider the problem of how the dynamics of electrons and holes as well as electron–hole pairs can be influenced and controlled by the nonlinear dynamics of an embedding system (a nonlinear lattice). We find that lattice deformations are capable of trapping excess electrons or electron–hole pairs, thus forming novel bosonic quasiparticle compounds.

I. INTRODUCTION: HOW TO CONTROL THE DYNAMICS OF TINY CHARGES

In this paper, we extend previous work on nonlinear excitations on a lattice and, in particular, bound states of electrons, holes, and solitons in bilayers of triangular lattices.[2] The focus of our interest is charged particles, which are able to move in lattices,

i.e., electrons and holes. Because of their intrinsic quantum character, electrons and holes are microscopic objects that are difficult to localize and to control. The reason is that under free conditions, these charges, represented by quantum probability densities, tend to delocalize so that after a few seconds, they are out of control. Controlling microscopic charges is one of the basic problems of quantum mechanics, as well as of many applications in modern quantum technologies. A special and even more difficult task is the control of the dynamics of bosonic electron–hole pairs in bilayers, which might become relevant for future electronics. As a matter of fact, the traditional electronics is mostly built on controlling the motion of charges by wires; however, modern microelectronic devices are essentially based on generating and controlling quite complicated paths of charge carriers between well-defined sources and sinks on micro-scale chips. The earliest microfabrication processes were used for integrated circuit design. Electrons move in microelectronic devices in potential channels etched in semiconductor "masks." The miniaturization and design of such devices presents challenges in many areas of science and engineering: physics, chemistry, materials science, computer science, ultra-precision engineering, fabrication processes, and equipment design. This also gives rise to various kinds of interdisciplinary research. The technological concepts of microfabrication include microlithography, doping, etching, bonding, and polishing. The final aim of all these tools is the control of the path of charges according to a highly sophisticated scheme.

Here, we propose a different method to control the path of electrons or electron–hole pairs on lattices not based on given masks but on the possibilities of nonlinear dynamics. This is a field where the school of Vadim Anishchenko in Saratov in close collaboration with the school of Lutz Schimansky-Geier in Berlin have made fundamental contributions, in particular, with respect to the highly relevant stochastic aspects of these problems.[1]

We will show here how nonlinear lattice dynamics opens up the chance for a completely different way of controlling the path of charges on a microscopic scale. Our central idea is the following: In order to control the path of an electron in the lattice, we should create a kind of channel in the lattice. In microelectronics, the channel is created by etching of the lattice, i.e., a local distortion of the lattice. What we propose here is to create the channel for controlling the charges by a local dynamic lattice compression, i.e., by excited solitons. In order to understand the interaction between a charge and the compression wave, we have to remember that each compression of the lattice leads to a local polarization effect, which interacts with charges in the spirit of the classical Landau–Pekar polaron effect. The interaction between the lattice soliton and the charge is described by the polarization potential of polarizable atoms, which represents a moving potential well in which the electron may be trapped.[3] For sufficiently deep wells, bound states, where the electron wave function is localized ("solectrons"), may be formed.[3,4]

Let us consider a two-dimensional (2D) crystal lattice layer with units interacting via anharmonic Morse potentials. Furthermore, let us consider that we can inject an excess electron whose evolution is governed by the Schrödinger equation [here in the tight binding approximation (TBA)]. In parallel, we consider the dynamics of an electron–hole pair on a lattice bilayer. It is known that when lattice solitons are excited in such layers or bilayers, those lattice deformations can trap excess electrons or electron–hole pairs and carry them along with them with a supersonic initial velocity, thus forming localized quasi-particle compounds. In contrast, as free particles, the electrons evolve according to the Schrödinger equation and tend to quickly disperse over the whole lattice. Hence, with appropriate rescaling, the overall dimensionless Hamiltonian H for the bilayer system incorporating the Morse lattice dynamics, the electron and hole dynamics, and the associated interactions of lattice and charges, and Coulomb attraction between electrons and holes can be set up.[2] The dynamical system we study here is treated as a mixed classical-quantum system since the lattice atoms follow Newtonian mechanics, while the charges obey the Schrödinger quantum evolution. This mixed approach allows for novel applications in materials science at the nanolevel, for instance, graphene technology.

In the previous work, we have studied the coupled dynamics of injected electrons and nonlinear 2D atomic lattice excitations (acoustic solitons), leading to the effect of "electron surfing."[3,4] In particular, we included effects of degeneracy[5] and transport[6] and focused on possible applications of controlling electron transport.[7,8] Our computer simulations have been compared successfully with experimental data on polydiacetylene crystals.[9] The lattice solitons may be excited thermally or by mechanical or electrical shocks, e.g., by pressing the tip of an electron field microscope onto the crystal lattice layer. In our computer simulations, we typically considered a few hundred atoms in a plane forming triangular[3,7] or hexagonal lattices.[8] Furthermore, we have investigated two coupled triangular lattices separated by a small distance, where bound states of electrons, holes, and solitons may arise.[2] Such bilayer lattice configurations with interlayer separations between 2 and 10 nm can be experimentally realized by graphene layers, and even barrier thicknesses as small as $d \simeq 1$ nm have been obtained, for instance, by hexagonal boron nitride dielectrics with a dielectric constant $\varepsilon \simeq 3$.[10]

The purpose of this paper is to extend our previous work by considering solitons that occupy not only single-site rods but are laterally extended over several adjacent lattice sites (M-solitons). With this, it is possible to increase the lifetime of the solitons and hence also of the trapped electrons or electron–hole pairs. In order to provide a sufficiently long trajectory before the lattice excitation is destroyed due to lattice defects or radiation of energy to lateral rods along the trajectory of the soliton, it is advantageous to excite the soliton simultaneously in M adjacent lattice sites. Then, the soliton travels along several adjacent rods for a much longer distance; therefore, also, the transported electron or electron–hole pair lives much longer before delocalizing. In this paper, we consider this effect both for single layers and for bilayers. Two coupled triangular lattices separated by a small distance are shown in Fig. 1. Both crystal lattices are doped, the top layer with electrons (red dot) and the bottom layer with holes (green dot), and the electron and the hole interact via Coulomb forces and can form a bound quantum state (dashed ellipse). Such a configuration may be of interest for technological applications.[11–16] Although it has been known for a long time that the coupling of electrons injected into one layer to holes injected into the other layer might possibly lead to high-temperature electron–hole superfluidity and other interesting effects, only recent work has taken up these ideas,[17,18] which

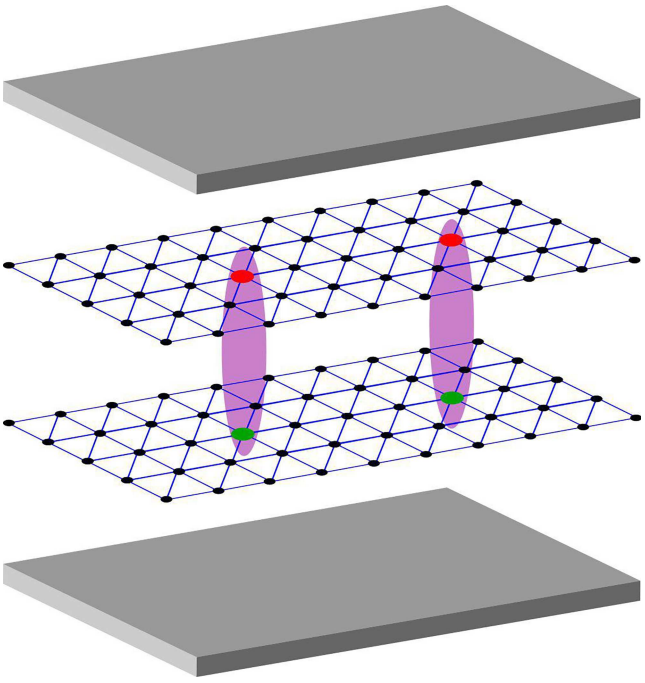

FIG. 1. Scheme of the bilayer architecture. Two coupled triangular crystal lattices with an insulating layer in between, all placed between two metal gates, are shown. An electron is injected in the upper layer (red dot), and a hole is created in the bottom layer (green dot). The color shaded ellipses indicate the electric potential responsible for the bound electron–hole pair.

might also be relevant in coupled quantum dots forming mesoscopic bilayers.[19–22] In particular, double bilayer graphene heterostructures and other two-dimensional atomic crystals that can form electrically insulated, conducting bilayers, one doped with electrons and the other with holes with tunable densities,[23–25] open up exciting new perspectives.

This paper is organized as follows. Section II deals with soliton-assisted transport of electrons and electron–hole pairs. Details of the model for a bilayer architecture are discussed. Section III presents the results of our simulations, followed by a discussion of conditions on how solitons assist the formation and transport of electron–hole-pairs eventually ending in a boson-like quasiparticle (e–h–soliton). Finally, in Sec. IV, we conclude, highlighting the significance of our results for novel applications.

II. SOLITON-ASSISTED TRANSPORT OF ELECTRONS AND ELECTRON–HOLE PAIRS ON TRIANGULAR LATTICE LAYERS AND BILAYERS

A. Bound states of electrons and electron–hole pairs in polarization wells

As shown in Fig. 1, we consider two layers $s = 1$ (upper layer, doped with electrons) and $s = 2$ (lower layer, doped with holes), separated by a dielectric medium, with equal density of electrons and holes.

The nonlinear lattice interaction in each layer is given by the Morse potential

$$V(r^s) = D\left\{\exp[-2b(r^s - \sigma)] - 2\exp[-b(r^s - \sigma)]\right\}, \quad (1)$$

where $r^s = |\mathbf{r}_n^s - \mathbf{r}_j^s|$ is the relative distance of atoms $\mathbf{r}_n^s, \mathbf{r}_j^s$ and with constants $D = 1/2$, $b = 4$, $\sigma = 1$ (nondimensionalized equilibrium lattice constant). We impose a potential cutoff at 1.5σ in order to avoid unphysical cumulative interactions.[3] By a strong kick at one of the atoms at the boundary, we excite traveling compression waves in the form of supersonic solitons moving along a crystallographic axis.[26] Other similar localized traveling excitations are discrete breathers (also known as intrinsic localized modes) and crowdions.[27,28]

The compression connected with a soliton creates a polarization potential[3]

$$U(r) = -U_e \frac{h^4}{(r^2 + h^2)^2} \approx -U_e\left(1 - \frac{2}{h^2}r^2\right), \quad (2)$$

where r is the distance of the electron from the soliton, h is the range of polarization interaction, and the constant $U_e \sim$ polarizability α is in the range $0.05, \ldots, 0.1$ eV. A bound state of the electron may be formed in this approximately parabolic potential well.

Next, we consider the Coulomb interactions between electrons and holes in two coupled lattices; see Fig. 1. When the two layers are sufficiently close, boson-like electron–hole pairs may form.[11] Assuming low electron and hole densities in the two layers, we neglect their Coulomb repulsion, and the Coulomb potential between an electron in layer 1 and a hole in layer 2 is given by[19–21,29]

$$V_{eh}(R, d) = -\frac{e^2}{\epsilon(R^2 + d^2)^{1/2}}, \quad (3)$$

where $R^2 = (x_i^1 - x_j^2)^2 + (y_i^1 - y_j^2)^2$; $x_i^1, y_i^1, x_j^2, y_j^2$ are the electron and hole coordinates in the upper and lower layer, respectively; $\varepsilon \simeq 3, \ldots, 6$ is the effective dielectric constant; and $d \simeq 0.1, \ldots,$ 10 nm is the interlayer spacing. In the following simulations, the electrons and holes are treated quantum mechanically, while the anharmonic lattice dynamics is described classically; the electrons tend to follow the trajectories of soliton-like excitations, forming *solectrons*. The same applies to holes resulting in *hole solectrons*. For small enough horizontal distances R, Eq. (3) can be expanded in parabolic approximation

$$V_{eh}(R, d) \approx \frac{e^2}{\epsilon d}\left(-1 + \frac{R^2}{2d^2}\right), \quad (4)$$

which corresponds to a two-dimensional harmonic oscillator with frequency ω_{eh}, defined by $m_{eh}\omega_{eh}^2 = e^2/(\epsilon d^3)$. The quantum-mechanical bound states of the electron–hole $(e - h)$ pair are given by[30,31]

$$E_{n_x,n_y} = -\frac{e^2}{\epsilon d} + \hbar\omega_{eh}\left[1 + n_x + n_y\right], \quad (5)$$

with oscillator quantum numbers n_x, n_y, and the ground state wave function of the $e - h$ pair is

$$\psi_0(R) \sim \exp\left[-\frac{R^2}{2a_{eh}^2}\right], \quad (6)$$

with the horizontal width

$$a_{eh} = \frac{\hbar}{m_{eh}\omega_{eh}}.$$

A condition for the existence of discrete bound states is that the layer spacing d is sufficiently small, yet not too small, in order to rule out interlayer tunneling. Detailed conditions were derived in Ref. 2. For example, for $\varepsilon \simeq 3$ and $d \simeq 1$ nm, binding energies are in the range of $10^{-2}, \ldots, 10^{-1}$ eV. Note that Perali *et al.*[16] estimate for graphene an effective Bohr radius for the electron–hole pair $a^B_{eh} \simeq 8$ nm with a binding energy of 30 meV.

B. Bosonic electron–hole pairs moving with solitons

The next step is to consider the dynamics of a hole moving under the simultaneous influence of an electron and a soliton localized at the same position and at horizontal distance r from that solectron. The total potential acting on the hole is in parabolic approximation

$$V^*_{eh}(R,d) = -\frac{e^2}{\epsilon\sqrt{R^2+d^2}} - U_e\frac{h^4}{(R^2+h^2)^2} \tag{7}$$

$$\approx -\left(U_e + \frac{e^2}{\epsilon d}\right) + \frac{1}{2}a^*_{eh}R^2 + \cdots, \tag{8}$$

with

$$a^*_{eh} = m^*_{eh}(\omega^*_{eh})^2 = \frac{e^2}{\epsilon d^3} + \frac{4U_e}{h^2}. \tag{9}$$

Here, m^*_{eh} is the effective relative mass of the "dressed" $e-h$ pair in the presence of a soliton and ω^*_{eh} is the oscillator frequency. Due to the decrease of the interatomic distances in the compression wave defining the soliton, the effective mass is decreased by the soliton influence: $m^*_{eh} < m_{eh}$. In harmonic approximation, the ground state levels of the "dressed" electron–hole pair are modified by two effects: (i) The potential well is deepened by the compression of the lattice, and (ii) the effective mass is decreased. In other words, the nonlinear lattice compression favors the formation of electron–hole pairs.

C. Dynamics of electron–hole–soliton quasi-particles

The theoretical basis of the boson-like electron–hole–soliton quasi-particle dynamics has been described in more detail in Refs. 2–4 and 6. The lattice atoms follow Newtonian mechanics, while the electrons and electron–hole pairs obey the Schrödinger quantum evolution.

The quantum-mechanical description of the electron–hole pairs in the presence of lattice deformations is given by a Hamiltonian with several components. The atomic Hamiltonian of each separate triangular lattice (denoted by superscripts $s = 1$ and $s = 2$) is

$$H_a = H^1_a + H^2_a,$$

$$H^s_a = \frac{1}{2m}\sum_n (\mathbf{p}^s_n)^2 + \frac{1}{2}\sum_{n,j} V(\mathbf{r}^s_n, \mathbf{r}^s_j), \tag{10}$$

where $\mathbf{p}^s_n$ is momentum and $V(\mathbf{r}^s_n, \mathbf{r}^s_j)$ is the Morse interaction potential. The subscripts n denote the atoms, all with equal mass m, and the summations over n, j run from 1 to N in the upper and lower lattice (superscript s), respectively. The electron–hole Hamiltonian for the bilayer architecture of coupled lattices consists of three parts for electrons (subscript e), holes (h), and the interaction V_{eh},

$$H_{eh} = H_e + H_h + V_{eh}. \tag{11}$$

The electron and hole Hamiltonians H_e and H_h, respectively, are given in tight binding approximation, thus restricting our consideration to hopping processes along lattice sites.[2] The contribution V_{eh} from Eq. (3) describes the Coulomb interaction between the electron in the upper lattice 1 and the hole in the lower lattice 2. In second quantization, the electron Hamiltonian is

$$H_e = \sum_n E^1_n c^{1+}_n c^1_n + \sum_{n,n'} t_{n,n'}(\mathbf{r}^1_{n'} - \mathbf{r}^1_n) c^{1+}_{n'} c^1_n, \tag{12}$$

where E^1_n is the single-electron energy; c^{1+}_n, c^1_n are electron creation and annihilation operators, respectively; and $t_{n,n'}$ is the hopping transition matrix. The analogous contribution of the holes is

$$H_h = \sum_n E^2_n c^{2+}_n c^2_n + \sum_{n,n'} t_{n,n'}(\mathbf{r}^2_{n'} - \mathbf{r}^2_n) c^{2+}_{n'} c^2_n. \tag{13}$$

Here, we assume only one quantum state of electrons/holes per lattice site and transitions between positions $\mathbf{r}_n$ and $\mathbf{r}_{n'}$.

The coordinates of the lattice point of the atoms at each time and the interaction of lattice deformations with the electrons and holes are calculated by solving Newton's equations of motion for each lattice point under the influence of all forces. The spatial coordinates are rescaled by σ as a length unit. Time is normalized to the inverse frequency of linear oscillations near the minimum of the Morse potential well, ω_0^{-1}, whereas energy is scaled with $2D$. The initial condition is given by the equilibrium state corresponding to the minimum of potential energy for a triangular lattice $80\sigma \times 20\sigma$. For visualization, the lattice atoms are considered little spheres consisting of the atomic "core" represented by a corresponding Gaussian density distribution of width λ centered at each lattice site. Using the trajectories of solitons $Z^s_n(t)$ in layer s and their velocities, we can calculate the lattice atomic core density $\rho(Z^s, t)$. For convenience, we use complex coordinates $Z^s = x^s + iy^s$, where x^s and y^s are Cartesian coordinates. In the 2D triangular Morse lattice, the velocity of sound is slightly above unity in our dimensionless units, corresponding to soliton-like behavior. Indeed, the solitons move for a few picoseconds without change in their shape, and this robustness is the reason why we can visualize them as described. Losses due to scattering and radiation of linear waves are quite low, and the dynamics is almost integrable. Note that the 2D solitons observed here are similar to the lump solutions of the Kadomtsev–Petviashvili equation.[3]

The electron and hole dynamics follow the $2N$ Schrödinger equations for the complex time-dependent coefficients c^s_n of the wave-functions in the upper and lower lattice layer,[2] where $|c^s_n|^2$ is the probability of finding an electron ($s = 1$) or hole ($s = 2$) at lattice site n. We are using the tight binding approximation as in Ref. 2. In our simulations, we assume that the electron dynamics does not influence the lattice dynamics; i.e., we neglect the polaron effect. This means that the mechanical excitations dominate the time and space evolution of the bilayer.

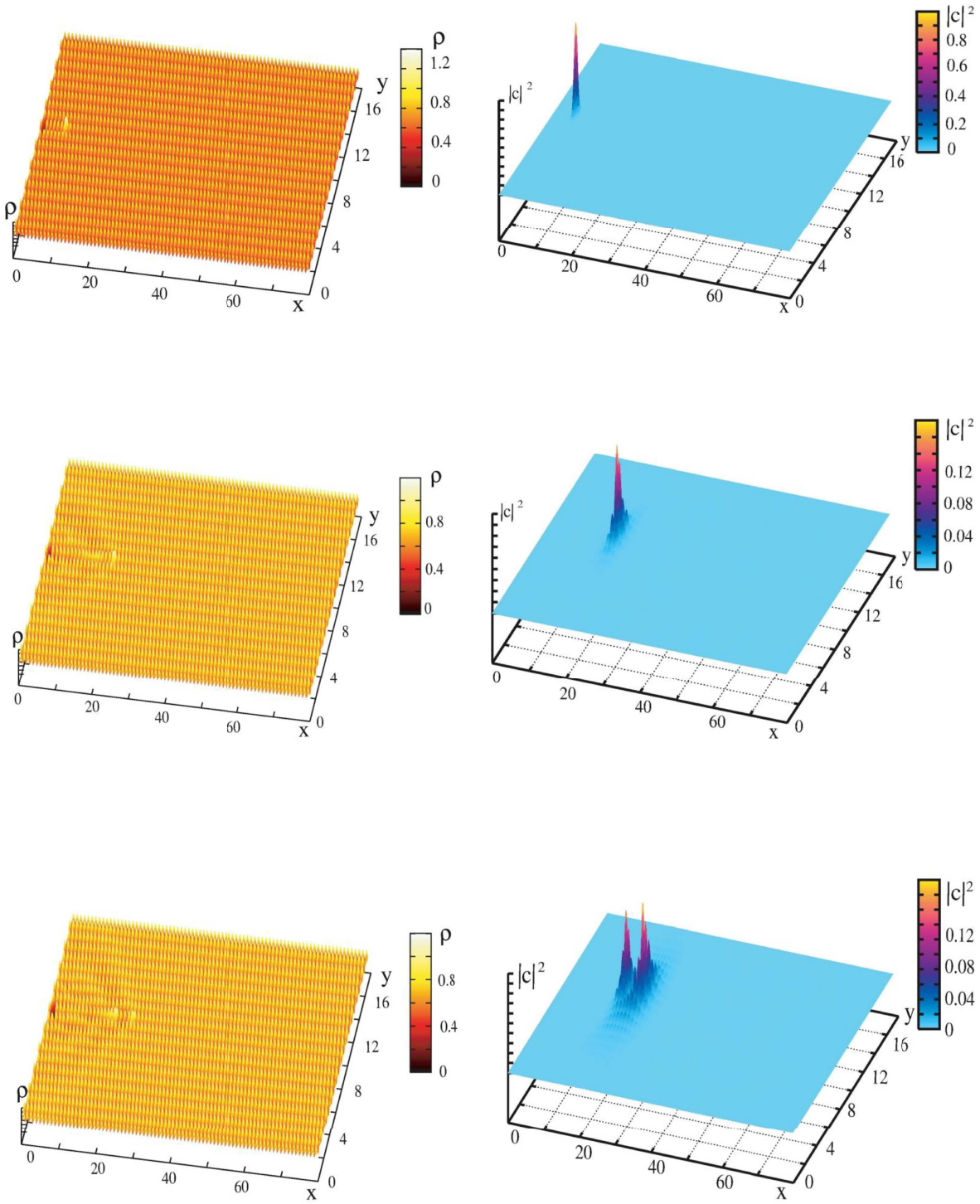

FIG. 2. Transport of an electron by a soliton along one rod of a single-layer triangular Morse lattice with $N = 80 \times 20$, $D = 1/2$, $b = 4$, $\sigma = 1$, $A_{sol} = 2$ at three subsequent times: top panel $t = 0$, middle panel $t = 5$, and bottom panel $t = 7$. Left column: atomic core density ρ of the lattice. Right column: electron probability density distribution $|c|^2$ (right column). Initially, the electron is located at the center of a soliton moving from left to right.

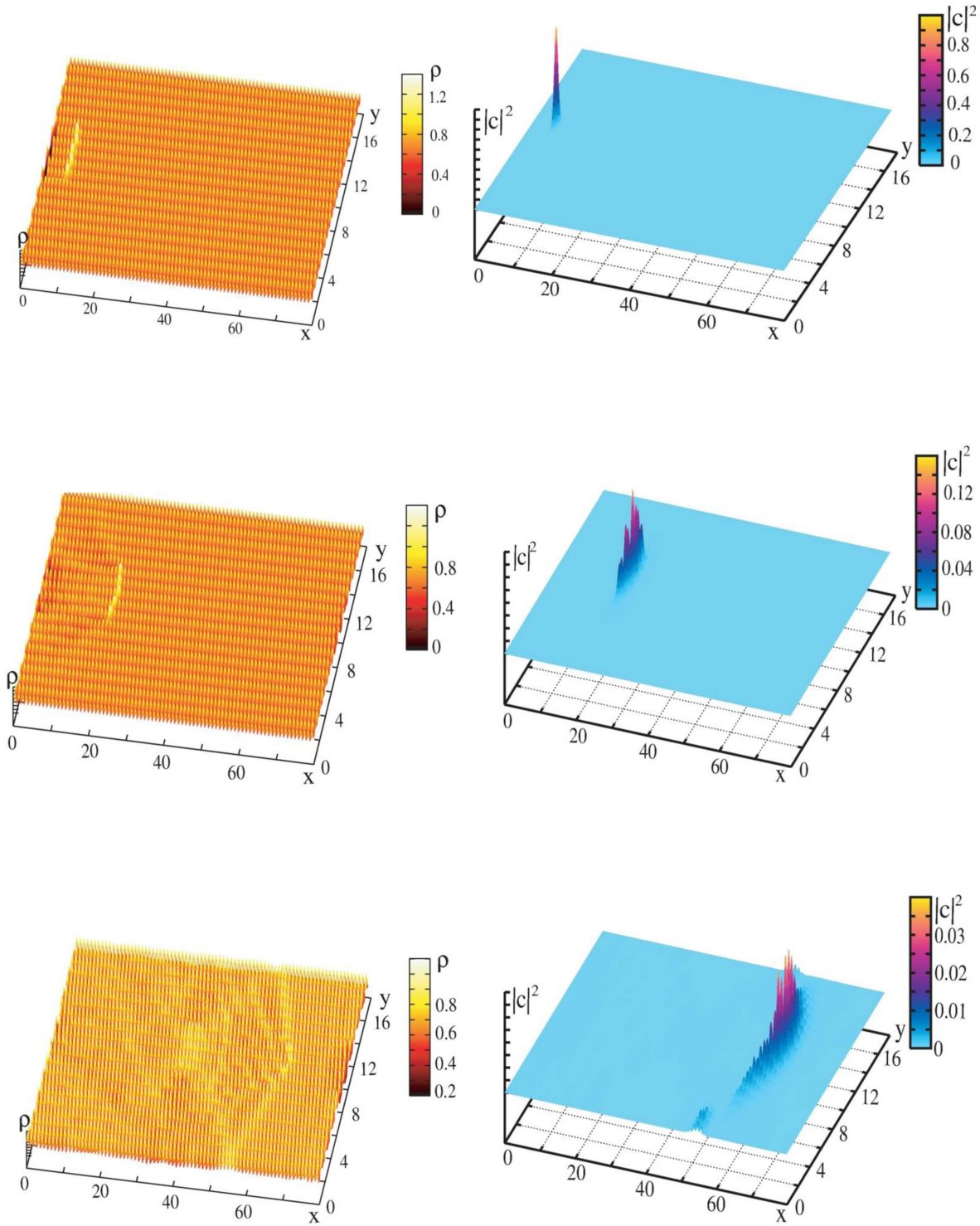

FIG. 3. Transport of an electron by an M-soliton along M = 5 rods of a single-layer triangular Morse lattice with $N = 80 \times 20, D = 1/2, b = 4, \sigma = 1, A_{sol} = 2$ at three subsequent times: top panel $t = 0$, middle panel $t = 5$, and bottom panel $t = 25$. Left column: atomic core density ρ of the lattice. Right column: electron probability density distribution $|c|^2$ (right column). Initially, the electron is located at the center of a soliton moving from left to right.

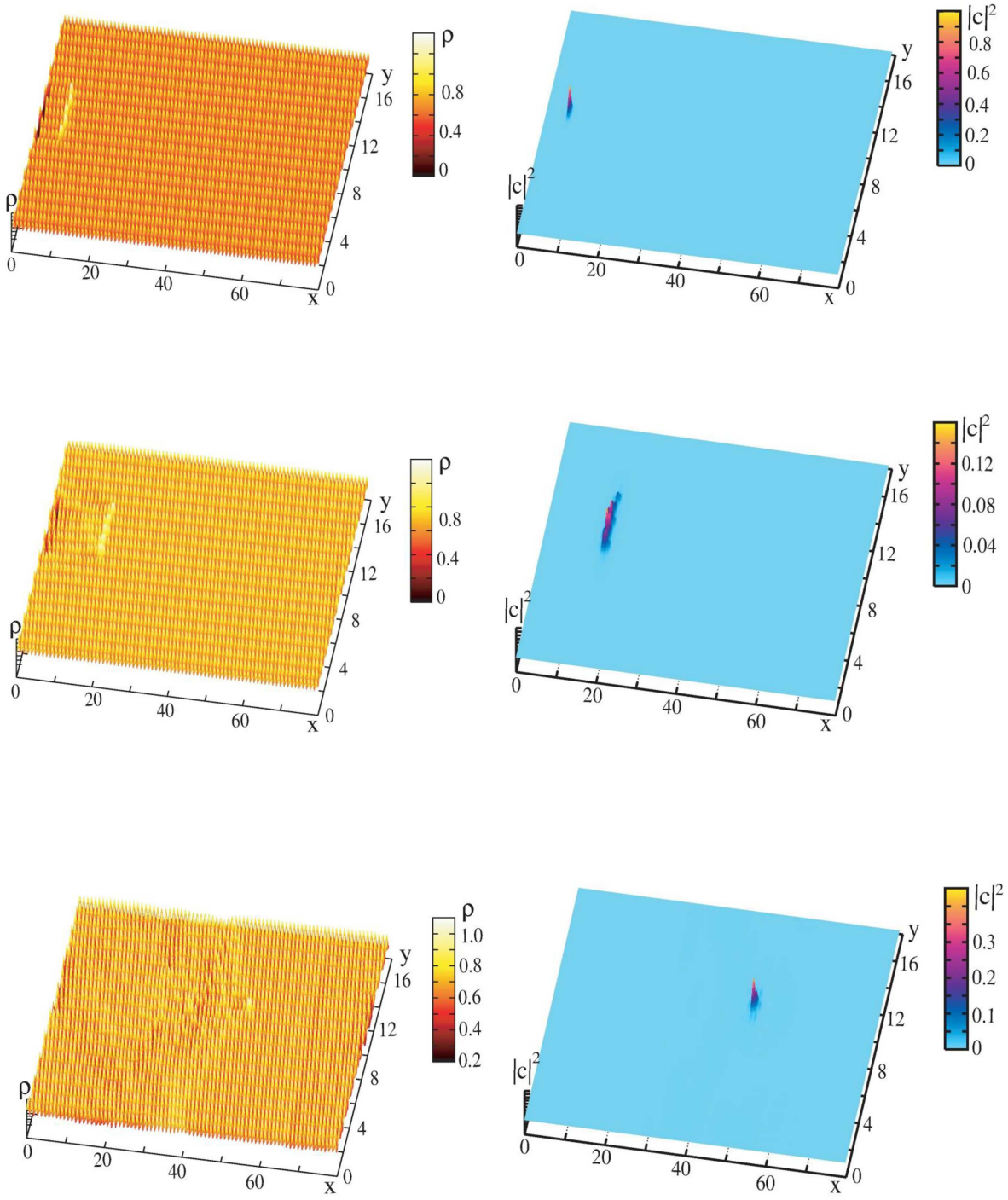

FIG. 4. Transport of an electron–hole pair by an M-soliton excited in one layer of a bilayer Morse lattice ($M = 5$ and $A_{sol} = 2$) at three subsequent times: top panel $t = 0$, middle panel $t = 5$, and bottom panel $t = 20$. Left column: atomic core density ρ of the lattice in the layer where the soliton is excited. Right column: electron probability density distribution $|c|^2$ in the same layer. The hole probability density in the other layer is similar and not shown here. Initially, an electron is localized at the center of an M-soliton moving from left to right in one layer, and a hole is located at the same position in the other layer. Parameters: $N = 80 \times 20, D = 1/2, b_s = 4, \sigma = 1$.

III. RESULTS OF NUMERICAL SIMULATIONS

In this section, we present results of numerical simulations of transport of electrons and electron–hole pairs carried by lattice solitons. First, we show simulations of the transport of an electron by a soliton moving in a rod of atoms along one of the main crystallographic axes of a single-layer triangular Morse lattice with $N = 80 \times 20$ lattice sites. The Morse potential parameters in Eq. (10) are $D = 1/2, b = 4, \sigma = 1$. The soliton is excited initially with an amplitude $A_{sol} = 2$. Initially, the electron is localized at the center of a lattice soliton moving from left to right. Figure 2 presents the distribution of the atomic core density ρ of the lattice (left column) and the electron probability distribution $|c|^2$ (right column) at three subsequent times: $t = 0$ (top), $t = 5$ (middle), and $t = 7$ (bottom). Note that the high-energy soliton loses some energy and splits into two parts as it travels, and as the electron is carried along with the lattice soliton, it also disperses and forms a two-peak probability distribution (bottom panel).

The results of our simulations show that the amplitude of the soliton in one rod must be strong enough to provide a sufficiently long trajectory before the lattice excitation is destroyed due to radiation of energy to lateral rods along the trajectory of the soliton. In this case, the traveling localized excitation has a special form called crowdion as it captures an additional particle because strong initial excitation kicks out an atom from its equilibrium state. As a result, a vacancy forms at the initial position and an interstitial particle gets stuck when the crowdion is destroyed. Such a soliton is able to trap an electron and draw it if the wave function is localized initially close to the soliton (crowdion). The trajectory of the electron is not very long because a crowdion transforms first into a soliton leaving an interstitial particle behind and then it is destroyed losing energy. An electron may be localized in a lattice compression arising due to an interstitial particle for some time, but then the wave function is dispersed over the lattice. Tunneling between potential wells of the soliton and the "interstitial compression" may be observed sometimes before delocalization of the wave function. Unfortunately, defects are formed in a 2D lattice after passing a crowdion. This makes it difficult to re-use soliton/crowdion propagation.

However, it is possible to excite soliton-like deformations in a few (M > 3) adjacent rods, creating an M-soliton[32] and avoiding formation of defects in the lattice. In practice, the M-soliton trajectory grows in length almost proportionally to M.[32] Interstitial particles are not trapped by such solitons, small lattice perturbations caused by a passing soliton vanish since energy dissipation in the lattice is low, and the soliton–electron may propagate in the lattice over long distances. Transport of an electron by an M-soliton, i.e., a soliton that is extended across M adjacent rods of a triangular Morse layer, is shown in Fig. 3 for M = 5. Here, $N = 80 \times 20$, $b = 4$, and $A_{sol} = 2$. Initially, the electron is localized at the center of the M-soliton moving from left to right ($t = 0$). The electron probability density distribution (right column) spreads laterally over several lattice sites as the electron moves along with the soliton, i.e., the wavepacket delocalizes.

We have performed similar simulations also for bilayers. The dynamics of an electron–hole–soliton, which represents a novel kind of quasi-particles, is shown in Fig. 4 for bilayers consisting each of $N = 80 \times 20$ sites. Each layer has the parameters $D = 1/2, b = 4, \sigma = 1$. An M-soliton ($M = 5$) is excited in the first layer with amplitude $A_{sol} = 2$. Initially, an electron is localized at the center of an M-soliton moving from left to right in one layer, and a hole is distributed around the same position in the other layer. Again, the soliton traps an electron forming a traveling quasi-particle in one layer, which is bound to the hole in the other layer. Thus, a neutrally charged quasi-particle emerges. It travels for a long time along the lattice axis, but the distance traveled before the quasi-particle is destroyed is shorter than in the case of a single layer because the soliton has to carry two charged particles in the bilayer case. Note that a localized electron–hole pair can exist in the absence of a soliton only when the Coulomb binding is strong enough.

IV. CONCLUSIONS

We have shown that control of electron transport by soliton-assisted electron surfing is possible in single layers of a crystal lattice as well as in two coupled layers. We have considered injection of an excess electron in one layer and of a hole in the other layer such that electron–hole pairs form, provided that the Coulomb attraction is strong enough and the interlayer spacing is sufficiently narrow, albeit large enough to prevent tunneling. In such a bilayer configuration, if a soliton is excited in one of the layers (here in the upper n-doped layer), then the lattice deformation can trap the electron and carry it together with the hole in the lower (p-doped) layer. Experimentally, the lattice may be excited by short laser pulses at the lattice edge or by a wave generator using an inverse piezoelectric effect. If, e.g., a sinusoidal wave is applied, due to the balance between nonlinearity and dispersion in the dynamics of the lattice, it eventually becomes a sequence of soliton-like pulses. Our bilayer structure may be used for gating the flowing current; i.e., it may be considered mimicking an electronic chip in which the electron can be injected at three different points in one lattice layer (sources in the transistor language),[33] and there are three additional points in the other layer where holes may be created.

Furthermore, we have shown by the example of the interaction of an M-soliton (i.e., a soliton extending over M lattice sites) with an electron–hole pair that quite complicated dynamical quasi-particle compounds may be formed, involving multiple lattice deformations besides the electron and the hole. The complicated quantum-classical dynamics of these compounds as well as possible prospects for novel nano-electronics technologies still remain to be explored.

AUTHORS' CONTRIBUTIONS

A.P.C. did the numerical simulations. All authors designed the study and contributed to the preparation of the manuscript. All authors have read and approved the final manuscript.

DEDICATION

This work is dedicated to the memory of our old friends Vadim Anishchenko and Lutz Schimansky-Geier. Both friends worked together for almost 40 years, jointly developed concepts, wrote books and tutorials, organized conferences, and visited each other innumerable times with their families. We will never forget these

unique personalities, gifted teachers, full of temperament and passion, who formed a generation of talented scholars. Like good friends, they passed away only a few days after each other.

ACKNOWLEDGMENTS

This work was supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation, Project No. 163436311—SFB 910 "Control of self-organizing nonlinear systems: Theoretical methods and concepts of application") and by RFBR and DFG according to the research project (No. 20-52-12004).

DATA AVAILABILITY

The simulation data that support the findings of this study are available within the article.

REFERENCES

[1] V. S. Anishchenko, V. Astakhov, T. Vadivasova, A. Neiman, and L. Schimansky-Geier, *Nonlinear Dynamics of Chaotic and Stochastic Systems: Tutorial and Modern Developments*, Springer Series in Synergetics (Springer Nature, Berlin, 2020).
[2] A. Chetverikov, W. Ebeling, E. Schöll, and M. G. Velarde, Eur. Phys. J. B **92**, 122 (2019).
[3] A. P. Chetverikov, W. Ebeling, and M. G. Velarde, Eur. Phys. J. B **70**, 217 (2009); **80**, 137 (2011).
[4] A. P. Chetverikov, W. Ebeling, G. Röpke, and M. G. Velarde, Contrib. Plasma Phys. **51**, 814 (2011).
[5] A. P. Chetverikov, W. Ebeling, and M. G. Velarde, "Toward a theory of degenerated solectrons in doped lattices," in *Without Bounds a Scientific Canvas of Nonlinearity and Complex Dynamics*, edited by R. G. Rubio *et al.* (Springer, Berlin, 2013).
[6] A. P. Chetverikov, W. Ebeling, G. Röpke, and M. G. Velarde, Eur. Phys. J. B **87**, 153 (2014); **87**, 153 (2014).
[7] A. P. Chetverikov, W. Ebeling, and M. G. Velarde, Eur. Phys. J. B **89**, 196 (2016).
[8] A. P. Chetverikov, W. Ebeling, E. Schöll, and M. G. Velarde, Int. J. Dyn. Control **6**, 1376 (2018).
[9] M. G. Velarde, A. P. Chetverikov, J.-P. Launay, W. Ebeling, and E. G. Wilson, J. Chem. Phys. **153**, 044117 (2020).
[10] R. V. Gorbachev, A. K. Geim, M. I. Katsnelson, K. S. Novoselov, and T. Tudorovskiy, Nat. Phys. **8**, 896 (2012).
[11] Y. E. Lozovik and V. I. Yudson, JETP Lett. **22**, 274 (1975); Sov. Phys. JETP **44**, 389 (1976).
[12] S. Shevchenko, Sov. J. Low Temp. Phys. **2**, 251 (1976).
[13] Y. E. Lozovik and O. L. Berman, Pis'ma Zh. Eksp. Teor. Fiz. **64**, 526 (1996); JETP **84**, 1027–1035 (1997); Zh. Eksp. Teor. Fiz. **111**, 1879 (1997); JETP **84**, 1027 (1997).
[14] P. Eisenstein and A. H. MacDonald, Nature **432**, 691 (2004).
[15] D. K. Efimkin, V. A. Kulbachinskii, and Y. E. Lozovik, JETP Lett. **93**, 219 (2011).
[16] A. Perali, D. Neilson, and A. R. Hamilton, Phys. Rev. Lett. **110**, 146803 (2013).
[17] S. Conti, A. Perali, F. M. Peeters, and D. Neilson, "Multicomponent electron-hole superfluidity and the BCS-BEC crossover in double bilayer graphene," arXiv:1706.07672 (2017).
[18] A. Perali, S. Conti, F. M. Peeters, and N. Neilson, in *Contribution to SCCS-17, 474 Kiev, August 2017* (57, 2017).
[19] A. Filinov, M. Bonitz, and Y. E. Lozovik, Phys. Rev. Lett. **86**, 3851; Phys. Status Solidi B **221**, 231 (2000).
[20] M. Bonitz, V. Golubnychij, A. V. Filinov, and Y. E. Lozovik, Microelectron. Eng. **63**, 141 (2002).
[21] P. Ludwig, A. V. Filinov, M. Bonitz, and Y. E. Lozovik, Contrib. Plasma Phys. **43**, 285 (2003).
[22] J. Schleede, A. Filinov, M. Bonitz, and H. Fehske, Contrib. Plasma Phys. **52**, 10 (2012).
[23] A. K. Geim and I. V. Grigorieva, Nature **499**, 419 (2013).
[24] K. Lee, J. Xue, D. C. Dillen, K. Watanabe, T. Taniguchi, and E. Tutuc, Phys. Rev. Lett. **117**, 046803 (2016).
[25] J. Li, T. Taguchi, K. Watanabe, J. Hone, A. Levchenko, and C. Dean, Phys. Rev. Lett. **117**, 046802 (2016).
[26] M. Toda, *Theory of Nonlinear Lattices*, 2nd ed. (Springer-Verlag, New York, 1989).
[27] M. G. Velarde, A. P. Chetverikov, W. Ebeling, S. V. Dmitriev, and V. D. Lakhno, Eur. Phys. J. B **89**, 233 (2016).
[28] S. V. Dmitriev, E. A. Korznikova, Y. A. Baimova, and M. G. Velarde, Phys. Usp. **59**, 446 (2016).
[29] A. Chetverikov, W. Ebeling, G. Röpke, and M. G. Velarde, Contrib. Plasma Phys. **47**, 465 (2007).
[30] W. Ebeling, V. E. Fortov, and V. Filinov, *Quantum Statistics of Dense Gases and Nonideal Plasmas* (Springer, Berlin, 2017).
[31] A. V. Savin, Z. S. Kivshar, and R. Hu, Phys. Rev. B **82**, 195422 (2010).
[32] A. P. Chetverikov, S. V. Dmitriev, W. Ebeling, E. A. Korznikova, and M. G. Velarde, Mater. Phys. Mech. **35**, 16 (2018).
[33] M. G. Velarde and E. G. Wilson, "Solectron field effect transistor and inverter," UK patent serial number GB 2533105 (20 May 2020).