

OBJETIVE OF THE PROJECT

The objective of this project is to use Matlab to represent the evolution of shock waves based on different parameters, such as the speed of the emitter, the medium of dispersion...

EXPLANATION OF THE PHYSICAL PHENOMENON

When an object (emitter) travels faster than the speed of sound in a medium (i.e. Mach number)

$$M = \frac{v_E}{c} > 1$$

it generates a **shock wave** — a high-amplitude pressure front resulting from the overlapping of individual sound wavefronts. This occurs because the object travels faster than the waves it emits, leading to **constructive interference** along a conical surface known as the **Mach cone**. The shock wave is analogous to a **supersonic Doppler effect**, where all wavefronts add coherently at the envelope. The cone angle and shape depend on the emitter's speed and the medium's sound speed. The following equations describe the wave emission and shock formation and are used for the MATLAB simulation.

Mach angle:

$$\sin \mu = \frac{c}{v_E} = \frac{1}{M}$$

where c is the sound speed and v_E the emitter speed. The sound speed is not always constant, it depends on variables such as density or pressure.

Individual wavefronts (emitted at time τ) satisfy

$$(x - v_E \tau)^2 + y^2 = c^2(t - \tau)^2$$

Envelope of all wavefronts (i.e. the shock front) in polar coordinates (r, θ) at time t :

$$r(\theta) = \frac{ct}{1 - M \cos \theta}$$

These equations will be used in MATLAB to draw each circular wavefront centered at $x - v_E \tau$ with radius $c(t - \tau)$, and to plot their common envelope (the Mach cone).

EXPLANATION OF COMPUTATIONAL PART

To create the plots of the shock waves using Matlab, first we set all the parameters that we were going to need. These were: the speed of sound, we range of values for the emitter speed, the number of wavefronts and the time we were going to let our shock waves travel before plotting them.

Then, we started our video writer, which creates a short animation by passing through all the plots of our selected range.

After that, we start creating the figure itself. First, we create the wavefronts, which are just circumferences equally spaced from the starting point ($x = 0$).

Then, when the emitter speed is bigger than the speed of sound we plot the Mach cone, showing the value of the cone angle as well.

Finally, we label the axis, set its limits and we end the video writer.

```
for idx= 1:length(vE_range)
    vE= vE_range(idx);
    clf; % we delete the figure created before
    hold on
    axis equal
    title(sprintf('Emitter speed v_E = %d m/s (Mach %.2f)', vE, vE/v));

    t_emit= linspace(0,T,N_front);
    cmap= jet(length(t_emit)); % this is a colormap for seeing each
    % circumference in a different colour
    for k= 1:length(t_emit)
        r= v*(T - t_emit(k)); % radius of circumference
        cx= vE * t_emit(k); % center of the circumference
        x= r*cos(theta) + cx;
        y= r*sin(theta);
        plot(x,y,'LineWidth',1.5,'Color',cmap(k,:));
    end
end
```

SHOCK WAVES VISUAL REPRESENTATION

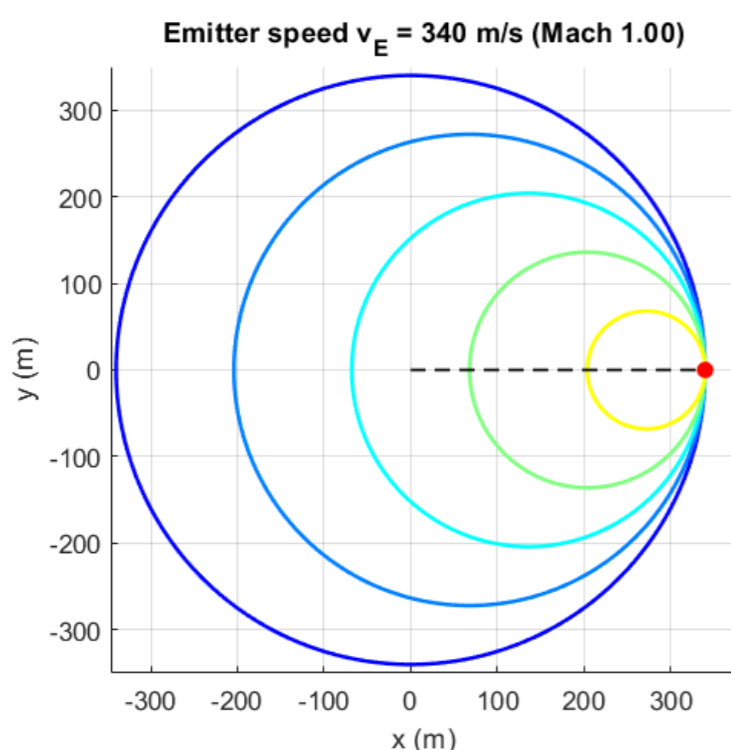


Figure 1. Shock waves when the emitter speed is the same as the speed of sound

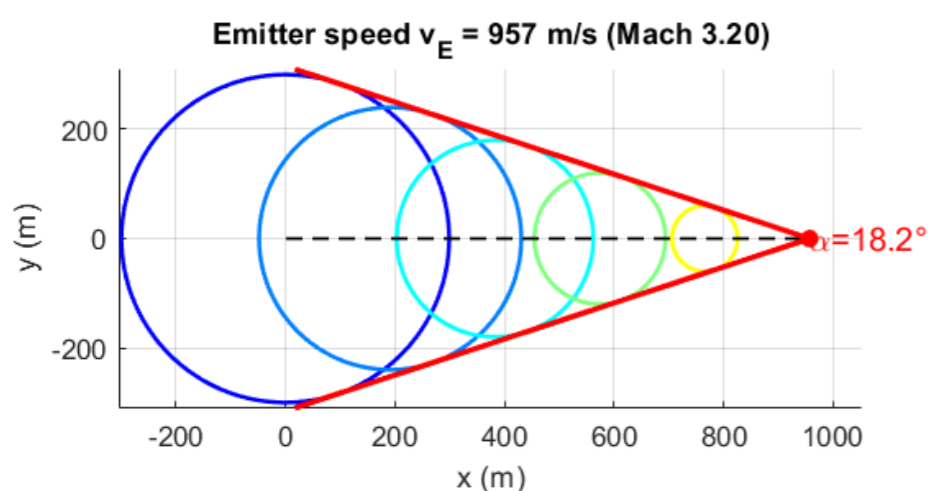


Figure 2. Shock waves created by the Lockheed SR-71 Blackbird, the fastest military aircraft ever created, which travels at a sustained Mach 3.2 speed and at an altitude of around 25 000 meters.

EXPLANATION OF THE FIGURES

In the figures we can see the evolution of shock waves when we modify several parameters, such as the emitter speed or the Mach relation.

- **FIGURE 1:** Shock waves created when the emitter travels at the speed of sound.
- **FIGURE 2:** Shock waves generated by the Lockheed SR-71 Blackbird, the fastest aircraft ever created.
- Beware that, because of altitude, the speed of sound is not 340 m/s, but closer to 300 m/s.
- **FIGURES 3 AND 4:** Shock waves created by the same emitter speed (500 m/s) but changing the medium:
- In figure 3 the medium is air at 1 atm ($v = 340$ m/s)
- In figure 4 the medium is seawater ($v = 1500$ m/s)

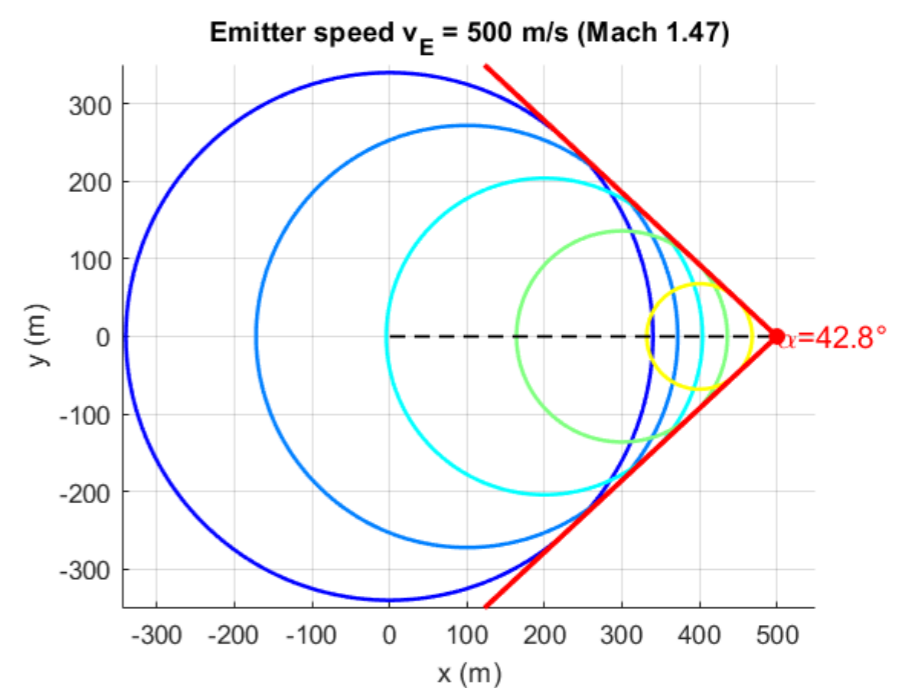


Figure 3. Shock waves and shock wave cone created by an emitter speed of 500 m/s in air

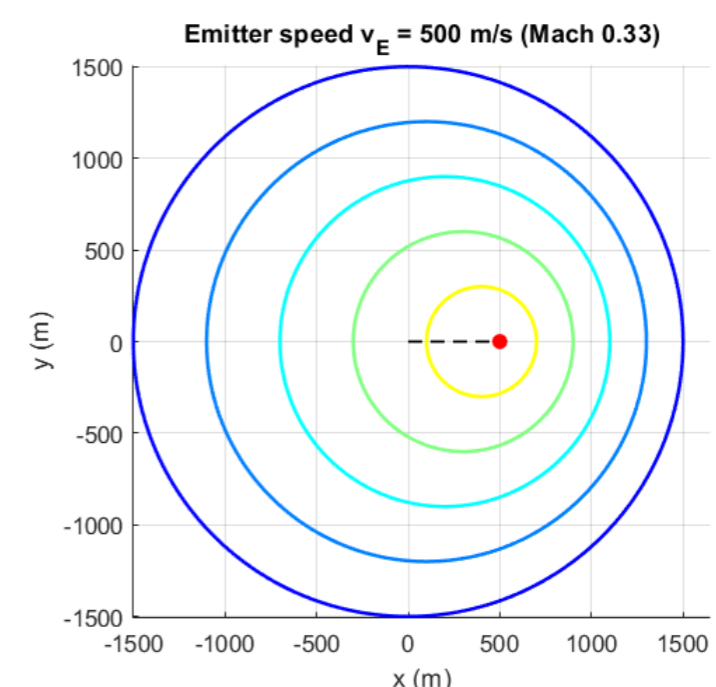


Figure 4. Shock waves created by an emitter speed of 500 m/s in water (speed of sound in water ≈ 1500 m/s)

REFERENCES

- Sears, F. W. & Zemansky, M. W. (1949). University Physics. Pearson Education.
- National Museum of the United States Air Force Webpage.

ACKNOWLEDGEMENTS

This work is part of the INNOVA project "Integration of scientific computing in physics education (INCENFIS) (ref 344).