INTERFERENCE OF HARMONIC WAVES

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OBJETIVE OF THE PROJECT

The aim of this poster is to analyze the interference between two harmonic waves, so constructive and destructive interference are observed.

EXPLANATION OF THE PHYSICAL PHENOMENON

When two waves with the same frequency, amplitude and wave number but phase difference **interfere,** the new wave can be the result of a:

CONSTRUCTIVE INTERFERENCE. It occurs when both waves are **in phase** (ϕ=*0,* meaning peaks and troughs of both waves align). As a result, the new wave has the same frequency and wave number, but its amplitude increases.

DESTRUCTIVE INTERFERENCE. It occurs when both waves are **out of phase** (ϕ=*π,* meaning peaks of one wave align with troughs of the other). This leads to a zero resultant amplitude.

$$\boldsymbol{y_1(x,t) = A sin(kx - \omega t)}$$
$$\boldsymbol{y_2(x,t) = A sin(kx - \omega t + \phi)}$$

A = amplitude [m]
k = wave number [m^{-1}]
ω = angular frequency [rad/s]
ϕ = ***phase difference*** *[rad]*

The **resultant wave** from the interference is:

$$y(x,t) = y_1 + y_2 = Asin(kx - \omega t) + Asin(kx - \omega t + \phi)$$

and using the trigonometric identity:

$$\sin(\alpha) + \sin(\beta) = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

The following expression is obtained:

$$\boldsymbol{y(x,t) = 2Acos(\frac{\phi}{2})sin(kx - \omega t + \frac{\phi}{2})}$$

So, the resulting **amplitude of the interference pattern** depends on $cos(\frac{\phi}{2})$, showing how the phase difference **directly controls the interference intensity** and determines the constructive or destructive nature of the interference.

EXPLANATION OF COMPUTATIONAL PART

First of all, the wave parameters are defined.
The chosen values are: amplitude ***A=1 m*** and wavelength ***λ=2π m***, so that the wave number is ***k=1 m***$^{-1}$, which helps to clearly see when the waves add up or cancel out. The angular frequency is set to ***ω=2π rad/s***, corresponding to a frequency ***v=1 Hz***.
However, this is not critical since the waves are fixed at time ***t=0 s***, (this is a spatial study, not a dynamic one).

The x-variable is defined to display **4 periods** of the wave (***from 0 to 4λ m***), and the waves are represented as smooth curves using **1000 points**.

The expression of a sinusoidal wave is $y = Asin(kx - \omega t + \phi)$. Since wave 1 has no phase shift, it is defined as $y_1 = Asin(kx - \omega t)$.The phase of wave 2 varies, so for its expression a loop is created with a variable *"*ϕ*"* starting at **ϕ *= 0 rad*** (0º) up to **ϕ *= 7π/4 rad*** (315º) in ***steps of π/4 rad*** (45º). We do not go up to ϕ *= 2π rad* (360º) since it is the same situation as for ϕ *= 0 rad* (0º). Inside the loop, we type the wave 2 expression: $y_2 = Asin(kx - \omega t + \phi)$; the interference wave, as the sum of y_1 and y_2 : y_{sum}=y_1+y_2; and the analytical expression to verify the accuracy of the interference wave:

$$y_{analytical} = 2Acos(\frac{\phi}{2})sin(kx - \omega t + \frac{\phi}{2}).$$

Finally, the four waves are plotted together, adding a title, axis labels and a legend.

```
y1 = A*sin(k*x-w*t);                                        % wave 1
for phi=0:pi/4:7*pi/4
    y2=A*sin(k*x-w*t+phi);                                  % wave 2
    y_sum=y1 + y2;                                          % interference
    y_analytical=2*A*cos(phi/2).*sin(k*x-w*t+phi/2);         % analytical expression
    figure;
    plot(x,y1,"c","LineWidth", 1);
    hold on;
    plot(x,y2,"b","LineWidth",1);
    plot(x,y_sum,"m","LineWidth",2);
    plot(x,y_analytical,"--k","LineWidth",1)
    title(['Wave Interference (Phase = ' num2str(phi, '%.2f') ' rad)']);
    xlabel("x (m)");
    ylabel("Amplitude (m)");
    legend("Wave 1", "Wave 2", "Interference", "Analytical Expression");
    xlim([0 26]);
    grid on;
end
```

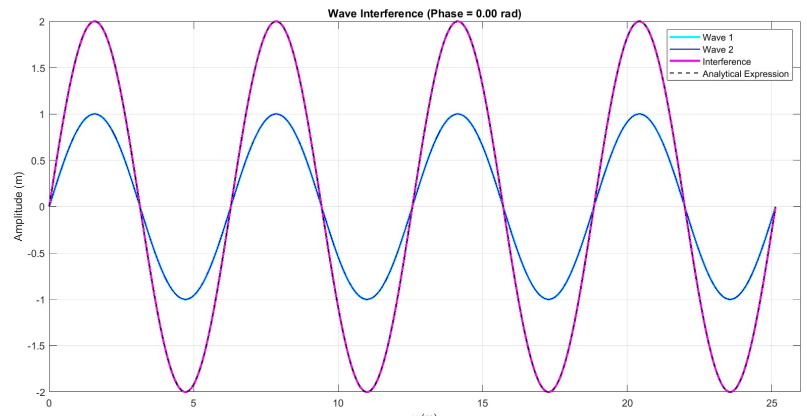

Figure 1. Phase=0.00 rad. Fully constructive interference, both waves are perfectly in phase resulting in maximum amplitude.

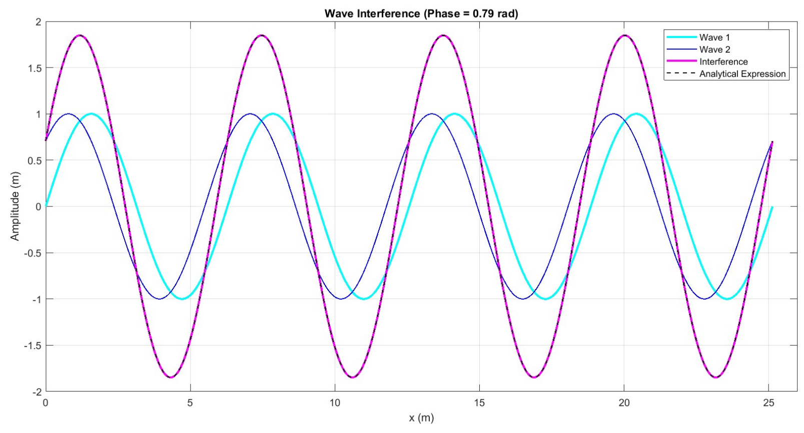

Figure 2. Phase=0.79 rad(≈ π/4). Partially constructive interference. Slight increase in amplitude.

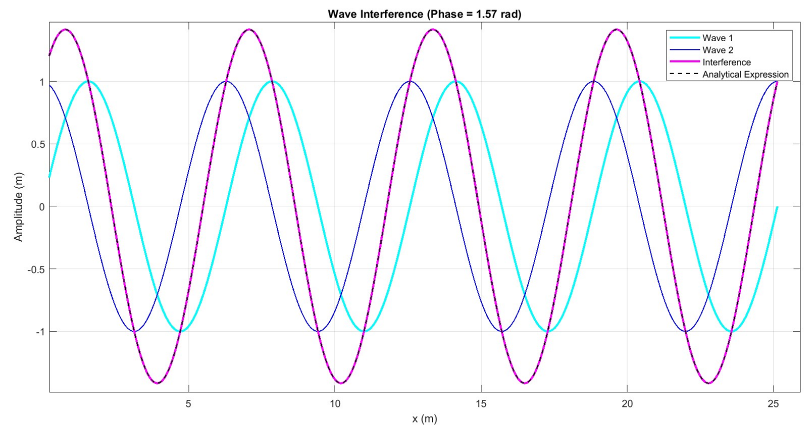

Figure 3. Intermediate interference. The waves are a quarter of a cycle out of phase. Significant reduction in amplitude.

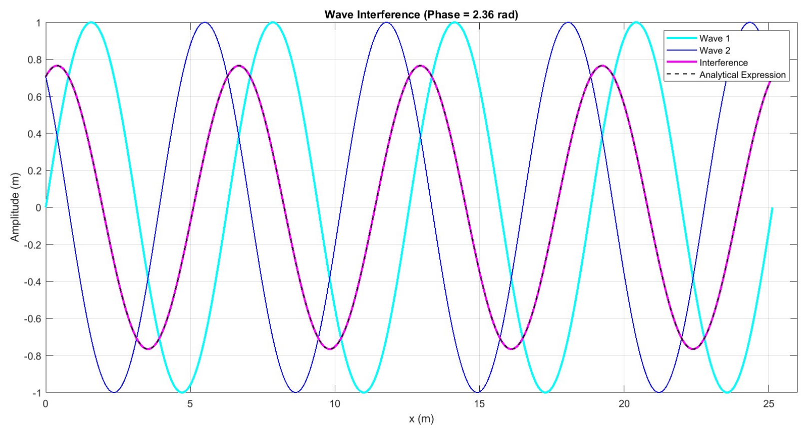

Figure 4. Phase= 2.36 rad. Partial interference, resulting in intermediate amplitude, meaning neither constructive nor destructive.

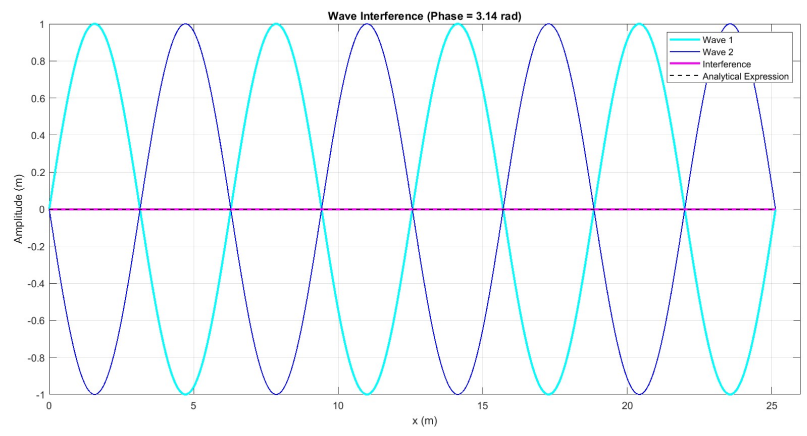

Figure 5.Phase=3.14 rad (π).Completely destructive interference. The waves are half a cycle out of phase, resulting in total cancellation. Amplitude is near zero.

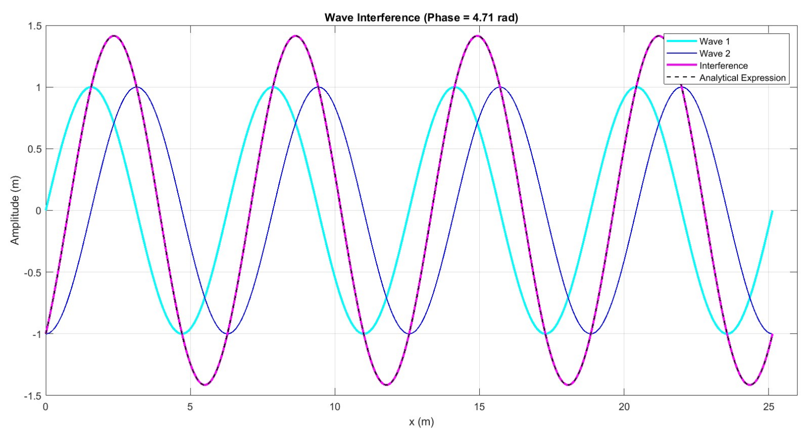

Figure 6. Intermediate interference. The waves are slightly more than half a cycle out of phase. Significant reduction in amplitude.

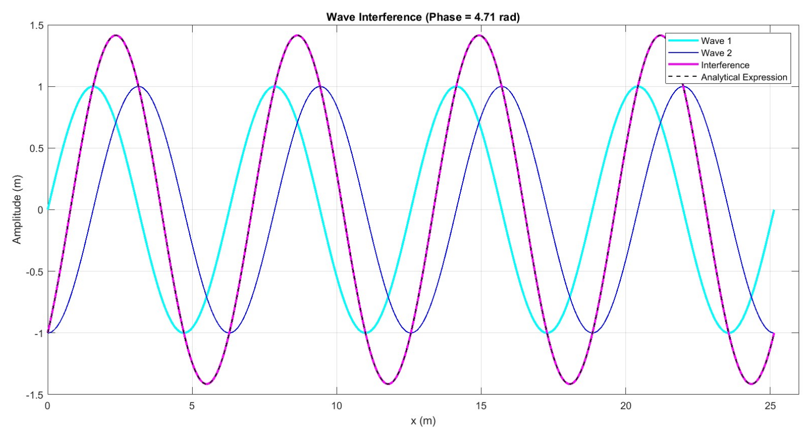

Figure 7.Phase=4.71 rad (3π/2). Partial interference. The waves are three-quarters of a cycle out of phase. Amplitude is reduced but not cancelled.

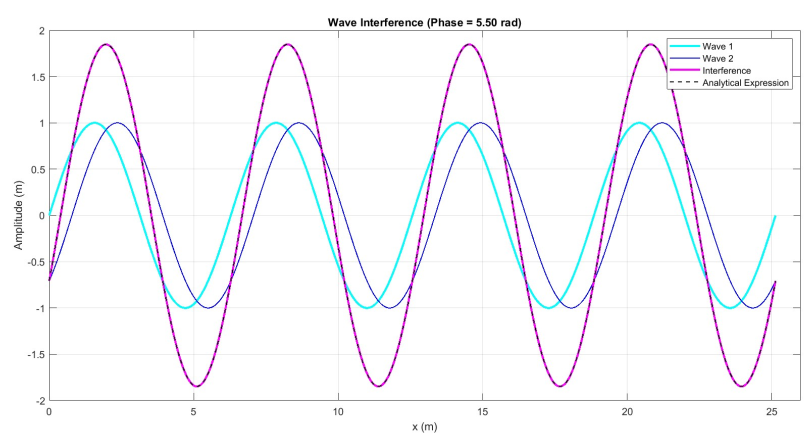

Figure 8. Phase=6.28 rad (2π). Fully constructive interference. The waves are in phase, resulting in maximum amplitude.

RFERENCES

Physics for Scientists and Engineers, Raymond A.Serway and John W. Jewett, 2018.