BOOK OF ABSTRACTS

WORKSHOP
TOPOLOGICAL AND ALGEBRAIC GENERICITY III

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A brief introduction to the study of the unit sphere in polynomial spaces and its applications

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`Abstract.` We shall present a brief (although global) perspective on the geometry of spaces of polynomials. More particularly, we will focus on polynomial spaces of dimension 3, providing, in that case, a graphical representation of the unit ball. Also, the extreme points in the unit ball of several polynomial spaces are characterized. A number of applications to obtain sharp classical polynomial inequalities will be presented as well.

This work is partly based on the recent research monograph [**1**], which is the first ever complete account on the geometry of the unit ball of polynomial spaces. Nowadays there are many research papers on this topic and this work gathers the state of the art of the main and/or relevant results up to now, some of which were originally started by authors such as R. M. Aron, Y. S. Choi, or M. Klimek, among others. We shall also comment on some new results from several ongoing joint works with M. Han, E. D'Aniello, S.K. Kim, M. Maiuriello, and G.A. Muñoz ([**2,3**]).

[**1**] J. Ferrer, D. García, M. Maestre, G. A. Muñoz-Fernández, D. L. Rodríguez-Vidanes, and J. B. Seoane-Sepúlveda, J.B., *Geometry of the Unit Sphere in Polynomial Spaces*, SpringerBriefs in Mathematics, Springer (2023).

[**2**] M. Han, S. K. Kim, G. A. Muñoz-Fernández, and J. B. Seoane-Sepúlveda, *Geometry of quadratic polynomials on hexagonal domains with applications to Bernstein and Markov inequalities*, Work in progress (2024).

[**3**] E. D'Aniello, M. Maiuriello, G. A. Muñoz-Fernández, and J. B. Seoane-Sepúlveda, *Supremum norms for 2-homogeneous polynomials on non symmetrix domains*, Work in progress (2024).

General spaceability criteria for complements of unions, and practical applications

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Abstract. In this talk we provide two general spaceability criteria, and we apply them to several specific topological vector spaces. As a consequence of our main results, we can extend and verify several classical results of this theory, as well as we can ensure the existence, except for zero, of large algebraic structures within, for example, spaces of Lebesgue measurable or continuous functions, of several sequence spaces, of Hölder or Sobolev spaces, spaces of non-absolutely summing operators or even in the space of functions of bounded variation. This is a joint work with Anderson Barbosa, Anselmo Raposo Jr. and Geivison Ribeiro.

M-ideals of compact operators and norm attaining operators

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Abstract. A closed subspace J of a Banach space X is called an M-ideal if $X^* = J^* \oplus_1 J^\perp$. It is clear that if X is the ℓ_∞-sum of J and its complement, then J is an M-ideal. However, the converse does not hold in general. In particular, the space $\mathcal{L}(X,Y)$ of linear operators cannot be represented as such an ℓ_∞-sum with the space $\mathcal{K}(X,Y)$ of compact operators if $\mathcal{K}(X,Y)$ is proper. Nevertheless, $\mathcal{K}(X,Y)$ is an M-ideal for certain pairs of Banach spaces (X,Y). In this talk, I will introduce various known properties that hold in these cases and present new results that connect this study to norm-attaining operators.

Expanding the landscape of twisted Hilbert spaces: singularities, duality, and bi-Lipschitz constructions

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Abstract. We explore the construction and properties of a new class of singular twisted Hilbert spaces defined by bi-Lipschitz maps. We demonstrate that these spaces are not only isomorphic to their duals but distinctively not to their conjugate duals. This structure provides a broader understanding of twisted Hilbert spaces beyond the Kalton-Peck space, particularly by establishing an infinite-dimensional cone of these bi-Lipschitz maps, a step towards addressing a conjecture by Cabello Sánchez and Castillo. Additionally, we discuss the unique stability features of these spaces, including their behavior under normalized basic sequences and their association with the canonical bases of Orlicz sequence spaces.

This is a joint work with Willian Corrêa and Sheldon Dantas [**1**].

[**1**] W. Corrêa, S. Dantas, and D. L. Rodríguez-Vidanes, *Twisted Hilbert spaces defined by bi-Lipschitz maps*, Preprint (2024), arXiv:2408.07827.

Genericity problems in continuous and non-continuous polynomials in Banach spaces

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Abstract. In this talk we will revise some common notions on polynomials in infinite dimensional spaces. Then we will discuss a couple of genericity problems in connection with polynomials. On the one hand we will focus on the algebraic size of the set of non-continuous polynomials in infinite dimensional spaces. On the other hand we will consider a well-know characterization of continuity. The fact that a continuous function on $\mathbb{R}$ maps compact sets to compact sets and connected sets to connected sets is an elementary topic covered in the most basic courses on real variable analysis. However, it is not as widely known–although it is equally accessible to a first-year student–that a function which maps compact sets to compact sets and connected sets to connected sets is necessarily continuous. In this talk, we will discuss the algebraic size of the sets of discontinuous functions that satisfy only one of the two mentioned conditions. We will also show what happens if we consider these two properties in an infinite-dimensional space of polynomials.