Q.E.D.

I ColoQuio Estival de Doctorandos

Facultad de Ciencias Matemáticas UCM

Red de Doctorandos Matemáticas UCM

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Organising committee

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Plenary session

Aula Miguel de Guzmán, 10:00

High-Tech Algebraic Topology

From Topological Invariants to Quantum Field Theories

Ángel González Prieto, UCM

Algebraic topology is the branch of mathematics devoted to addressing topological problems through algebraic methods, typically by developing algebraic invariants that capture and characterize geometric features. It is a highly fruitful and active research area, with deep connections and applications to differential geometry, algebraic geometry, knot theory, and many other fields. However, this viewpoint is, in some sense, static: a rigid invariant is assigned to a space, but such an invariant is often unable to capture the space's deformations or the processes relating different spaces. In recent years, a new perspective has emerged that seeks to incorporate dynamics into this picture by considering not only spaces themselves, but also bordisms between them.

In this idea, motivated by Witten's work on the quantization of the Jones polynomial of knots, Topological Quantum Field Theories (TQFTs) have arisen as powerful categorical tools. They allow ideas and techniques from theoretical physics and quantum field theory to be incorporated into a rigorous mathematical framework, thereby bringing algebraic topology to a higher level of effectiveness. In this talk, we will discuss the mathematical formulation of TQFTs and explain how they formalize the notion of a path integral in quantum field theory. We will analyze several examples, illustrating the power of these methods to compute highly non-trivial algebraic invariants in an efficient and conceptual way. Time permitting, we will also discuss applications of these constructions to algebraic geometry and knot theory.

Plenary session

Aula Miguel de Guzmán, 15:00

A^p_α classes in the Dirichlet range

Miguel Monsalve López, UCM

We study properties of A^p_α spaces in the Dirichlet range, recently defined by Brevig, Kulikov, Seip and Zlotnikov for $0 < \alpha < 1$ and $p > 0$. We answer in the negative two questions posed by Brevig et al. by showing that, if $p \neq 2$ and $p > \frac{1}{2}$, A^p_α is not a vector space and that the norm is in general not increasing in p. This is achieved by means of an equivalent description for A^p_α which is given in terms of the Poisson integral of the boundary function of its inhabitants. Such a norm also leads to a description of A^p_α functions in the Dirichlet range given in terms of their inner and outer factors. As a corollary, we show that A^1_α is contained in the weak product of a Dirichlet-type space.

This is a joint work with A. Dayan (UAB) and A. Llinares (UAM).

Parallel session

Aula Miguel de Guzmán, 11:30

Fixed points up to homotopy

Alejandro Moreno Becerra, UCM

Nielsen theory aims to estimate the minimum number of fixed points among all maps homotopic to a given one. To this end, it partitions fixed points into equivalence classes and asks which classes can be eliminated by a homotopy, assigning to each class an index that detects those which cannot be discarded. In this talk, we will discuss a theorem of Wecken showing that, for every connected compact topological manifold of dimension greater than 2, the number of non-discardable classes is exactly the minimum number of fixed points up to homotopy.

Parallel session

Aula Miguel de Guzmán, 12:00

Shub's Conjecture for Sphere Branched Coverings: A Prime Ends Approach

Malena Domínguez Sirgo, UCM

The starting point of this talk is Shub's conjecture, an open problem in topological dynamics that studies the growth rate of the number of periodic points of a map. Specifically, we will focus on the recent paper [1], which proves this conjecture for sphere branched coverings under certain conditions on a simply connected completely invariant region. This paper makes use of a highly useful tool in the context of topological dynamics: Carathéodory's prime ends.

Driven by this motivation, we will introduce the foundations of prime end theory from a topological perspective. This theory constructs a compactification of a simply connected domain into a closed unit disk, where ideal boundary points represent the directions of approach to the actual boundary. We will see how some maps on the sphere extend to this new boundary, facilitating the analysis of their dynamic behavior.

Finally, we will conclude the talk by showing how prime ends stand as a key tool for obtaining the main theorem in [1]. Using them, we will sketch the idea of an alternative proof to the one presented in the paper, which aims to open the way for potential generalizations of the result.

References

[1] J. Iglesias, A. Portela, A. Rovella, and J. Xavier. *Sphere branched coverings and the growth rate inequality.* Nonlinearity, (2020).

Parallel session

Aula Miguel de Guzmán, 12:30

Abelian vortices in complex geometry

Diego Ruiz Cases, UCM-ICMAT

The abelian vortex equation on the real plane is a partial differential equation for a connection on a Hermitian line bundle (i.e., an electromagnetic field) and a Higgs field whose solutions are the absolute minima of the abelian Yang-Mills-Higgs functional. These solutions, called vortices, appear naturally in the Ginzburg-Landau theory of superconductivity. If we substitute the real plane for a compact Riemann surface, the vortex equation is related to the moduli problem of classifying pairs given by a holomorphic line bundle together with a holomorphic section in complex geometry via a Hitchin-Kobayashi correspondence. The aim of this talk is to illustrate the surprising connection between Yang-Mills theory and some moduli problems in algebraic and complex geometry through the example of the vortex equation.

Parallel session

Aula 115, 16:30

Parabolic Higgs Bundles and the Nilpotent Cone

Lucía Baena Ligero, UCM

Parabolic vector bundles were introduced by Mehta and Seshadri in 1980 to extend the Narasimhan–Seshadri correspondence to punctured Riemann surfaces. Subsequently, these notions were generalized to the setting of parabolic Higgs bundles through the works of Simpson and Yokogawa.

In this talk I will give an overview of some aspects of the geometry of moduli spaces of parabolic Higgs bundles. These spaces admit a Hitchin fibration hα, which endows them with the structure of an integrable system. In particular, I will focus on the fiber over the origin $h_\alpha^{-1}(0)$, known as the parabolic nilpotent cone, whose geometry plays an important role in the global geometry of the moduli space. Finally, I will mention the notions of wobbly and very stable parabolic bundles, whose connection with the nilpotent cone arises naturally from the existence of nilpotent Higgs fields.

Parallel session

Aula 115, 17:00

Higher Open Books and Geometric structures

Wei Zhou, UCM

An open book is a way of describing a manifold as a family of hypersurfaces, all meeting along a common submanifold, and arranged like the pages of a book around its spine. Equivalently, away from this common core, the manifold fibers over a circle. This classical picture has a natural higher analogue: one can replace the circle by a higher-dimensional sphere, leading to the notion of a k-open book.

On the other hand, contact geometry can be seen as the odd-dimensional counterpart of symplectic geometry. A fundamental link between topology and geometry is given by the Thurston–Winkelnkemper construction and Giroux correspondence: open books provide a natural framework for producing and understanding contact structures.

This talk is based on joint work in progress with Eduardo Fernández and Abhijeet Ghanwat, where we ask what kind of geometric structures should be associated with higher open books. The first interesting examples suggest that, beyond contact geometry, one is naturally led to higher-corank analogues such as fat distributions. I will explain the motivating examples and the possibility of a Thurston–Winkelnkemper-type construction in this broader setting.

An introduction to support vector machines for functional data classification

Manuel Gonzalo Carvajal, UCM

The classification problem is one of the most studied tasks in machine learning since it is very simple to formulate while also applicable to a wide variety of situations. There exist several strategies to solve this problem like neural networks, kernel methods, logistic regression, etc. In this talk we will focus on one of them: support vector machines.

Support vector machines, or SVMs for short, are machine learning algorithms based on a geometrical interpretation of the classification problem where the solution is obtained as a hyperplane that separates the different classes of the data in an adequate vector space. This is accomplished by solving a simple quadratic optimization problem that inherits some convenient characteristics, such as sparsity or interpretability, thanks to the geometrical properties of the problem.

SVMs, like most machine learning algorithms, were originally designed to be applied to finite dimensional data ($\in \mathbb{R}^d$). However, in many applications it is common to find data with functional nature. These cases could be treated directly by naively discretizing (for example by evaluating on a finite number of points) and then working on finite dimension, but this approach is not usually optimal. To obtain better results it is necessary to take into consideration properties and concepts exclusive to functional data such as derivatives or regularity.

In this talk we will introduce the basic SVM formulations and show how they can be almost directly adapted to infinite dimensional input data, and how the used of kernel functions adapted to the infinite dimensional case leads to better performance.

References

[1] F. Rossi and N. Villa. *Support Vector Machine for Functional Data Classification.* Neurocomputing, **69** (2006).

[2] P. Trunfio. *Support Vector Machines.* Data Mining and Knowledge Discovery Series, Chapman and Hall/CRC, (2012).

Parallel session

Aula 115, 12:00

How to fish for points using the volume: Connecting convex geometry and number theory

Jorge Santiago Ibáñez Marcos, UCM

Convex geometry is not just a theory of objects in Euclidean space; it is also a tool that connects very different areas of mathematics. In this talk, we will explore one of its bridges to number theory: Minkowski's Theorem. We will use volume as a fishing net to capture integer points, and examine how this idea allows us to tackle arithmetic problems and Diophantine equations, illustrating how questions about integers can be transformed into geometric problems.

Parallel session

Aula 115, 12:30

Fractional Diffusion Equations: Smoothing Effects and Perturbation Techniques

Pablo Molina Benito, UCM - ICMAT

This work studies evolution equations driven by fractional powers A^μ, with $0 < \mu < 1$, of uniformly elliptic operators. A well-known example of these kind of operators is the fractional Laplacian. These fractional operators arise naturally in the study of anomalous diffusion and other nonlocal phenomena, and can be viewed as a generalization of the classical diffusion operator.

Our main goal is to understand the evolution equation

$$u_t + A^\mu u = 0,$$

and the semigroup generated by $-A^\mu$. In particular, we study how solutions of this problem evolve over time and how the diffusion process enhances their regularity.

We also investigate the effect of adding lower-order perturbations to the equation, including operators with low-regularity coefficients. Under suitable assumptions, we show that the main qualitative properties of the fractional diffusion problem are preserved.

This project is being developed in collaboration with Aníbal Rodríguez-Bernal (UCM–ICMAT).

References

[1] H. Amann. *Linear and Quasilinear Parabolic Problems, Vol. I: Abstract Linear Theory.* Springer, (1995).

[2] J. W. Cholewa and A. Rodriguez-Bernal. *Sharp estimates for homogeneous semigroups in homogeneous spaces. Applications to PDEs and fractional diffusion in* $\mathbb{R}^N$. Comm. Contemp. Math., (2022).

[3] J. W. Cholewa and A. Rodriguez-Bernal. *Linear higher order parabolic problems in locally uniform Lebesgue's spaces.* J. Math. Anal. Appl., **449** (2017), 1–45.

[4] A. Rodríguez-Bernal. *Perturbation of analytic semigroups in scales of Banach spaces and applications to linear parabolic equations with low regularity data.* Sema J., **53** (2011), 3–54.

Parallel session

Aula Miguel de Guzmán, 16:30

The Busemann–Petty problem

Isabel María Moreno Cuadrado, UCM-ICMAT

The Busemann–Petty problem is one of the most influential questions in modern convex geometry. Formulated by Herbert Busemann and Clinton M. Petty in 1956, it poses the following question: if two origin-symmetric convex bodies have the property that all central sections of one have a smaller volume than the corresponding sections of the other, does it necessarily follow that its total volume is also smaller?

The answer to this problem turned out to be surprisingly dependent on the dimension of the space: it is affirmative in low dimensions and negative in high dimensions. The complete resolution of the problem required the development of new tools from harmonic analysis. In particular, the works of Alexander Koldobsky showed that the Fourier transform of distributions provides the appropriate framework to understand the phenomenon, revealing a deep connection between the analytic and geometric properties of convex bodies.

This talk will present the formulation of the problem, the main known results, and the fundamental ideas that allow it to be approached using techniques from harmonic analysis and the Fourier transform.

Parallel session

Aula Miguel de Guzmán, 17:00

Free products of Banach lattices

Gonzalo Martínez Fernández, UCM

We introduce free products, that is, coproducts, in the category of Banach lattices and contractive lattice homomorphisms. We give a concrete construction of the free product of an arbitrary family of Banach lattices as a quotient of a free Banach lattice. For compact Hausdorff spaces K_1 and K_2 we identify $C(K_1) * C(K_2)$ lattice isomorphically with $C(K_1 * K_2)$, where $K_1 * K_2$ denotes join of topological spaces. We further discuss free factors of free Banach lattices, and exploit the existence of non-trivial homology spheres to show that a free Banach lattice can have free factors which are not isomorphic to free Banach lattices.