Jumps in Commodity Prices. New Approaches for Pricing Options.

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PhD in Quantitative Finance and Economics "QF", UCM
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Outline

Outline

In Context

- Doctoral Student: Carme Frau
- Title of PhD Thesis (provisional):
  "Quantitative methods on energy commodity derivatives, new term-structure models aimed for option pricing"
- Program: PhD in Quantitative Finance and Economics "QF"
- Universidad Complutense de Madrid
- Supervisor: John Crosby (University of Maryland)
- Tutor @ UCM: Lola Robles

Outline

Motivation

- Findings $\Rightarrow$ lack of model presenting all desired stylised facts per different energy asset:
  - ✓ mean-reversion in spot prices
  - ✓ stochastic spot price and cost of carry curve
  - ✓ Samuelson effect[1]
  - ✓ stochastic volatility
  - ✓ jumps and/or spikes
  - ✓ seasonality
- Objectives:
  #1 $\Rightarrow$ model energy price dynamics and option pricing
  #2 $\Rightarrow$ beat the state-of-the-art model (accuracy of results)
  #3 $\Rightarrow$ analytic solution
- Action $\Rightarrow$ provide better adapted models and test performance

[1] The volatility of futures prices increases as contracts approach expiration.

Outline

1. Introduction
   - In Context
   - Motivation
   - Structure
2. The New Oil Model
   - Theory
   - Data
   - Results
   - Wrap up & Conclusions
   - Next steps in PhD thesis
   - References
3. Q&A

Structure

- PhD thesis in Quantitative Finance presenting new term-structure models on energy commodities aimed for option pricing.
- Underlying assets are key energy assets (spot/futures prices).
- Structure of the PhD thesis:
  - ✓ introductory chapter - expanded state-of-the-art survey
  - ✓ asset #1 - oil (WTI[2] light crude oil)
  - ✓ asset #2 - natural gas (HH[3])
  - ✓ asset #3 - electricity (TBD)
- Format of publishable articles per asset.
- Presentation uniquely focused on the oil chapter (preliminar version).

[2] West Texas Intermediate.
[3] Henry Hub.

Outline

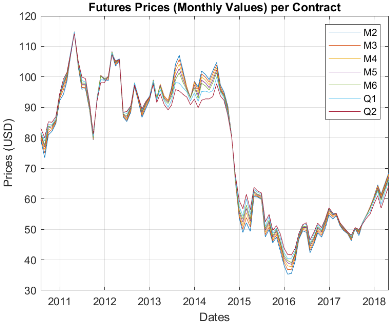
Theory

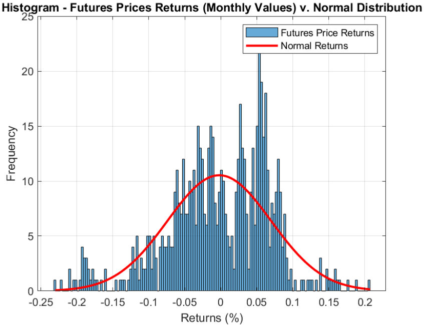
- $F(t,T) \equiv S_t e^{\int_t^T r(t,u)du - \int_t^T cy(t,u)du} = S_t e^{\int_t^T y(t,u)du}$
  - ✓ $r(t,T)$: interest rate
  - ✓ $cy(t,T)$: convenience yield[4]
  - ✓ $y(t,T)$: cost of carry[5]
- Stylised facts in oil markets:
  - ✓ mean-reversion in spot prices ($\rho_{S_t, cy(t,T)} > 0$).
  - ✓ stochastic spot prices and convenience yields
  - ✓ stochastic volatility of futures prices
  - ✓ volatility of futures prices increases as contracts approach expiration
  - ✓ jumps in spot prices and in convenience yields
  - ✓ futures prices with distant maturities jump less than those closer to it
- Are oil futures prices returns normally distributed? See slide+1.

[4] It is the benefit that comes from holding a physical good as inventory rather than a futures contract. It applies to consumption assets rather than those held as investments. An example of a product with a high convenience yield is oil.

[5] Interest rate minus convenience yield).

Theory (c'ed)



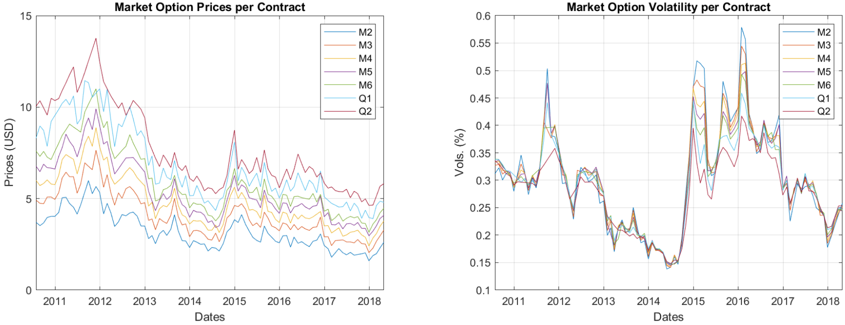


Contracts	Average	Std. Deviation	Skewness	Kurtosis	JB Statistic	JB p-Value	JB Test
M2	0.342%	8.684%	-0.292	4.339	12.452	0.002	FALSE
M3	0.304%	8.405%	-0.340	4.291	12.412	0.002	FALSE
M4	0.270%	8.179%	-0.371	4.237	12.130	0.002	FALSE
M5	0.238%	7.976%	-0.394	4.214	12.215	0.002	FALSE
M6	0.211%	7.789%	-0.411	4.211	12.494	0.002	FALSE
Q1	0.180%	7.587%	-0.412	4.313	14.022	0.001	FALSE
Q2	0.113%	7.085%	-0.473	4.456	17.585	0.000	FALSE
ALL	0.226%	7.980%	-0.374	4.263	12.566	0.002	FALSE

NOTES: JB accounts for the Jarque-Bera normality test. The null hypothesis refers to futures prices returns being normally distributed. The statistic critical value related to a significance level of 0.05 is 5.991.

Theory (c'ed)

- Are oil option volatilities constant, deterministic or stochastic?
- See that $\sigma_t^{Q2} < \sigma_t^{M2}$ (Samuelson effect).



(The plots above refer to ATM call option prices.)

Theory (c'ed)

- STATE-OF-THE-ART MODEL: Trolle and Schwartz (2009, RFS) SV1[6].
- Dynamics of state variables under $\mathbb{Q}$:

$$dS_t/S_t = y_t dt + \sigma_S \sqrt{v_t} dW_t^S$$
$$dy(t,T) = \mu_y(t,T)dt + \sigma_y(t,T)\sqrt{v_t} dW_t^y$$
$$dv_t = \kappa(\theta - v_t)\,dt + \sigma_v \sqrt{v_t} dW_t^v$$

with $\sigma_y(t,T) \equiv \alpha_t e^{-\gamma_t(T-t)}$.

- Futures price dynamics under $\mathbb{Q}$ are martingale:

$$\frac{dF(t,T)}{F(t,T)} = \sqrt{v_t}\left(\sigma_S dW_t^S + \sigma_Y(t,T) dW_t^y\right)$$

- Characteristic function[7] (CF) of futures prices:

$$\psi_t(iu,t,T_{Opt},T) = e^{A(T_{Opt}-t)+B(T_{Opt}-t)v_t+iu\log F(t,T)}$$

[6] Three-factor version of the term-structure model, TS09 hereafter.
[7] It is a Fourier transform of the probability density function (pdf).

Theory (c'ed)

- Option pricing:
  ✓ It is possible to price plain vanilla options if the CF of the terminal price is known analitically.
  ✓ The authors indicate that the CF cannot be solved algebraically, thus the need to resort to NUMERICAL METHODS.
  ✓ They use the Fourier inversion theorem - it needs to solve 2 integrals numerically (see Duffie, Pan and Singleton (2000)):

$$\mathscr{C}(t, T_{Opt}, T, K) = P(t, T_{Opt}) \Big(G_{1,1}(\log K) - K G_{0,1}(\log K) \Big),$$

$$G_{a,b}(y) = \frac{\psi_t(a, t, T_{Opt}, T)}{2} - \frac{1}{\pi} \int_0^{\infty} \frac{\Im \left[\psi_t(a + iub, t, T_{Opt}, T) e^{-iuy} \right]}{u} du.$$

Theory (c'ed)

- OUR MODEL is an expansion of TS09 which includes jumps:

$$dS_t/S_t = \left(y_t - \lambda_t E_t^{\mathbb{Q}}\left[e^{J_S} - 1\right]\right) dt + \sigma_S \sqrt{v_t} dW_t^S + \left(e^{J_S} - 1\right) dN_t$$

$$dy(t,T) = \left(\mu_y(t,T) - \lambda_t E_t^{\mathbb{Q}}\left[J_y(t,T)\right]\right) dt + \sigma_y(t,T)\sqrt{v_t} dW_t^y + J_y(t,T) dN_t$$

$$dv_t = \kappa(\theta - v_t)\, dt + \sigma_v \sqrt{v_t} dW_t^v$$

- Jumps inspired in Crosby (2008).[8]
- Futures price dynamics under $\mathbb{Q}$ are martingale:

$$\frac{dF(t,T)}{F(t,T)} = \sqrt{v_t}\left(\sigma_S dW_t^S + \sigma_Y(t,T) dW_t^y\right) - \lambda_t E_t^{\mathbb{Q}}\left[e^{J_S + J_Y(t,T)} - 1\right] dt + \left(e^{J_S + J_Y(t,T)} - 1\right) dN_t$$

- Characteristic function (CF) of futures prices:

$$\psi_t(iu, t, T_{Opt}, T) = e^{A(T_{Opt}-t) + B(T_{Opt}-t)v_t + C(T_{Opt}-t)\lambda_t + iu \log F(t,T)}$$

[8] Term-structure model with jumps in the futures prices and deterministic volatility.

Theory (c'ed)

- Subspecifications based on jump assumptions:

Ass.	Factor	Condition	Expression
a	S_t	J_s iid	$J_s \sim (\mu_{J_S}, \sigma^2_{J_S})$
$b1$	$y(t,T)$	a_t iid, $b_t = 0$	$J_y \sim (\mu_{J_y}, \sigma^2_{J_y})$
$b2$	$y(t,T)$	a_t const., $b_t \geq 0$	$J_y(t,T) = a_t e^{-b_t(T-t)}$
$a, b1$	$S_t, y(t,T)$	J_s iid a_t iid, $b_t = 0$	$J_s \sim (\mu_{J_S}, \sigma^2_{J_S})$ $J_y \sim (\mu_{J_y}, \sigma^2_{J_y})$
$a, b2$	$S_t, y(t,T)$	J_s iid a_t const., $b_t \geq 0$	$J_s \sim (\mu_{J_S}, \sigma^2_{J_S})$ $J_y(t,T) = a_t e^{-b_t(T-t)}$

Theory (c'ed)

- Option pricing:
  ✓ Sitzia (2018) derived a closed-form representation of the CF of the futures prices following TS09.
  ✓ We obtain ANALYTIC EXPRESSIONS for each term in our CF.
  ✓ Alternatively, we use the Fast Fourier transform (FFT) algorithm - it only needs to solve 1 integral numerically (see Carr and Madan (1999)):

$$\mathscr{C}(t, T_{Opt}, T, K) = P(t, T_{Opt}) \frac{e^{-\alpha \log(K)}}{\pi} \int_0^\infty \Re \left[\frac{e^{-iu\log(K)} \psi_t(u - i(\alpha+1), t, T_{Opt}, T)}{\alpha^2 + \alpha - u^2 + iu(2\alpha+1)} \right] du.$$

- ACHIEVEMENTS re. COMPUTATION TIME:
  ✓ significant reduction due to the analytic solution to the CF
  ✓ additional reduction due to the FFT algorithm

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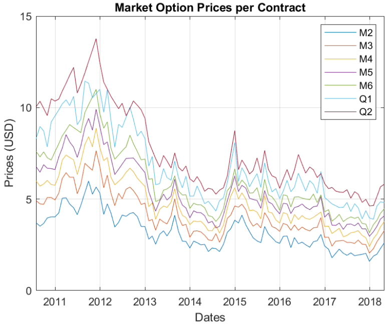
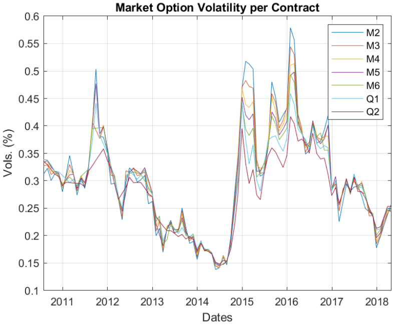
Data

- Preliminary results being presented today.
- Underlying energy commodity: WTI light crude oil.
- Data set consists of 6720 option prices spanning >8Y (01/2010-05/2018):
  - ✓ 96 montly observations
  - ✓ 7 futures contracts (maturities)[9]
  - ✓ 10 moneyness levels (ATM and OTM options)
- Calibration of parameters to market data (futures and option prices).
- Compare the fit of our model to that of nested cases (extant models).
- See prices of ATM call options and Black volatilities in slide+1.

[9] Based on liquidity reasons (open interest).

Data (c'ed)


Market Option Prices per Contract
Prices (USD)
Dates
M2
M3
M4
M5
M6
Q1
Q2
0
5
10
15
2011
2012
2013
2014
2015
2016
2017
2018


Market Option Volatility per Contract
Vols. (%)
Dates
M2
M3
M4
M5
M6
Q1
Q2
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.55
0.6
2011
2012
2013
2014
2015
2016
2017
2018

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Results

- Can we price options more accurately than extant (nested) models? (5) Trolle (2014), TS09 (SV1), Bates (1996), Heston (1993), Merton (1973).

Model	S_t	$y(t,T)$		v_t		Jumps		Count	**Analytic**
	Stoch.	Const.	Stoch.	Const.	Stoch.	S_t	$y(t,T)$		**Solution**
BLACK-SCHOLES	✓	✓	–	✓	–	–	–	1	✓
MERTON	✓	✓	–	✓	–	✓	–	4	✓
HESTON	✓	✓	–	✓	✓	–	–	4	✓
BATES	✓	✓	–	✓	✓	✓	–	7	✓
TS09 (SV1)	✓	–	✓	–	✓	–	–	9	–
TROLLE	✓	–	✓	–	✓	✓	–	12	–
OUR MODEL	✓	–	✓	–	✓	✓	✓	12,14	✓

- Cross-model estimated parameters, MAE[10], computation time in slide+1.
- ACHIEVEMENTS:
  ✓ First work using analytic solutions to TS09 - over 30 times quicker
  ✓ Our multi-factor jump model presents analytic solutions
  ✓ Our model is more accurate than TS09 (77.60% v 78.79%)

[10] MAE represents the absolute mean of differences between volatilities predicted by the model $\hat{\sigma}_t$ and empirical values σ_t observed from $t = 1, \ldots, N$:

$$MAE = \frac{\sum_{t=1}^{N} |\hat{\sigma}_t - \sigma_t|}{N}. \quad (1)$$

Results

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	S_t	$y(t,T)$		v_t		Jumps		Count	**Analytic**
Model	Stoch.	Const.	Stoch.	Const.	Stoch.	S_t	$y(t,T)$		**Solution**
BLACK-SCHOLES	✓	✓	–	✓	–	–	–	1	✓
MERTON	✓	✓	–	✓	–	✓	–	4	✓
HESTON	✓	✓	–	✓	✓	–	–	4	✓
BATES	✓	✓	–	✓	✓	✓	–	7	✓
TS09 (SV1)	✓	–	✓	–	✓	–	–	9	–
TROLLE	✓	–	✓	–	✓	✓	–	12	–
OUR MODEL	✓	–	✓	–	✓	✓	✓	12,14	✓

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$$MAE = \frac{\sum_{t=1}^{N} |\hat{\sigma}_t - \sigma_t|}{N}. \quad (1)$$

Results (c'ed)

	FC(J_s, J_y^{tdf})	FC(J_s, J_y^{iid})	FC(J_y^{tdf})	FC(J_y^{iid})	FC(J_s)	TS	Bates	Heston	Merton
σ_S	1	1	1	1	1	1	1	1	0.2679
α	3.4526	3.3466	3.3908	3.3836	3.3836	3.0048	–	–	–
γ	4.5239	4.4522	4.5812	4.4797	4.4797	4.8169	–	–	–
κ	1.6711	1.7063	1.6549	1.7045	1.7045	1.3902	0.5402	0.3221	–
θ	0.0100	0.0100	0.0100	0.0100	0.0100	0.0159	0.0100	0.0736	–
σ_v	0.1824	0.1848	0.1812	0.1840	0.1840	0.2084	0.1039	0.2178	–
ρ_{Sy}	0.1295	0.1355	0.1213	0.1305	0.1305	0.1652	–	–	–
ρ_{Sv}	-0.8409	-0.8404	-0.8337	-0.8459	-0.8459	-0.8449	-1.0000	-1.0000	–
ρ_{yv}	-0.9751	-0.9753	-0.9892	-0.9608	-0.9608	-1.0000	–	–	–
λ	0.0056	0.0058	0.0063	0.0064	0.0064	–	0.0301	–	0.2366
μ_{J_S}	-0.6549	-0.7877	–	–	-1.2888	–	-0.8434	–	-0.0573
σ_{J_S}	0.0642	0.0179	–	–	0.0115	–	0.0007	–	0.0040
μ_{J_y}	–	-0.7877	–	-1.2888	–	–	–	–	–
σ_{J_y}	–	0.0179	–	0.0115	–	–	–	–	–
a_t	-1.3130	–	-1.5103	–	–	–	–	–	–
b_t	0.0000	–	0.0000	–	–	–	–	–	–
Count	14	14	12	12	12	9	7	4	4
MAE	0.7778	0.7765	0.7769	0.7756	0.7756	0.7879	0.8615	0.9091	4.3526
Calc. t (N)	268	241	284	125	109	64.8	10.8	4.81	1.83
Calc. t (A)	7.80	10.8	6.60	10.2	8.40	2.15	1.80	1.08	0.60

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Wrap up & Conclusions

- WRAP UP:
  ✓ We present a new term-structure model expanding the stat-of-the-art model TS09 with jumps in S_t and $y(t, T)$.
  ✓ We provide an analytical solution to our model (CF).
  ✓ We price options on WTI crude oil futures prices.
  ✓ 1st article in the literature performing option pricing using analytical solutions to TS09.
  ✓ We test our model's and extant ones' performance.
- CONCLUSIONS:
  ✓ All nested models considered (even TS09) have been beaten - MAE.
  ✓ Computation time is considerably lower than using previous technology.
  ✓ Objectives achieved.

Outline

Next steps in PhD thesis

- CRUDE OIL:
  - ⇒ Update market data to 12/2019.
  - ⇒ Include deeper-OTM options.
  - ⇒ Granularity from monthly to daily.
  - ⇒ Completion estimated by Q1-Q2 2020.
- NATURAL GAS:
  - ⇒ WIP.
  - ⇒ Completion estimated by Q4 2020.
- ELECTRICITY:
  - ⇒ International PhD in London Q2 2021 (TBC).
  - ⇒ Completion estimated by Q4 2021.

Outline

References

- Black, F. and Scholes, M. (1973). The Pricing of Options and Corporate Liabilities. *Journal of Political Economy*, 81(3), 637-654.
- Bates, D. (1996). Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options. *Review of Financial Studies*, 9:69-107.
- Carr, P.P. and Madan, D.B. (1999). "Option Evaluation Using the Fast Fourier Transform", Journal of Computational Finance, 2:61-73.
- Crosby, J. (2008). A Multi-Factor Jump-Diffusion Model for Commodities. *Quantittive Finance*, 8(2):181-200.
- Duffie, D., Pan, J. and Singleton, K. (2000). Transform Analysis and Asset Pricing for Affine Jump-Diffusions. *Econometrica*, 68:1343-1376.
- Heston, S. (1993). A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *Review of Financial Studies*, 6:327-344.
- Merton, R.C. (1976). Option Pricing When Underlying Stock Returns Are Discontinuous. *Journal of Financial Economics*, 3:125-144.
- Sitzia, N. (2018). The Analytical Solution of Trolle-Schwartz Model. MSc. Thesis, École Polytechnique Fédérale de Lausanne.
- Trolle, A. B. (2014). Efficient Pricing of Energy Derivatives. Chapter in "Energy Pricing Models: Recent Advances, Methods, and Tools" (2014). Ed. Marcel Prokopczuk.
- Trolle, A. B. and Schwartz, E. S. (2009). Unspanned Stochastic Volatility and the Pricing of Commodity Derivatives. *Review of Financial Studies*, 22(11):4423-4461.

Q&A

MANY THANKS FOR YOUR TIME AND ATTENTION TODAY

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