Satisficing Matching

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Abstract

We bring the notion of "satisficing" to matching theory and develop a weaker notion of stability. Satisficing behavior is an alternative to maximizing behavior, where instead of always going for the best, satisficers settle on something that is "good enough". We say that a matching is satisficing if every agent is matched to an achievable partner. (An agent is achievable to another if they are matched in some stable matching.)

We show that satisficing matchings have the following properties: (i) They are Pareto efficient; (ii) All women (men) weakly prefer the woman-optimal (man-optimal) stable matching to any satisficing matching; (iii) For any two satisficing matchings, there exist two couples, each formed under one of the two matchings, such that the agents in each couple have opposite preferences over the two matchings (weak decomposition); (iv) If agents are asked to vote between a stable matching and a satisficing matching, the two matchings tie; (v) Using as operator an extension, via the join, of the least upper bound of the common order of one gender, the set of satisficing matchings forms an abelian semigroup whose ideal is the set of stable matching; (vi) Truthtelling is a rationalizable strategy for satisficing mechanisms.

We also extend the results to the college admission model.

JEL Classification Codes: C78, D71, D90

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The rationality paradigm in economics assumes that individual decision-makers make optimal decisions no matter how complicated their choice problem is. In his very influential work, Herbert Simon (see Simon, 1956) postulated that, in reality, agents abandon the search for optimality when the problem is perceived to be too complicated. Instead, Simon suggested, an overwhelmed decision-maker is happy to settle for an option that she deems good enough, given the circumstances. To describe this less demanding behavioral principle, Simon coined the term *satisficing*, a portmanteau of *satisfying* and *sufficing*.

The dominant equilibrium concept in matching theory, stability, follows the rationality paradigm. In a marriage market, a couple blocks a matching if the woman prefers the man to her assigned match, and he prefers her to his. Under well-understood assumptions, stable matchings exhibit a series of very interesting properties. They are the only matchings in the market's core, and the preferences of the two genders conflict when comparing stable matchings, for example. Notably, the set of all stable matchings can be partially ordered according to the shared preferences of one of the two genders, and any two stable matchings possess an infimum and a supremum under those preferences. Unfortunately, the conflict of preferences implies that the agents have incentives to misreport their preferences to any mechanism that tries to implement a stable matching.

Our point of departure in this paper is that (say, if a marriage market is large and preferences are heterogeneous) the behavioral premise behind the definition of stable matchings is at odds with Simon's postulate. Suppose that, after all the women have dated all the men, and vice versa, and after everybody has experienced being alone, there comes the time when the couples are going to commit. Suppose each of these couples goes on one last date where they will make the commitment decision. If the solution concept is stability, before saying yes, each person in the couple may need to excuse themselves and start calling/texting many other potential partners to see what proposals they have received and whether they are accepting them. Of course, each of these people will need to communicate with others, who will need to do the same, and so on. That seems (unromantic and) overwhelming.

Instead, we propose a definition of satisficing matching that relaxes the rationality requirement of the solution. Suppose a couple is at the definitive proposal date. We propose that the couple will commit if both can envision a state of affairs in which each is happy to form a couple with the other. They don't know who else is committing to whom, but if they can think of a situation under which they are mutually best for each other, they conclude that the other person is in any case good enough and agree to the commitment. They will reject the proposal if, no matter what is happening on other dates, they cannot convince themselves that they are mutually happy with the match.

Of course, weakening the solution concept implies that some of its properties may be lost. Our central claim is that our definition of satisficing matching exhibits at least weak versions of the properties of stable matchings.

Some empirical observations

The idea that not all decision-makers maximize in all their choices has found large support in psychology. The literature is abundant, but the interested reader can find a comprehensive motivation and coverage in Schwartz (2003) and Schwartz and Ward (2004). The evidence robustly suggests that decision-makers have different approaches to choice, ranging from the "maximizer" types, who always want to make the optimal choice, no matter how complex the setting, to "satisficers, who look for a good enough option. The other clear regularity seems to be that maximizers are much more prone to exhibit ex-post regret over their choices, even though they exerted more effort trying to "get it right," than satisficers.[1]

The observation of satisficing behavior is not exclusive of pure decision problems. It extends to strategic situations in general, and to matching problems in particular. For example, Yang and Chiou (2010) show that maximizers evaluate more profiles for potential partners before contacting one through on-line dating platforms, while Buri et al. (2008) show that they are more likely to terminate their relationships than satisficers.[2] Iyengar et al. (2006) report that when searching for jobs, maximizers search more aggressively but find themselves less happy about their job outcomes than satisficers, even though they typically secure better jobs.[3] Interestingly, Rong et al. (2025) propose a satisficing algorithm for matching riders and drivers in ridesharing platforms that is meant to gain significant efficiency over maximizing algorithms.

Related literature

In a similar context, others have also explored alternative solution concepts to (pairwise) stability. For example, Klijn and Massó (2003) proposed weakly stable matchings where none of the blocking pairs is the most preferred blocking pair for both of the involved agents. Richter and Rubinstein (2024) proposed unilateral stability where the equilibria are immune to a single agent unilaterally initiating contact to another agent and investigated the solution concept under different restrictions[4] As a counterpart to the idea that one agent alone can object to a matching, Doe (2024) proposes the notion of the agreeable core, where all affected agents have to agree to object to a matching. Pittel and Rudov (2024) study stability with switching costs under cardinal utilities, where agents block only if the benefit from blocking is larger than the switching cost. Our paper, like Klijn and Massó (2003), Richter and Rubinstein (2024), and Doe (2024), assumes ordinal preferences, as in Gale and Shapley (1962).

[1] See Bruine de Bruin et al. (2007) for a detailed analysis of the behavioral differences between maximizers and other types of decision makers.

[2] Tellingly, Horowitz et al. (2019) report that about one third of the participants in their sample survey identify convenience as a reason to enter into a relationship or to stay on in.

[3] See, also, the experimental results of Caplin et al. (2011).

[4] When some couples are permissible and some are taboo; when an agent can only approach agents who are lower-ranked than himself; when an agent can only approach agents who are lower-ranked than his partner; and when a power hierarchy is determined which dictates who can approach whom.

Less directly related solution concepts are also discussed in contexts where stability clashes with other desirable properties or where a stable matching may not exist. In school choice problems[5] where stable matchings may not be Pareto efficient, Tang and Zhang (2021) and Reny (2022), for example, proposed weakly stable matching and priority-efficient matching (respectively). In roommate problems where a stable matching may not exist, Abraham et al. (2005), Morrill (2010), Klaus et al. (2010), and Sotomayor (2011), for example, proposed almost stable matching, Pareto efficient matching, stochastically stable matching, and Pareto-simple matching (respectively). In many-to-one and many-to-many matching where the core may be empty, Echenique and Yenmez (2007) and Echenique and Oviedo (2006) discuss relevant solution concepts.

We are also related to the literature on matching mechanisms/algorithms. Following the earlier discussion on strategic properties of stable mechanisms (Roth and Sotomayor (1990) and Roth and Vande Vate (1991)), we show that truth-telling is a rationalizable strategy for all agents for stable mechanisms and satisficing mechanisms. The school choice literature also has extensive discussion on the use of unstable mechanisms for the sake of efficiency, for example the Top Trading Cycle adopted from Shapley and Scarf (1974) by Abdulkadiroğlu and Sönmez (2003), the efficiency-adjusted deferred acceptance mechanism by Kesten (2010), and so on. The computer science literature considers near-stable algorithms that address certain welfare and computational complexity considerations (see Caragiannis et al. (2019) and Chen et al. (2021)).

Incidentally, our paper provides a microfoundation for the sets of acquaintances (with sets of achievable partners) used in Halmos and Vaughan (1950) and in the Hall's Marriage Theorem (Knuth, 1976). The combinatorial objective is to marry each man to one of his acquainted woman. To achieve this, a necessary and sufficient condition is that every finite set of men is, collectively, acquainted with at least an equal number of women. When there are at least as many women as men, letting the set of acquaintances for each man to be the set of his achievable partner is sufficient for the Hall's marriage problem.

1 Satisficing matchings

Using the notation in Roth and Sotomayor (1990), RS henceforth, a *marriage market* is a triple (W, M, P) that consists of: a set of women, W; a set of men, M; and for each agent a, a strict order P_a over the agents of the other gender and themself. We will write $b >_a c$ to denote that, according to P_a, agent a strictly prefers agent b over agent c. The expression $b \geq_a c$ means that either $b >_a c$ or $b = c$.

Given a marriage market, a *matching* is a bijection $\mu : W \cup M \to W \cup M$ such that: (1) for every woman, $\mu(w) \in M \cup \{w\}$; (2) for every man, $\mu(m) \in W \cup \{m\}$; and (3) for every agent, $\mu(\mu(a)) = a$. Matching μ is *individually rational* if $\mu(a) \geq_a a$ for every agent. It is *stable* if it is individually rational and if there is no couple $\{w, m\}$ for whom $m >_w \mu(w)$ and $w >_m \mu(m)$.

[5] Schools' preferences are only priorities of students and are irrelevant for welfare considerations.

Recall that an agent is said to be *achievable* to another if a stable matching exists at which they are paired (RS, p. 32). The solution concept we propose is the following: a matching is satisficing if every agent is paired with an achievable partner. Formally,

DEFINITION 1. *Matching μ is* satisficing *if for every agent a, there exists a stable matching λ such that $\lambda(a) = \mu(a)$.*

The idea is that each person in a couple finds their partner to be "good enough" if there is a conceivable situation (λ) in which they would not object to being paired.

Of course, every stable matching is satisficing, but the opposite is not true. The following is an example:

EXAMPLE 1. Suppose there are four women and four men. Their preferences are:

$$
\begin{aligned}
P(w_1) &= m_4, m_1, m_2, m_3, w_1 & \qquad P(m_1) &= w_1, w_2, w_3, w_4, m_1 \\
P(w_2) &= m_3, m_4, m_1, m_2, w_2 & \qquad P(m_2) &= w_2, w_3, w_1, w_4, m_2 \\
P(w_3) &= m_2, m_3, m_1, m_4, w_3 & \qquad P(m_3) &= w_3, w_4, w_1, w_2, m_3 \\
P(w_4) &= m_1, m_3, m_4, m_2, w_4 & \qquad P(m_4) &= w_4, w_2, w_3, w_1, m_4
\end{aligned}
$$

Note that

$$
\lambda_1 : \begin{matrix} m_1 & m_2 & m_3 & m_4 \\ w_1 & w_2 & w_3 & w_4 \end{matrix} \qquad
\lambda_2 : \begin{matrix} m_1 & m_2 & m_3 & m_4 \\ w_1 & w_3 & w_4 & w_2 \end{matrix} \qquad
\lambda_3 : \begin{matrix} m_1 & m_2 & m_3 & m_4 \\ w_4 & w_3 & w_2 & w_1 \end{matrix}
$$

are (the only) stable matchings. It follows that

$$
\mu : \begin{matrix} m_1 & m_2 & m_3 & m_4 \\ w_1 & w_3 & w_2 & w_4 \end{matrix}
$$

is satisficing because each matched pair is achievable. However, μ is not stable because (w_4, m_3) blocks it.

2 Welfare properties

In this section, we study the extent to which the main welfare implications of stability for matchings weaken for the case of satisficing matchings. We focus on the welfare bounds, efficiency, popularity, and the conflict of interests between the two sides of the market.

2.1 Bounds

The following notation is from RS. Given two matchings μ and ν, $\mu \geq_W \nu$ denotes that $\mu(w) \geq_w \nu(w)$ for every woman w. If the latter is true, and $\mu(w) >_w \nu(w)$ for at least one woman, then write

$\mu >_W \nu$. A similar notation applies to the men. μ_W and μ_M denote, respectively, the W-optimal and M-optimal stable matchings (RS, p. 32). Using this notation, for any stable matching μ, $\mu_W \geq_W \mu \geq_W \mu_M$ and $\mu_W \leq_M \mu \leq_M \mu_M$.

Our first result is that these bounds continue to apply to satisficing matchings.

PROPOSITION 1. *For any satisficing matching* μ, $\mu_W \geq_W \mu \geq_W \mu_M$ *and* $\mu_W \leq_M \mu \leq_M \mu_M$.

Proof. Fix an arbitrary woman w. By definition, $\mu(w)$ is achievable for w, so there exists a stable matching λ for which $\lambda(w) = \mu(w)$. By definition of μ_W and μ_M, $\mu_W(w) \geq_w \lambda(w) \geq_w \mu_M(w)$, since λ is stable.

An identical argument applies to the men. □

2.2 Efficiency

It is well known that a matching is stable if, and only if, it is in the core of the marriage market (RS, Theorem 3.3).[6] Here, we rule out the possibility of inefficient satisficing matchings.

Before the argument, it is illustrative to note that no stable matching can dominate a satisficing matching. To see why, suppose, by way of contradiction, that μ is satisficing, while ν is stable and dominates μ:

$$\forall a\,,\ \nu(a) \geq_a \mu(a); \tag{1}$$

$$\exists a : \nu(a) >_a \mu(a). \tag{2}$$

Suppose that $\nu(\bar{w}) >_{\bar{w}} \mu(\bar{w})$ Then, letting $\bar{m} = \mu(\bar{w})$, $\nu(\bar{m}) \neq \mu(\bar{m})$ and, by (1), $\nu(\bar{m}) >_{\bar{m}} \mu(\bar{m})$ since preferences are strict. We can then strengthen Eq. (2) to

$$\exists (\bar{w}, \bar{m}) : \nu(\bar{w}) >_{\bar{w}} \mu(\bar{w}) \ \text{ and } \ \nu(\bar{m}) >_{\bar{m}} \mu(\bar{m}). \tag{3}$$

Since μ is satisficing, there exists a stable λ such that $\lambda(\bar{w}) = \mu(\bar{w})$ and $\lambda(\bar{m}) = \mu(\bar{m})$. From Eq. (3), $\nu(\bar{w}) >_{\bar{w}} \lambda(\bar{w})$ and $\nu(\bar{m}) >_{\bar{m}} \lambda(\bar{m})$, which is impossible, since ν and λ are both stable, by RS, Corollary 2.21.

Through a similar logic, we now argue that satisficing matchings are efficient.

Since preferences are strict, an agent who is single in one stable matching remains single in all stable matchings, by RS, Theorem 2.22. We refer to those people as *hopeless*. By definition, hopeless agents remain single in any satisficing matching.

LEMMA 1. *Let* μ *be satisficing, and let* ν *dominate* μ*. If agent* a *is hopeless, then* $\nu(a) = a$.

Proof. Suppose that man $\bar{m}$ is hopeless, yet $\nu(\bar{m}) = \bar{w} \in W$. Since preferences are strict, $\bar{w} >_{\bar{m}}$

[6] For the readers' convenience, we include the results we invoke from RS in an appendix.

$\bar{m} = \mu_M(\bar{m})$, where the last equality follows from the fact that $\bar{m}$ is hopeless. Since μ_M is stable and preferences are strict, $\bar{m} <_{\bar{w}} \mu_M(\bar{w})$. By Corollary 1, since μ is satisficing, $\mu_M \leq_W \mu$, so $\nu(\bar{w}) = \bar{m} <_{\bar{w}} \mu(\bar{w})$. Thus, ν cannot dominate μ □

Before proving our efficiency result, it is helpful to adapt a theorem by Gusfield (1988) to our setting.

Lemma 2 (Gusfield). *There exists a sequence $\langle \lambda_i \rangle_{i=1}^I$ of stable matchings, with $\lambda_1 = \mu_M$ and $\lambda_I = \mu_W$, such that*

$$\lambda_1 <_W \lambda_2 <_W \ldots <_W \lambda_I,$$

$$\lambda_1 >_M \lambda_2 >_M \ldots >_M \lambda_I,$$

and if matching μ is satisficing, then for every woman w there exists an index i such that $\lambda_i(w) = \mu(w)$ — and, therefore, $\lambda_i(m) = w$ for $m = \mu(w)$.

Proof. This follows immediately from the definition of satisficing matching by RS Corollary 2.21 and Theorem 3.20. □

Proposition 2 (Efficiency). *Any satisficing matching is efficient.*

Proof. Given Lemma 1, for this argument, we assume that there are the same number of men and women and that no agent is hopeless.

Let μ be satisficing, and suppose ν dominates μ. Define $E = \{a \in W \cup M \mid \nu(a) = \mu(a)\}$, and note that $E^c \neq \varnothing$ and for all $a \in E^c$, $\nu(a) >_a \mu(a)$, since preferences are strict. Note also that if $a \in E^c$, then $\mu(a) \in E^c$ too.

Using the sequence of stable matchings of Lemma 2, let i be the largest index for which $\lambda_i(w) = \mu(w)$ for some woman in E^c, which is possible thanks to the Lemma. Let w be one such woman, and denote $m = \mu(w)$.

For $\bar{m} = \nu(w)$, since $w \in E^c$, $\bar{m} >_w m$. If $\bar{m} \in E$, then $w = \nu(\bar{m}) = \mu(\bar{m})$, which would imply that $\bar{m} = m$, an impossibility. It follows that $\bar{m} \in E^c$ and, hence, $w >_{\bar{m}} \mu(\bar{m})$.

Since λ_i is stable, $\bar{m} >_w m = \lambda_i(w)$ implies that $\lambda_i(\bar{m}) \geq_{\bar{m}} w$. Then, $\lambda_i(\bar{m}) >_{\bar{m}} \mu(\bar{m})$. Since μ is satisficing, there exists another index j such that $\lambda_j(\bar{m}) = \mu(\bar{m})$. The preference $\lambda_i(\bar{m}) >_{\bar{m}} \lambda_j(\bar{m})$ requires $j > i$, which is impossible by our choice of i (as the largest index). □

2.3 Decomposition

Comparing ν and λ_3 in Example 1, we can see that the decomposition lemma in Knuth (1976), Corollary 2.21 in RS, does not extend to satisficing matchings. Still, a stronger version of Lemma 2 yields a weak decomposition result for satisficing matchings.

Lemma 3. *Let ν be a stable matching. A sequence $\langle\lambda_i\rangle_{i=1}^I$ of stable matchings with the properties of that in Lemma 2 can be constructed such that $\lambda_{i^*} = \nu$ for some $1 \le i^* \le I$.*

Proof. This follows from Theorem 4.1 in Gusfield (1988). See the comment after the proof of Theorem 3.20 in RS. □

Proposition 3 (Weak decomposition). *Let μ be satisficing, and let $\nu \neq \mu$.*

1. *If ν is stable, then for all couples formed under μ, if one of the two agents prefers μ the other prefers ν.*
2. *If ν is satisficing, there exist two couples, each formed under one of the two matchings, such that one of the agents in the couple prefers μ and the other one (in the same couple) prefers ν.*

Proof. Let W' be the set of women for whom $\mu(w) \neq \nu(w)$.

For the first claim, since ν is stable, by Lemma 3 there is a sequence $\langle\lambda_i\rangle_{i=1}^I$ of stable matchings, with $\lambda_1 = \mu_M$, $\lambda_{i^*} = \nu$ and $\lambda_I = \mu_W$, such that $\lambda_1 <_W \ldots <_W \lambda_I$, $\lambda_1 >_M \ldots >_M \lambda_I$, and for every woman w there exists an index $i(w)$ such that $\lambda_{i(w)}(w) = \mu(w)$.

Consider any couple $\{w, m\}$ such that $\nu(w) \neq \mu(w) = m$. By construction, $m = \lambda_{i(w)}(w)$ and $w = \lambda_{i(w)}(m)$. If $\lambda_{i^*}(w) = \nu(w) >_w m$, it must be, by Lemma 3, that $i^* > i(w)$, so $w >_m \lambda_{i^*}(m) = \nu(m)$. If, alternatively, $m >_w \lambda_{i^*}(w) = \nu(w)$, then $i^* < i(w)$ and $\lambda_{i^*}(m) = \nu(m) >_m w$.

For the second claim, fix the sequence $\langle\lambda_i\rangle_{i=1}^I$ of Lemma 2. For the first couple, define, for each $w \in W'$,

(a) $i^*(w)$ as the largest index i for which $\lambda_i(w) = \mu(w)$;

(b) $j^*(w)$ as the largest index j for which $\lambda_j(w) = \nu(w)$;

(c) $i^* = \max\{i^*(w) : w \in W'\}$;

(d) $j^* = \max\{j^*(w) : w \in W'\}$; and

(e) $k = \max\{i^*, j^*\}$.

With no loss of generality, suppose that for woman w^*, $\lambda_k(w^*) = \mu(w^*) = m^*$. By construction, for some $i, j < k$, $\lambda_j(w^*) = \nu(w^*)$ and $\lambda_i(m^*) = \nu(m^*)$. $j < k$ implies $\lambda_j <_W \lambda_k$, while $i < k$ implies $\lambda_i >_M \lambda_k$. It follows that $\nu(w^*) <_{w^*} \mu(w^*)$ while $\nu(m^*) >_{m^*} \mu(m^*)$.

To construct the second pair, use the other end of the sequence of Lemma 2: (a) $i_*(w)$ is the lowest i for which $\lambda_i(w) = \mu(w)$; (b) $j_*(w)$ is the lowest j for which $\lambda_j(w) = \nu(w)$; (c) $i_* = \min\{i_*(w) : w \in W'\}$; (d) $j_* = \min\{j_*(w) : w \in W'\}$; and (e) $\ell = \min\{i_*, j_*\}$. Again, we can find some woman w_* for whom, with no loss of generality, $\lambda_\ell(w_*) = \nu(w_*) = m_*$. Then, for some $i, j > \ell$ we must have $\lambda_i(w_*) = \mu(w_*)$ and $\lambda_j(m_*) = \mu(m_*)$. Thus, $\mu(w_*) >_{w_*} \nu(w_*)$ while $\nu(m_*) >_{m_*} \mu(m_*)$. □

It is worth noting that the two couples in the second statement of the previous proposition may share a member. For instance, it can happen that $\mu(w) >_w \nu(w)$, $\nu(\mu(w)) >_{\mu(w)} w$, and

$\mu(\nu(w)) >_{\nu(w)} w$.

2.4 Voting

A matching is said to be a *majority assignment* if no alternative matching is preferred by a majority.[7] Gärdenfors (1975) showed that any stable matching is a majority assignment.

Corollary 1 (Weak majority). *Let matching μ be satisficing, and suppose that ν is stable. The number of agents who prefer μ is the same as those who prefer ν.*

Proof. This follows immediately from the first claim in Proposition 3. □

The following example shows that the same result does not hold between two satisficing matchings.

Example 2. Suppose there are three women and three men, and consider the following matchings:

$$\lambda_1 : \begin{matrix} m_1 & m_2 & m_3 \\ w_1 & w_3 & w_2 \end{matrix}, \qquad \lambda_2 : \begin{matrix} m_1 & m_2 & m_3 \\ w_3 & w_2 & w_1 \end{matrix}, \qquad \lambda_3 : \begin{matrix} m_1 & m_2 & m_3 \\ w_2 & w_1 & w_3 \end{matrix}.$$

If we endow $\Lambda = \{\lambda_1, \lambda_2, \lambda_3\}$ with the order $\lambda_1 < \lambda_2 < \lambda_3$, the structure $(\Lambda, <)$ is a complete lattice. By the theorem in Blair (1984), namely Theorem 3.9 in RS, there exist preferences for the six agents so that Λ is the set of stable matchings, $\lambda_1 <_W \lambda_2 <_W \lambda_3$, and $\lambda_1 >_M \lambda_2 >_M \lambda_3$.

Now, the following matchings are satisficing:

$$\mu : \begin{matrix} m_1 & m_2 & m_3 \\ w_1 & w_2 & w_3 \end{matrix}, \qquad \nu : \begin{matrix} m_1 & m_2 & m_3 \\ w_2 & w_3 & w_1 \end{matrix}$$

Note that μ forms the pairs $\{w_1, m_1\} = \{w_1, \lambda_1(w_1)\}$, $\{w_2, m_2\} = \{w_2, \lambda_2(w_2)\}$, and $\{w_3, m_3\} = \{w_3, \lambda_3(w_3)\}$, while ν has $\{w_1, \lambda_2(w_1)\}$, $\{w_2, \lambda_3(w_2)\}$, and $\{w_3, \lambda_1(w_3)\}$. Women w_1 and w_2 then prefer ν, whereas w_3 prefers μ.

On the other hand, $m_1 = \lambda_1(w_1) = \lambda_3(w_2)$, $m_2 = \lambda_2(w_2) = \lambda_1(w_3)$, and $m_3 = \lambda_3(w_3) = \lambda_2(w_2)$. Then, the pairs under ν can be re-written as $\{\lambda_3(m_1), m_1\}$, $\{\lambda_1(m_2), m_2\}$, and $\{\lambda_2(m_3), m_3\}$. For man m_1, $\mu(m_1) = \lambda_1(m_1) >_{m_1} \lambda_3(m_1) = \nu(m_1)$. However, $\nu(m_2) = \lambda_1(m_2) >_{m_2} \lambda_2(m_2) = \mu(m_2)$ and $\nu(m_3) = \lambda_2(m_3) >_{m_3} \lambda_3(m_3) = \mu(m_3)$.

In this case, agents w_1, w_2, m_2, and m_3 prefer ν over μ.

The argument in the example can be illustrated with the graph in Fig. 1. Each column orders the three stable matchings. The higher the dot, the more preferred the matching from the women's point of view. The first column corresponds to satisficing matching μ, with one couple forming according to each of the three stable matchings. Satisficing matching ν is the second column, again

[7] A matching ν is *preferred by a majority* to matching μ if the number of men and women who prefer ν to μ is greater than the number who prefer μ to ν.

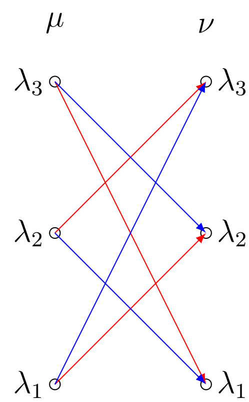

Figure 1

with one couple according to each stable matching. The red arrows represent the re-matching of women from μ to ν, while the blue arrows do the same for men. A red arrow going up represents a woman who prefers ν. A man who prefers ν is a blue arrow going down. Each node has one arrow of each color going out in the left column and one coming in in the right column. The two red arrows going up mean that women w_1 and w_2 prefer ν. The two blue arrows going down represent the same for men m_2 and m_3. Four of the six agents vote for ν over μ.

2.5 Decomposition and voting revisited

Example 2 also reiterates that we cannot strengthen the second claim in Proposition 3 to all couples. When comparing the two satisficing matchings, the two couples formed under λ_2 internally agree on the ranking: both agents would rather be in the other matching.

For the first claim in Proposition 3, suppose that matching ν is one of the three stable matchings. In Fig. 2, all the arrows departing from the nodes in the left column must arrive in the same node to the right, for instance λ_2. Couple by couple, both arrows go up or down, representing opposing preferences for the two members of that couple.

Using this type of graphical argument, one can see that the two pairs in the second statement of Proposition 3 can share a member. Suppose there are six stable matchings ordered as $\lambda_1 <_W \lambda_2 <_W \ldots <_W \lambda_6$ and the satisficing matchings μ and ν form as in Fig. 3. The red arrow going up represents one woman who prefers ν. The three blue arrows going up mean that all the men prefer μ. In both satisficing matchings, the woman who prefers ν is matched to men who prefer μ. That woman is the only agent in the market who prefers ν.

Unfortunately, this observation implies that Proposition 3 has no implications for the results on majority voting. The strongest conclusion in this regard follows directly from Proposition 2:

Corollary 2 (No unanimous agreement). *Let matching μ be satisficing and let ν be another matching. There is no consensus that ν is better than μ.*

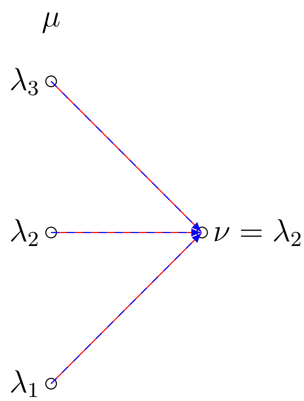

Figure 2

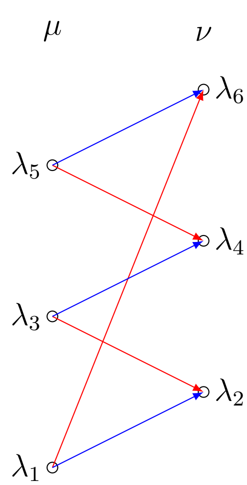

Figure 3

3 Algebraic structure

If μ and ν are stable matchings, RS denote by $\mu \wedge_W \nu$ and $\mu \vee_W \nu$, respectively, their greatest lower bound and least upper bound according to $\geq_W$. We simplify this notation by writing just $\mu \wedge \nu$, $\mu \vee \nu$, and $\geq$. The algebraic structure of the set of stable matchings is Conway's celebrated theorem (Knuth, 1976; RS, Theorem 3.8): this set is a distributive lattice under $\geq$. For this reason, from now on we denote the set of stable matchings as L.

As with the efficiency, decomposition, and voting properties, the satisficing matchings display a weaker property. Unfortunately, the operation $mu \wedge \nu$ is not well defined for unstable satisficing matchings. Our next result is that the set of satisficing matchings is an abelian semigroup and that L is the ideal that generates it.

First, note that one can see the lattice of stable matchings as two abelian semigroups $(L, \vee)$ and $(L, \wedge)$. We will use the former, but a dual semigroup of satisficing matchings can be constructed with the latter. To avoid confusion, we write $\mathcal{L} = (L, *)$ instead of $(L, \vee)$. Importantly, $(L, *)$ is a monoid with identity μ_M.

Second, if Λ is a collection of stable matchings, we denote their greatest lower bound and their least upper bound, respectively, by

$$\inf \Lambda := \bigwedge_{\lambda \in \Lambda} \lambda \quad \text{and} \quad \sup \Lambda := \bigvee_{\lambda \in \Lambda} \lambda.$$

Note that these two matchings are also stable (RS, Theorem 3.8).

Third, we extend the semigroup operator to the set of all satisficing matchings, which we denote by S. If $\mu \in L$, we let $\Lambda(\mu) := \{\mu\}$; for any satisficing but unstable matching $\mu \in S \setminus L$, denote by $\Lambda(\mu)$ the collection of stable matchings that make μ satisficing:

$$\Lambda(\mu) := \{\lambda \in \mathcal{L} \mid \exists a \in W \cup M : \lambda(a) = \mu(a)\}.$$

With these two definitions at hand, define extend operation $*$ to any two satisficing matchings μ and ν: $\mu * \nu = \sup \Lambda(\mu) \vee \sup \Lambda(\nu)$. Note that $\mu * \nu$ is a stable matching, so it is satisficing. From the viewpoint of all women, it is the least desirable stable matching that leaves them at least as well off as at both μ and ν.

Proposition 4 (Semigroup). *$\mathcal{S} = (S, *)$ is an abelian semigroup and $\mathcal{L} \preccurlyeq \mathcal{S}$ is the ideal generated by S.*

Proof. It follows by construction that $*$ maps $S \times S$ onto L. Operator $*$ is associative and commutative, since so is $\vee$, so $\mathcal{S}$ is a semigroup. In addition, if $\mu, \nu \in L$, then $\mu * \nu = \mu \vee \nu \in L$, so $\mathcal{L}$ is a subsemigroup of $\mathcal{S}$.

Define the function $\varphi : S \to L$ by $\varphi(\mu) = \mu * \mu$. By construction, $\varphi(\mu * \nu) = (\mu * \nu) * (\mu * \nu) =$

$\varphi(\mu) * \varphi(\nu)$, so φ is a morphism from S to L. Since $\varphi(\mu) = \mu$ for all $\mu \in L$, it follows that φ is surjective. By definition, then, $\mathcal{L} \preccurlyeq \mathcal{S}$.

Let $S^1 = S \cup \{1\}$, and define $\mu * 1 = \mu$ and $1 * \mu = \mu$. Since $S^1 * S * S^1 = L$, $\mathcal{L}$ is the ideal generated by S. □

4 Computational issues

It is well known that the deferred acceptance algorithm of Gale and Shapley (1962) finds μ_W in $O(|W \times M|)$ time. Let W' and M' be the sets of women and men that remain after excluding all the agents that are single in μ_W, and denote $n = |W'| = |M'|$. By Theorem 2.22 in RS, any agent who is single in μ_W remains single in all satisficing matchings, so the algorithm in Gusfield (1988) finds the set of all achievable pairs in $O(n^2)$ time. Let A denote that set.

Construct the bipartite graph $G = (W', M', A)$, with W' and M' as sets of vertices, and A as the set of edges. For any graph, a *perfect matching* is a subset of edges such that every vertex in the graph is adjacent to exactly one edge in the subset. For graph G, a perfect matching is $B \subseteq A$ such that for each woman $w \in W'$ there exists one, and only one, man $m \in M'$ for which $(w, m) \in B$. By construction, each perfect matching of the graph G defines a satisficing matching of the marriage market (W', M', P'), where P' is the restriction of preferences P to $W' \cup M'$. It is in this sense that this paper microfounds the sets of acquaintances used in Halmos and Vaughan (1950) and in Hall's Marriage Theorem.

Luckily, the equivalence of perfect and satisficing matchings yields algorithms for finding the latter. The number of satisficing matchings can be determined using the algorithm of Björklund (2012) in $O(1.415^{2n})$ time. Let this number be c. The algorithm in Fukuda and Matsui (1994), finally, finds the set of all satisficing matchings in $O(c(n + |A|) + n^{2.5})$ time.

5 Implementation

For the purposes of this section, keep the sets of women and men fixed, but not their preferences. If agent a's preferences are P_a, we now write bP_ac to mean that a prefers b to c.

Let $\mathcal{P}$ be the set of all (strict) preference profiles and $\mathcal{M}$ the set of all matchings. A *mechanism* is a function $f : \mathcal{P} \to \mathcal{M}$. A correspondence $F : \mathcal{P} \rightrightarrows \mathcal{M}$ will be called a *pre-mechanism*. If for every profile P, matching $f(P)$ satisfies some property, then so does the mechanism f. The same applies for pre-mechanism F if every matching in $F(P)$ satisfies the property.

We will say that agent a's preferences P_a are *smitten with agent* b if they rank b at the top, followed by a,[8] and then by the other agents of the other gender than a, in any order. Using this language, we obtain the following property:

[8] If $b = a$, then this second requirement is void.

Lemma 4. *Suppose mechanism f is stable, and let $P \in \mathcal{P}$ and $\mu = f(P)$. Suppose further that $Q \in \mathcal{P}$ is such that for every man, Q_m is smitten with his respective $\mu(m)$,*[9] *while $Q_w = P_w$ for every woman. Then, $f(Q) = \mu$.*

Proof. Note first that stable matchings satisfy the following version of Maskin's monotonicity: if μ is stable for P and the strict preference profile $R \in \mathcal{P}$ is such that $\mu(a)P_a b$ implies $\mu(a)R_a b$ for all agents, then μ is stable for R.[10] This implies that μ is stable for Q, so we only need to argue that no other matching is stable under the preference profile Q. To see that this is the case, just note that for each m, only $\mu(m)$ is acceptable under Q_m. □

Corollary 3. *Suppose mechanism f is stable, and let $P \in \mathcal{P}$ and $\mu = f(P)$. Suppose further that $Q \in \mathcal{P}$ is such that for every woman, Q_w is smitten with $\mu(w)$, while $Q_m = P_m$ for every man. Then, $f(Q) = \mu$.*

Proposition 5. *Truthtelling is a rationalizable strategy for all agents if a stable mechanism is used.*

Proof. We prove the result for an arbitrary woman, $\bar{w}$. The argument for a man is identical, *mutatis mutandis.*

Let P be the real profile of preferences, and denote $\mu = f(P)$. Let $\bar{w}$'s first-order beliefs be the profile $\hat{Q}_{\neg\bar{w}}$ defined as follows: $\bar{w}$ thinks that all men will play preferences $\hat{Q}_m$ that are smitten with their respective $\mu(m)$, while all women will report truthfully, $\hat{Q}_w = P_w$.

We first argue that reporting $Q_{\bar{w}} = P_{\bar{w}}$ is a best response, under $P_{\bar{w}}$, to $\hat{Q}_{\neg\bar{w}}$. If she reports $P_{\bar{w}}$, by Lemma 4 she is matched with $\mu(\bar{w})$, which is at least as good for her as being unmatched, since μ is individually rational under P. If, alternatively, $\bar{w}$ misreports preferences $Q_{\bar{w}}$ and forces the matching $\nu = f(Q_{\bar{w}}, \hat{Q}_{\neg\bar{w}})$, she cannot be matched to a man $m \neq \mu(\bar{w})$, since $\hat{Q}_m$ reports that only $\mu(m) \neq \bar{w}$ is acceptable to him and ν is individually rational under $(Q_{\bar{w}}, \hat{Q}_{\neg\bar{w}})$. It follows that if $\nu(\bar{w}) \neq \mu(\bar{w})$, then $\nu(\bar{w}) = \bar{w}$ and $\mu(\bar{w}) >_{\bar{w}} \bar{w}$, since preferences are strict.

To complete the argument, we need to define $\bar{w}$'s higher-order beliefs and argue that her k^{th}-order beliefs are justified by her $(k+1)^{\text{st}}$-order beliefs. Let the second-order beliefs be, simply, that every other agent shares their first-order beliefs. By the same argument that applies to her, $\bar{w}$'s first-order belief that other women will disclose their preferences truthfully is sustained by her second-order beliefs. However, we still need to argue that the same is true for men. Suppose that man $\bar{m}$ holds the beliefs $\hat{\hat{Q}}_{\neg\bar{m}}$ according to which every other man will play preferences $\hat{\hat{Q}}_m$ that are smitten with $\mu(m)$, while every woman will disclose her preferences truthfully, $\hat{\hat{Q}}_w = P_w$. If $\bar{m}$ discloses preferences smitten with $\mu(\bar{m})$, by Corollary 3 he is matched with $\mu(\bar{m})$. Alternatively, he can disclose other preferences $\hat{Q}_{\bar{m}}$ and force $\nu = f(\hat{Q}_{\bar{m}}, \hat{\hat{Q}}_{\neg\bar{m}})$. Obviously, $\bar{m}$ does not benefit

[9] That is, of the form $\hat{Q}_m = [\mu(m), m, \ldots]$.

[10] Note that $\mu(a)$ is at least as highly ranked by a under R_a as under P_a. This means that fewer objections arise to μ under R than under P.

from this if $\nu(\bar{m}) = \mu(\bar{m})$. Since mechanism f is stable, given $\hat{\hat{Q}}_{\neg\bar{m}}$ he can only force a different match for himself if it is to a woman w for whom $\bar{m}\hat{\hat{Q}}_w\mu(w)$, given that $\mu(w)$ declared to be smitten with w. The latter would imply $\bar{m}P_w\mu(w)$. For $\bar{m}$ to benefit from misreporting, that woman must satisfy $wP_w\mu(m)$, which is impossible since μ is stable for P.

Finally, for any $k \geq 3$, let $\bar{w}$'s k^{th}-order beliefs be that everybody else has the same $(k-1)^{\text{st}}$-order beliefs. By the previous arguments, the latter rationalizes the former. □

Pre-mechanisms fail to define a game form and are therefore insufficient for implementation. To overcome this issue, suppose that an extra agent is introduced who selects a matching in $F(\cdot)$ after all the agents have declared their preferences. Formally, this agent is a *non-deterministic mechanism without prior*. For brevity, we call it the *Chaperone*.

PROPOSITION 6. *Let pre-mechanism F be the correspondence that maps each preference profile $P \in \mathcal{P}$ into the set of all the matchings that are satisficing for P. If the Chaperone is indifferent between all the matchings, then truthtelling is a rationalizable strategy for all women and men.*

Proof. The argument is similar to the proof of Proposition 5, except that $\bar{w}$ must form beliefs on the matching that the Chaperone will choose.

Suppose the preferences are P, and let $\mu \in F(P)$. By definition, μ is satisficing so there exists a matching λ that is stable for P and such that $\lambda(\bar{w}) = \mu(\bar{w})$. Augment $\bar{w}$'s beliefs in the proof of Proposition 5 by the belief that the Chaperone will play λ, that every other agent thinks that the Chaperone will play λ, and so on. Since λ is stable for P, by the same argument as before, $Q_{\bar{w}} = P_{\bar{w}}$ is a best response for $\bar{w}$ to these augmented beliefs. The beliefs remain valid since $\lambda \in F(P)$ and the Chaperone is indifferent. □

6 A weaker solution concept

Fix the profile of preferences P again. An alternative definition is that a matching is weakly satisficing if every agent is paired with someone at least as desirable as an achievable partner. That is

DEFINITION 2. *A matching μ is* weakly satisficing *if for every a, there is a stable matching λ such that $\lambda(a) \leq_a \mu(a)$.*

Immediately, μ is weakly satisficing if, and only if, for every woman w, $\mu(w) \geq_w \mu_M(w)$, while $\mu(m) \geq_m \mu_W(m)$ for every man m.

Since every satisficing matching is weakly satisficing, it follows, again, that every stable matching is weakly satisficing.

Example 1 shows that satisficing matchings need not be stable. The following example shows that weakly satisficing matchings need not be satisficing.

Example 3. Using the same market as in Example 1, note that

$$\nu : \begin{matrix} m_1 & m_2 & m_3 & m_4 \\ w_2 & w_3 & w_4 & w_1 \end{matrix}$$

is weakly satisficing, since each agent is matched to someone at least as good as their worst achievable partner, but not satisficing because (w_2, m_1) is not an achievable pair.

Weakly satisficing matchings admit the same welfare bounds as satisficing and stable matchings.

Proposition 7. *For any weakly satisficing matching μ, $\mu_W \geq_W \mu \geq_W \mu_M$ and $\mu_W \leq_M \mu \leq_M \mu_M$.*

Proof. By definition of weakly satisficing matching, for each w there exists a stable matching λ for which $\lambda(w) \leq_w \mu(w)$. By definition of μ_M, $\mu_M(w) \leq_w \lambda(w)$, since λ is stable.

Suppose now that $\mu(w) >_w \mu_W(w)$. Assuming that $\mu(w) = m \in M$, it follows a stable matching ν such that $\nu(m) \leq_m w$. Since ν is stable, $\mu_W(m) \leq_m \nu(m)$. It follows that $w \geq_m \mu_W(m)$. Moreover, $\mu(w) >_w \mu_W(w)$ implies that $\mu(w) \neq \mu_W(w)$ and hence, $w \neq \mu_W(m)$. But since preferences are strict, $w >_m \mu_W(m)$, contradicting the fact that μ_W is stable.

To complete the proof, note that if $\mu(w) = w$, then $w \geq_w \lambda(w)$ in some stable matching. Since preferences are strict, this requires that $w = \lambda(w)$ in that stable matching2, w is unmatched in all stable matchings, so $\mu_W(w) = \mu_M(w) = \mu(w)$ and the result is trivial. □

On the other hand, other properties of satisficing matchings fail to extend.

Example 4. Suppose there are four women and four men. Their preferences are:

$$P(w_1) = m_1, m_3, m_4, m_2, w_1 \qquad P(m_1) = w_2, w_3, w_4, w_1, m_1$$
$$P(w_2) = m_2, m_4, m_3, m_1, w_2 \qquad P(m_2) = w_1, w_4, w_3, w_2, m_2$$
$$P(w_3) = m_3, m_1, m_2, m_4, w_3 \qquad P(m_3) = w_4, w_1, w_2, w_3, m_3$$
$$P(w_4) = m_4, m_2, m_1, m_3, w_4 \qquad P(m_4) = w_3, w_2, w_1, w_4, m_4$$

Note that

$$\mu_M : \begin{matrix} m_1 & m_2 & m_3 & m_4 \\ w_2 & w_1 & w_4 & w_3 \end{matrix}, \qquad \mu_W : \begin{matrix} m_1 & m_2 & m_3 & m_4 \\ w_1 & w_2 & w_3 & w_4 \end{matrix},$$

In this case, the matchings

$$\nu : \begin{matrix} m_1 & m_2 & m_3 & m_4 \\ w_4 & w_3 & w_2 & w_1 \end{matrix}, \qquad \lambda : \begin{matrix} m_1 & m_2 & m_3 & m_4 \\ w_3 & w_4 & w_1 & w_2 \end{matrix},$$

are both weakly satisficing, but λ strongly dominates ν.

While weakly satisficing, matching ν is a pretty bad arrangement: *all* couples agree that λ is better, so ν loses to λ *unanimously*.

Moreover, comparing ν with all the matchings that belong to the set of stable matchings in Example 1, we can see that not every weakly satisficing matching is a *majority assignment* as defined in Gärdenfors (1975): for ν, we can find a matching, λ_2, that is preferred by a majority to ν.

7 The college admissions problem

Let the set of applicants be A and the set of colleges be C. Each college c has a maximum number q_c of applicants it can admit. For the sake of simplicity, we assume that every college is acceptable for every applicant and vice versa, and that $|A| = \sum_{c \in C} q_c$. These assumptions imply that every applicant is accepted by some college and every college seat is filled. By Theorems 5.12 and 5.13 in RS, our results are robust to lifting the assumptions, as we argue in an appendix.

Each applicant a has strict preferences over the set of colleges, P_a, and each college c has preferences P_c over subsets of applicants. The structure of the latter is as follows. For any distinct pair of applicants, $b, d \in A$, P_c is strict, and we write $b >_c d$ to mean that college c prefers applicant b over applicant d.[11]. For any subsets $B, D \subseteq A$ with the same cardinality, $B >_c D$ means strict preference for B over D, but we also allow for weak preference $B \geq_c D$ even when the two sets are different. Importantly, we assume that preferences P_c are *responsive* in the following sense: for any subset $D \subseteq A$ and any $a, b \in A \setminus D$,

$$\{a\} \cup D >_c \{b\} \cup D \Leftrightarrow a >_c b.$$

This property implies that if $b >_c d$ for every $(b, d) \in B \times D$ and $|B| = |D|$, then $B >_c D$.

A *matching* is a mapping that assigns to each applicant a an element $\mu(a) \in C$, and to each college c a set $\mu(c) \subseteq A$ with $|\mu(c)| = q_c$, with the requirement that $\mu(a) = c$ if, and only if, $\mu(c) \ni a$.

Under our assumption of complete mutual acceptability, every matching is individually rational. Matching μ is *stable* if there does not exist a pair $\{a, c\}$ of an applicant and a college such that $c >_a \mu(a)$ and $a >_c b$ for some $b \in \mu(c)$.

We will say that college c is *achievable for applicant* a if there exists a stable matching μ for which $\mu(a) = c$. For colleges, there are two definitions of the property: applicant by applicant, or in subsets of applicants. We will say that *applicant a is achievable for college c* if there is a stable matching μ such that $\mu(c) \ni a$. Alternatively, *a subset B is achievable* for the college if there is a stable μ for which $\mu(c) \supseteq B$.[12]

Note that part of the definition of satisficing matchings is redundant in the case of a marriage market. We could have required that all women be matched to achievable partners, and the requirement would have been immediate for men, by Theorem 2.22 in RS. The same is not true

[11] The formal notation would be $\{b\} >_c \{d\}$, which seems unnecessary. Also, $b \geq_c d$ means that either $b >_c d$ or $b = d$, as before.

[12] Obviously, if B is achievable and $a \in B$, then a is achievable.

in the college admissions problem, so we have two definitions of satisficing matchings: one where we require that all applicants be allocated to an achievable college, and one where the condition is that *the set of applicants* allocated to each college be achievable to it.

DEFINITION 3. *Matching μ is:*

1. satisficing *if for each applicant a, there exists a stable matching λ such that $\lambda(a) = \mu(a)$, and for each college c and each $b \in \mu(c)$ there exists a stable matching λ such that $\lambda(c) \ni b$; and,*
2. strongly satisficing *if the same requirement is satisfied for each applicant while for each college c there exists a stable matching λ such that $\lambda(c) = \mu(c)$.*

Again, due to Roth's Rural Hospital Theorem, part of each definition is redundant. For the weak definition, imposing the condition only on the applicants suffices, by Theorem 5.13 in RS. Theorem 5.12 in RS implies that the condition imposed on colleges is enough for the strong definition.

The stronger definition of satisficing matching would be the only valid one in situations where the colleges (hospitals, or firms) have preferences that exhibit externalities between admitted applicants (or interns, or workers). In the case of responsive preferences considered here, both definitions have merit.

7.1 Connection to the marriage problem

Using the construction of Subsection 5.3 in RS, each matching μ in the college admissions problem is associated with a matching μ' in the marriage problem where each college c is replicated q_c times and each replica can admit only one applicant.

LEMMA 5. *Matching μ is satisficing if, and only if, the associated matching μ' is satisficing in the marriage market.*

Proof. It suffices that we develop the argument for the applicants.

For sufficiency, fix an applicant a. Since μ is satisficing, there exists a stable λ for which $\lambda(a) = \mu(a)$. By Lemma 5.6 in RS, the associated matching λ' is stable in the marriage problem, and $\lambda'(a) = \lambda(a)$. It follows that $\lambda'(a) = \mu(a) = \mu'(a)$.

For necessity, again let a be an applicant. Since μ' is satisficing in the associated marriage market, there is a matching λ', stable in that same market, such that $\lambda'(a) = \mu'(a)$. The matching λ that generates λ' is stable in the college admissions problem, and $\lambda(a) = \lambda'(a)$. As before, $\lambda(a) = \mu(a)$. □

Completing the argument of the if part with its redundant portion is instructive. Let $c \in C$ and $b \in \mu(c)$. By assumption, there is a stable matching λ for which $\lambda(c) \ni b$. Again, by RS Lemma 5.6, matching λ' is stable in the marriage market and $\lambda'(c) = b$. Since c and $b \in \mu(c)$ were arbitrary, we have that for all $c \in C$ and all $b \in \mu(c)$, there exists λ', stable in the marriage market, such that $\lambda'(c) \ni b$.

However, if for the same c we pick $d \in \mu(c) \setminus \{b\}$, the fact that μ is satisficing implies that for some stable ν, we have that $\nu(c) \ni d$. Without a property that guarantees that $\nu = \lambda$, the argument would be insufficient for the claim that if μ' is satisficing, then μ is strongly satisficing.

Of course, the only if part continues to hold under the strong definition of satisficing matchings:

Corollary 4. *Matching μ is strongly satisficing only if the associated matching μ' is satisficing in the marriage market.*

7.2 Efficiency

The following is an extension of Lemma 2.

Lemma 6. *There exists a sequence $\langle \lambda_i \rangle_{i=1}^{I}$ of stable matchings such that*

$$\lambda_1 <_A \lambda_2 <_A \ldots <_A \lambda_I,$$

$$\lambda_1 \geq_C \lambda_2 \geq_C \ldots \geq_C \lambda_I,$$

and:

1. *if μ is satisficing, then for every applicant a there is an index i for which $\lambda_i(a) = \mu(a)$; while*
2. *if μ is strongly satisficing, then for every college c there is an index i for which $\lambda_i(c) = \mu(c)$.*

Proof. This follows from Lemma 5.6 and Theorems 3.20 and 5.23 in RS. □

As in the case of the marriage market, this result allows us to obtain efficiency properties for satisficing matchings.

Proposition 8 (Efficiency). *Any strongly satisficing matching is efficient.*

Proof. Let μ be strongly satisficing and suppose that ν dominates μ. Due to our assumption that everyone is acceptable and the Rural Hospital Theorem, it suffices to restrict our attention to ν where all applicants are matched and no school has empty seats. As in the proof of Proposition 2, let $E = \{b \in A \cup C \mid \nu(b) = \mu(b)\}$, and note that if $c \in C \setminus E$, then $\mu(c) \setminus E \neq \varnothing$.

Using the sequence of stable matchings of Lemma 6, let i be the largest index for which $\lambda_i(c) = \mu(c)$ for some college $c \in C \setminus E$, and let c be one such college.

Let $a \in \mu(c) \setminus E$ and $\bar{c} = \nu(a)$. Since ν dominates μ, $a \in A \setminus E$, and P_a is strict, then $\bar{c} >_a c$. Since λ_i is stable, it must be true that

$$\forall b \in \lambda_i(\bar{c}), b \geq_{\bar{c}} a. \tag{4}$$

Invoking again the fact that μ is satisficing, let j be such that $\lambda_j(\bar{c}) = \mu(\bar{c})$. Since both λ_i and λ_j are stable and $\lambda_i(\bar{c}) \neq \lambda_j(\bar{c})$, Theorems 5.26 and 5.27 in RS imply that either

$$\forall b \in \lambda_i(\bar{c}) \setminus \lambda_j(\bar{c}), \forall d \in \lambda_j(\bar{c}) \setminus \lambda_i(\bar{c}), b >_{\bar{c}} d \tag{5}$$

or

$$\forall b \in \lambda_i(\bar{c}) \setminus \lambda_j(\bar{c}), \forall d \in \lambda_j(\bar{c}) \setminus \lambda_i(\bar{c}), d >_{\bar{c}} b. \tag{6}$$

By construction, $a \in \nu(\bar{c})$, while by dominance of ν over μ, we further have that $\nu(\bar{c}) \geq_{\bar{c}} \lambda_j(\bar{c})$. Together with responsiveness of $P_{\bar{c}}$ and Eq. (4), this implies that Eq. (5) cannot be true, so Eq. (6) must be true. Hence $\lambda_i(\bar{c}) >_{\bar{c}} \lambda_j(\bar{c})$.

The latter requires, again by construction, that $i < j$, which is impossible since i was maximal and $\bar{c} \in C \setminus E$. □

If a matching is only (weakly) satisficing, there is no guarantee that the set of applicants allocated to a college corresponds to the set that would be allocated under some stable matching, so Theorems 5.26 and 5.27 in RS cannot be invoked, and the argument fails. Still,

PROPOSITION 9 (Weak efficiency). *Any satisficing matching is weakly efficient.*

Proof. Suppose that μ is satisficing and strongly dominated by ν. Using the sequence of Lemma 6, let i be the smallest index such that $\lambda_i(c) \ni \mu(c)$ for some college, and let c be one such college.

By assumption $\nu(c) \cap \mu(c) = \varnothing$ and $\nu(c) >_c \mu(c)$, since ν is strongly dominant. Since P_c satisfies responsiveness, there must exists $\bar{a} \in \nu(c)$ such that $\bar{a} >_c b$ for all $b \in \mu(c)$. Fix one such $\bar{a}$, and note that, since $\mu(c) \cap \lambda_i(c) \neq \varnothing$ and λ_i is stable, it must be true that $\lambda_i(\bar{a}) >_{\bar{a}} c$.

Using again the fact that μ is satisficing, there must exist an index j such that $\lambda_j(\bar{a}) = \mu(\bar{a})$, and since ν dominates μ, $\nu(\bar{a}) >_{\bar{a}} \mu(\bar{a})$. Since $\nu(\bar{a}) = c$, the latter implies that $\lambda_i(\bar{a}) >_{\bar{a}} \lambda_j(\bar{a})$. This requires $j < i$, again an impossibility. □

7.3 Decomposition

As with the efficiency properties, the stronger definition of satisficing matching displays a stronger decomposition property. On one hand,

PROPOSITION 10 (Weak decomposition). *Let μ and ν be distinct strongly satisficing matchings. There exist two pairs $\{c, B\}$, with $c \in C$ and $B \in \{\mu(c) \setminus \nu(c), \nu(c) \setminus \mu(c)\}$, such that c prefers one of the two matchings and all the applicants in B prefer the other.*

Proof. The argument resembles the proof of Proposition 3. Let C' be the set of colleges for which $\mu(c) \neq \nu(c)$. By assumption, this set contains at least two elements.

Using again the sequence of Lemma 6, define, for each $c \in C'$: (1) $i^*(c)$ as the largest i for which $\lambda_i(c) = \mu(c)$; (2) $j^*(c)$ as the largest j for which $\lambda_j(c) = \nu(c)$; (3) $i^* = \max\{i^*(c) : c \in C'\}$; (4) $j^* = \max\{j^*(c) : c \in C'\}$; and (5) $k = \max\{i^*, j^*\}$. With no loss of generality, suppose that for college c^*, $\lambda_k(c^*) = \mu(c^*)$. By construction, for some $j < k$, $\lambda_j(c^*) = \nu(c^*)$. By theorem 5.26, since $j < k$, it must be true that $\nu(c^*) >_{c^*} \mu(c^*)$. If we let $a \in \mu(c^*) \setminus \nu(c^*)$, there must be some $i < k$ such that $\lambda_i(a) = \nu(a)$. Since $i < k$, $\lambda_i <_A \lambda_k$, so it follows that $\mu(a) >_a \nu(a)$.

The second pair can be constructed using the other end of the sequence, as in the proof of Proposition 3 and Proposition 11 below. □

On the other hand,

PROPOSITION 11 (Weaker decomposition). *Let μ and ν be distinct satisficing matchings. There exist two pairs $\{a, c\}$, with $a \in A$ and $c \in \{\mu(a), \nu(a)\}$, such that a prefers one of the two matchings and c prefers the applicant that lost the seat to a in that matching.*

Proof. Let A' be the set of applicants for whom $\mu(a) \neq \nu(a)$ and, invoking once again the sequence of Lemma 6, define: (1) $i_*(a)$ as the lowest i for which $\lambda_i(a) = \mu(a)$; (2) $j_*(a)$ as the lowest j for which $\lambda_j(a) = \nu(a)$; (3) $i_* = \min\{i_*(a) : a \in A'\}$; (4) $j_* = \min\{j_*(a) : a \in A'\}$; and (5) $\ell = \max\{i_*, j_*\}$. Again, we can find some woman a_* for whom, with no loss of generality, $\lambda_\ell(a_*) = \nu(a_*)$. Then, for some $i > \ell$ we must have $\lambda_i(a_*) = \mu(a_*)$, and hence $\mu(a_*) >_{a_*} \nu(a_*)$. Letting $c_* = \nu(a_*)$, we must have that $a_* \in \nu(c_*) \setminus \mu(c_*)$ and, hence, for some $b \in \mu(c_*) \setminus \nu(c_*)$ and some $j > \ell$, $\lambda_j(c_*) \ni b$. Then, $\lambda_j \leq_C \lambda_\ell$ and $\lambda_j(c_*) \neq \lambda_\ell(c_*)$ imply, by Theorem 5.26 in RS, that $b <_{c_*} a_*$.

Once again, the other pair can be constructed using the higher sequence indexes, as in the proofs of Proposition 3 and Proposition 10. □

8 Concluding remarks

We have proposed a novel solution concept for matching problems that weakens the rationality premise of stable matchings in the direction promoted by Simon (1956). There is empirical evidence that justifies our exercise, but one could worry that the elegant and useful structure imposed by stability may be lost. We argue that this is not the case, since our satisficing matchings display weaker versions of the properties exhibited by stable matchings. Table 1 summarizes these properties for the case of one-to-one matchings (with strict preferences).

In the case of many-to-one problems, two definitions of satisficing matchings are conceivable. Under the stronger one, where the concept is applied to the whole set of agents matched to the parties, the same efficiency and decomposition properties of the one-to-one case hold (Propositions 8 and 10). A weaker definition of satisficing matching further weakens these properties (Propositions 9 and 11). We leave for future research to explore whether these results can extend when we impose weaker conditions than responsiveness, such as substitutability or quota-separabilty restrictions that have been widely used in this literature.

Table 1: The properties of stable and satisficing matchings in marriage markets

PROPERTY	*For stable matchings*	*For satisficing matchings*
EXISTENCE	Gale and Shapley (1962).	Immediately implied by Gale and Shapley (1962).
BOUNDS	μ_W and μ_M (Knuth, 1976)	μ_W and μ_M, by Proposition 1
WELFARE	Matching μ is stable if, and only if, it is in the core (RS, Theorem 3.3)	Matching μ is satisficing only if it is efficient, by Proposition 2
DECOMPOSITION	If μ and ν are stable, all couples disagree in how they rank them (RS, Lemma 2.20).	If μ is satisficing and ν is stable, all couples formed under μ disagree in their ranking; if ν is only satisficing, at least two couples do. See Proposition 3
VOTING	If μ is stable, then it is a majority assignment (Gärdenfors, 1975).	If μ is satisficing and ν is stable, the same number of people vote for each; if ν is only satisficing, no unanimous choice between μ and ν is possible. See Corollaries 1 and 2.
STRUCTURE	The set of stable matchings is a distributive lattice with respect to the common order of the women (Conway; see Knuth, 1976).	The set of satisficing matchings is an abelian semigroup when equipped with the sup operator of the women; moreover, the set (monoid) of stable matchings is the ideal it generates. See Proposition 4.
IMPLEMENTATION	No mechanism that implements stable matchings induces truthtelling in Nash equilibrium (Roth, 1986); such a mechanism exists in rationalizable strategies, however (Proposition 5).	The satisficing correspondence together with an impartial selector (our Chaperone) induces truthtelling in rationalizable strategies, by Proposition 6.

Appendix: Algebraic structures

A *partially ordered set*, or *poset*, is an algebraic structure $(X, \geq)$ consisting of a set X and a partial order $\geq$ on X.

A poset is a *lattice* if each two-element subset $\{x, y\}$ has a least upper bound, $x \vee y$, and greatest lower bound, $x \wedge y$. Equivalently, a lattice is an algebraic structure $\mathcal{L} = (L, \vee, \wedge)$ consisting of a set L and two binary, commutative, and associative operations $\vee$ and $\wedge$ on L such that for any two $x, y \in L$, $x \vee (x \wedge y) = x$ and $x \wedge (x \vee y) = x$. A lattice is *distributive* if for all x, y, z in L, $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$.

A *semigroup* is an algebraic structure $\mathcal{S} = (S, *)$ consisting of a set S and an associative binary operation $*$ on S. If the operation is commutative, the semigroup is said to be *abelian.* A semigroup is a *monoid* if it has an *identity*:

$$\exists 1 \in S : \forall x \in S, x * 1 = 1 * x = x.$$

If a semigroup is a monoid, the set S^1 is defined to be equal to S; otherwise, it is $S^1 = S \cup \{1\}$. (A *group* is a monoid in which each element has an *inverse.*[13])

The semigroup operation induces an operation on the collection of its subsets: given subsets X and Y of S, $X * Y = \{x * y \mid x \in X, y \in Y\}$. A subset X is a *subsemigroup* if $X * X \subseteq X$. A subset I is an *ideal* if $I * S = I$ and $S * I = I$. The *ideal generated by subset* X is the set $S^1 * X * S^1$. It is the smallest ideal of $\mathcal{S}$ containing X.

If $\mathcal{S} = (S, *)$ and $\mathcal{T} = (T, \times)$ are semigroups, a mapping $f : S \to T$ is a *semigroup morphism* if for every $x, x' \in S$, $f(x * x') = f(x) \times f(x')$. Semigroup $\mathcal{T}$ is a *quotient* of $\mathcal{S}$ if there exists a surjective semigroup morphism $f : S \to T$. Also, $\mathcal{T}$ *divides* $\mathcal{S}$, denoted $\mathcal{T} \preccurlyeq \mathcal{S}$, if $\mathcal{T}$ is a quotient of a subsemigroup of $\mathcal{S}$.

Appendix: The college admissions problem with unacceptable agents

Each applicant a has strict preferences over the set of colleges and not going to college, P_a, so we write $c >_a a$ or $a >_a c$ to denote whether applicant a prefers attending c over not going to college, or the other way around. Each college c has preferences P_c over the set of applicants and leaving seats unfilled: $a >_c c$ means that c prefers admitting a to leaving a seat unfilled, while and $c >_c b$ means that c prefers leaving a seat unfilled over admitting b.

Abusing notation slightly, let $\{c\}^q$ be the multiset that (only) contains c with multiplicity q.[14] A

[13] $\forall x \in S, \exists x^{-1} \in S : x^{-1} * x = x * x^{-1} = 1$.

[14] This is that RS call a *unordered family of elements*: $\{c\}^q = \{c, c, \ldots, c\}$ with cardinality q.

matching is a mapping that assigns to each applicant a an element $\mu(a) \in C \cup \{a\}$, and to each college c a multiset $\mu(c) \subseteq A \cup \{c\}^{q_c}$ such that: (1) $|\mu(c)| = q_c$ and for every $a \in \mu(c) \cap A$ the multiplicity is 1; and (2) $\mu(a) = c$ if, and only if, $\mu(c) \ni a$.

Matching μ is *individually rational for applicant* a if $\mu(a) \geq_a a$, and *for college* c if $a \geq c$ for all $a \in \mu(c)$. It is *individually rational* if it is so for all agents, and *stable* if, in addition, there do not exist a pair $\{a, c\}$ of an applicant and a college such that $c >_a \mu(a)$ and $a >_c b$ for some $b \in \mu(c)$.

Theorem 5.12 in RS guarantees that the same set of applicants is admitted and the same set of college seats is filled at all stable matchings. Theorem 5.13 further states that if a college fails to fill all its seats at one stable matching, then that college is allocated precisely the same set of applicants at all stable matchings.

Applicants who are not accepted and schools that fail to fill all their seats at some matching play the same role as hopeless agents in a marriage market: they do not affect the satisficing-ness of a matching. If an applicant is not admitted in some matching, she will remain so at all satisficing matchings. A college that does not fill all its seats at some stable matching will receive precisely the same set of applicants at all satisficing matchings. It is in this sense that the assumptions imposed in Section 7 are without loss.

Appendix: Results from Roth and Sotomayor (1990)

Except for notation and context, the following results are from RS:

1. Lemma 2.20: Let μ and ν be stable matchings in a marriage market where all preferences are strict. Let $W(\mu)$ be the set of women who prefer μ to ν, and let $M(\nu)$ be the set of men who prefer ν to μ. Then μ and ν map $W(\mu)$ onto $M(\nu)$.

2. Corollary 2.21: Under the premises of Lemma 2.20, further define $W(\nu)$ as the set of women who prefer ν to μ, and $M(\mu)$ as the set of men who prefer μ to ν. Then μ and ν map $W(\mu)$ onto $M(\nu)$ and $W(\nu)$ onto $M(\mu)$.

3. Theorem 2.22: In a marriage market with strict preferences, the set of people who are single is the same for all stable matchings.

4. Theorem 3.3: The core of a marriage market equals the set of stable matchings.

5. Theorem 3.8: When preferences are strict, the set of stable matchings of a marriage market is a distributive lattice under the common order of the women, dual to the common order of the men.

6. Theorem 3.9: Every finite distributive lattice equals the set of stable matchings of some marriage market.

7. Theorem 3.20: In a marriage market, let $\mu_M = \lambda_1 <_W \lambda_2 <_W \ldots <_W \lambda_I = \mu_W$ be the sequence

of stable matchings obtained by the algorithm in Gusfield (1988).[15] Then every achievable pair appears in at least one of the matchings in the sequence.

8. Lemma 5.6: A matching of the college admissions problem is stable if, and only if, the corresponding matchings of he related marriage market are stable.

9. Theorem 5.12: In the college admissions problem with strict preferences, the set of applicants admitted and seats filled is the same at every stable matching.

10. Theorem 5.13: In the college admissions problem with strict preferences, any college that does not fill its quota at some stable matchings is assigned precisely the same set of applicants at every stable matching.

11. Theorem 5.26: In the college admissions problem with strict preferences, if μ and ν are stable matchings, then college c is indifferent between $\mu(c)$ and $\nu(c)$ only if $\mu(c) = \nu(c)$.

12. Theorem 5.27: In the college admissions problem with strict preferences, if μ and ν are stable matchings, then for every college c, $\mu(c) >_c \nu(c)$ only if $a >_c b$ for every applicant $a \in \mu(c)$ and every $b \in \nu(c) \setminus \mu(c)$.

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[15] If μ is a stable matching, $P(\mu)$ is the profile of *reduced preferences* obtained by moving in each woman w's preferences all the men such that $\mu(w) >_w m >_w w$ and placing them below w, and doing the same for men. The algorithm is stated as follows: *Step 1:* Find μ_W and μ_M using the deferred acceptance algorithm of Gale and Shapley (1962), and find $P(\mu)$; *Step k:* For each profile Q of reduced preferences obtained in step $(k-1)$, find one corresponding cycle (if none exists, stop) and obtain the corresponding μ for Q. Then obtain the profile of reduced preferences $P(\mu)$.

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