Modified TMD Factorization and Sub-leading Power Corrections

Sergio Leal Gómez
work in preparation with Massimiliano Procura
(University of Vienna)

Madrid, 11.05.2022

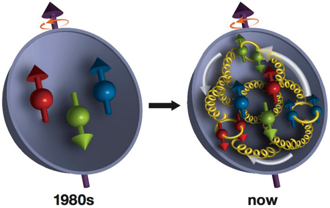
∫dkΠ Doktoratskolleg Particles and Interactions
FWF Der Wissenschaftsfonds.
universität wien

OUTLINE

1. INTRODUCTION
2. COLLINEAR FACTORISATION
3. TMDPDF FACTORISATION
4. POWER CORRECTIONS
5. SUMMARY & OUTLOOK

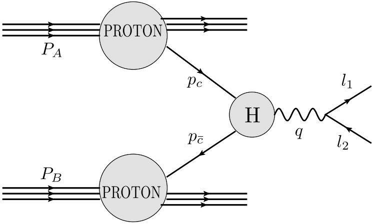
QCD as theory of hadrons

- So far QCD seems to be our best theory to describe hadrons.
- Its non-perturbative nature makes tough the comparison between theory and experiments.
- We do not know how the proton, neutron or pion wave function looks like.
- Extremely hard to compute the proton mass or spin starting from quarks and gluons.



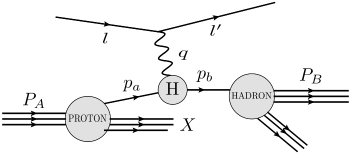
QCD AS THEORY OF HADRONS

Drell-Yan



$$d\sigma \sim H_{\text{DY}} \otimes \mathcal{F} \otimes \mathcal{F}$$

Semi-Inclusive Deep Inelastic Scattering

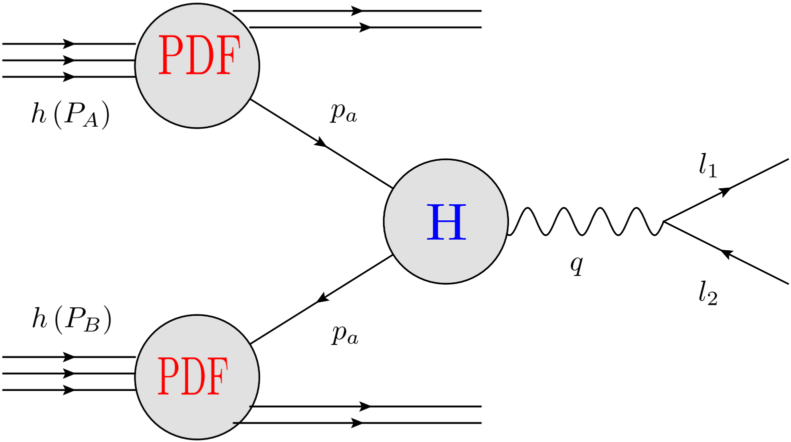


$$d\sigma \sim H_{\text{SIDIS}} \otimes \mathcal{F} \otimes \mathcal{D}$$

FACTORISATION THEOREMS

- Factorisation theorems are a powerful to split the large distance contributions (non-perturbative) and the short distances contributions (perturbative)
- The non-perturbative contribution is encode in distributions $\mathcal{F}$ and $\mathcal{D}$, which we extract from experimental data.
- The distributions $\mathcal{F}$ and $\mathcal{D}$ could be scheme dependent, differing in a perturbative finite amount. A standard choice is the $\overline{\mathsf{MS}}-$scheme.
- The perturbative contribution can be computed in perturbation theory using the standard QCD rules.
- So far there are two approach: factorisation at large $\mathbf{q}_T^2/Q^2$ or Collinear Factorisation and small $\mathbf{q}_T^2/Q^2$ or TMD factorisation

Collinear Factorization



$$\frac{d\sigma_{h_A h_B \to l l' X}}{dQ^2 dy dq_T^2} \sim H\left(Q^2, \alpha_s\left(\mu^2\right)\right) \otimes f_{c \leftarrow h_A}\left(\alpha_s\left(\mu^2\right), z_a\right) \otimes f_{\bar{c} \leftarrow h_B}\left(\alpha_s\left(\mu^2\right), z_b\right)$$

COLLINEAR FACTORIZATION

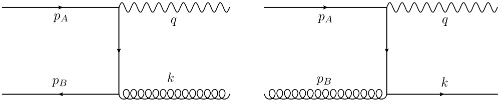
From the pioneering work of Sterman et al. Nucl. Phys. B **250** (1985), 199-224; Davies et al. Nucl. Phys. B**256** (1985), 413; Collins et al. Adv. Ser. Direct. High Energy Phys. **5** (1989), 1-91; Catani et al. Nucl. Phys. B **596** (2001), 299-312· · ·

$$\frac{d\sigma_{h_A h_B \to ll' + X}}{dQ^2 dy dq_T^2} = \sum_{a,b,c} \int_{x_A}^{1} \frac{dz_a}{z_a} \int_{x_B}^{1} \frac{dz_b}{z_b} H_{c\leftarrow a, \bar{c}\leftarrow b}\left(\alpha_S\left(Q^2\right), q_T^2, \frac{x_A}{z_a}, \frac{x_B}{z_b}\right)$$
$$\times f_{a\leftarrow h_A}\left(\alpha_S\left(Q^2\right), z_a\right) f_{b\leftarrow h_B}\left(\alpha_S\left(Q^2\right), z_b\right)$$

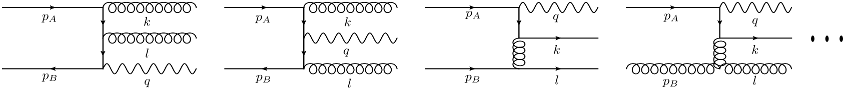
- $f_{c\leftarrow a}\left(\alpha_S\left(Q^2\right), z_a\right)$ is the PDF (Parton Distribution Function).
- $H_{c\leftarrow a, \bar{c}\leftarrow b}$ is hard scattering coefficient, it encodes the short distant behaviour.
- The renormalization scale μ^2 is set to Q^2.
- The PDFs need to be fitted to experimental data, while hard scattering coefficient can be computed in perturbative QCD
- $z_{a(b)}$ is the ratio between hadron and parton momentum.
- $x_{A(B)} = \sqrt{Q^2}/\sqrt{s}e^{\pm y}$

COLLINEAR FACTORIZATION

NLO



NNLO



COLLINEAR FACTORIZATION

In what kinematic regime can we expect collinear factorization to work?

- The lepton invariant mass $\sqrt{Q^2}$ has to be much bigger that the hadron mass M (or Λ_{QCD}).
- The remnants from both colliding hadrons are boosted and well separated in rapidity regions, i.e. we could expect that the interaction between them is negligible.
- We assume that the momentum of the emerging partons is parallel to the momentum of the colliding hadrons.
- The emerging partons are on-shell.
- At NNLO and higher orders the phases space integral are extremely cumbersome, only analytical calculation in the limit $q_T^2/Q^2 \ll 1$ are known.

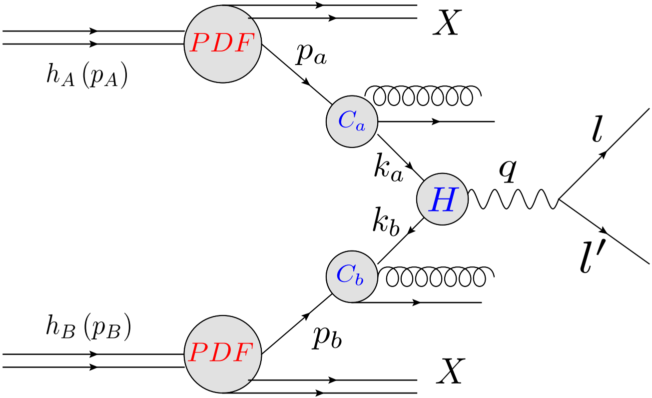
COLLINEAR FACTORIZATION

- Large value of Q^2 allow us to compute $H_{c\bar{c}}\left(\alpha_S\left(\mu^2\right)\right)$ in powers of $\alpha_S\left(\mu^2\right)$.
- However $q_T^2/Q^2 \ll 1$ spoils the convergences pattern of the perturbative series, i.e.

$$H = 1 + H^{[1,1]}\alpha_S\left(Q^2\right)\ln\frac{Q^2}{q_T^2} + \alpha_S\left(Q^2\right)H^{[1,2]}\ln^2\frac{Q^2}{q_T^2} + \alpha_S\left(Q^2\right)^2 H^{[2,4]}\ln^4\frac{Q^2}{q_T^2} + \cdots$$

- Even if $\alpha_S\left(Q^2\right)$ is small, $\alpha_S\left(Q^2\right)^n\ln^k\frac{Q^2}{q_T^2}$ could be large. We need a way to rearrange all the logarithmic terms (resummation of large logs) such that they do not spoil the convergence.

COLLINEAR FACTORIZATION



$$\frac{d\sigma_{h_A h_B \to l l' X}}{dQ^2\, dy\, dq_T^2} = \sum_{c,a,b} \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{b}_T\cdot\mathbf{q}_T}\, S\, C_{c\leftarrow a} \otimes f_{a\leftarrow h_A}\, C_{\bar{c}\leftarrow b} \otimes f_{b\leftarrow h_B}$$

COLLINEAR FACTORIZATION: RESUMMATION

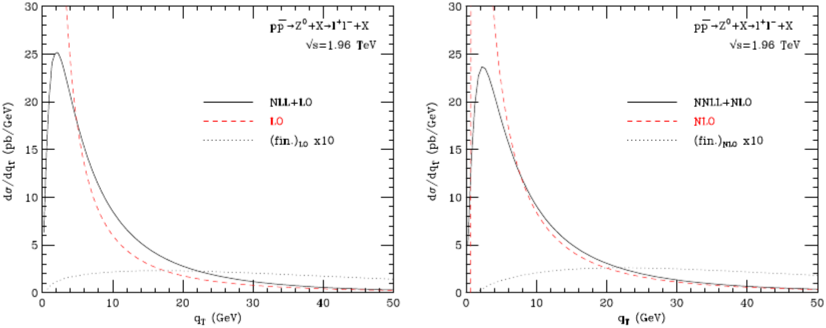
It is well know that at the limit $q_T^2/Q^2 \ll 1$ the cross section can be written as Sterman et al. Nucl. Phys. B **250** (1985), 199-224; Davies et al. Nucl. Phys. B**256** (1985), 413, Catani et al. Nucl. Phys. B **596** (2001), 299-312$\cdots$

$$
\begin{aligned}
\frac{d\sigma_{h_A h_B \to l l' X}}{dQ^2 dy dq_T^2} = \sum_{a,b,c} \int_{x_A}^{1} \frac{dz_a}{z_a} \int_{x_B}^{1} \frac{dz_b}{z_b} \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{b}_T \cdot \mathbf{q}_T} \\
\times \exp\left\{ -\int_{\mu_i^2}^{\mu_f^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \Gamma_{\text{cusp}}\left(\alpha_S\left(\bar{\mu}^2\right)\right) \ln\frac{Q^2}{\bar{\mu}^2} + B\left(\alpha_S\left(\bar{\mu}^2\right)\right) \right\} \\
\times C_{c\leftarrow a}\left(\alpha_S\left(\mu_i^2\right), \frac{x_A}{z_a}\right) C_{\bar{c}\leftarrow b}\left(\alpha_S\left(\mu_i^2\right), \frac{x_B}{z_b}\right) \\
\times f_{a\leftarrow h_A}\left(\alpha_S\left(\mu_i^2\right), z_a\right) f_{b\leftarrow h_B}\left(\alpha_S\left(\mu_i^2\right), z_b\right)
\end{aligned}
$$

where $\mu_i^2 = 4e^{-2\gamma_E}/b_T^2$ and $\mu_f^2 = Q^2$. The exponential factor is the well known Sudakov form factor $S\left(\mu_i^2, \mu_f^2\right)$. The factor H is absorbed into the coefficients C.

Collinear Factorization: Resummation

Catani et al. Phys. Lett. B **696** (2011), 207-213

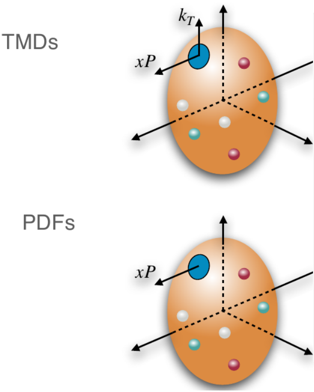


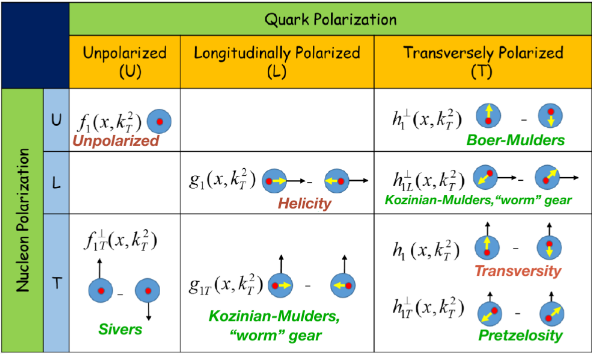
The resummation is responsible for the turn-around at small q_T.

COLLINEAR FACTORIZATION: LIMITATIONS

- The emerging partons are assumed to be parallel to the incoming hadrons.
- The partons are on-shell.
- Each element in the factorization formula correspond to a certain kinematic limit in the phase space.
- No field theoretic definition of the coefficients in the factorized cross section.
- Provide only have a 1-dimensional picture of the hadronic structure function, through the PDFs. A better understanding of hadronic properties like spin demands a 3-dimensional picture, E.Leader, M. Anselmino, Z. Phys. C40 (1988) 239.

TMDPDF





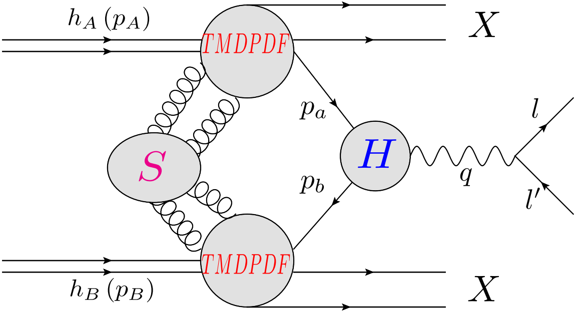
		Quark Polarization		
		Unpolarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1(x,k_T^2)$ - *Unpolarized*		$h_1^\perp(x,k_T^2)$ - *Boer-Mulders*
	L		$g_1(x,k_T^2)$ - *Helicity*	$h_{1L}^\perp(x,k_T^2)$ - *Kozinian-Mulders,“worm” gear*
	T	$f_{1T}^\perp(x,k_T^2)$ - *Sivers*	$g_{1T}(x,k_T^2)$ - *Kozinian-Mulders, “worm” gear*	$h_1(x,k_T^2)$ - *Transversity* $h_{1T}^\perp(x,k_T^2)$ - *Pretzelosity*

TMD FACTORIZATION IN SCET

Scimemi et al. JHEP **07** (2012), 002; Becher and Neubert, EPJC **71** (2011), 1665
Soft Collinear Effective Theory allowed us to derive a factorization formula where

- The emerging partons are not parallel to the incoming hadron and are off-shell.
- The partons from the TMDPDFs have a non-negligible transverse momentum $\mathbf{p}_{Ta(b)}$
- All ingredients can be written as matrix element of QFT operators
- The transverse momentum has to be smaller than the collinear component of the emerging parton, i.e. we are in the limit $q_T^2/Q^2 \ll 1$.

TMD FACTORIZATION IN SCET



$$\frac{d\sigma_{h_A h_B \to l l' X}}{dQ^2\, dy\, dq_T^2} =$$

$$\sum_c \sigma^{\text{Born}} H\left(\alpha_S\left(Q^2\right)\right) \int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{b}_T\cdot\mathbf{q}_T} F_{c\leftarrow h_A}\left(\alpha_S\left(Q^2\right), b_T^2, x_A\right) F_{\bar{c}\leftarrow h_B}\left(\alpha_S\left(Q^2\right), b_T^2, x_B\right)$$

TMD FACTORIZATION IN SCET

Scimemi et al. JHEP **07** (2012), 002; JHEP **07** (2015), 158; Phys. Rev. D **90** (2014) no.1, 014003; Phys. Lett. B **726** (2013), 795-801; EPJC **73** (2013) no.12, 2636; JHEP **09** (2016), 004

$$\frac{d\sigma_{h_A h_B \to l l' X}}{dQ^2 dy dq_T^2} = \sum_c \sigma^{\text{Born}} H\left(\alpha_S\left(Q^2\right)\right)$$
$$\int \frac{d^2\mathbf{b}_T}{(2\pi)^2} e^{i\mathbf{b}_T\cdot\mathbf{q}_T} F_{c\leftarrow h_A}\left(\alpha_S\left(Q^2\right), b_T^2, x_A\right) F_{\bar{c}\leftarrow h_B}\left(\alpha_S\left(Q^2\right), b_T^2, x_B\right)$$

where $x_{A(B)} = \sqrt{Q^2+q_T^2}/\sqrt{s}e^{\pm y}$. The factor $S\left(\alpha\left(Q^2\right), b_T^2\right)$ is absorbed by the TMDPDF.

$$\frac{dF_{a\leftarrow h_A}\left(\alpha_S\left(\mu^2\right), b_T^2, x_A, \mu^2, \zeta\right)}{d\ln\mu^2} = \frac{1}{2}\gamma_q\left(\alpha_S\left(\mu^2\right), \mu^2, \zeta\right) F_{a\leftarrow h_A}\left(\alpha_S\left(\mu^2\right), b_T^2, x_A, \mu^2, \zeta\right)$$
$$\frac{dF_{a\leftarrow h_A}\left(\alpha_S\left(\mu^2\right), b_T^2, x_A, \mu^2, \zeta\right)}{d\zeta} = -\mathcal{D}\left(\alpha_S\left(\mu^2\right), \mu^2, b_T^2\right) F_{a\leftarrow h_A}\left(\alpha_S\left(\mu^2\right), b_T^2, x_A, \mu^2, \zeta\right)$$

TMD factorization in SCET: Advantages

- The emerging partons have transverse momentum.
- TMDPDFs are universal non-perturbative objects.
- The rearranged of the logarithmically-enhanced contributions (resummation) through RGEs (Renormalization Group Equations).
- The TMDPDF describes the non-perturbative source of transverse momentum.
- TMDPDF gives the probability to find a parton inside a hadron with a given transverse momentum $\mathbf{p}_{Ta(b)}$ and a certain amount $x_{A(B)}$ of the of the collinear momentum of the parent hadron.
- TMDPDF offers a 3-dimensional picture crucial to understand the hadron spin and phenomenology that involves polarised hadrons and partons.

Power corrections in Drell-Yan

Partonic cross section in Drell-Yan process

$$\frac{d\sigma}{dQ^2 dy d\mathbf{q}_T^2} = \sigma^{\text{Born}} +$$

$$\frac{1}{\mathbf{q}_T^2} \sum_{n=1} \alpha_s^n \frac{d\sigma^{[n,-1]}}{dQ^2 dy d\mathbf{q}_T^2} + \delta^{(2)}(\mathbf{q}_T) \sum_{n=1} \alpha_s^n \frac{d\sigma^{[n,0]}}{dQ^2 dy d\mathbf{q}_T^2} + \frac{1}{Q^2} \sum_{n=1} \left(\frac{\mathbf{q}_T^2}{Q^2} \right)^m \alpha_s^n \frac{d\sigma^{[n,m]}}{dQ^2 dy d\mathbf{q}_T^2}$$

- $\frac{d\sigma^{[n,-1]}}{dQ^2 dy d\mathbf{q}_T^2}$ and $\frac{d\sigma^{[n,0]}}{dQ^2 dy d\mathbf{q}_T^2}$ are leading power contributions. Well studied in TMD factorization Scimemi et al. JHEP **07** (2012), 002; Becher and Neubert, EPJC **71** (2011), 1665
- $\frac{d\sigma^{[n,m]}}{dQ^2 dy d\mathbf{q}_T^2}$ are the power suppressed corrections: kinematic, Operator Product Expansion, SCET factorization

Sources of power corrections

A lot of work done so far in power corrections: Balitsky et al. JHEP **05** (2018), 150; Balitsky et al. JHEP **05** (2021), 046; Nefedov et al. Phys. Lett. B **790** (2019), 551-556; Ebert et al. 2112.07680 [hep-ph]; Luke et al. Phys. Rev. D **104** (2021) no.7, 076018, Beneke et al. JHEP **03** (2018), 001, Mulders et al. Nucl. Phys. B **667** (2003), 201-241…

- Kinematic corrections due to the definition of relevant variables for the process

$$\text{DY:} \quad x_{A(B)} = \sqrt{\frac{Q^2+\mathbf{q}_T^2}{s}}\, e^{\pm y}, \quad \text{SIDIS:} \quad \mathbf{q}_T^2 = \frac{p_\perp^2}{z^2}\,\frac{1+\gamma^2}{1-\gamma^2\frac{p_\perp^2}{z^2Q^2}}$$

- Matching TMDPDF(FF) onto PDF(FF)
  Vladimirov et al. Eur. Phys. J. C **78** (2018) no.10, 802

$$F_{a\leftarrow h_A}(\mathbf{b}_T, x) = \sum_{r,n}\left(\frac{\mathbf{b}_T}{M^2}\right)^n C^n_{a\leftarrow r}\left(\ln \mathbf{b}_T^2\mu^2, x\right)\otimes f_{r\leftarrow h_A}(x)$$

- Corrections to the TMD factorization included in the $Y-$term
  Collins et al. Nucl. Phys. B **250** (1985), 199-224; Collins et al. Phys. Rev. D **94** (2016) no.3, 034014

APPROACH

We use ideas from q_T-subtraction method: Catani et al. Nucl. Phys. B **596** (2001), 299-312; Catani et al. Phys. Lett. B **696** (2011), 207-213; Catani et. al. Phys. Rev. Lett. **98** (2007), 222002

$$d\sigma = \lim_{q_T\to 0} d\sigma + \left[d\sigma - \lim_{q_T\to 0} d\sigma \right]$$

- In our case the first term is well described by TMD factorization.
- It contains large logs (due to the expansion) that need to be resummed. TMD formalism is quite convenient for this task.
- The second term includes our power corrections as the difference at partonic level and fixed order.
- Typically the second term is computed using Monte-Carlo event generators. We provide an analytical computation at NLO+NLL.
- We modified the TMD factorization formula for DY to include this second term.

COMPUTATION AT NLO+NLL

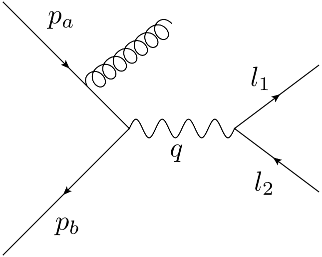
We compute

$$\frac{d\sigma_{h_A h_B \to l l' X}}{dQ^2 dy d\mathbf{q}_T^2} = \frac{d\sigma^{\text{TMD}}_{h_A h_B \to l l' X}}{dQ^2 dy d\mathbf{q}_T^2} + \left[\frac{d\sigma_{h_A h_B \to l l' X}}{dQ^2 dy d\mathbf{q}_T^2} - \frac{d\sigma^{\text{TMD}}_{h_A h_B \to l l' X}}{dQ^2 dy d\mathbf{q}_T^2} \right]$$

- The first term contains large logs due to the expansion in $\mathbf{q}_T^2 / \left(Q^2 + \mathbf{q}_T^2\right)$.
- We perform a NLO+NLL analytic computation of the second term.
- No need to regularized divergences using +-distributions
- The logarithmically enhanced contributions cancel out order by order in α_s.
- We seek for a modified factorization formula that at fixed order reproduces powers behaviour.

PARTONIC CROSS SECTION

Let us start considering the process $q(p_a) + \bar{q}(p_b) \to l(l_1) + \bar{l}(l_2) + g(k)$



- q and $\bar{q}$ are massless quark and anti-quark.
- l and $\bar{l}$ is a pair lepton anti-lepton.
- g corresponds to real gluon.
- The differential partonic cross section at small $\mathbf{q}_T^2/Q^2$

$$\frac{d\sigma}{dQ^2\, dy\, d\mathbf{q}_T^2} \sim \alpha_s\left(\mu^2\right) \frac{x_a^2 + x_b^2}{\mathbf{q}_T^2} \delta\left((1-x_a)(1-x_b) - \frac{\mathbf{q}_T^2}{(p_a+p_b)^2} \right)$$

where $p_a^\mu = p_a^- n^\mu$, $p_b^\mu = p_b^+ \bar{n}^\mu$, $Q^2 = q^2$, $y = \frac{1}{2} \log \frac{q^-}{q^+}$, $x_{a(b)} = \frac{\sqrt{Q^2+\mathbf{q}_T^2}}{\sqrt{s}} e^{\pm y}$, $q^- = \frac{q^0+q^3}{\sqrt{2}}$, and $q^+ = \frac{q^0-q^3}{\sqrt{2}}$. Notice $(p_a+p_b)^2 = \frac{Q^2+\mathbf{q}_T^2}{x_a x_b}$

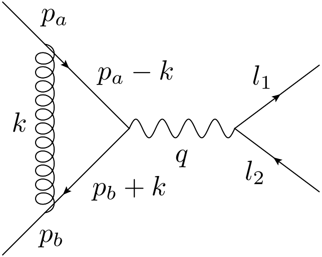
Partonic cross section

The virtual correction has the following structure

$$\frac{d\sigma}{dQ^2 dy d\mathbf{q}_T^2} \sim \alpha_S\left(\mu^2\right)\left[\frac{4}{\epsilon_{\mathrm{IR}}^2}+\frac{1}{\epsilon_{\mathrm{IR}}}\left(6+4\log\frac{\mu^2}{Q^2}\right)\right]$$

$$+\alpha_S\left(\mu^2\right)\delta^{(2)}\left(\mathbf{q}_T\right) f\left(Q^2,\log\frac{\mu^2}{Q^2}\right)$$

- The IR are nothing but the collinear and soft singularities, which appear due to we are considering massless initial states.
- The function $f\left(Q^2,\log\frac{\mu^2}{Q^2}\right)$ is finite. It has logarithmic contributions that fixing $\mu^2 = Q^2$ vanish.
- The IR from the virtual contribution has to cancel out with IR poles from the real emission contribution.
- We see explicitly this cancellation in impact parameter space, $\mathbf{b}_T$.



RESUMMING LOGS

- Taking the limit $\frac{\mathbf{q}_T^2}{(p_a+p_b)^2} \ll 1$ we reproduce the TMDPDF factorization results at partonic level and fixed order.
- We obtain contributions of the form $\frac{1}{1-x_{a(b)}}$, that could be regularized using $+-$distributions
- This regularization process will lead to logarithmic contributions of the form $\log \frac{\mathbf{q}_T^2}{Q^2}$
- The logarithmic contributions at higher powers are suppressed by a factor $\left(\frac{\mathbf{q}_T^2}{Q^2}\right)^n$.
- Doing the Fourier transform to impact parameter space $\mathbf{b}_T$ we obtain logarithmically enhanced contributions which can be resummed using TMDPDF evolutions equations.

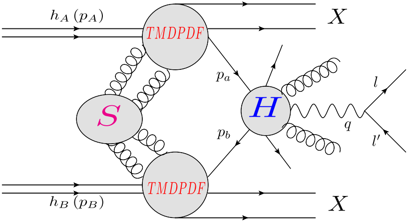
MODIFIED FACTORIZATION FORMULA

$$\frac{d\sigma_{h_A h_B \to l l' X}}{dQ^2 dy d\mathbf{q}_T^2} =$$

$$\sum_{a,b,c} \sigma_c^{\text{Born}} \int d^2\mathbf{p}_{Ta} d^2\mathbf{p}_{Tb} d^2\mathbf{q}'_T \delta^{(2)}\left(\mathbf{q}_T - \mathbf{p}_{Ta} - \mathbf{p}_{Tb} - \mathbf{q}'_T\right) \int_{x_A}^{1} \frac{dz_a}{z_a} \int_{x_B}^{1} \frac{dz_b}{z_b} \theta\left(\frac{(z_a - x_A)(z_b - x_B)}{x_A x_B} - \frac{\mathbf{q}_T^{2\prime}}{Q^2 + \mathbf{q}_T^2}\right)$$

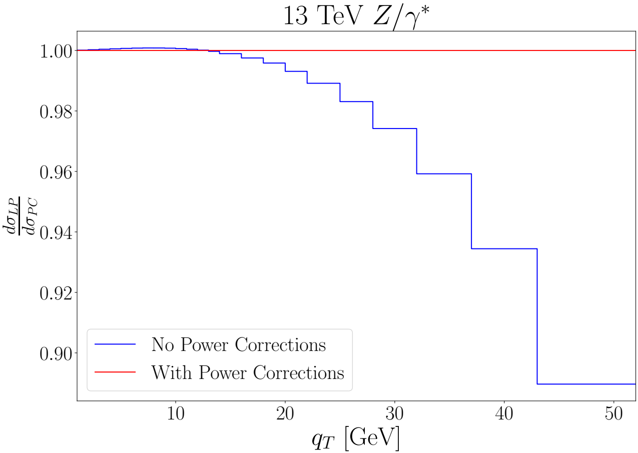
$$\tilde{H}_{c\leftarrow a, \bar{c}\leftarrow b}\left(\alpha_s, Q^2, \frac{x_A}{z_a}, \frac{x_B}{z_b}, \mathbf{q}_T^{2\prime}, \mathbf{q}_T^2\right) F_{a\leftarrow h_A}\left(\alpha_s, z_a, \mathbf{p}_{Ta}^2\right) F_{b\leftarrow h_B}\left(\alpha_s, z_a, \mathbf{p}_{Tb}^2\right)$$

- The origin of θ is pure kinematic.
- The coefficient $\tilde{H}$ is free of large logarithm contributions. All of them are absorbed by the TMDPDF.
- The TMD operators are unchanged, its evolution remains the same. $\zeta = \mu_F^2 = \mu_R^2 = \frac{\left(Q^2 + \mathbf{q}_T^2\right) z_{a(b)}}{x_{A(B)}}$
- $x_{A(B)} = \sqrt{\frac{Q^2 + \mathbf{q}_T^2}{s}} e^{\pm y}$



Power corrections vs Leading power

Preliminary



$0.4 \leq |y| \leq 0.8$, $76.18\text{GeV} \leq Q \leq 106.19\text{GeV}$.
Bigger than electroweak corrections Grazzini et al. Phys. Rev. Lett. **128** (2022) no.1, 012002; Sborlini et al. JHEP **08** (2018), 165

POWER CORRECTIONS VS LEADING POWER

Next steps in the numerical analysis

- The convolution in $\mathbf{p}_{Ta}$ and $\mathbf{p}_{Tb}$ is done from $-\infty$ to ∞. We need to introduce a cut off to perform the numerical integral.
- The code is to be optimized for this cut-off such that the numerical contribution above is truly negligible.
- So far we have used $p_{Ta}^{\text{cut-off}} = p_{Tb}^{\text{cut-off}} = \sqrt{Q^2 + \mathbf{q}_T^2} \cdot 0.25$. A better choice would be $p_{Ta}^{\text{cut-off}} = p_{Tb}^{\text{cut-off}} = \sqrt{Q^2 + \mathbf{q}_T^2}\sqrt{\frac{z_{a(b)}}{x_{A(B)}}} \cdot 0.25$

SUMMARY & OUTLOOK

SUMMARY

- At small $\mathbf{q}_T^2/Q^2$ our factorization formula reproduces TMD factorization.
- At $|\mathbf{q}_T| = Q \cdot 0.30$ we start to appreciate the effects of power corrections.
- The power corrections increase the cross section at large q_T, making it more close the experimental data.
- Electroweak corrections are subleading compared to power corrections Grazzini et al. Phys. Rev. Lett. **128** (2022) no.1, 012002; Sborlini et al. JHEP **08** (2018), 165

OUTLOOK

- Improvement of the code for integration in $\mathbf{p}_T$ of the TMDPDF.
- Extension to e^+e^- to jets/hadrons.
- Study of polarized processes.
- New extraction of TMDPDFs.
- Inclusion of power suppressed terms in the matching of TMDs onto PDFs.

THANK YOU FOR YOUR ATTENTION

Backup

Large logs

- Momentum Space $\mathbf{q}_T$

$$\frac{d\sigma}{dQ^2 dy d\mathbf{q}_T^2} \sim c_1^{[1]} \frac{\alpha_s}{\mathbf{q}_T^2} \log \frac{Q^2}{\mathbf{q}_T^2} + \frac{\alpha_s^2}{\mathbf{q}_T^2} \left(c_1^{[2]} \log \frac{Q^2}{\mathbf{q}_T^2} + c_2^{[2]} \log^2 \frac{Q^2}{\mathbf{q}_T^2} + c_3^{[2]} \log^3 \frac{Q^2}{\mathbf{q}_T^2} \right) + \cdots$$

- Impact parameter space $\mathbf{b}_T$

$$\frac{d\sigma}{dQ^2 dy d\mathbf{b}_T^2} \sim \alpha_s \left(c_0^{[1]} \log \frac{Q^2\mathbf{b}_T^2}{4e^{-2\gamma_E}} + c_1^{[1]} \log^2 \frac{Q^2\mathbf{b}_T^2}{4e^{-2\gamma_E}} \right) +$$

$$\alpha_s^2 \left(c_0^{[2]} \log \frac{Q^2\mathbf{b}_T^2}{4e^{-2\gamma_E}} + c_1^{[2]} \log^2 \frac{Q^2\mathbf{b}_T^2}{4e^{-2\gamma_E}} + c_2^{[2]} \log^3 \frac{Q^2\mathbf{b}_T^2}{4e^{-2\gamma_E}} + c_3^{[2]} \log^4 \frac{Q^2\mathbf{b}_T^2}{4e^{-2\gamma_E}} \right) + \cdots$$

Small $\mathbf{q}_T$ expansion at NLO.

Using the methods presented in Bacchetta et al. JHEP **08** (2008), 023; Soper et al. Phys. Rev. D **54** (1996), 1919-1935

$$\delta\left((p_a - p_b - q)^2\right) =$$

$$\frac{1}{Q^2 + \mathbf{q}_T^2}\left[\frac{1}{(1-x_a)_+}\delta(1-x_b) + \frac{1}{(1-x_a)_+}\delta(1-x_a) - \delta(1-x_a)\,\delta(1-x_b)\ln\frac{\mathbf{q}_T^2}{Q^2+\mathbf{q}_T^2}\right] + \mathcal{O}\left(\frac{\mathbf{q}_T^2}{Q^2}\right)$$

θ FACTOR

$$(p_a + p_b)^2 + (p_a - q)^2 + (p_b - q)^2 = Q^2 + s_2$$

where

$$s_2 = \left(\sum_i k_i\right)^2 \geq 0; \quad k_i^2 = 0$$

Therefore

$$(p_a + p_b)^2 + (p_a - q)^2 + (p_b - q)^2 - Q^2 \geq 0$$