Effective field theories for heavy quarkonium phenomenology

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Based on work in collaboration with A. Pineda, M. Stahlhofen, J. Segovia, M. Beneke and D. van Dyk

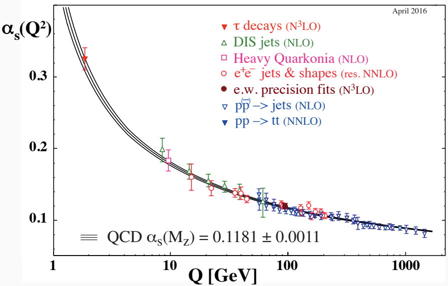
Outline

1. Motivation
2. The EFTs: NRQCD and pNRQCD
3. The Heavy quarkonium spectrum
    - Heavy quarkonium at N^3LO
    - Heavy quarkonium at N^3LL
4. Top production near threshold
5. Final Remarks

Motivation

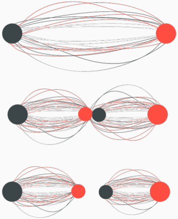
QCD describes the interaction of quarks and gluons

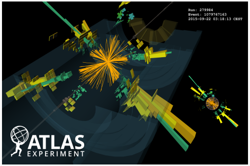
- It is asymptotically free: predictability at high energies



QCD describes the interaction of quarks and gluons

- It is asymptotically free: predictability at high energies
- The strong interaction grows at large distances: confinement



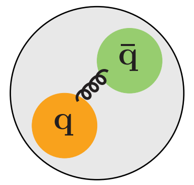


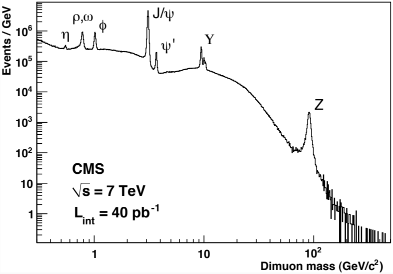
Need for **precision** in QCD parameters: α_s, quark masses, CKM matrix elements

Quarkonium

narrow QCD resonances:

Discovery of J/ψ '74, Υ '76



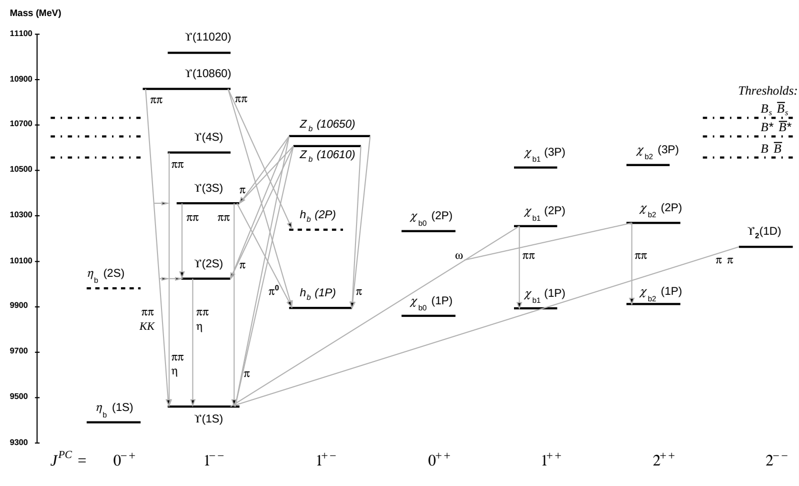


- Tower of $q\bar{q}$ **bound states**:

e.g. J/ψ is ground-state charmonium ($c\bar{c}$)

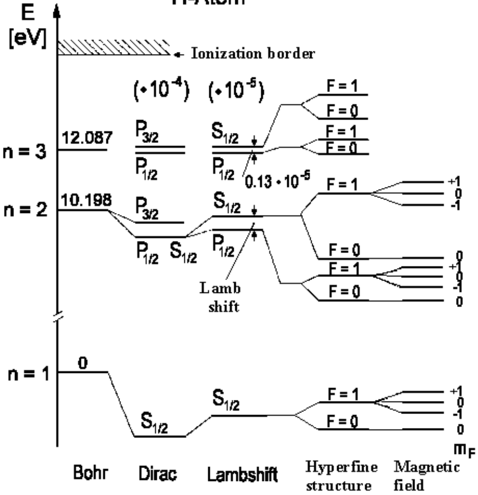
Υ is ground-state bottomonium ($b\bar{b}$)

The bottomonium system



PDG '19

H-Atom


E
[eV]
Ionization border
(•10⁻⁴)
(•10⁻⁵)
n = 3
12.087
P3/2
P1/2
S1/2
P1/2
0.13 • 10⁻⁵
F = 1
F = 0
F = 1
F = 0
n = 2
10.198
P3/2
P1/2
S1/2
S1/2
P1/2
Lamb shift
F = 1
+1
0
-1
F = 0
0
F = 1
+1
0
-1
F = 0
0
n = 1
0
S1/2
S1/2
F = 1
+1
0
-1
F = 0
0
mF
Bohr
Dirac
Lambshift
Hyperfine structure
Magnetic field

Bound states

Hydrogen

is successfully described by quantum mechanics.

- The **Schrödinger Equation** (1926)

describes **nonrelativistic** systems such as bound states

$$\boxed{\left(i\partial_0 - \frac{\mathrm{p}^2}{2m_r} - V(r)\right)\psi(r) = 0}$$

In the hydrogen atom:

$V(r) = V_C(r) +$ relativistic corrections

$$V_C(r) = -\tfrac{\alpha}{r} \text{ (Coulomb)}$$

⇒ develop similar framework for **Quarkonium** with $\boxed{\alpha \to \alpha_s}$

Multi-scale problem

- Bound state scales:

  Hard: m_r, **Soft:** $|\mathrm{p}| \sim \frac{1}{r} \sim m_r v$, **Ultrasoft:** $E \sim \frac{\mathrm{p}^2}{2m_r} \sim m_r v^2$

- Confinement scale:

  QCD: Λ_{QCD}

- Non-relativistic systems fulfill the relation: $\boxed{m_r \gg |\mathrm{p}| \gg E}$
- **Heavy quark:** $\boxed{m_r \gg \Lambda_{\mathrm{QCD}}}$

There is a hierarchy of scales:

We can integrate out the hard scale to obtain **NRQCD**

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  Strong coupling regime: $|\mathrm{p}| \sim \Lambda_{\mathrm{QCD}}$

  Weak coupling regime: $|\mathrm{p}| \gg \Lambda_{\mathrm{QCD}}$, Coulomb-like potential

Multi-scale problem

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- Confinement scale:

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- Non-relativistic systems fulfill the relation: $\boxed{m_r \gg |\mathrm{p}| \gg E}$
- **Heavy quark:** $\boxed{m_r \gtrsim |\mathrm{p}| \gg \Lambda_{\mathrm{QCD}}}$

There is a hierarchy of scales: $\alpha_s \sim v$

We can integrate out the hard and soft scales to obtain **pNRQCD**

EFTs for bound states

- Tools to describe **low energy QCD**
  - **Model independence**: only input is QCD
  - **Simplicity**: effective action for all observables
  - **Systematic**: power counting rules
  - No need for **perturbation theory**: Wilson loops, lattice

The EFTs: NRQCD and pNRQCD

NRQCD

- $m_1 \sim m_2 \sim m_r \gg |\mathrm{p}|, \Lambda_{\mathrm{QCD}}$ Caswell, Lepage, '86

$$\delta\mathcal{L}_{\mathrm{NRQCD}} = \psi_1^\dagger \left\{ \frac{c_F^{(1)}}{2m_1} g\boldsymbol{\sigma}\cdot \mathrm{B} + \frac{c_D^{(1)}}{8m_1^2} g[\boldsymbol{\nabla}\cdot \mathrm{E}] + \frac{c_S^{(1)}}{8m_1^2} ie\boldsymbol{\sigma}\cdot\{\mathrm{D}\times \mathrm{E}\} \right\} \psi_1 + (1 \to 2,\, g \to -g)$$

The bilinear **Wilson coefficients** are related to the form factors:

$$\begin{aligned} c_F &= \tilde{F}_1 + \tilde{F}_2, \\ c_S &= \tilde{F}_1 + 2\tilde{F}_2, \\ c_D &= \tilde{F}_1 + 2\tilde{F}_2 - 8\tilde{F}_1' \end{aligned}$$

- renormalization scheme and scale dependent
- contain **all** the information of the hard scale

two-loop computation Gerlach, Mishima, Steinhauser '19

pNRQCD

- $m_1 \sim m_2 \sim m_r$ and strict weak coupling regime $E \sim mv^2 \gg \Lambda_{\rm QCD}$

Strict weak coupling regime $mv^2 \gg \Lambda_{\rm QCD}$

$$\left.\begin{array}{l} \left(i\partial_0 - \frac{\mathrm{p}^2}{2m_r} - V^{(0)}(r)\right)\phi(\mathrm{r}) = 0 \\ + \text{ corrections to the potential} \\ + \text{ interaction with other low-energy degrees of freedom} \end{array}\right\} \text{pNRQCD.}$$

The pNRQCD (singlet) Lagragian

$$\mathcal{L}_{\rm pNRQCD} = \int d^3\mathrm{r}\ \mathrm{Tr}\left\{\mathrm{S}^\dagger\left(i\partial_0 - \frac{\mathrm{p}^2}{2\,m_r} - \frac{\mathrm{P}_\mathrm{R}^2}{2\,M} - V_s(\mathrm{r},\mathrm{p},\mathrm{P}_\mathrm{R},\mathrm{S}_1,\mathrm{S}_2)\right)\mathrm{S}\right\}$$

$$+\,V_A(r)\mathrm{Tr}\left\{\mathrm{O}^\dagger\mathrm{r}\cdot g\mathrm{E}\,\mathrm{S} + \mathrm{S}^\dagger\mathrm{r}\cdot g\mathrm{E}\,\mathrm{O}\right\} - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu\,a} + \sum_{i=1}^{n_f}\bar{q}_i\,i\not{D}\,q_i$$

The QCD singlet potential

$$V_s(\mathrm{p},\mathrm{r}) = V^{(0)}(r) + \frac{V^{(1,0)}(r)}{m_1} + \frac{V^{(0,1)}(r)}{m_2} + \frac{V^{(2,0)}(\mathrm{p},\mathrm{r})}{m_1^2} + \frac{V^{(0,2)}(\mathrm{p},\mathrm{r})}{m_2^2} + \frac{V^{(1,1)}(\mathrm{p},\mathrm{r})}{m_1 m_2}$$

Basis of operators:

$$\text{e.g. } V^{(2,0)}(\mathrm{p},\mathrm{r}) = \frac{1}{2}\left\{\mathrm{p}^2, V^{(2,0)}_{\mathrm{p}^2}\right\} + V^{(2,0)}_{\mathrm{L}^2}\frac{\mathrm{L}^2}{r^2} + V^{(2,0)}_r + V^{(2,0)}_{LS}\mathrm{L}\cdot\mathrm{S}_1$$

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Symmetries of the potentials:

- charge conjugation: $\psi \leftrightarrow \chi_c$
- mass exchange: $m_1 \leftrightarrow m_2$
- Poincare symmetry

The QCD singlet potential

$$V_s(\mathrm{p},\mathrm{r}) = V^{(0)}(r) + \frac{V^{(1,0)}(r)}{m_1} + \frac{V^{(0,1)}(r)}{m_2} + \frac{V^{(2,0)}(\mathrm{p},\mathrm{r})}{m_1^2} + \frac{V^{(0,2)}(\mathrm{p},\mathrm{r})}{m_2^2} + \frac{V^{(1,1)}(\mathrm{p},\mathrm{r})}{m_1 m_2}$$

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Wilson coefficients of the EFT:

- **matching** and **renormalization** scheme dependent
- related by a **field redefinition** CP, Pineda, Stahlhofen

The Heavy quarkonium spectrum

$$\textbf{Physical systems:}\ \begin{cases} \text{Bottomonium (1S):} & |\mathrm{p}| \sim m_b v \sim 1.3\ \text{GeV} \\ B_c\ \text{(1S):} & |\mathrm{p}| \sim 2 m_r v \sim 0.85\ \text{GeV} \\ \text{Charmonium (1S):} & |\mathrm{p}| \sim m_c v \sim 0.68\ \text{GeV} \end{cases}$$

Can we use the results from weak coupling ($|\mathrm{p}| \gg \Lambda_{QCD}$)?
Is the US scale perturbative ($E \gg \Lambda_{QCD}$)?

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Approach: obtain maximum improvement in weak coupling and **assess**

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Approach: obtain maximum improvement in weak coupling and **assess**

1. Achieve high orders in PT: N^{3}LO / N^{3}LL

History of spectrum computations

Pineda, Yndurain (1998)	$\mathcal{O}(m\alpha_s^4)$	$m_1 = m_2$
Brambilla et al. (2000)	$\mathcal{O}(m\alpha_s^4)$	$n = 1, m_1 \neq m_2$
Brambilla et al. (1999)	$\mathcal{O}(m\alpha_s^5 \ln(\alpha_s))$	
Pineda (2001)	$\mathcal{O}(m\alpha_s^{4+m} \ln^m(\alpha_s))$	
Penin, Steinhauser (2002)	$\mathcal{O}(m\alpha_s^5)$	$n = 1, m_1 = m_2$
Penin et al. (2004)	$\mathcal{O}(m\alpha_s^{5+m}) \ln^m(\alpha_s))$	spin-dependent
Beneke et al., Penin et al. (2005)	$\mathcal{O}(m\alpha_s^5)$	S-wave, $m_1 = m_2$
Brambilla et al. (2009)	$\mathcal{O}(m\alpha_s^{5+m}) \ln^m(\alpha_s))$	static
Pineda (2011)	$\mathcal{O}(m\alpha_s^{5+m}) \ln^m(\alpha_s))$	ultrasoft
Kiyo, Sumino (2014)	$\mathcal{O}(m\alpha_s^5)$	$m_1 = m_2$
CP, Pineda, Stahlhofen (2015)	$\mathcal{O}(m\alpha_s^5)$	$m_1 \neq m_2$
CP, Pineda, Segovia (2018)	$\mathcal{O}(m\alpha_s^{5+m}) \ln^m(\alpha_s))$	P-wave, $m_1 \neq m_2$
Anzai, Pineda, Moreno (2018)	$\mathcal{O}(m\alpha_s^{5+m}) \ln^m(\alpha_s))$	$m_1 \neq m_2$ (almost)
Gerlach, Mishima, Steinhauser (2019)	$\mathcal{O}(m\alpha_s^{5+m}) \ln^m(\alpha_s))$	$m_1 \neq m_2$ (almoster)

Physical systems:

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Is the US scale perturbative ($E \gg \Lambda_{QCD}$)?

Approach: obtain maximum improvement in weak coupling and **assess**

1. Achieve high orders in PT: N^3LO / N^3LL
2. Accelerate convergence
   - thershold masses
   - resummation of large logarithms
   - alternative computation schemes (see 1806.05197,1809.09124)

Convergence of the perturbative series (I)

$$\mathcal{L} = \sum_i \frac{1}{m_q^i} C_i \mathcal{O}_i, \qquad C_i(\nu) = \tilde{C}_i + \sum_{n=0}^{\infty} C_{i,n} \alpha_s^{n+1}$$

- Wilson coefficients are **asymptotic** : $\boxed{C_{i,n} \sim n!}$

$\Rightarrow$ **BUT** comply the OPE: $m_q = m_{os} + \tilde{\Lambda}_{QCD}$ is renormalon free

$$m_{os} = m_{\overline{\rm MS}} \left(1 + B_1 \alpha_s + B_2 \alpha_s^2 + \dots\right), \qquad B_n \sim n!$$

- Redefine the mass such that C_i is **not asymptotic**:

Threshold masses

Convergence of the perturbative series (II)

Reference scales:

Hard: $\nu_H \sim m_r$, **Soft:** $\nu_S \sim m_r \alpha_s$, **Ultrasoft:** $\nu_{US} \sim m_r \alpha_s^2$

Multiscale computations give rise to Large Logarithms

$$\langle \alpha_s \ln(mr) \rangle \sim \alpha_s \ln \frac{\nu_H}{\nu_S} \sim \alpha_s \ln \frac{1}{\alpha_s} \sim \mathcal{O}(1)$$

⇒ breakdown of power counting

Solution: solve the **R**enormalization **G**roup **E**quation to resume large logs

$$\frac{dC(\mu)}{d \ln \mu} = \Gamma C(\mu) \;\&\; C(\nu_h) = C_0 \Rightarrow \boxed{C(\nu_S, \nu_H) \sim \frac{\alpha_s(\nu_H)}{\alpha_s(\nu_S)} \sim \sum_n \alpha_s^n \ln \frac{\nu_H}{\nu_S}}$$

Resummation of Large Logarithms

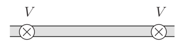
The Heavy quarkonium spectrum

Heavy quarkonium at N^{3}LO

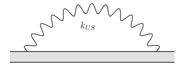
Energy levels at N^{3}LO

$$E(n,l,s,s^-,j) = E_n^C \left(1 + \frac{\alpha_s}{\pi} P_1(L_\nu) + \left(\frac{\alpha_s}{\pi}\right)^2 P_2(L_\nu) + \left(\frac{\alpha_s}{\pi}\right)^3 P_3(L_\nu)\right),$$

1. expectation value of the potential $\langle V_s \rangle$
2. quantum-mechanical perturbation theory to third order



3. contribution of US gluons : perturbative for $mv^2 \gg \Lambda_{\rm QCD}$



Energy levels at N^3LO

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- Nonperturbative effects: parametrically $\sim \Lambda^3_{\rm QCD}\langle r^2\rangle$
- **Renormalon subtracted schemes:**

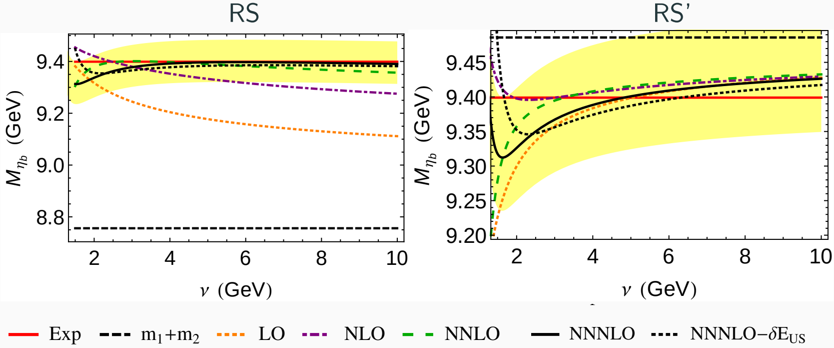
$$m_{\rm RS(')} = m_{OS} - N_m \pi \nu_f \sum_{N=0(1)}^{\infty}\left(\frac{\beta_0}{2}\right)^N\left(\frac{\alpha(n_l,\nu_f)}{\pi}\right)^{N+1}\sum_{n=0}^{\infty}\tilde{\chi}_n\frac{\Gamma(b+N+1-n)}{\Gamma(b+1-n)}$$

Pineda

⇒ Introduction of a new scale $\nu_f \sim m_r\alpha_s$

The bottom quark mass

Fit $m_{b,RS(')}$: $M_{\eta_b(1S)} = 2m_{b,RS(')} + E(1,0)$ at $\nu = 5$ GeV, $\nu_f = 2$ GeV



At $\nu = 5$ GeV:

$$\overline{m}_b(\overline{m}_b) = 4186(37)\mathrm{MeV}$$

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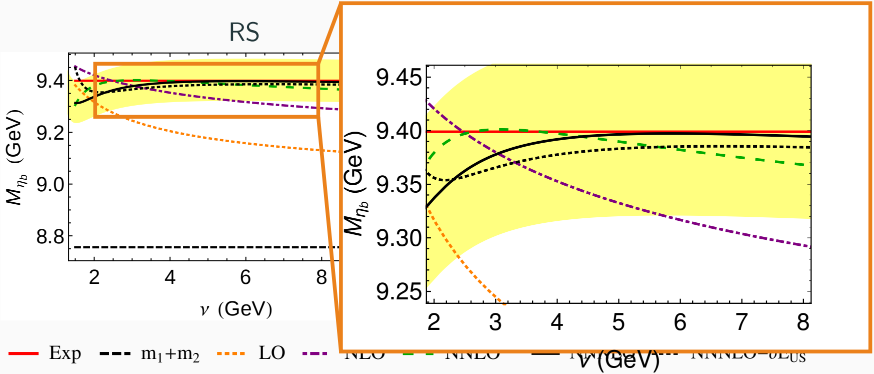

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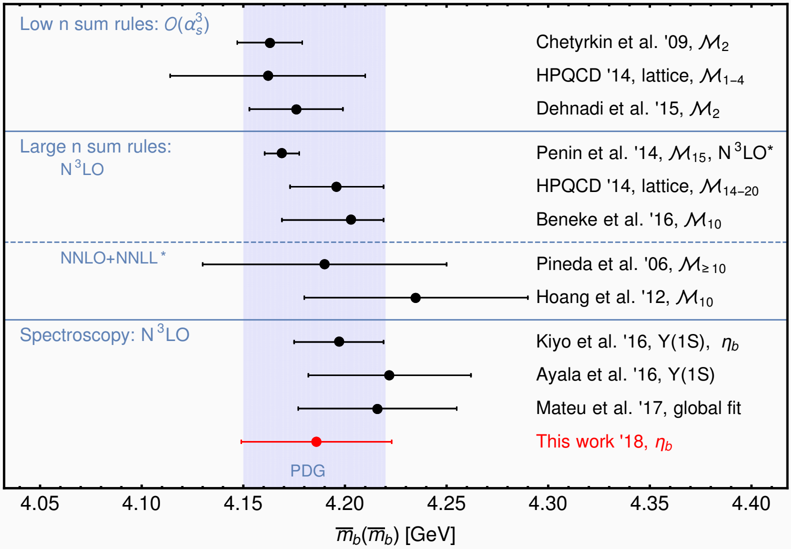
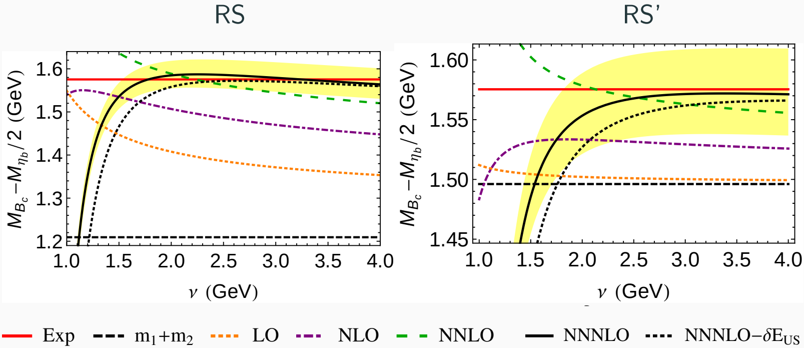
Other determinations of the bottom mass



The charm quark mass

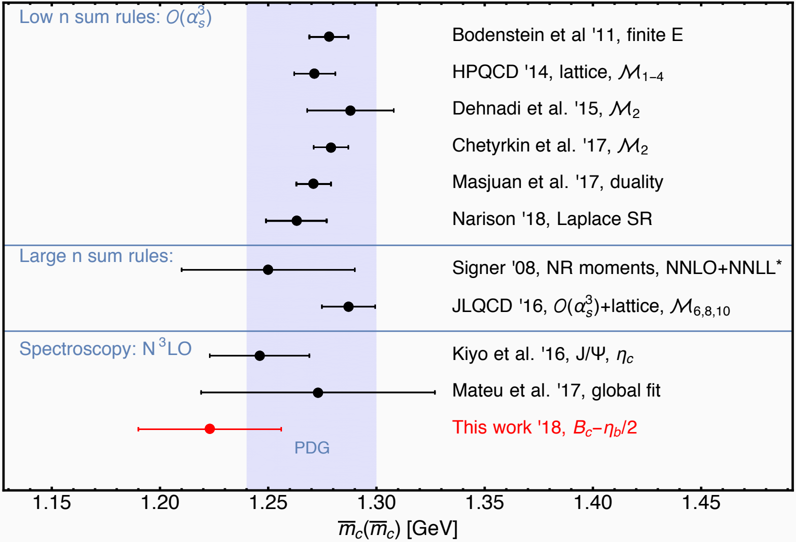
Fit $m_{c,RS(')}$: $M_{B_c} - M_{\eta_b}/2 = m_{c,RS(')} + E(1,0)$ at $\nu = 2.5$ GeV, $\nu_f = 1$ GeV



At $\nu = 2.5$ GeV:

$$\overline{m}_c(\overline{m}_c) = 1223(33)\mathrm{MeV}$$

Other determinations of the charm mass

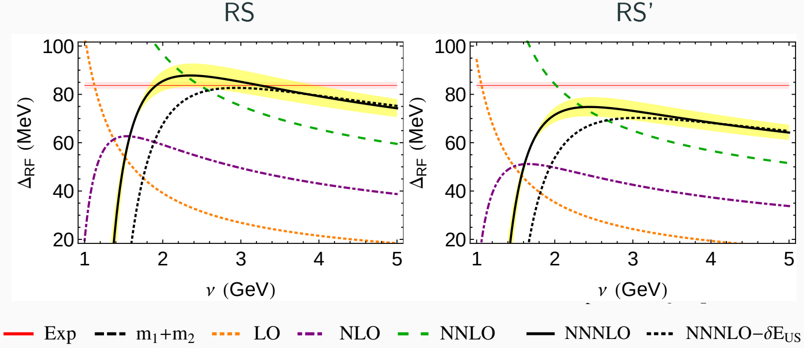


Determination of α_s

Consider the renormalon free shift:

$$\Delta_{\mathrm{RF}} = M_{B_c} - M_{\eta_b}/2 - M_{\eta_c}/2 = E(1,0)|_{B_c} - E(1,0)/2|_{\eta_b} - E(1,0)/2|_{\eta_c}$$

with $\nu_f = 1$ GeV


RS

RS'

Determination of α_s

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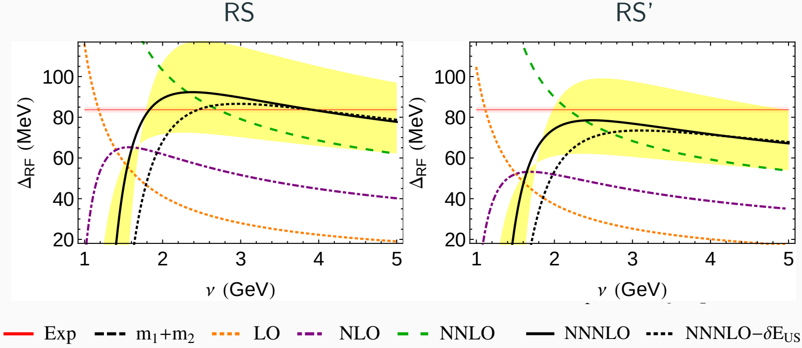


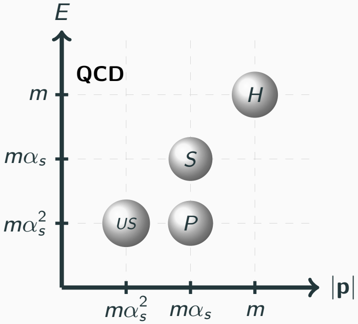
$$\alpha_s(M_z) = 0.1195(53)$$

The Heavy quarkonium spectrum

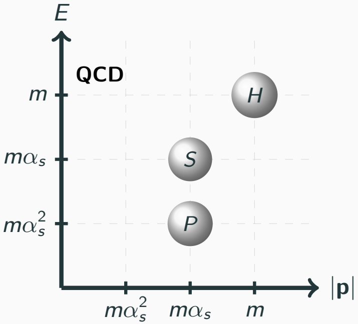
Heavy quarkonium at N^{3}LL

The N^{3}LL framework

- **QCD:**



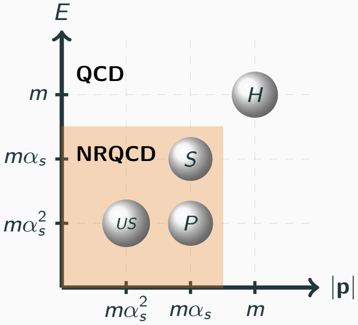
Gluons



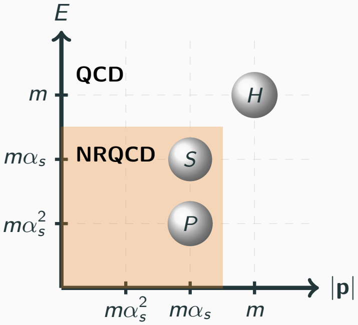
Heavy quarks

The N^{3}LL framework

- **NRQCD:** $E, |\mathrm{p}|, \Lambda_{\mathrm{QCD}} \ll \nu_S \ll \nu_H \sim m$



Gluons



Heavy quarks

The N^{3}LL framework

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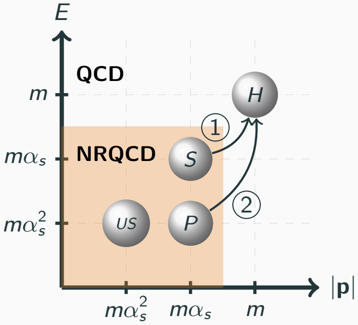
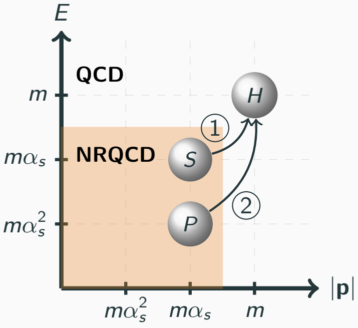
① Soft running & ② Potential running



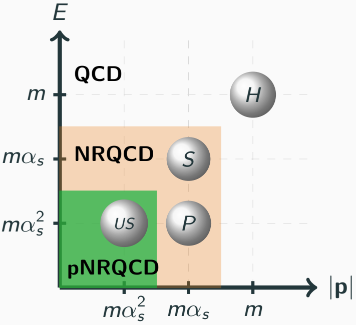
Gluons



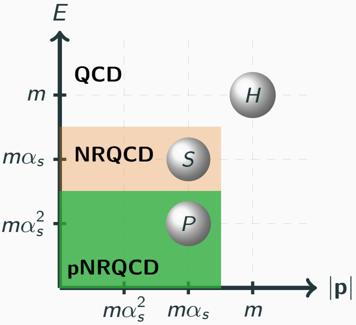
Heavy quarks

The N^{3}LL framework

- **pNRQCD:** $E, \Lambda_{\text{QCD}}, \frac{\mathrm{p}^2}{m} \ll \nu_{US} \ll |\mathrm{p}| \ll \nu_P, \nu_S \ll \nu_H \sim m$



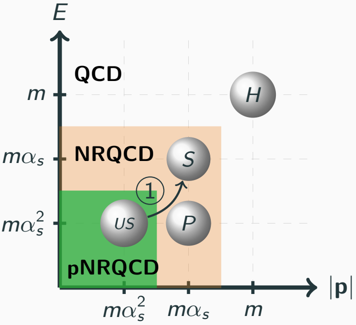
Massless d.o.f.



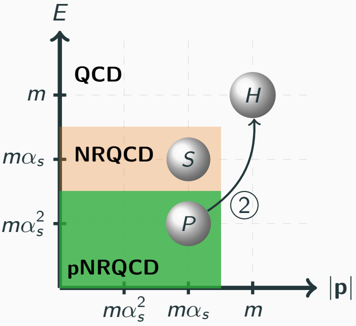
Heavy quarks

The N^{3}LL framework

- **pNRQCD:** $E, \Lambda_{\rm QCD}, \frac{\mathrm{p}^2}{m} \ll \nu_{US} \ll |\mathrm{p}| \ll \nu_P, \nu_S \ll \nu_H \sim m$
  ① **Ultrasoft running**
  ② **Potential running:** needs correlation of scales



Massless d.o.f.



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① **Ultrasoft running**

② **Potential running:** needs correlation of scales

$$V_s(\nu_H, \nu_S, \nu_P, \nu_{US}) = V_s(c(\nu_H, \nu_S, \nu_P), \nu_H, \nu_P, \nu_{US})$$

Energy levels

$$E(n, l, s, s^-, j) = E_n^C \left(1 + \frac{\alpha_s}{\pi} P_1 + \left(\frac{\alpha_s}{\pi}\right)^2 P_2^{\mathrm{NNLL}} + \left(\frac{\alpha_s}{\pi}\right)^3 P_3^{\mathrm{N}^3\mathrm{LL}}\right),$$

State-of-the-art of N^{3}LL computations

① **Ultrasoft running:**

–1 and 2-loop self energy with an ultrasoft gluon ✓

② **Soft running:**

– Momentum-dependent $\frac{1}{m^2}$-potentials ✓

– delta-like potentials ✗

③ **Potential running:**

– Only in delta-like potentials ✓

State-of-the-art of N^{3}LL computations

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③ **Potential running:**
  - Only in delta-like potentials ✓

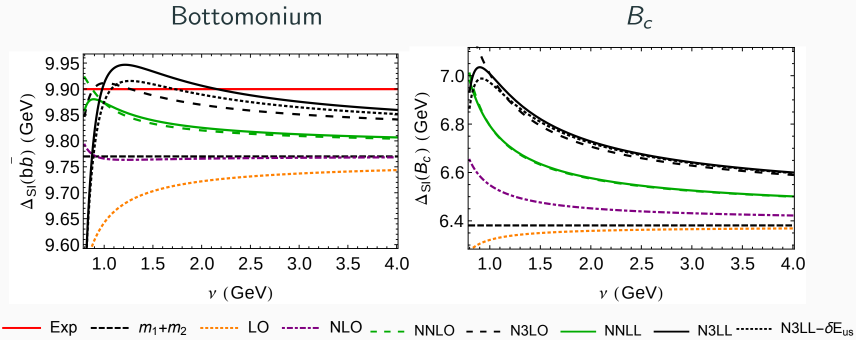
⇒ **P-wave @N^{3}LL**

Spin-averaged p-wave splitting

Only sensitive to ultrasoft scale

$$\Delta_{\mathrm{SI}} = \frac{1}{12}\left(5M_{\chi_{b2}} + 3M_{\chi_{b1}} + M_{\chi_{b0}}\right) + \frac{1}{4}M_{h_b}$$

- RS' masses at $\nu_f = 1$ GeV, at $\nu_h = 2m_r$ $\nu_S = \nu$ and $\nu_{US} = 1$ GeV



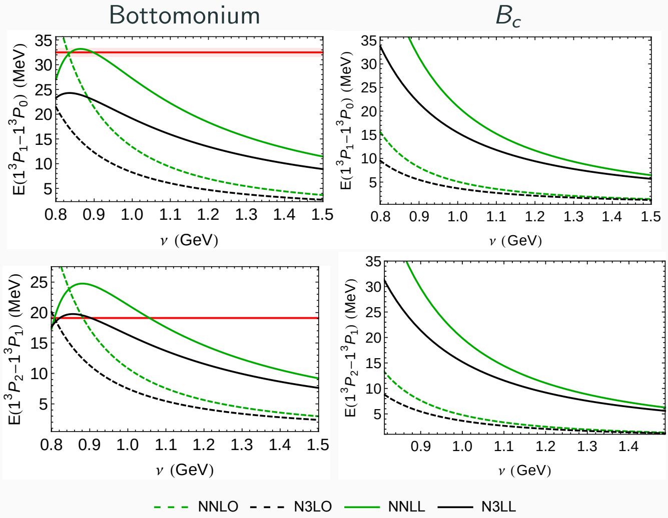
Bottomonium

B_c

Fine p-wave splitting

Only sensitive to hard scale

- RS' masses at $\nu_f = 1$ GeV, at $\nu_h = 2m_r$ $\nu_S = \nu$ and $\nu_{US} = 1$ GeV

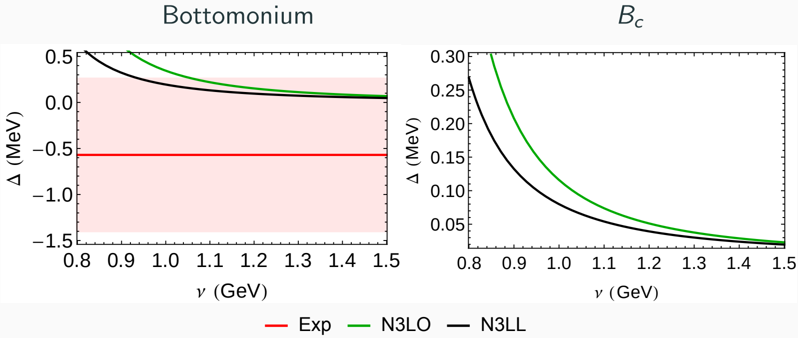


Hyperfine p-wave splitting

Only sensitive to hard scale

$$\Delta = M_{h_b} - \frac{1}{9}\left(5M_{\chi_{b2}} + 3M_{\chi_{b1}} + M_{\chi_{b0}}\right)$$

- RS' masses at $\nu_f = 1$ GeV, at $\nu_h = 2m_r$ $\nu_S = \nu$ and $\nu_{US} = 1$ GeV



Bottomonium

B_c

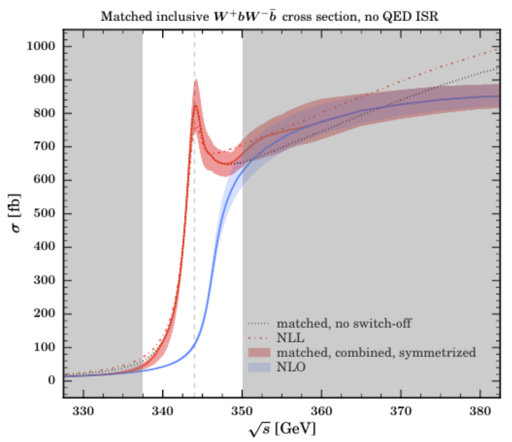
Top production near threshold

Top threshold region of $e^+e^- \to W^+W^-\bar{b}bX$:

- Extremely precise determination of m_t:

  $\boxed{\delta\bar{m}_t(\bar{m}_t)} \sim 50$ MeV.

- Sensitive to Γ_t, α_s, Y_t



[Bach et al. 2017]

EFT set-up

Vector current:

$$j^{(v)\mu}_{\text{QCD}} = \bar{\psi}\gamma^\mu\psi$$

$$j^{(v)i}_{\text{NRQCD}} = c_v(\nu_H,\nu_S)[\psi^\dagger\sigma^i\chi]_{\text{NRQCD}} + \frac{d_v(\nu_H,\nu_S)}{6m^2}[\psi^\dagger\sigma^i \mathrm{D}^2\chi]_{\text{NRQCD}}$$

$$j^{(v)i}_{\text{pNRQCD}} = c_v(\nu_H,\nu_S,\nu_{US})[\psi^\dagger\sigma^i\chi]_{\text{pNRQCD}} + \frac{d_v(\nu_H,\nu_S,\nu_{US})}{6m^2}[\psi^\dagger\sigma^i \mathrm{D}^2\chi]_{\text{pNRQCD}}$$

Scattering ratio:

$$R = \frac{\sigma(e^+e^- \to t\bar{t}X)}{\sigma(e^+e^- \to \mu^+\mu^-)} = 12\pi e_t^2 \text{Im}\left[\frac{2N_c}{s}c_v\left(c_v - \frac{E}{m}d_v\right)G(E)\right]$$

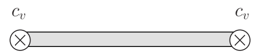
- The coefficient c_v known to N^{3}LO (Beneke, Kiyo, Schuller '13) and N^{2}LL (CP, Beneke, in preparation)*
- d_v is known at LO and LL (Pineda, Signer '06)

*(Hoang, Stahlhofen '12 in vNRQCD)

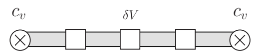
The Green's function to N^{3}LO

$$G(E) = G_0(E) + \delta_1 G(E) + \delta_2 G(E) + \delta_3 G(E)$$

1. two insertions of the vector current

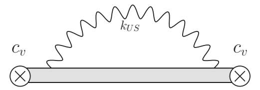


2. quantum-mechanical perturbation theory to third order



only need LL potentials

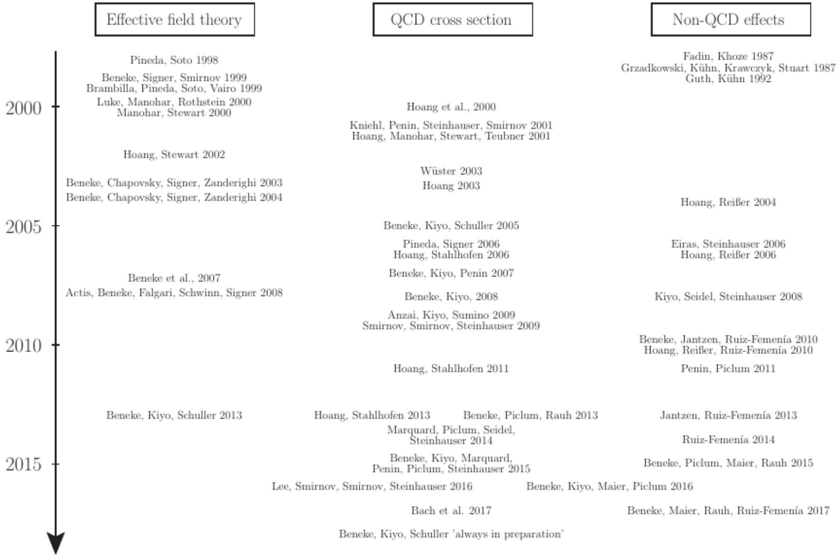
3. contribution of US gluons : perturbative for $mv^2 \gg \Lambda_{\mathrm{QCD}}$



+refinements: Γ_t, mass scheme, pole-resummation

QQbar_threshold

Work beyond NNLO QCD



Effective field theory

Pineda, Soto 1998

Beneke, Signer, Smirnov 1999
Brambilla, Pineda, Soto, Vairo 1999

Luke, Manohar, Rothstein 2000
Manohar, Stewart 2000

Hoang, Stewart 2002

Beneke, Chapovsky, Signer, Zanderighi 2003

Beneke, Chapovsky, Signer, Zanderighi 2004

Beneke et al., 2007

Actis, Beneke, Falgari, Schwinn, Signer 2008

Beneke, Kiyo, Schuller 2013

QCD cross section

Hoang et al., 2000

Kniehl, Penin, Steinhauser, Smirnov 2001
Hoang, Manohar, Stewart, Teubner 2001

Wüster 2003

Hoang 2003

Beneke, Kiyo, Schuller 2005

Pineda, Signer 2006
Hoang, Stahlhofen 2006

Beneke, Kiyo, Penin 2007

Beneke, Kiyo, 2008

Anzai, Kiyo, Sumino 2009
Smirnov, Smirnov, Steinhauser 2009

Hoang, Stahlhofen 2011

Hoang, Stahlhofen 2013

Beneke, Piclum, Rauh 2013

Marquard, Piclum, Seidel,
Steinhauser 2014

Beneke, Kiyo, Marquard,
Penin, Piclum, Steinhauser 2015

Lee, Smirnov, Smirnov, Steinhauser 2016

Beneke, Kiyo, Maier, Piclum 2016

Bach et al. 2017

Beneke, Kiyo, Schuller 'always in preparation'

Non-QCD effects

Fadin, Khoze 1987
Grzadkowski, Kühn, Krawczyk, Stuart 1987
Guth, Kühn 1992

Hoang, Reißer 2004

Eiras, Steinhauser 2006
Hoang, Reißer 2006

Kiyo, Seidel, Steinhauser 2008

Beneke, Jantzen, Ruiz-Femenía 2010
Hoang, Reißer, Ruiz-Femenía 2010

Penin, Piclum 2011

Jantzen, Ruiz-Femenía 2013

Ruiz-Femenía 2014

Beneke, Piclum, Maier, Rauh 2015

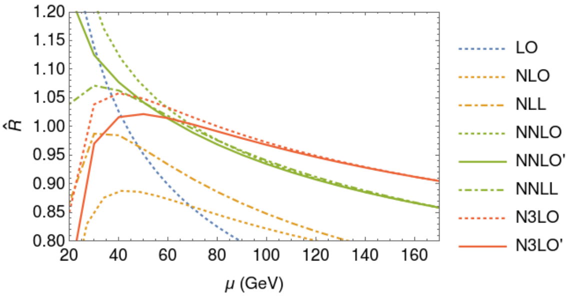
Beneke, Maier, Rauh, Ruiz-Femenía 2017

borrowed from T. Rauh

The $t\bar{t}$-scattering ratio

- Missing NLL δ-like potentials to achieve N^3LL

N^3LO'$=$ N^3LO fixed order$+$ NNLL resummation



PRELIMINARY

Reduced scale dependence from NNLL & N^3LO to N^3LO'!

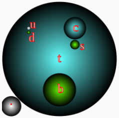
N^3LO and NNLL scale uncertainy $\sim$ 5%

Non QCD effects $\sim$15%

Final Remarks

Further applications of NREFTs

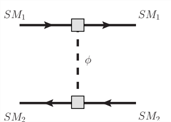
B_c light cone distribution amplitudes in NRQCD



- Form factors, semileptonic decays
- Light-cone sum rules

P. Lüghausen, C.P and D. van Dyk, in preparation

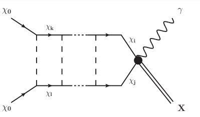
Dark sectors in atomic spectroscopy



- Precision QED computations:
  hydrogen, muonic hydrogen, positronium, muonium, etc.
- New physics seraches in precision experiments:
  bounds on a wide range of new physics models

C. Frugiuele and C.P.

Dark matter annihilation



- NREFT for enhancement effects:
  Sommerfeld and Sudakov logarithms

M. Beneke, C. Hasner, K. Urban, M. Vollmann

M. Beneke, S. Lederer and C.P., in preparation

Summary and conclusions

- We compute the **N^3LO spectrum** in pNRQCD for **different masses**
  - The potentials obtained are valid for $mv \gg \Lambda_{QCD}$
  - The US contribution is valid for $mv^2 \gg \Lambda_{QCD}$

From M_{η_b} and $M_{B_c} - M_{\eta_b}/2$ @N^3LO we obtain

$$\overline{m}_b(\overline{m}_b) = 4186(37)\mathrm{MeV} \qquad \overline{m}_c(\overline{m}_c) = 1223(33)\mathrm{MeV}$$

From $\Delta_{\mathrm{RF}} = M_{B_c} - M_{\eta_b}/2 - M_{\eta_c}/2$ @N^3LO

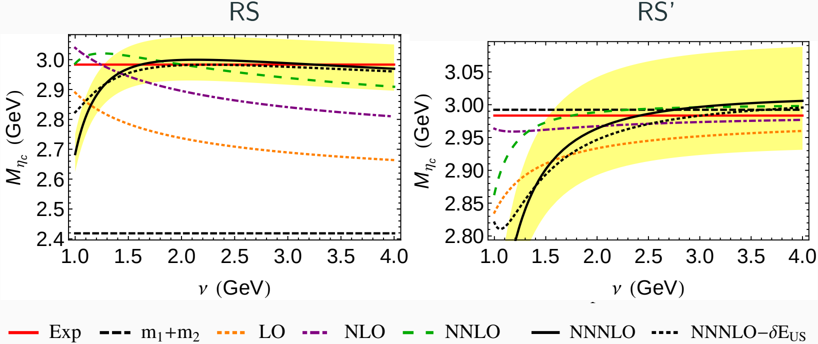
$$\alpha_s(M_z) = 0.1195(53)$$

- We compute the **N^3LL spectrum** in pNRQCD for **p-waves**
  - Curves are flatter but convergence is marginal
  - Prediction for B_c hyperfine splitting
- The ultrasoft effects look small
- We compute the **N^3LO' $t\bar{t}$ production** ratio in pNRQCD
  - Improves scale dependence (work in progress)

Thank you!

The charm quark mass

Fit $m_{c,RS(')}$: $M_{\eta_c(1S)} = 2m_{c,RS(')} + E(1,0)$ at $\nu = 2.5$ GeV, $\nu_f = 1$ GeV



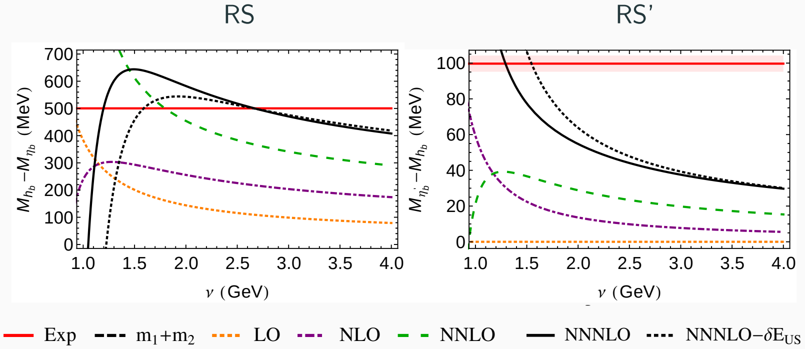
At $\nu = 1.5$ GeV:

$$\overline{m}_c(\overline{m}_c) = 1220(45)\mathrm{MeV}$$

Other renormalon free shifts

Consider also $n = 2$: $M_{h_b} - M_{\eta_b(1S)}$, $\quad M_{\eta_b(2S)} - M_{h_b}$

with $\nu_f = 1$ GeV



Further applications

- Use precision spectroscopy to bound possible new physics models

New vector or scalar coupling to leptons

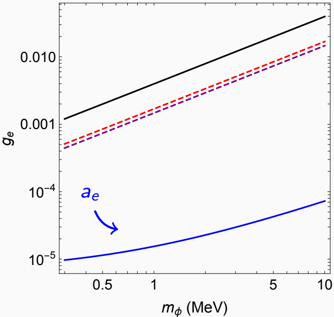
$$V^{ij}_{\mathrm{SI}}(r) = -\frac{g_i g_j}{4\pi}\frac{e^{-m_\phi r}}{r}$$

From 1S-2S, 2P-2S and Rydberg: $\Delta E^{\mathrm{BSM}}_{a\to b} \leq |\Delta E^{\mathrm{exp}}_{a\to b} - \Delta E^{\mathrm{the}}_{a\to b}| \lesssim 2\sigma_{\mathrm{Max}}$

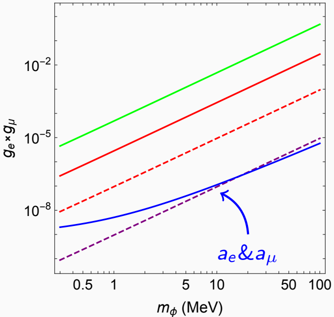
PRELIMINARY

Work in collaboration with C. Frugiuele and J. Pérez

Positronium (e^+e^-)



Muonium (μ^+e^-)



Further applications

- Use precision spectroscopy to bound possible new physics models

New pseudoscalar coupling to leptons

$$V^{ij}_{\mathrm{SD}}(r) = -\frac{g_i g_j}{12\pi m_i m_j}\left[\mathrm{S}_1\cdot\mathrm{S}_2\left(4\pi\delta^{(3)}(r) - \frac{m_\phi^2}{r}\right)\right]e^{-m_\phi r}$$

From of 2P-2S, 1S HFS and 2P HFS: $\Delta E^{\mathrm{BSM}}_{a\to b} \leq |\Delta E^{\mathrm{exp}}_{a\to b} - \Delta E^{\mathrm{the}}_{a\to b}| \lesssim 2\sigma_{\mathrm{Max}}$

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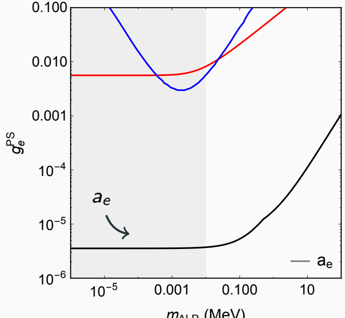

Muonium (μ^+e^-)