Seminar on Neutron Optics: Applications to Neutron Waveguides

Ignacio MOLINA DE LA PEÑA

Departamento de Óptica, Facultad de Ciencias Físicas, Universidad Complutense, 28040 Madrid, Spain

Topics

1. Neutron Optics

1. Neutron Optics: Basic Equations

- Neutrons propagating in matter are described by Schrödinger equation:

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V(x,t)\right]\psi(\mathrm{x},\mathrm{t}) = i\hbar\frac{\partial\psi(\mathrm{x},\mathrm{t})}{\partial t}$$

- First-order approximation; *t*-independent potential:

  Small neutron interaction, atoms are not displaced from their positions in lattice, nuclear absorption neglected…

$$\Psi(\mathbf{x},\mathrm{t}) = \varphi(\boldsymbol{x})\,\mathrm{exp}(-iEt/\hbar)$$

↓

stationary wave equation ⟶ $\left[\frac{-\hbar^2}{2m}\nabla^2 + V(\boldsymbol{x}) - E\right]\varphi(\mathbf{x}) = 0$

1.1. Neutron Optics: Basic Equations

stationary wave equation

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + V(\boldsymbol{x}) - E\right]\varphi(\boldsymbol{x}) = 0$$

- Eigenvalue problem

$$\boldsymbol{E} = \frac{\boldsymbol{2\pi^2\hbar^2}}{\boldsymbol{m\,\lambda_{db}}^2}$$

- Exploit analogies with geometrical optics

$$[\nabla^2 + \boldsymbol{k}^2]\varphi(\boldsymbol{x}) = 0$$

$$\boldsymbol{k}^2 = \frac{2m}{\hbar^2}[E - V(\boldsymbol{x})]$$

- Define a refractive index

$$n^2 = 1 - \frac{V(\boldsymbol{x})}{E}$$

1.2 Fermi Pseudopotential

- The potential depends on strong forces interaction (neutron-nuclei)
- Neutron Scattering:

$$V(\boldsymbol{x}) = \frac{2\pi\hbar^2}{m} b\delta(\boldsymbol{x})$$

- The media may be adequately described by a constant potential for each medium

$$\delta(\boldsymbol{x}) \rightarrow \rho$$

- Average material (with density $=\rho$) of order 10^{22} (nuclei/cm^3)

$$V = \frac{2\pi\hbar^2}{m} b\rho$$

$V \sim (10^{-7} - 10^{-8})$ eV

1.2 Fermi Pseudopotential

$$V = \frac{2\pi\hbar^2}{m} b\rho$$

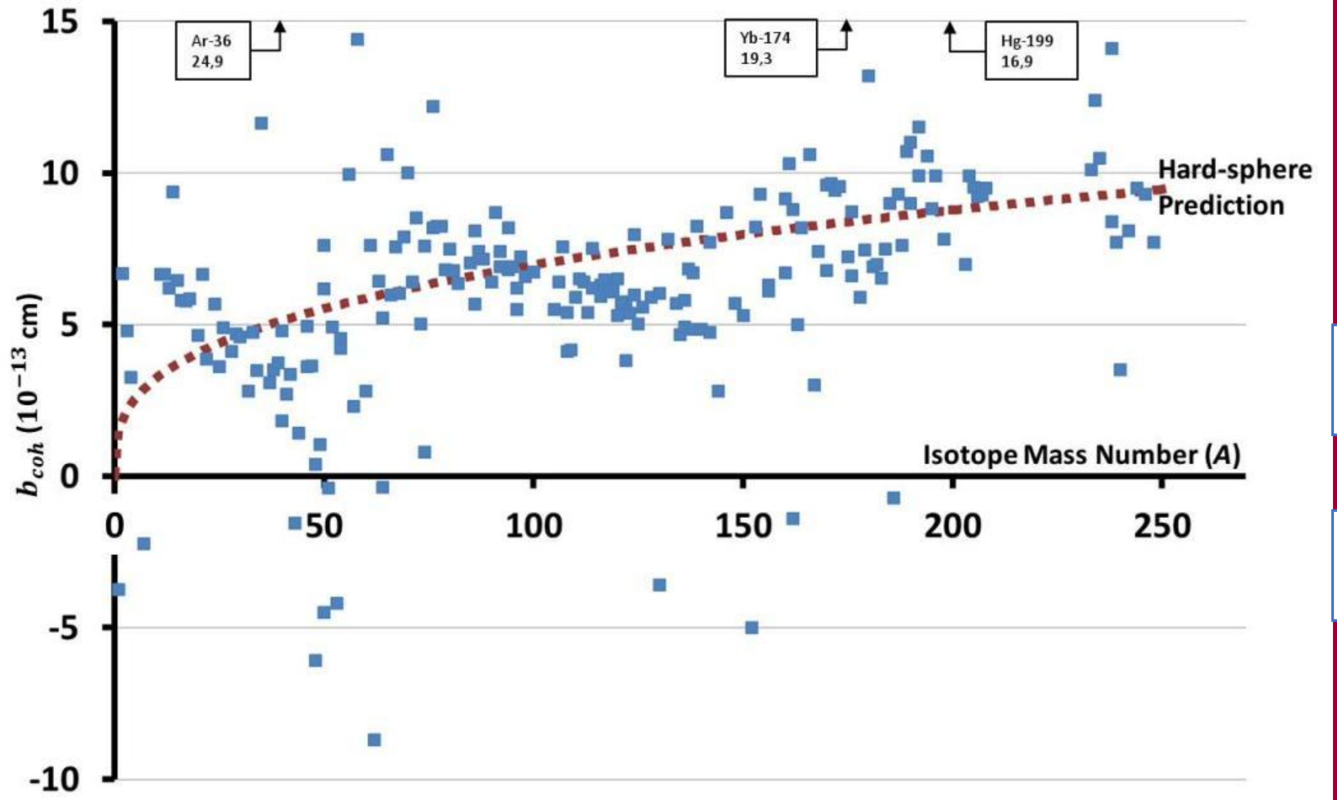
- b = scattering lenght (order of 10^{-13} cm) (coherent/incoherent/absorbing)
- b depends on the isotopes that forms the material
  - Different isotopes; average isotopes in compounds
  - Neutron-nuclei interaction is a scattering phenomenon:
    - First order approximation: Nuclei = Hard Spheres (radius=A)

$$b = 1.5x10^{-13} A^{\frac{1}{3}}$$

  - Resonant compounds neutron-nucleus and re-emission of neutron

$$b' = b - \frac{\Gamma_n^{(r)}}{2kE_r}$$

1.2 Fermi Pseudopotential



$$n^2 = 1 - \frac{2\pi b^\rho \hbar^2}{mE}$$

n<1
Less dense than vacuum

n>1
More dense than vacuum

Comparison with X-rays

Neutrons

- Strong forces governed
- Scattering in nuclei
- Isotropic
- Irregular refractive index
- Relatively low absorption
- Magnetic effects
- Change with isotopes

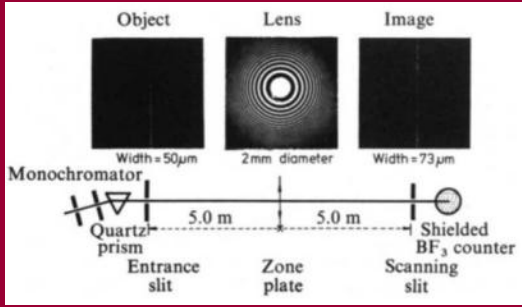
X-Rays

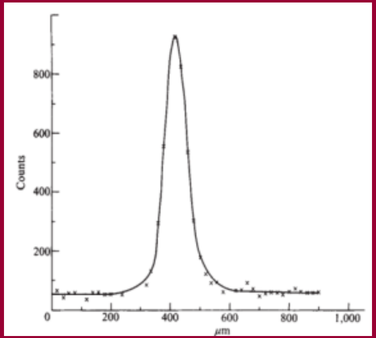
- Electromagnetical
- Distribution of electric charges
- Angular dependences
- Increases with A
- Absorption 10^2 to 10^3 cm^{-1}
- No magnetic effects
- Does not change replacing isotopes

1.3 Experiments: Wave Diffraction

- Arago-Poisson Spot:

  P.D. Kearney, A.G. Klein, G.I. Opat, and R. Gähler. Imaging and focusing of neutrons by a zone plate. NATURE, 287:313–314, 1984.





- Neutron Diffraction:

  A.G. Klein and A. Zeillinger. *Wave optics with cold neutrons*. Le Journal de Physique Colloques, 43 (C3): 239-242, 1984

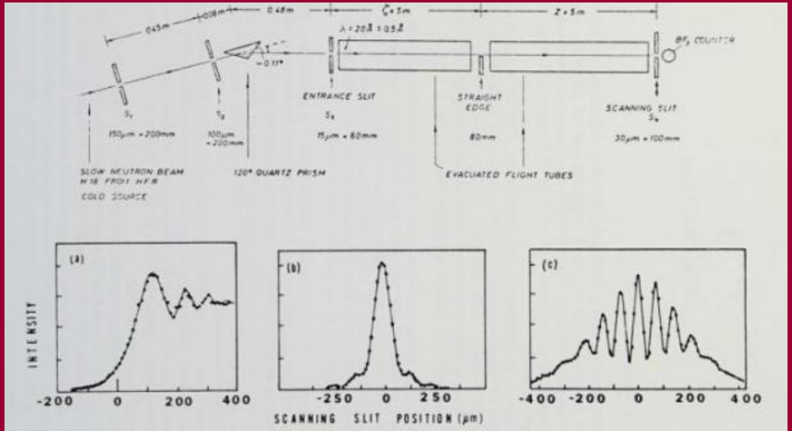


Fig. 1.1 Neutron diffraction by (a) a straight edge, (b) a single slit, and (c) a double slit (from Klein and Zeilinger, 1984).

1.3 Experiments

- Neutron Interferometry:

  H. Rauch, W. Treimer, and U. Bonse. *Test of a single crystal neutron interferometer*. Physics Letters A, 47(5):369–371, 1974.

- Sign Change with rotation:

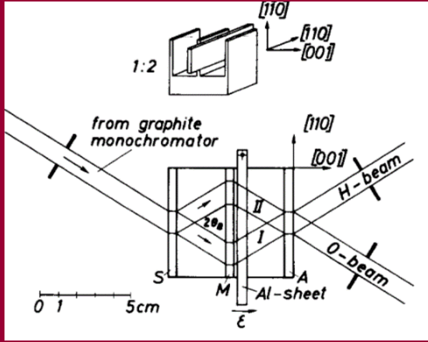
  S. A. Werner, R. Colella, A. W. Overhauser, and C. F. Eagen. *Observation of the phase shift of a neutron due to precession in a magnetic field*. Physical Review Letters, 35:1053–1055, 1975.

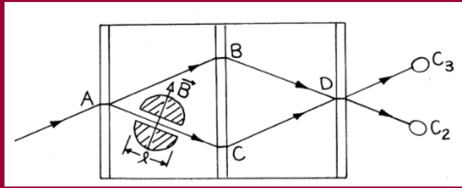
- Slow neutron Interaction with gravity:

  R. Colella, A. W. Overhauser, and S. A. Werner. *Observation of gravitationally induced quantum interference*. Physical Review Letters, 34:1472–1474, 1975.

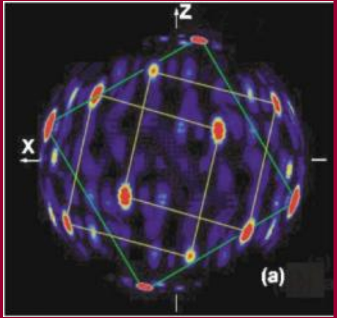
- Neutron Holography:

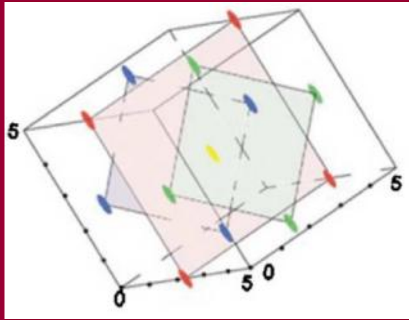
  L. Cser, G.y. Török, G. Krexner, I. Sharkov, and B. Faragó. *Holographic imaging of atoms using thermal neutrons*. Physical Review Letters, 89:175504, 2002.





$$\Psi \underset{2\pi}{\longrightarrow} -\Psi$$





1.3 Experiments: Birefringence

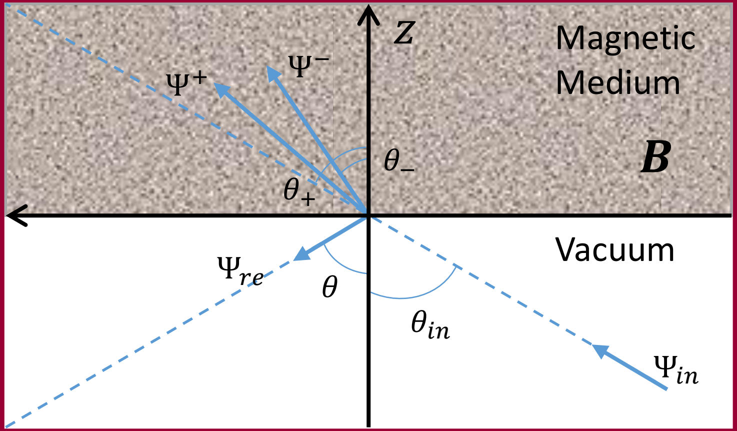
J.R. Dunning, P.N. Powers, and H.G. Beyer. *Experiments on the magnetic properties of the neutron.* Physical Review, 51:51–51, 1937.

- Neutron is a fermion

$$\Psi(\boldsymbol{x},t) = \begin{bmatrix} \Psi^{+}(\boldsymbol{x},t) \\ \Psi^{-}(\boldsymbol{x},t) \end{bmatrix} \qquad \boldsymbol{s} = \pm\frac{\hbar}{2}$$

- Fermi pseudopotential + Spin-magnetic field interaction

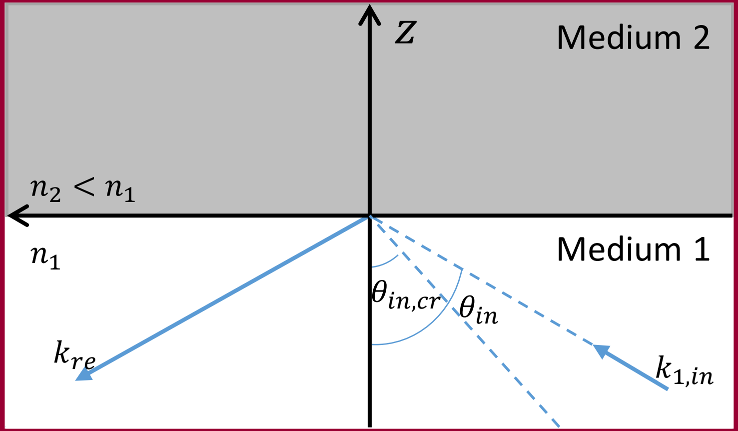
$$n_{\pm}^{2} = 1 - \frac{1}{E}\left(\frac{2\pi\hbar^{2} b\rho}{m} \mp \mu_{n}|\boldsymbol{B}|\right)$$



1.3 Experiments

- Total internal Reflection:

  E. Fermi and W.H. Zinn. *Reflection of neutrons on mirrors. Physical* Review, 70(1-2):103, 1946.



- Uses:

  Neutron Bottles (ultra cold neutrons)

  For example:

  - neutron lifetime S. Arzumanov, L. Bondarenko, S. Chernyavsky, P. Geltenbort, V. Morozov, V.V. Nesvizhevsky, Yu. Panin, and A. Strepetov. *A measurement of the neutron lifetime using the method of storage of ultracold neutrons and detection of inelastically up-scattered neutrons*. Physics Letters B, 745:79–89, 2015.
  - neutron dipole moment L.V. Groshev, V.N. Dvoretsky, A.M. Demidov, Y.N. Panin, V.I. Lushchikov, Y.N. Pokotilovsky, A.V. Strelkov, and Shapiro F.L. *Experiments with ultracold neutrons*. Physics Letters B, 234:293–295, 1971.

  Waveguides

2. Neutron Waveguides

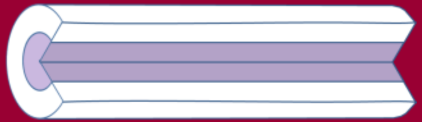
2. Neutron Waveguides

- Waveguide: central material (core) surrounded by other material (cladding or clad)

- Confined propagation of neutrons with energy *E* (mainly in core):
  - Waveguide confines neutrons if:
    - *bρ (core)<bρ (cladding)*
    - *n(core)>n(clad)*
  - Propagation mode solutions of Schrodinger equation

- Waveguide having air as core *(b=0)* and some material as cladding (ie: Macroscopic waveguides)

- Waveguide with *b(core)<0* and air as cladding. (Titanium)



Material	*b* (10^{-12} cm)	*V(x)* (eV)
Ti	-0,344	$-5{,}08 \cdot 10^{-8}$
Si	0,415	$5{,}40 \cdot 10^{-8}$
B	0,530	$1{,}89 \cdot 10^{-7}$
SiO_2	0,580 (oxygen)	$3{,}61 \cdot 10^{-8}$
Air	--	--

2. Neutron Waveguides (cont.)

- In geometrical optics approximation, neutron propagates confined in core of waveguide if :
  - *θ=incoming angle < acceptance angle θ_{cr}*
- For waveguide with *b(core<0)* $si^{n}\theta_{cr} = \sqrt{n(core)^2 - 1}$
- Typically (Ti..): θ_{cr}=10^{-2} to 10^{-3} radians
- Flux of neutrons entering into waveguide and propagating confined: $\boldsymbol{F_0\theta_{cr}^2}$
- Examples of waveguide having air as core

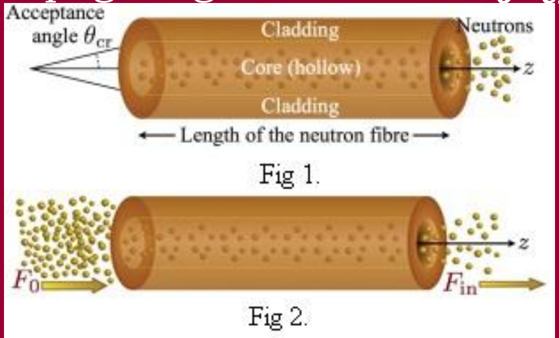


Fig 1.
Fig 2.

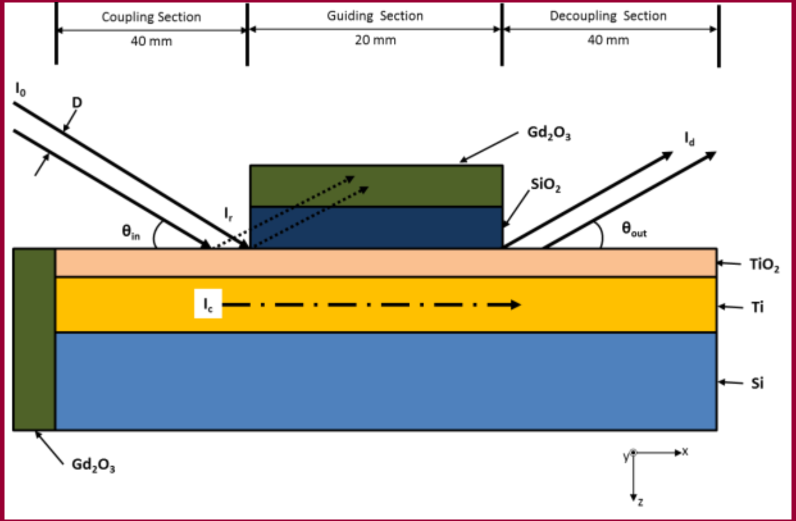
2.1 State of the Art: Thin Films

1. Theoretical Proposals

- R.E. DeWames and S.K. Sinha. *Possibility of guided-neutron-wave propagation in thin films.* Physical Review B, 7(3):917–921, 1973.
- R.F. Alvarez-Estrada and M.L. Calvo. *Neutron fibers. A possible application of neutron optics.* Journal Of Physics D-Applied Physics,17(3):475–502, 1984.
- M.L. Calvo and R.F. Alvarez-Estrada. Neutron fibers.2. *Some improving alternatives and analysis of bending losses.* Journal Of Physics D-Applied Physics, 19(6):957–973, 1986.

2. Experimental implementation

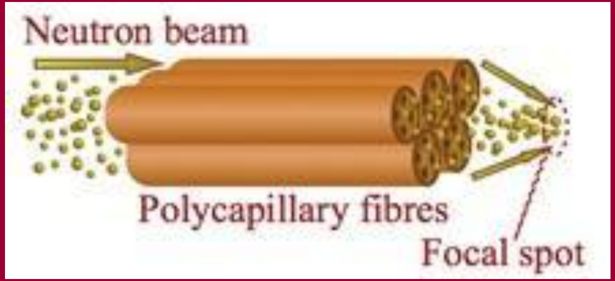
- Y.P. Feng, C.F. Majkrzak, S.K. Sinha, D.G. Wiesler, H. Zhang, and H.W. Deckman. *Direct observation of neutron-guided waves in a thin-film waveguide.* Physical Review B, 49(15):10814–10817, 1994.
- S. V. Kozhenikov, Y. N. Khaydukov, T. Keller, F. Ott, and F. Radu. *Polarized neutron channeling as a tool for the investigations of weakly magnetic thin films.* JETP Letters, (103):36–40, 2016.



2.1 State of the Art: PFGs

Polycapillary glass fibres (PGF)

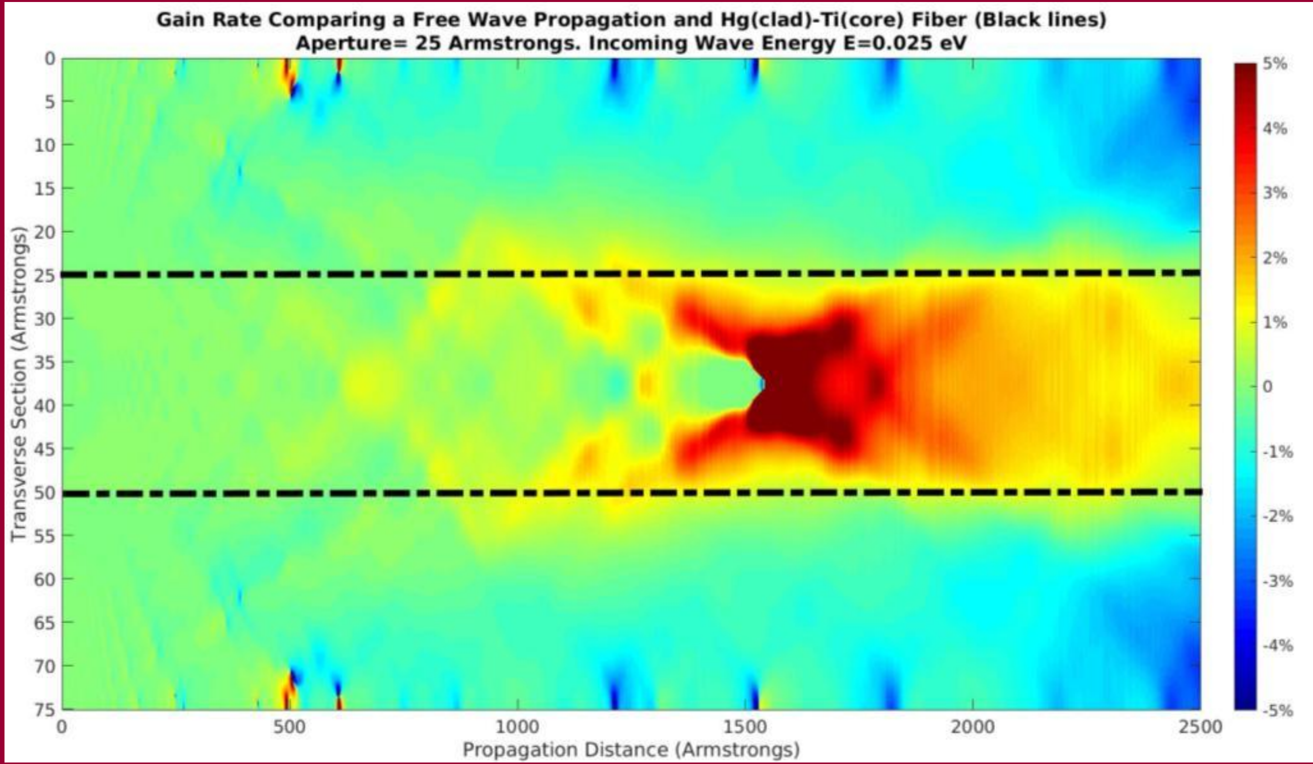
- M.A. Kumakhov and V.A. Sharov. *A neutron lens*. NATURE, 357(6377):390–391, 1992.
- 1 PGF: a few thousand hollow capillary channels (HCC) with length a few tens of cm
- 1 HCC: hollow channel (core) with diameter a few microns, surrounded by sillica glass (clad)



3. Mathematical characterization of neutron waveguides

Previous Results

- Use of FDTD simulations. High computational cost



Waveguide 25 Å aperture

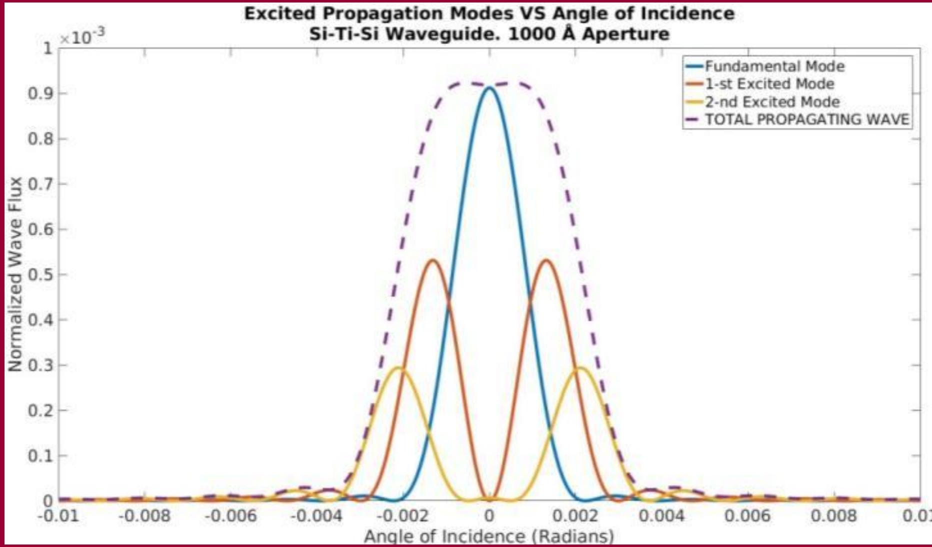
Simulation size: 75x2500 Å

Using FDTD: ¡80 hours!

Previous Results

- Analysis of the rise of propagation modes
  - Consider different propagation modes
  - From recipe Snyder and Love
    1. Compute refraction of φ_{in}
    2. Compute the rise of propagation modes

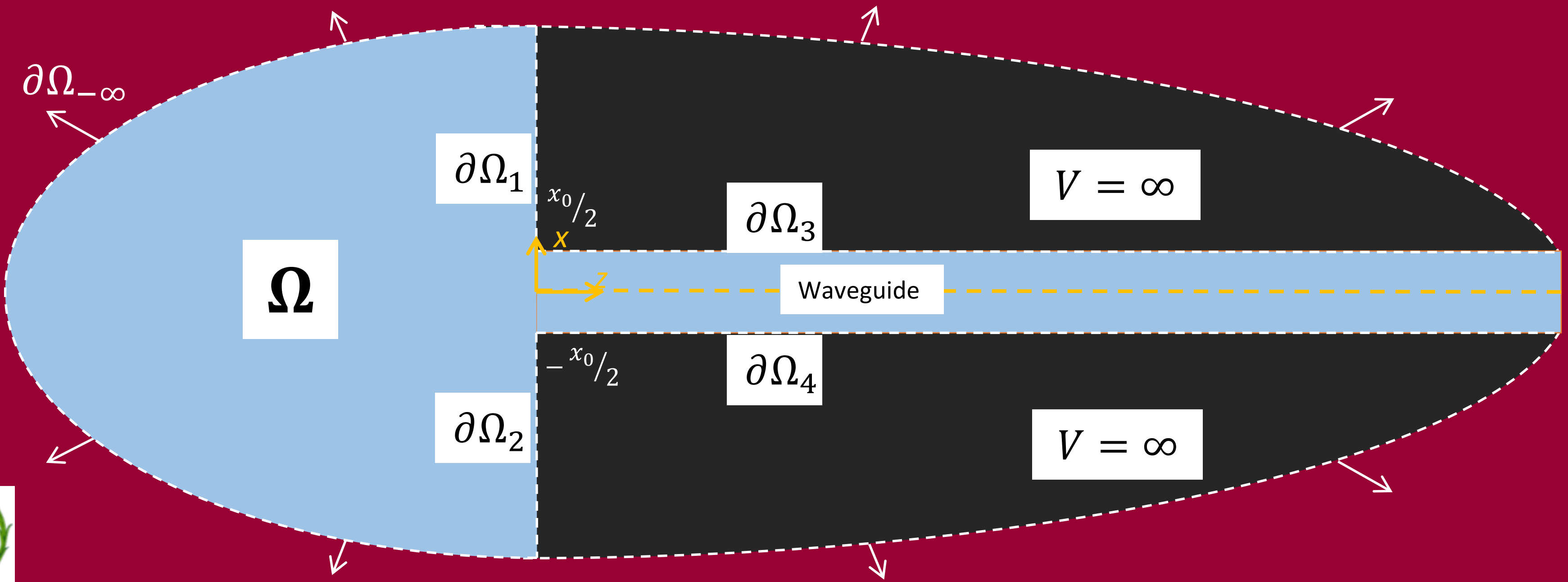
$$\int_{-\infty}^{+\infty} d\boldsymbol{x}\,\varphi_2 \widehat{\phi_{2,n}}^{*} = \begin{cases} A\left[\dfrac{\sin\left((K_x+\chi'_n)\frac{x_0}{2}\right)}{K_x+\chi'_n} + \dfrac{\sin\left((K_x-\chi'_n)\frac{x_0}{2}\right)}{K_x-\chi'_n}\right] (\mathit{even\ mode}) \\ \dfrac{A}{i}\left[\dfrac{\sin\left((K_x+\chi'_n)\frac{x_0}{2}\right)}{K_x+\chi'_n} - \dfrac{\sin\left((K_x-\chi'_n)\frac{x_0}{2}\right)}{K_x-\chi'_n}\right] (\mathit{odd\ mode}) \end{cases}$$



θ_{cr}

3.1 Dirichlet Boundary Conditions

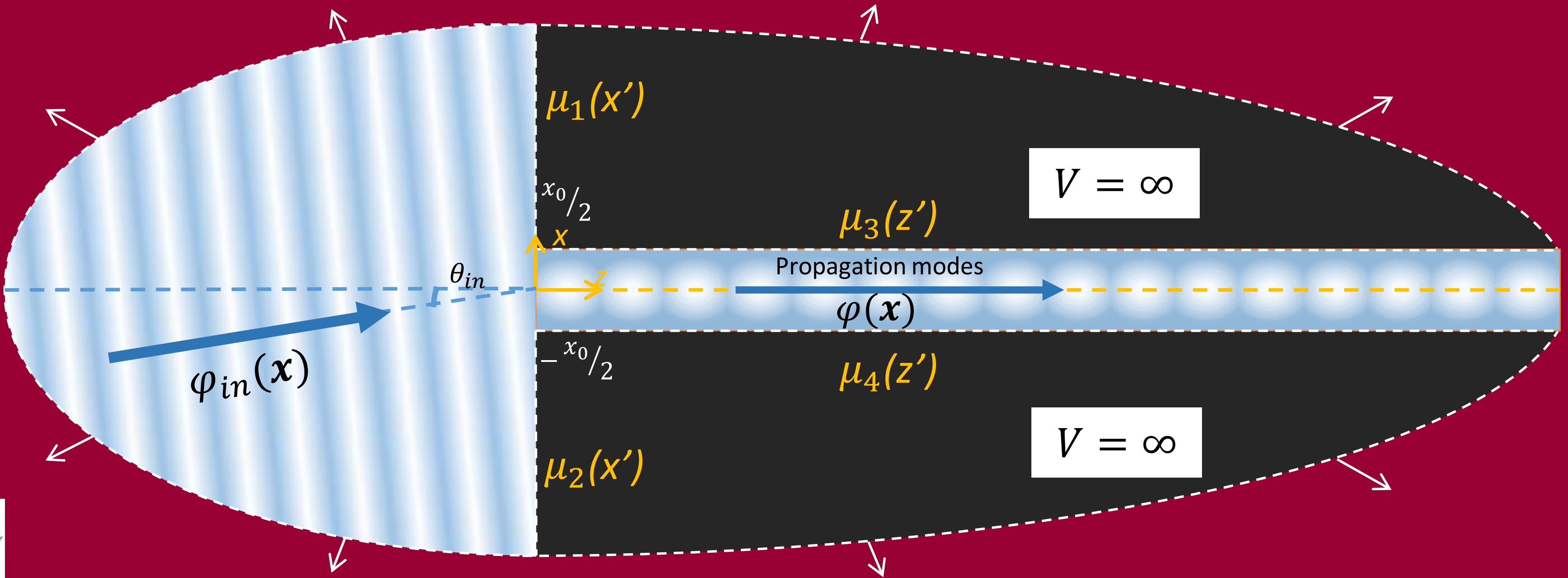
- Consider a region Ω and its boundaries $\partial\Omega_i$.
- Semiinfinite waveguide
- Repulsive (infinite potential)$\Rightarrow \varphi(\partial\Omega_i) = 0$ Dirichlet boundary conditions



3.1 Dirichlet Boundary Conditions

$$\varphi(\boldsymbol{x}) = \varphi_{in}(\boldsymbol{x}) + \sum_{i=1}^{n} \int_{\partial\Omega_i} d\Omega_i \left. \frac{\partial G(\boldsymbol{x} - \boldsymbol{x}_i')}{\partial \boldsymbol{n}_i} \right|_{\boldsymbol{x}_i' \in \partial\Omega_i} \mu_i(\boldsymbol{x}_i')$$

$$G(\boldsymbol{x} - \boldsymbol{x}') = -\int \frac{d^2\boldsymbol{K}'}{(2\pi)^2} \frac{exp[i\boldsymbol{K}'(\boldsymbol{x} - \boldsymbol{x}')]}{E + i\varepsilon - \frac{\hbar^2}{2m}\boldsymbol{K}'^2}$$



3.1 Dirichlet Boundary Conditions

- Obtain auxiliary functions $\mu_i(\boldsymbol{x'})$ Imposing DBC. Equations System

$$\varphi(\partial\Omega_i - \epsilon)\xrightarrow[\epsilon\to 0^+]{} 0 \quad \begin{cases} \frac{1}{2}\frac{2m}{\hbar^2}\mu_1(x) = -\varphi_{in}(x,0) + \int_0^\infty dz'\mu_3(z')\frac{\partial G(x-x',-z')}{\partial x'}\bigg|_{x'=\frac{x_0}{2}} - \int_0^\infty dz'\mu_4(z')\frac{\partial G(x-x',-z')}{\partial x'}\bigg|_{x'=-\frac{x_0}{2}}, for\ x \geq \frac{x_0}{2} \\ \frac{1}{2}\frac{2m}{\hbar^2}\mu_2(x) = -\varphi_{in}(x,0) + \int_0^\infty dz'\mu_3(z')\frac{\partial G(x-x',-z')}{\partial x'}\bigg|_{x'=\frac{x_0}{2}} - \int_0^\infty dz'\mu_4(z')\frac{\partial G(x-x',-z')}{\partial x'}\bigg|_{x'=-\frac{x_0}{2}}, for\ x \leq -\frac{x_0}{2} \end{cases}$$

$$\frac{1}{2}\frac{2m}{\hbar^2}\mu_3(z) = -\varphi_{in}\left(\frac{x_0}{2},z\right) + \int_{\frac{x_0}{2}}^\infty dx'\mu_1(x')\frac{\partial G\left(\frac{x_0}{2}-x',z-z'\right)}{\partial z'}\bigg|_{z'=0} + \int_{-\infty}^{-\frac{x_0}{2}} dx'\mu_2(x')\frac{\partial G\left(\frac{x_0}{2}-x',z-z'\right)}{\partial z'}\bigg|_{z'=0} - \int_0^\infty dz'\mu_4(z')\frac{\partial G\left(\frac{x_0}{2}-x',z-z'\right)}{\partial x'}\bigg|_{x'=-\frac{x_0}{2}}$$

$$\frac{1}{2}\frac{2m}{\hbar^2}\mu_4(z) = -\varphi_{in}\left(-\frac{x_0}{2},z\right) + \int_{\frac{x_0}{2}}^\infty dx'\mu_1(x')\frac{\partial G\left(\frac{-x_0}{2}-x',z-z'\right)}{\partial z'}\bigg|_{z'=0} + \int_{-\infty}^{-\frac{x_0}{2}} dx'\mu_2(x')\frac{\partial G\left(\frac{-x_0}{2}-x',z-z'\right)}{\partial z'}\bigg|_{z'=0} - \int_0^\infty dz'\mu_3(z')\frac{\partial G\left(\frac{-x_0}{2}-x',z-z'\right)}{\partial x'}\bigg|_{x'=\frac{x_0}{2}}$$

3.1 Approximations for analitical interpretation

$$\varphi(\boldsymbol{x}) = \varphi_{in}(\boldsymbol{x}) +$$

$$+\frac{2m}{\hbar^2}\frac{1}{2}\int \frac{dK_x'}{2\,\pi}\int_{x_0/2}^{\infty} dx'\mu_1(x')\, e^{iK_x'(x-x')}\mathrm{e}^{-izK_z'}$$

$$+\frac{2m}{\hbar^2}\frac{1}{2}\int \frac{dK_x'}{2\,\pi}\int_{-\infty}^{-x_0/2} dx'\mu_2(x')\, e^{iK_x'(x-x')}\mathrm{e}^{-izK_z'}$$

$$-\frac{2m}{\hbar^2}\frac{1}{2}\int \frac{dK_x'}{2\,\pi}\left[\int_{x_0/2}^{\infty} dx'\mu_1(x')e^{iK_x'x'} + \int_{-\infty}^{-x_0/2} dx'\mu_2(x')e^{iK_x'x'}\right] e^{ixK_x'}\mathrm{e}^{izK_z'} \approx$$

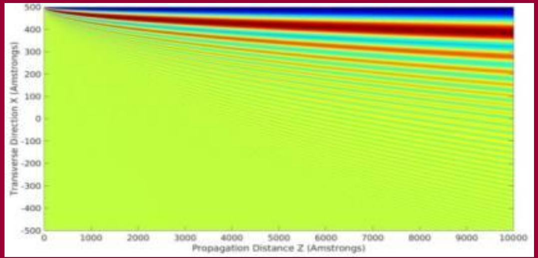
$$\approx -\left[\int_{-\infty}^{\infty} dx'\, e^{i(K_x-K_x')\,x'}\right]\int \frac{dK_x'}{2\,\pi}\, e^{iK_x'x}\mathrm{e}^{izK_z'} = -e^{iK_x x}e^{iz\sqrt{\frac{2mE}{\hbar^2}-K_x^2}}$$

Extintion of $\varphi_{in}(\boldsymbol{x})$

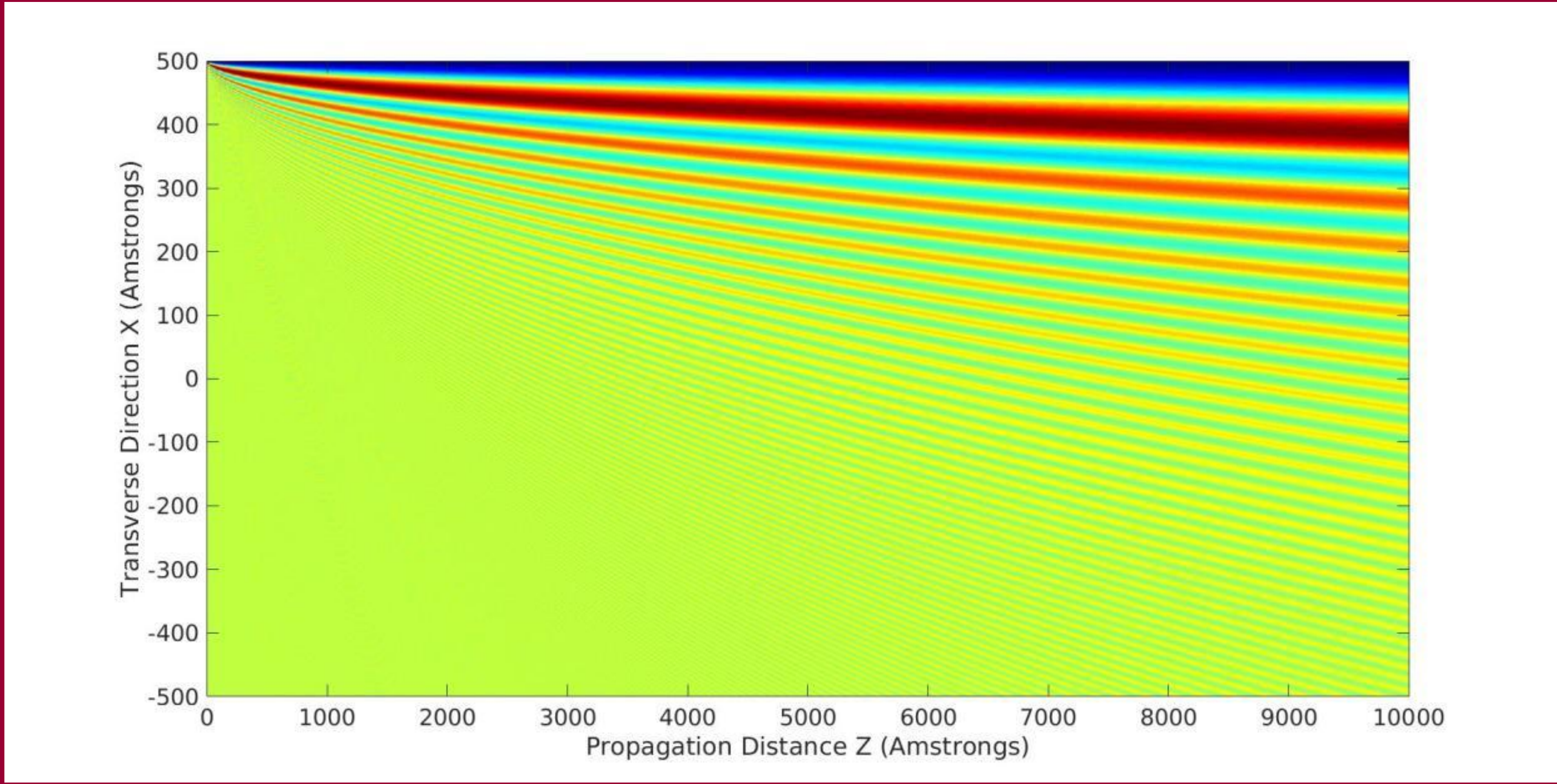
$$+\frac{2m}{\hbar^2}\frac{1}{2}\int \frac{dK_z'}{2\,\pi}\int_{0}^{\infty} dz'\mu_3(z')\, \mathrm{e}^{iK_z'(z-z')}e^{-iK_x'(x-x_0/2)}$$

$$+\frac{2m}{\hbar^2}\frac{1}{2}\int \frac{dK_z'}{2\,\pi}\int_{0}^{\infty} dz'\mu_4(z')\, \mathrm{e}^{iK_z'(z-z')}e^{iK_x'(x+x_0/2)}$$

$$\int \frac{dK_x'}{2\,\pi}\left[\int_{-x_0/2}^{x_0/2} dx'\mu(x')e^{iK_x'x'}\right] e^{ixK_x'}\mathrm{e}^{izK_z'}$$



Edge Diffraction



3.1 Approximations for analitical interpretation

$$\varphi(\boldsymbol{x}) = \varphi_{in}(\boldsymbol{x}) +$$

$$+\frac{2m}{\hbar^2}\frac{1}{2}\int \frac{dK'_x}{2\,\pi}\int_{x_0/2}^{\infty} dx'\mu_1(x')\, e^{iK'_x(x-x')}\mathrm{e}^{-izK'_z}$$

$$+\frac{2m}{\hbar^2}\frac{1}{2}\int \frac{dK'_x}{2\,\pi}\int_{-\infty}^{-x_0/2} dx'\mu_2(x')\, e^{iK'_x(x-x')}\mathrm{e}^{-izK'_z}$$

$$+\frac{2m}{\hbar^2}\frac{1}{2}\int \frac{dK'_z}{2\,\pi}\int_{0}^{\infty} dz'\mu_3(z')\, \mathrm{e}^{iK'_z(z-z')}e^{-iK'_x(x-x_0/2)}$$

$$+\frac{2m}{\hbar^2}\frac{1}{2}\int \frac{dK'_z}{2\,\pi}\int_{0}^{\infty} dz'\mu_4(z')\, \mathrm{e}^{iK'_z(z-z')}e^{iK'_x(x+x_0/2)}$$

$$\int \frac{dK'_z}{2\,\pi}\left[\int_0^{\infty} dz' e^{iqz'}e^{-iK'_z z'}\right] e^{iK'_z z} e^{\pm i(x\pm x_0/2)\sqrt{\frac{2mE}{\hbar^2}-K'^2_z}}$$

$$FT[ste\,P(z')]_{K'_z-q} = \pi\left(\frac{1}{i\pi(K'_z-q)} + \delta(K'_z-q)\right)$$

Fourier Transform

Hilbert Transform

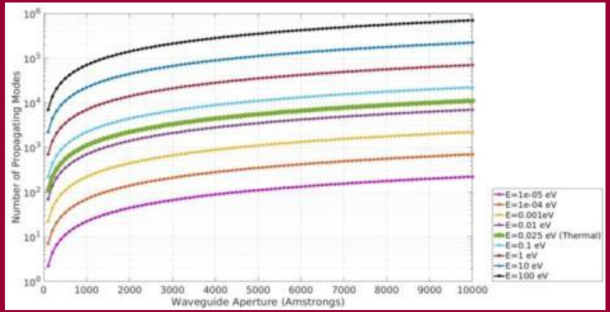
$$= e^{iq^z} e^{\pm i(x\pm x_0/2)\sqrt{\frac{2mE}{\hbar^2}-q^2}}$$

3.1 Approximations for analitical interpretation

- Compiling equations:

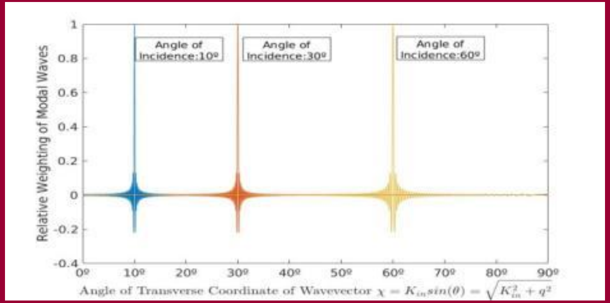
$$\varphi(x,z) = e^{iK_x x}e^{iK_z z} - \underbrace{e^{iK_z z}\left[e^{-i\left(x-\frac{x_0}{2}\right)K_x} + e^{i\left(x+\frac{x_0}{2}\right)K_x}\right]}_{K_x x_0 \ll 1 \quad \text{Extintion}} + e^{iqz}\left[\mu_+ e^{-i\left(x-\frac{x_0}{2}\right)\sqrt{\frac{2mE}{\hbar^2}-q^2}} + \mu_- e^{i\left(x+\frac{x_0}{2}\right)\sqrt{\frac{2mE}{\hbar^2}-q^2}}\right]$$

$$\varphi(x,z) = \begin{cases} -i\sin\left(\dfrac{2n\pi x}{x_0}\right) e^{iz\sqrt{\frac{2mE}{\hbar^2}-\left(\frac{2\pi n}{x_0}\right)^2}} \\ \cos\left(\dfrac{(2n+1)\pi x}{x_0}\right) e^{iz\sqrt{\frac{2mE}{\hbar^2}-\left(\frac{(2n+1)\pi}{x_0}\right)^2}} \end{cases}$$



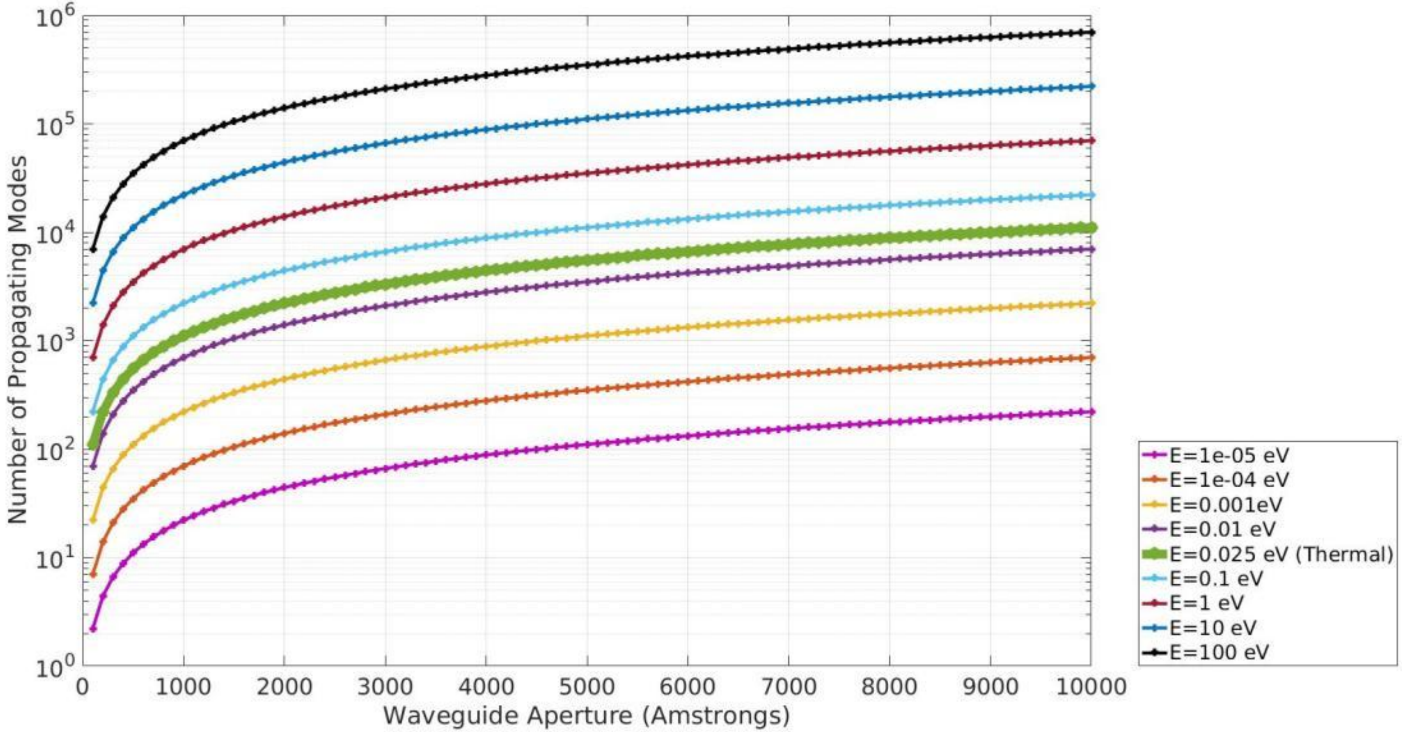
Number of allowed propagation modes

$$N_{max} \approx \frac{X_0}{\pi}\sqrt{\frac{2mE}{\hbar^2}}$$

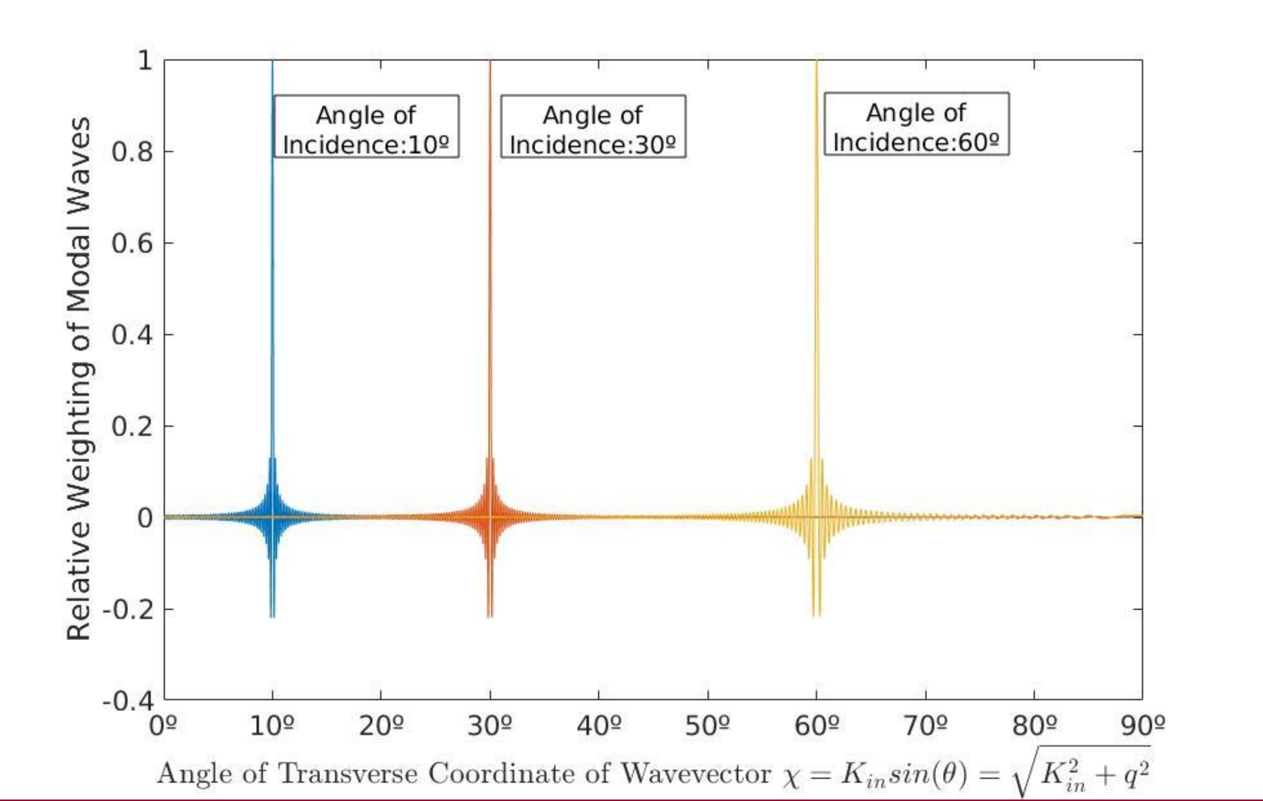


Distribution of Spatial Frequencies

Number of Allowed Levels



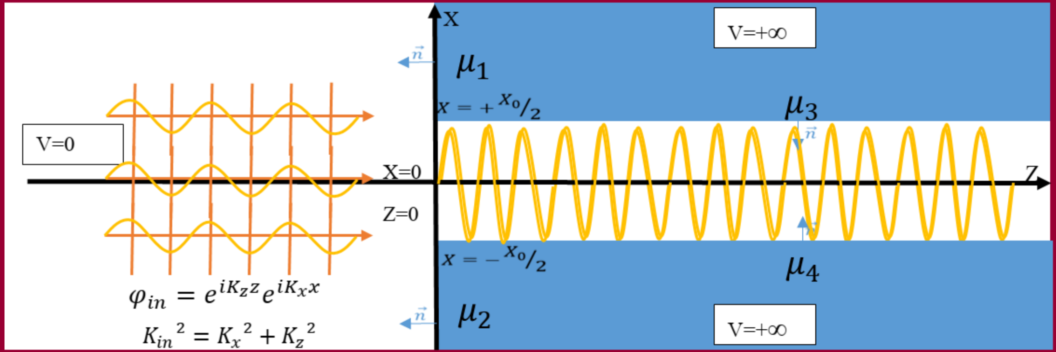
Spatial Frequencies



3.2 Recursive Algorithm

- Re-interpret Dirichlet Boundary Conditions

$$\begin{cases} \frac{1}{2}\frac{2m}{\hbar^2}\mu_1(x) = -\varphi(x, z=0)\,, for\ x \geq \frac{x_0}{2} \\ \frac{1}{2}\frac{2m}{\hbar^2}\mu_2(x) = -\varphi(x, z=0), for\ x \leq -\frac{x_0}{2} \\ \frac{1}{2}\frac{2m}{\hbar^2}\mu_3(z) = -\varphi\left(x = \frac{x_0}{2}, z\right) \\ \frac{1}{2}\frac{2m}{\hbar^2}\mu_4(z) = -\varphi\left(x = -\frac{x_0}{2}, z\right) \end{cases}$$

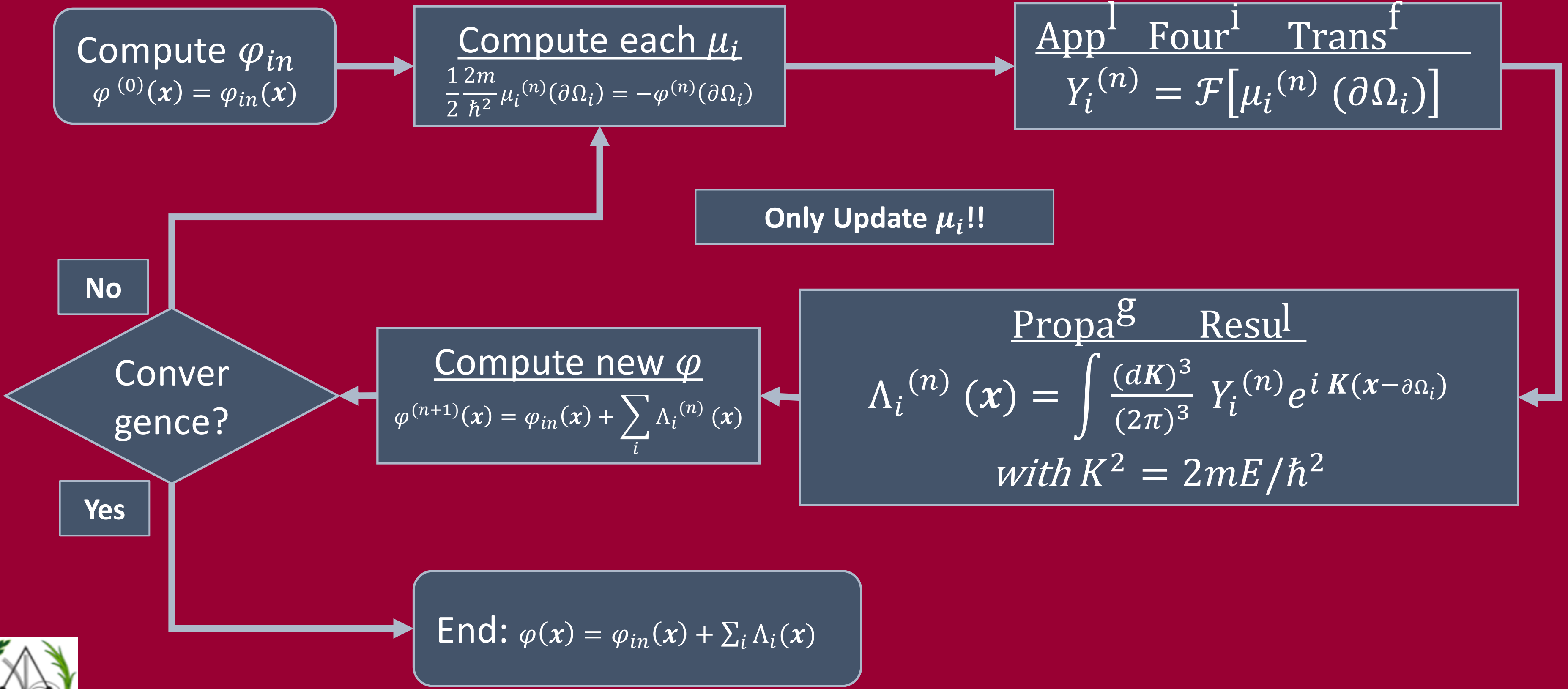


$$\frac{1}{2}\frac{2m}{\hbar^2}\mu_i(\partial\Omega_i) = -\varphi(\partial\Omega_i)$$

$$\varphi(\boldsymbol{x}) = \varphi_{in}(\boldsymbol{x}) + \sum_{i=1}^{n}\int \frac{(d\boldsymbol{K})^3}{(2\pi)^3}\,\mathcal{F}[\mu(\partial\Omega_i)]e^{i\,\boldsymbol{K}(\boldsymbol{x}-\partial\Omega_i)}$$

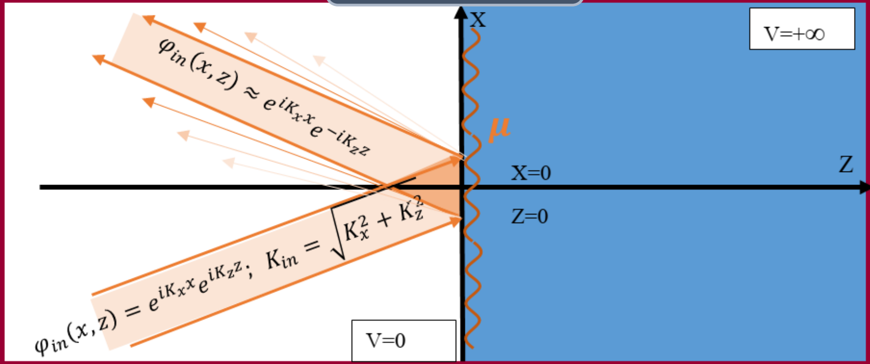
with $K^2 = K_{in}^2 = 2mE/\hbar^2$

3.2 Recursive Algorithm

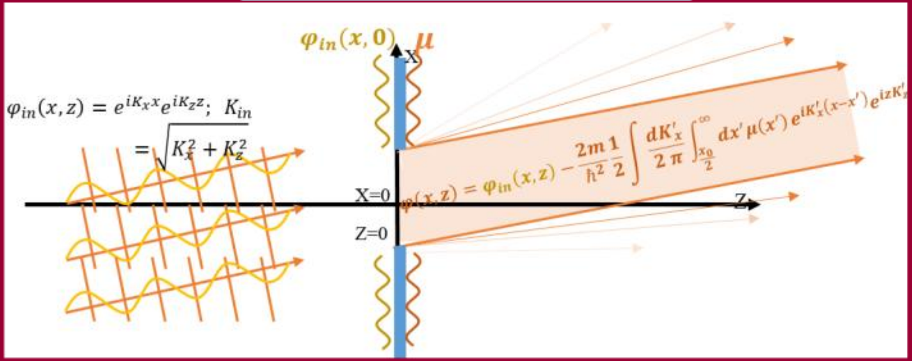


3.2 Simulations Results

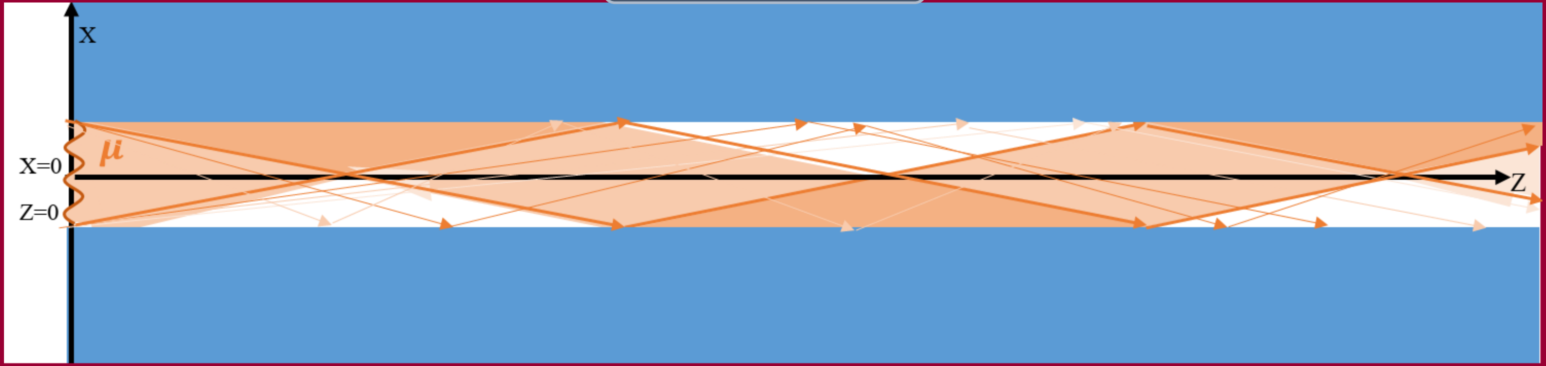
Reflection



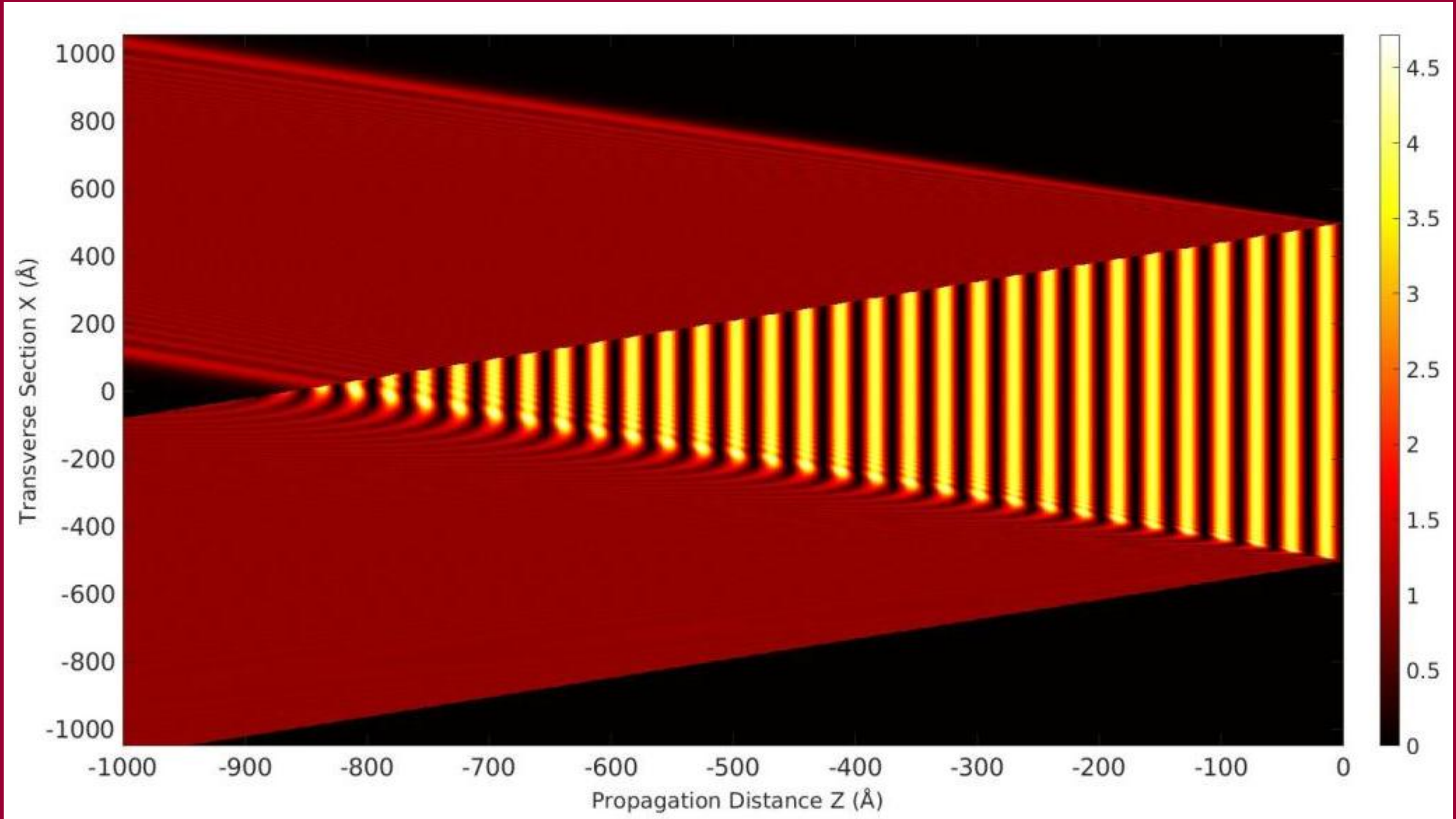
Aperture Diffraction



Waveguide

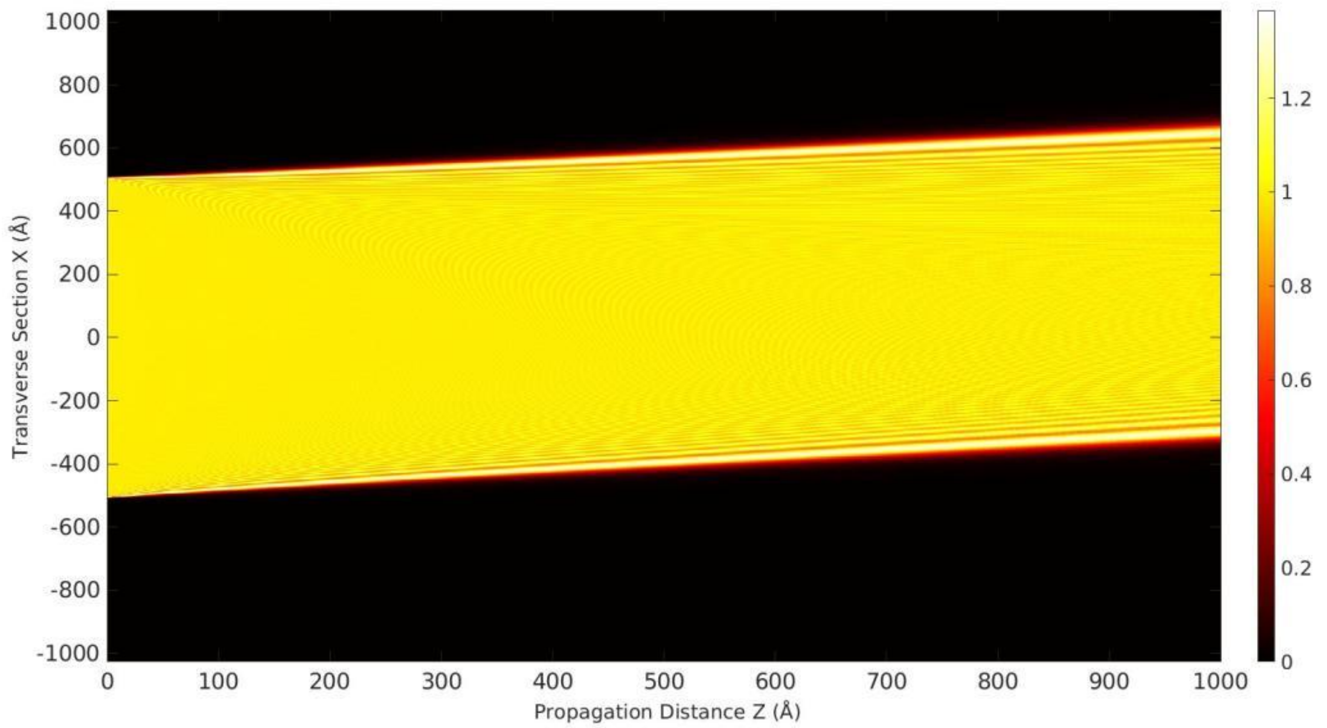


Reflection

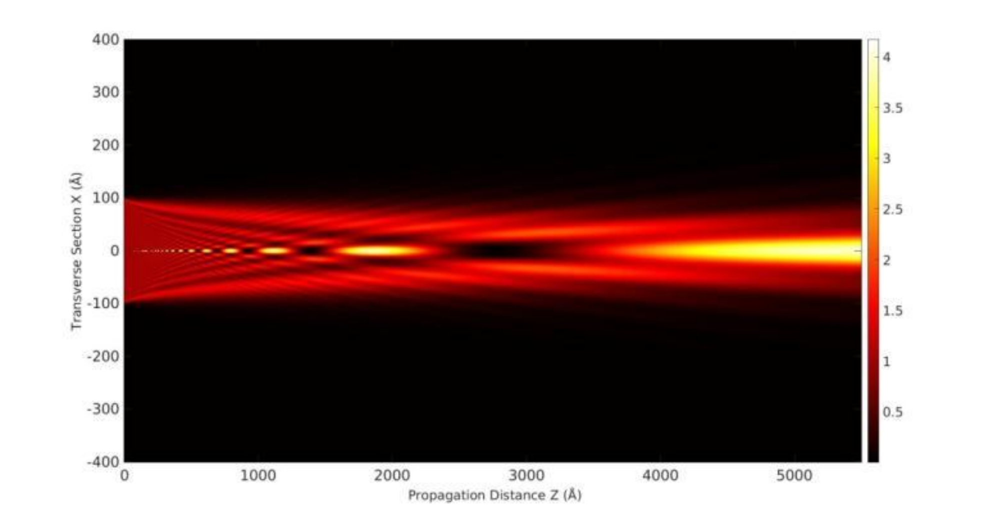


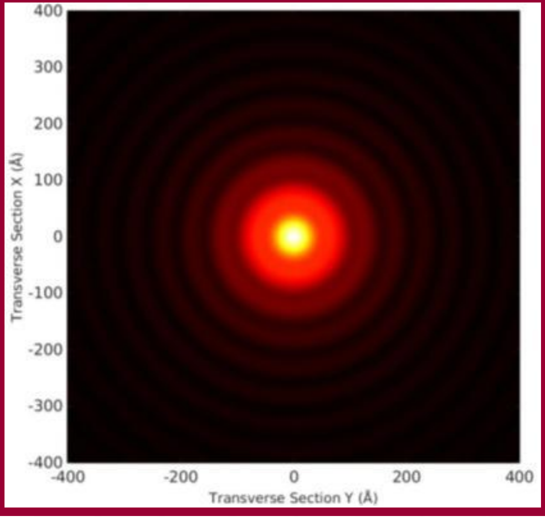
Transverse Section X (Å)
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
Propagation Distance Z (Å)
-1000
-900
-800
-700
-600
-500
-400
-300
-200
-100
0
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0

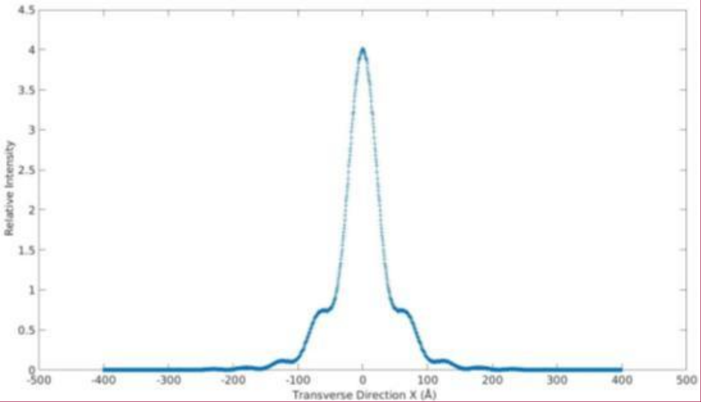
Difraction


1000
800
600
400
200
0
-200
-400
-600
-800
-1000
Transverse Section X (Å)
0
100
200
300
400
500
600
700
800
900
1000
Propagation Distance Z (Å)
1.2
1
0.8
0.6
0.4
0.2
0

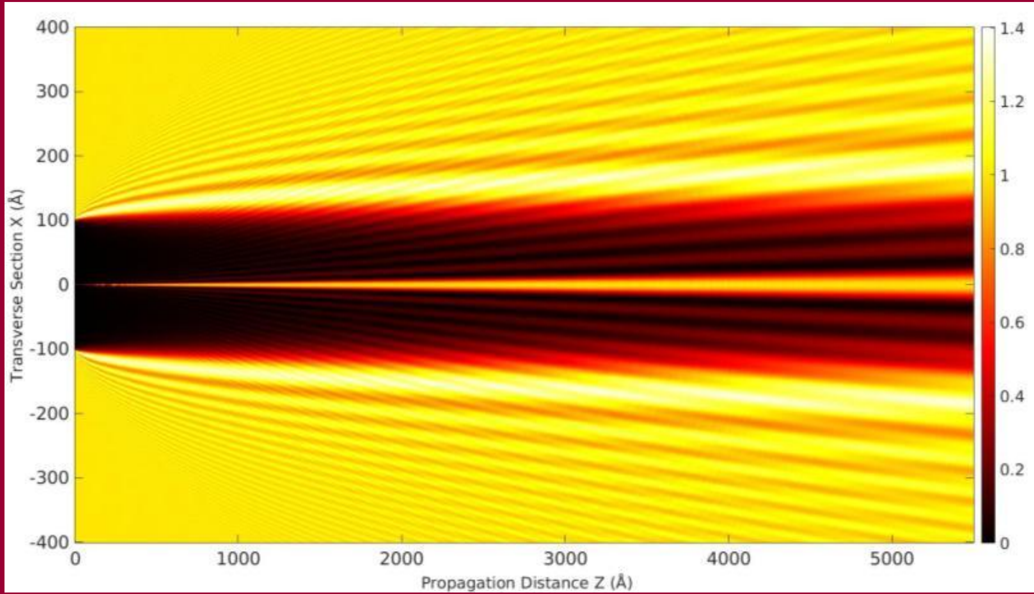
Fresnel Diffraction

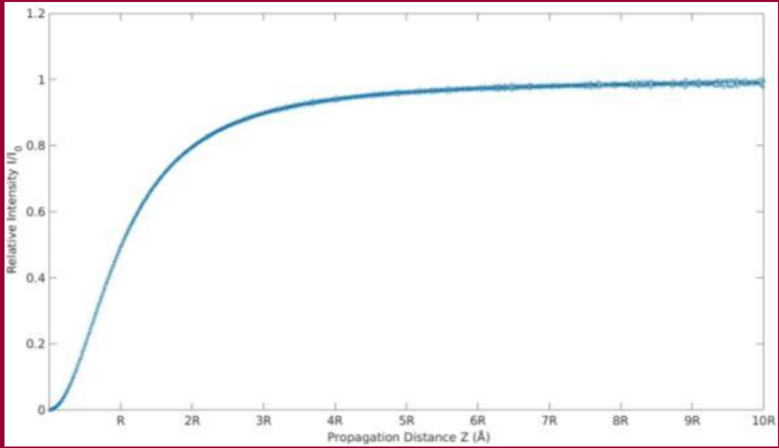




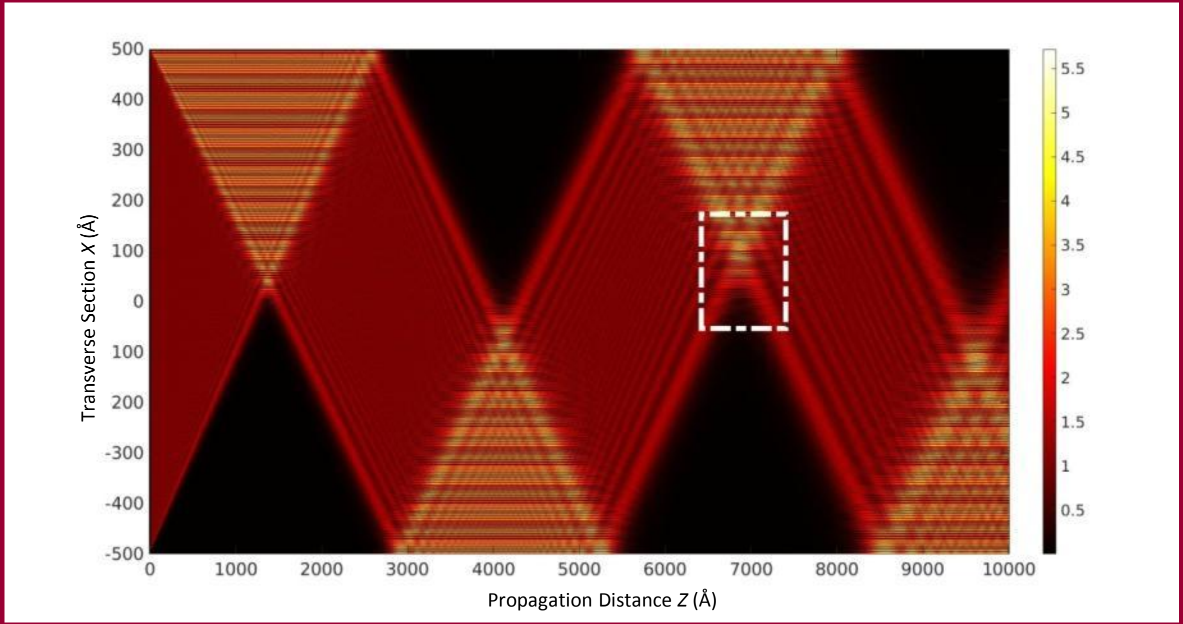


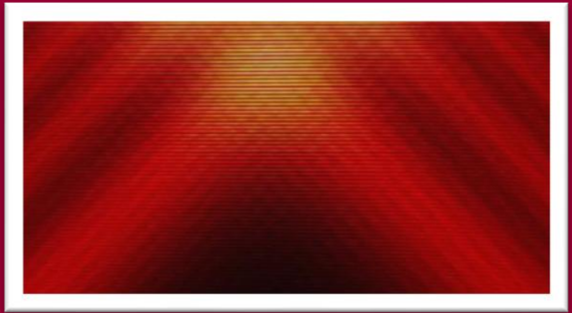
Arago-Poisson Spot



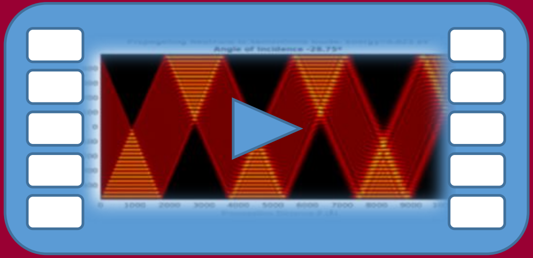


Waveguide





New Algorithm: 15 s aprox.
1,000 x 10,000 (53,3 x size)



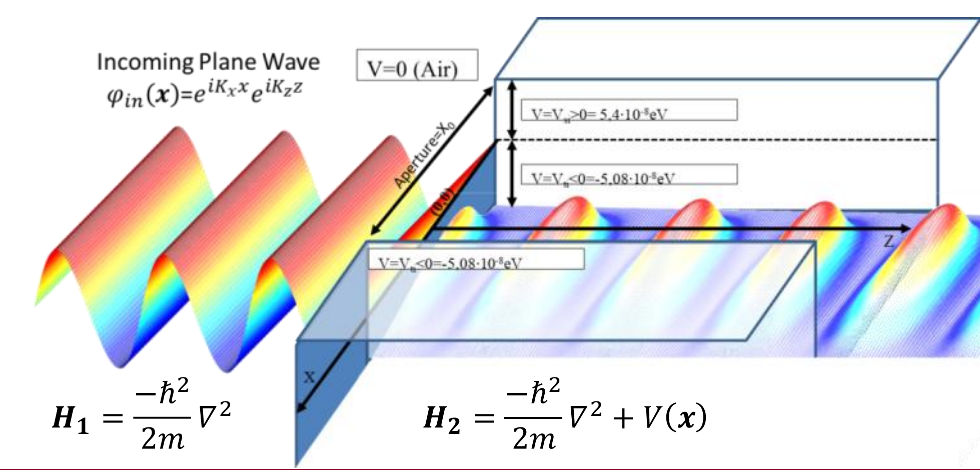
3.3 Finite Potential

- Neutrons do not obey Dirichlet Conditions:
  - Is assumed as a Zero-th order approximation (or case for ultracold neutrons)
  - Wave effects (aperture diffraction, rise of propagation modes,…)
- Waveguiding in Ti cores:
  - Beyond the Dirichlet approximation
  - Discrete guide propagation modes
  - Vanishing effects arise
  - Quantum propagator description
  - Lippmann-Schwinger

$$\Psi(\boldsymbol{x}) = \Psi_{in}(\boldsymbol{x}) + \int d^3\boldsymbol{x}_i\, G(\boldsymbol{x}-\boldsymbol{x}_i)V(\boldsymbol{x}_i)\Psi(\mathbf{x}_i)$$

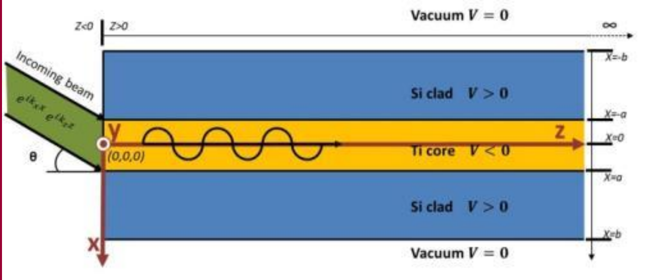
$$G_{1,2}(E) = \frac{1}{E + i\epsilon - H_{1,2}}$$

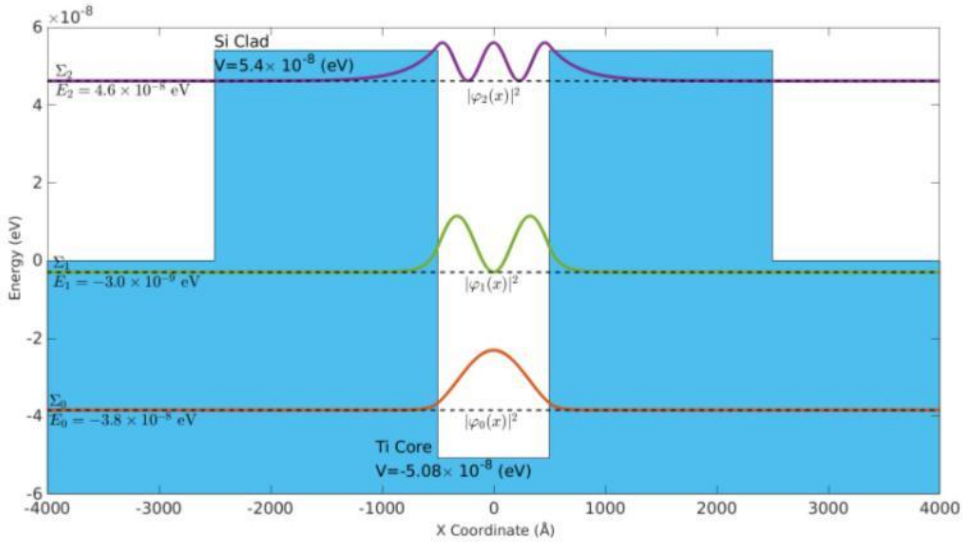
$$LS_1 \rightarrow LS_2 \rightarrow LS_1 \rightarrow LS_2 \rightarrow \cdots$$

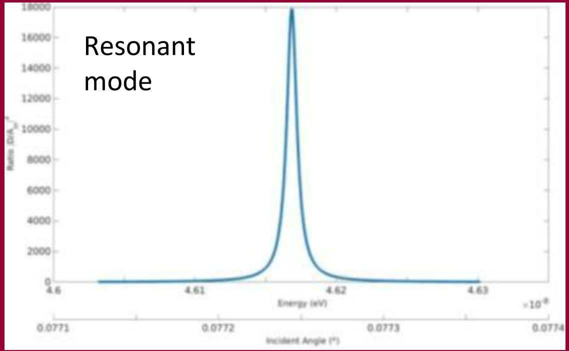


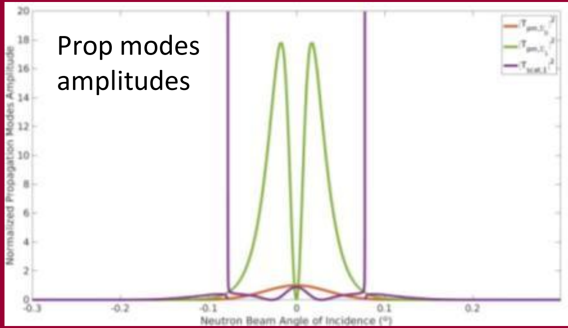
3.3 Finite Potential

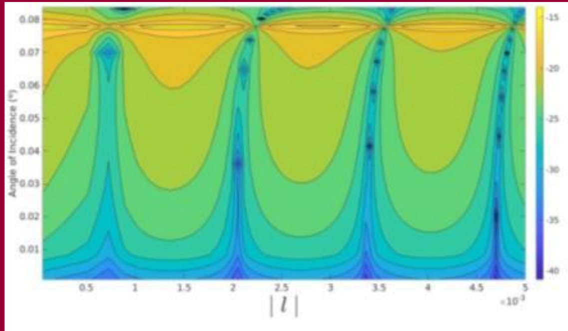
Ti-Si-Ti guide







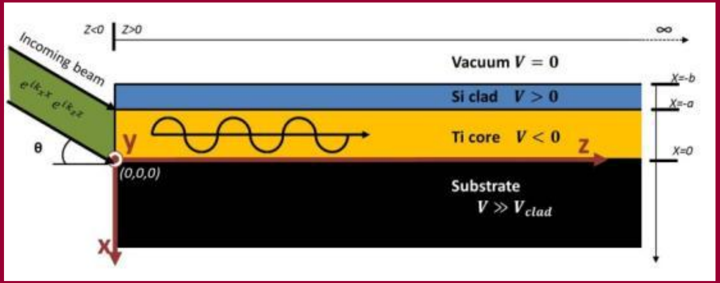


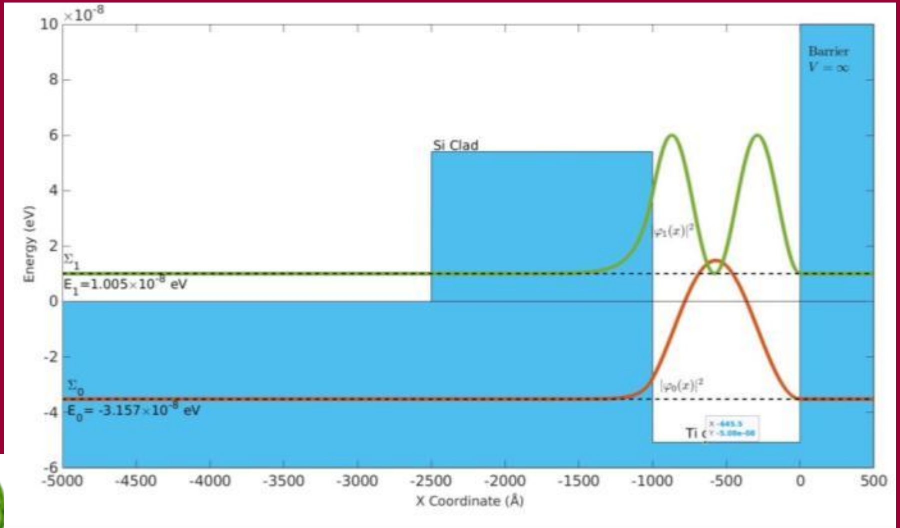


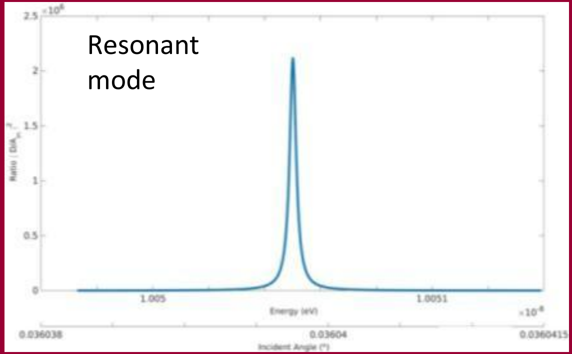
Reflection amplitudes

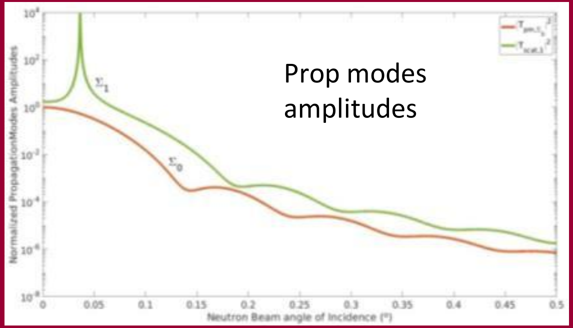
3.3 Finite Potential

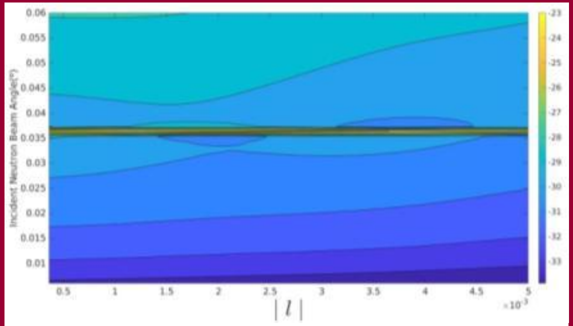
Ti-Si-Substrate guide









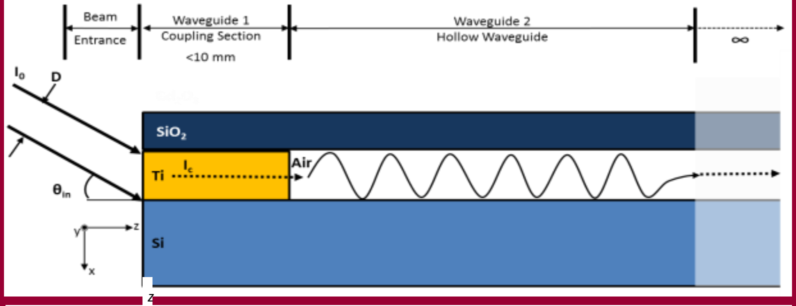


Reflection amplitudes

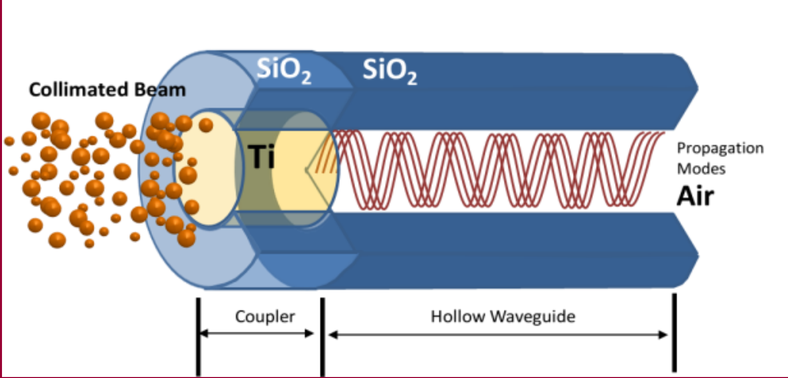
4. Possible Uses

4. Our Objective: Neutron Beam Couplers

- TCW1:
  - Two planar waveguides successively coupled along propagation direction:

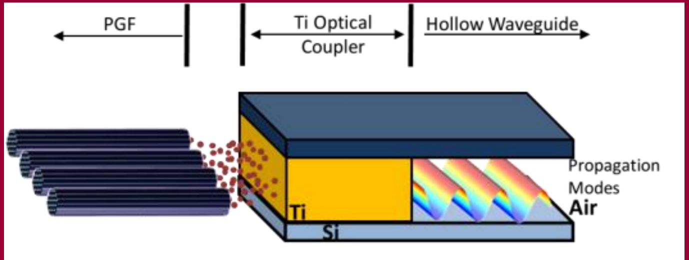


- TCW2:
  - Natural improvement of TCW1 involving the two transverse dimensions. Only one clad (Si or SiO_2)

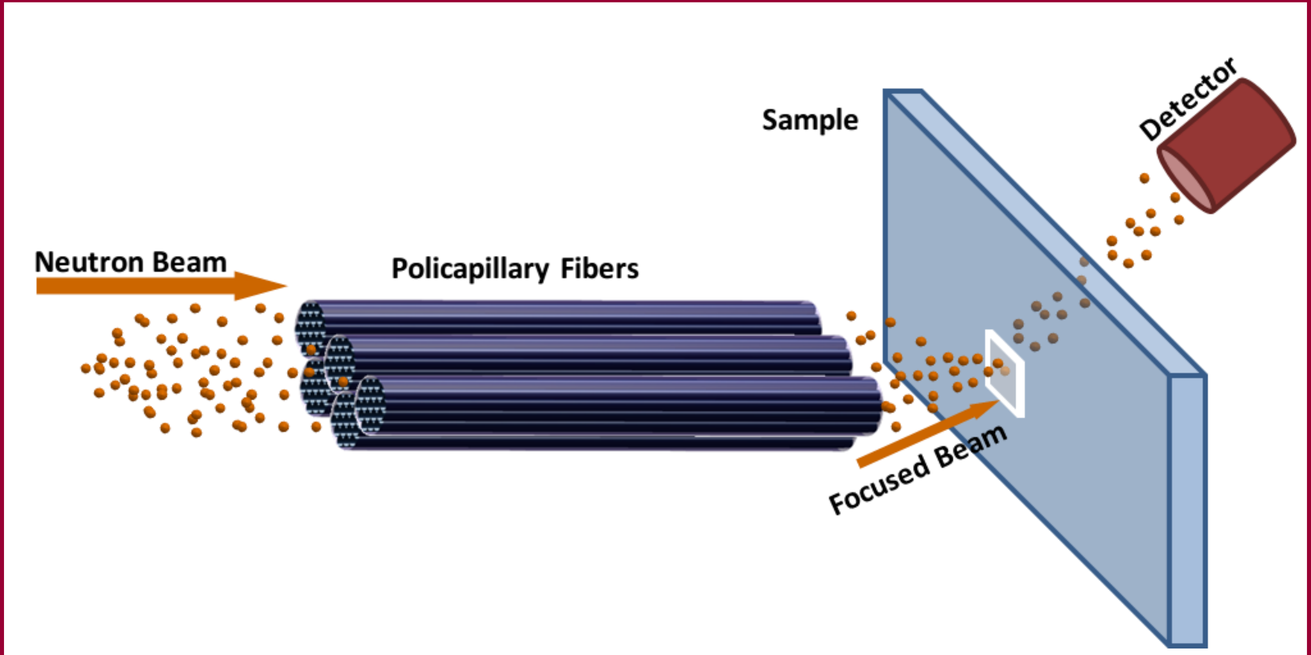


- Notice:
  - Both proposals involve Ti:
    - Value of $b<0$ and μ not so high

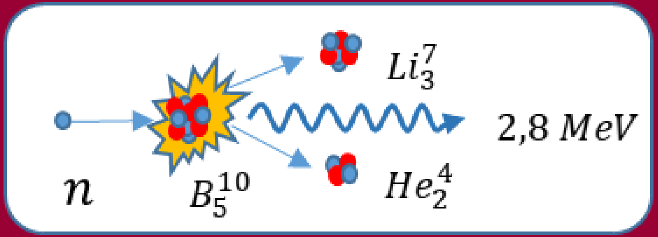
- CW3:
  - Three coupled waveguides



4. Analysis of Matter

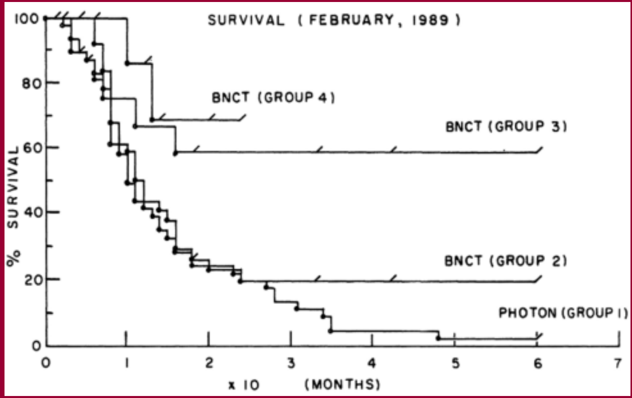


4. BNCT



- Boron Neutron Capture Therapy:
  - $n + B_5^{10} \rightarrow He_2^4 + Li_3^7 + 2.8\ MeV$
  - B is provided to tumour domains through compounds BSH and BPA
  - Boron: High absorbing cross section for thermal neutrons
  - He and Li nuclei released produce damage to tumour cells DNA
- Poor penetration for thermal neutrons
  - Craniotomy
  - Use of epithermal neutrons
    - Monte Carlo for Dose planning in patients

4. BNCT



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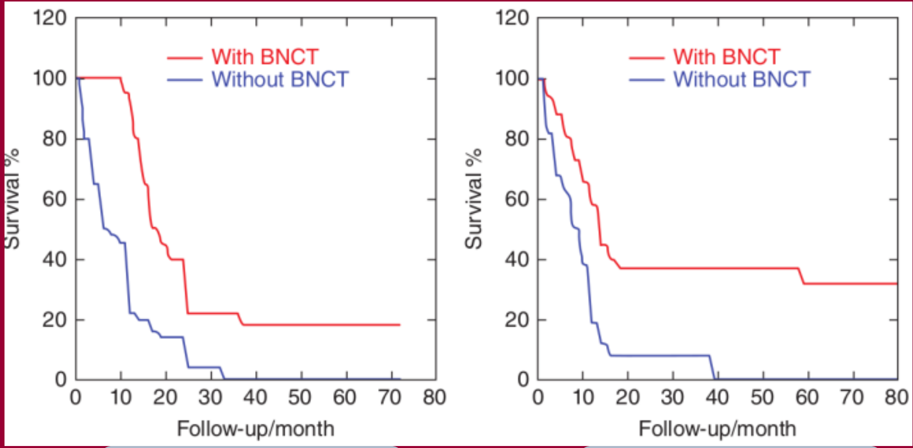
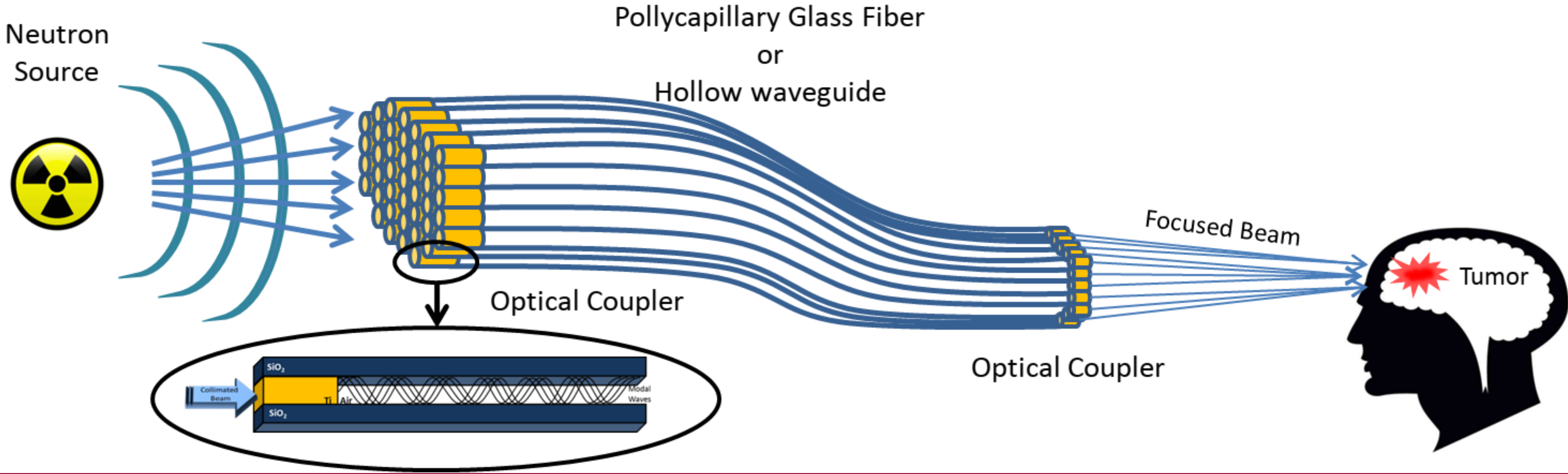


Glioblastoma

Neck Cancer

Nuclear Physics European Collaboration Committee (NUPECC), Nuclear Physics for Medicine Report, (2014) http://www.nupecc.org/pub/npmed2014.pdf

4. BNCT improvement



R.F. Alvarez-Estrada, I. Molina de la Peña, and M.L. Calvo. *Focalizing slow neutron beams at and below micron scales: Discussion on BNCT.* Phosphorus Sulfur And Silicon And The Related Elements, 193(2):64–73, 2018.

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Neutron Optics: Applications to Neutron Waveguides

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