UNIVERSITÄT BONN

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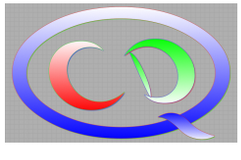
CENTER FOR SCIENCE & THOUGHT

JÜLICH FORSCHUNGSZENTRUM

New insights into the nucleon's electromagnetic structure

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VolkswagenStiftung

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CONTENTS

- Introductory remarks
- Theoretical framework: Dispersion relations
- Discussion of the spectral functions
- Fit procedure & theoretical uncertainties
- Results for space- and time-like ffs
- The proton radius controversy
- Summary and outlook

Introduction

FACETS/SIZES of the NUCLEONS

- Nucleons are basic objects of QCD in the strong coupling regime
- Various probes (electromagnetic, weak) see different facets/scales

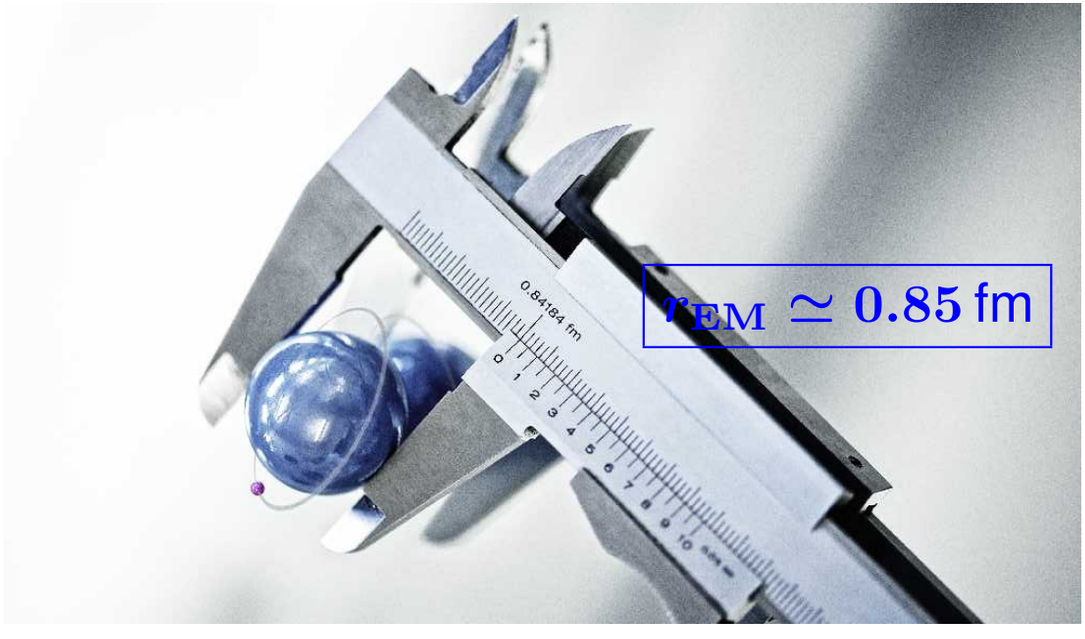


Fig. from Pohl et al., Nature 466 (2010) 213

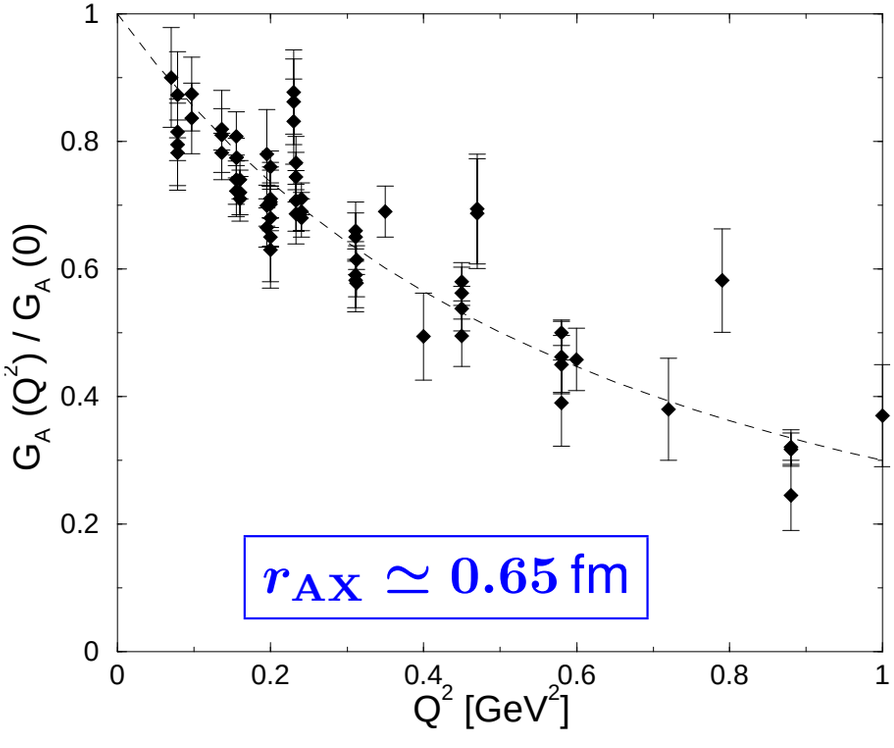


Fig. from Bernard, Elouadrhiri, UGM, J.Phys.G 28 (2002) R1

$\hookrightarrow$ learn more about the nucleon form factors & sizes

Theoretical framework

Review: Lin, Hammer, UGM, Eur. Phys. J. **A 57** (2021) 255

BASIC DEFINITIONS

- Nucleon matrix elements of the em vector current J^I_μ

$$\langle N(p')|J^I_\mu|N(p)\rangle = \bar{u}(p')\left[F^I_1(t)\,\gamma_\mu + i\,\frac{F^I_2(t)}{2m}\,\sigma_{\mu\nu}q^\nu\right]u(p)$$

⋆ isospin $I = S, V$ (isoScalar, isoVector) $[= (p \pm n)/2]$

⋆ four-momentum transfer $t \equiv q^2 = (p' - p)^2 \equiv -Q^2$

⋆ F_1 = Dirac form factor, F_2 = Pauli form factor

⋆ Normalizations: $F_1^V(0) = F_1^S(0) = 1/2$, $F_2^{S,V}(0) = (\kappa_p \pm \kappa_n)/2$

⋆ Sachs form factors: $G_E = F_1 + \frac{t}{4m^2}F_2$, $G_M = F_1 + F_2$

⋆ Nucleon radii: $F(t) = F(0)\,\left[1 + t\langle r^2\rangle/6 + \ldots\right]$ [except for the neutron charge ff]

WHY DISPERSION RELATIONS for the NUCLEON FFs ?

- Model-independent approach $\rightarrow$ important non-perturbative tool to analyze data
- Dispersion relations are based on fundamental principles: unitarity & analyticity
- Connect data from small to large momentum transfer
  as well as time- and space-like data
- Allow for a **simultaneous analysis** of all four em form factors
- Spectral functions encode perturbative and non-perturbative physics
  e.g. vector meson couplings, multi-meson continua, pion cloud, ...
- Spectral functions also encode information on the strangeness vector current
  $\rightarrow$ sea-quark dynamics, strange matrix elements
- Allow to extract nucleon electric and magnetic radii
- Can be matched to chiral perturbation theory [and not the other way around!]

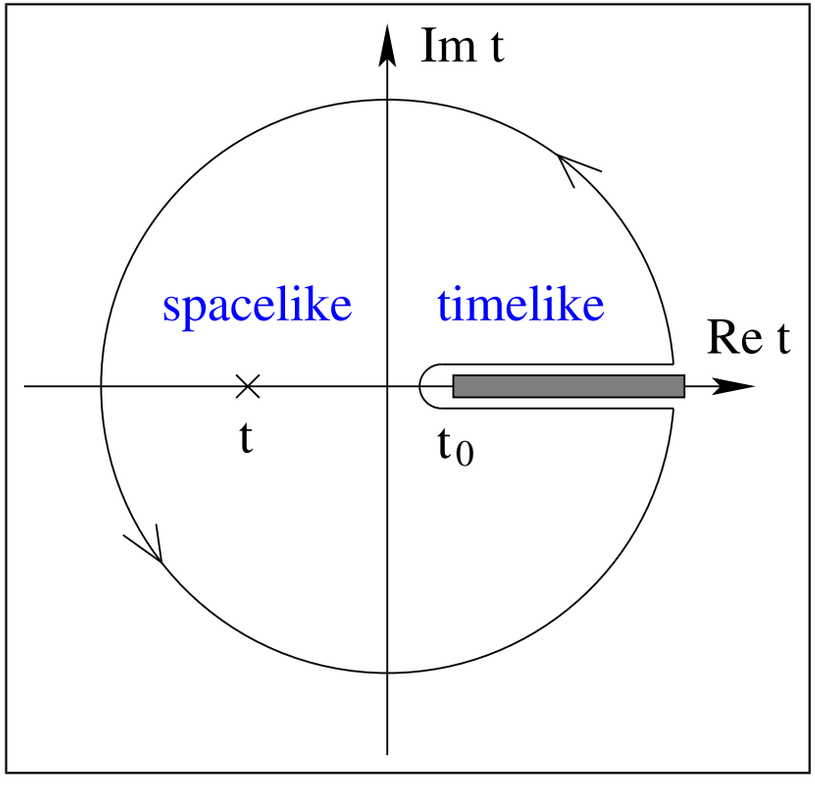
DISPERSION RELATIONS

Federbush, Goldberger, Treiman, Drell, Zachariasen, Frazer, Fulco, Höhler, . . .

- The form factors have cuts in the interval $[t_n, \infty[$ $(n = 0, 1, 2, \ldots)$ and also poles

$\Rightarrow$ Dispersion relations for $F_i(t)$ $(i = 1, 2)$:

$$F_i(t) = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \, \frac{\mathrm{Im}\, F_i(t')}{t' - t}$$



- no subtractions
  [only proven in perturbation theory]
- suppression of higher mass states
- central objects: spectral functions

$$\mathrm{Im}\, F_i(t)$$

  – cuts $\hat{=}$ multi-meson continua
  – poles $\hat{=}$ vector mesons

SPECTRAL FUNCTIONS – GENERALITIES

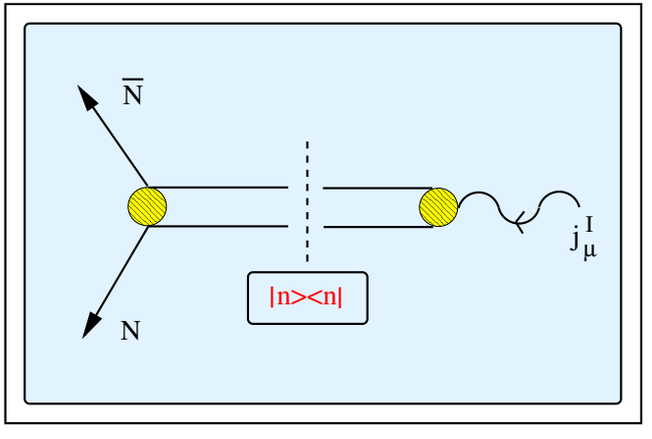
- Spectral decomposition: Chew, Karplus, Gasiorowicz, Zachariasen (1958)

$$\mathbf{Im}\,\langle \bar{N}(p')N(p)|J^I_\mu|0\rangle \sim \sum_{n}\langle \bar{N}(p')N(p)|n\rangle\langle n|J^I_\mu|0\rangle \Rightarrow \mathbf{Im}\,F$$

⋆ on-shell intermediate states

⋆ generates imaginary part

⋆ accessible physical states



- *Isoscalar* intermediate states: $3\pi, 5\pi, \ldots, K\bar{K}, K\bar{K}\pi, \pi\rho, \ldots +$ poles

  $\rightarrow t_0 = 9M_\pi^2$

- *Isovector* intermediate states: $2\pi, 4\pi, \ldots +$ poles $\rightarrow t_0 = 4M_\pi^2$

- Note that some poles are *generated* from the appropriate continua

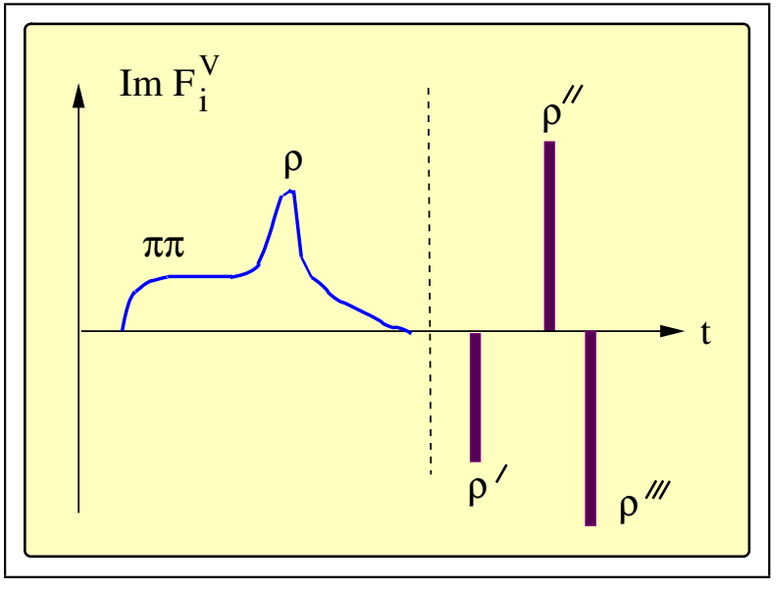
ISOVECTOR SPECTRAL FUNCTIONS

Frazer, Fulco, Höhler, Pietarinen, . . .

- exact 2π continuum is known from threshold $t_0 = 4M_\pi^2$ to $t \simeq 40\, M_\pi^2$

$$\boxed{\mathrm{Im}\, G^V_{E,M}(t) = \frac{q_t^3}{\sqrt{t}} F_\pi(t)^\star f^1_\pm(t)}$$

  - $\star$ $F_\pi(t)$ = pion vector form factor
  - $\star$ $f^1_\pm(t)$ = P-wave pion-nucleon partial waves in the t-channel



- Spectral functions inherit singularity on the second Riemann sheet in $\pi N \to \pi N$

$$\boxed{t_c = 4M_\pi^2 - M_\pi^4/m^2 \simeq 3.98\, M_\pi^2}$$ $\to$ strong shoulder $\to$ isovector radii

- This singularity can also be analyzed in CHPT

Bernard, Kaiser, UGM, Nucl. Phys. A **611** (1996) 429

- For a recent determination of the 2π continuum, see HKRHM, EPJA **52** (2016) 331

- Higher mass states represented by poles (not necessarily physical masses)

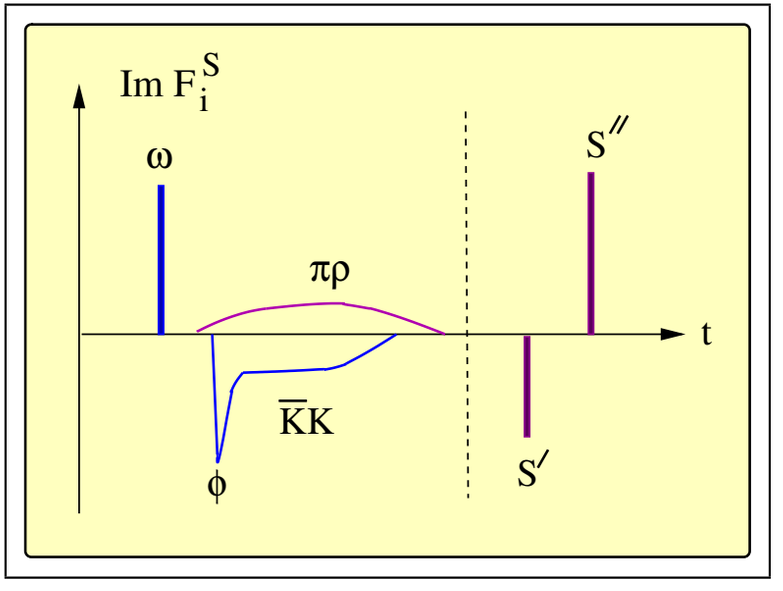
ISOSCALAR SPECTRAL FUNCTIONS

- $K\bar{K}$ continuum can be extracted from analytically cont. KN scattering amplitudes

  $\rightarrow$ analytic continuation must be stabilized

  $\rightarrow$ generates most of the ϕ contribution

  Hammer, Ramsey-Musolf, Phys. Rev. C **60** (1999) 045204, 045205



- Further strength in the ϕ-region generated by correlated $\pi\rho$ exchange

  $\rightarrow$ strong cancellations ($K\bar{K}$, K^*K, $\pi\rho$)

  $\rightarrow$ takes away sizeable strength from the ϕ

  UGM, Mull, Speth, van Orden, Phys. Lett. B **408** (1997) 381

- Spectral functions exhibit anomalous threshold (analyzed in 2-loop CHPT)

$$t_c = M_\pi^2 \left(\sqrt{4 - M_\pi^2/m^2} + \sqrt{1 - M_\pi^2/m^2}\right)^2 \simeq 8.9\, M_\pi^2$$

$\rightarrow$ effectively masked

Bernard, Kaiser, M, Nucl. Phys. A **611** (1996) 429

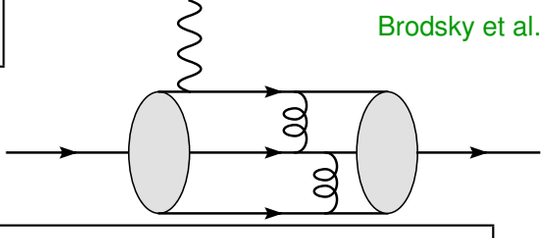
- Higher mass states represented by poles (with a finite width)

CONSTRAINTS ON THE SPECTRAL FUNCTIONS

- Normalizations: electric charges, magnetic moments
- Radii *not* imposed [except for the neutron charge radius] Filin et al., Phys. Rev. C **103** (2021) 024313
- Superconvergence relations $\cong$ leading pQCD behaviour

$$\boxed{F_1(t) \sim 1/t^2 \, , F_2(t) \sim 1/t^3 \;\; (\mathbf{helicity-flip})}$$

Brodsky et al.



$$\Rightarrow \boxed{\int_{t_0}^{\infty} \mathrm{Im}\, F_1(t)\, dt = 0 \, , \quad \int_{t_0}^{\infty} \mathrm{Im}\, F_2(t)\, dt = \int_{t_0}^{\infty} \mathrm{Im}\, F_2(t)\, t\, dt = 0}$$

- Various ways of implementing the asymptotic QCD behaviour

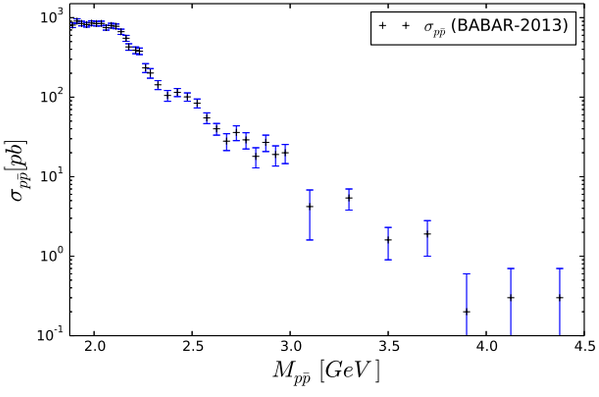
$\Rightarrow$ severely restricts the number of fit parameters

FORM FACTORS IN THE TIME–LIKE REGION

- Xsection for $e^+e^- \leftrightarrow \bar{p}p, \bar{n}n$ in the one-photon approximation

$$\sigma(s) = \frac{4\pi\alpha_{\rm EM}^2\beta}{3s}\, C(s)\, \left[|G_M(s)|^2 + \frac{2m_N^2}{s}\,|G_E(s)|^2\right], \quad \beta = \frac{k_p}{k_e}$$

$$= \frac{4\pi\alpha_{\rm EM}^2\beta}{3s}\, C(s)\, \left(1 + \frac{2m_N^2}{s}\right)\, |G_{\rm eff}|^2$$



- $G_{E,M}(s)$ are complex for $s \geq 4M_N^2$
- Threshold constraint: $G_E(4M_N^2) = G_M(4M_N^2)$
- Gamov-Sommerfeld factor (only for the proton):

$$C = \frac{y}{1 - e^{-y}}, \quad y = \frac{\pi\alpha_{\rm EM} m_p}{k_p}, \quad \sqrt{s} = 2\sqrt{m_p^2 + k_p^2}$$

- Data from $e^+e^- \to \bar{N}N$ & $\bar{N}N \to e^+e^-$: strong threshold enhancement & oscillations

SUMMARY: SPECTRAL & FIT FUNCTIONS

- Representation of the pole contributions: vector mesons
  [NB: can be extended for finite width]

$$\boxed{\mathrm{Im}\, F_i^V(t) = \sum_v \pi a_i^v\, \delta(t - M_v^2)\,, \quad a_i^v = \frac{M_v^2}{f_V} g_{vNN} \;\Rightarrow\; F_i(t) = \sum_v \frac{a_i^v}{M_v^2 - t}}$$

- *Isovector* spectral functions:

$$\mathrm{Im}\, F_i^V(t) = \mathrm{Im}\, F_i^{(2\pi)}(t) + \sum_{v=v_1,v_2,\ldots} \pi\, a_i^v\, \delta(t - M_v^2) + \sum_{V=V_1,\ldots} \mathrm{Im}\, F_i^{V_i} \qquad (i = 1, 2)$$

- *Isoscalar* spectral functions:

$$\mathrm{Im} F_i^S(t) = \mathrm{Im} F_i^{(K\bar{K})}(t) + \mathrm{Im} F_i^{(\pi\rho)}(t) + \sum_{v=\omega,\phi,s_1,s_2,\ldots} \pi\, a_i^v\, \delta(t - M_v^2) + \sum_{S=S_1,\ldots} \mathrm{Im}\, F_i^{S_i}$$

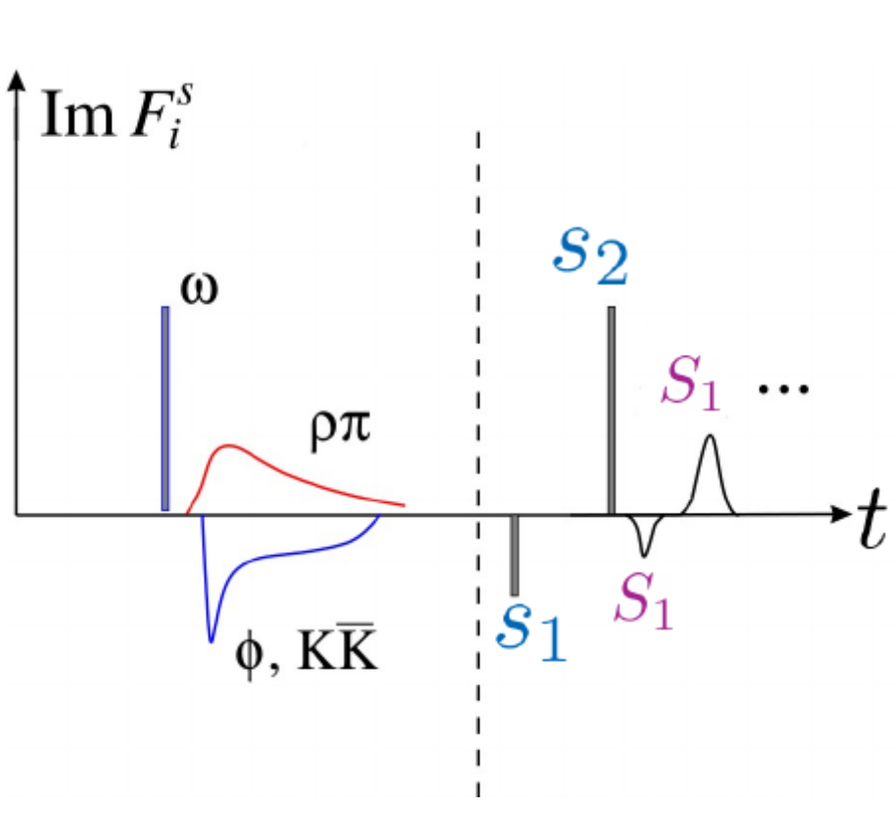
- Parameters: 2 for the ω, ϕ, 3 (4) for each other V-mesons minus # of constraints

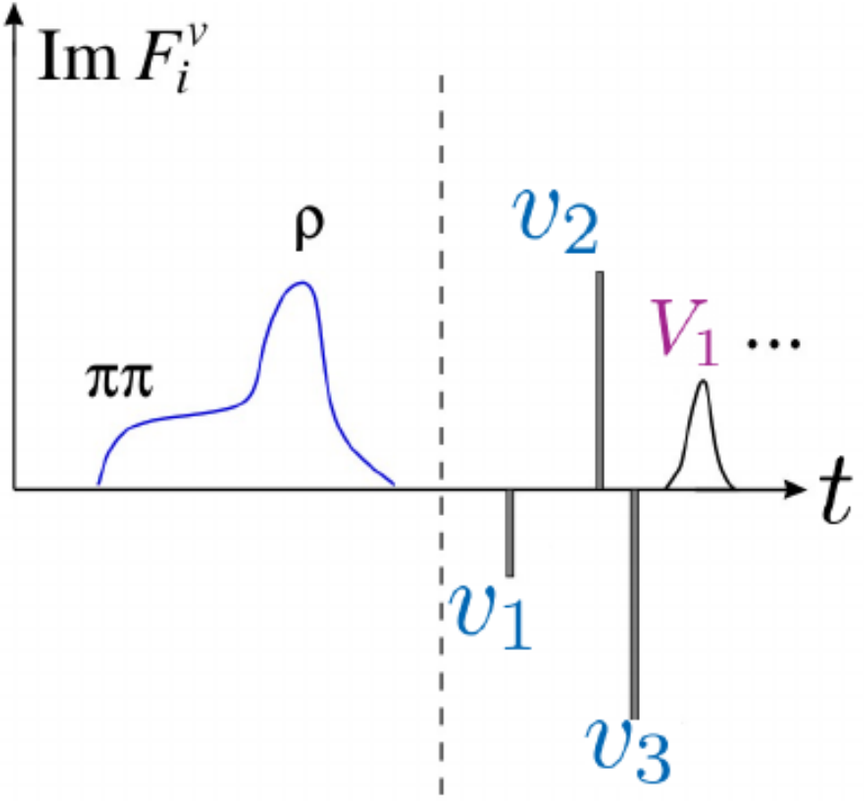
- Additional broad poles w/ 3 parameters above the $\bar{N}N$ threshold [structures in the time-like ffs]

- Ill-posed problem $\rightarrow$ extra constraint: minimal # of poles to describe the data

SUMMARY: SPECTRAL FUNCTIONS

- Cartoons of the isoscalar/isovector spectral functions:





Fit procedure & theoretical uncertainties

Review: Lin, Hammer, UGM, Eur. Phys. J. **A 57** (2021) 255

DATA BASIS

- Data basis:

Region	Observables	Coll.	$\|t\|$(GeV^2)	number
Spacelike($t<0$)	$d\sigma/d\Omega$	MAMI	0.00384–0.977	1422
		PRad	0.000215–0.058	71
	$\mu_p G_E^p/G_M^p$	JLab	1.18–8.49	16
	G_E^n	world	0.14–1.47	25
	G_M^n	world	0.071–10.0	23
Timelike($t>0$)	$\|G_{eff}^p\|$	world	3.52–20.25	153
	$\|G_{eff}^n\|$	world	3.53–9.49	27
	$\|G_E^p/G_M^p\|$	BaBar	3.52–9.0	6
	$d\sigma/d\Omega$	BESIII	3.52–3.80	10

- Number of data/fit parameters:

$$\#_{\rm data} = 1753\,, \quad \#_{\rm fitpara.} = \underbrace{4}_{\omega+\phi} + 3\underbrace{(N_s+N_v)}_{best\ fit:\ 3+5} + 4\underbrace{(N_S+N_V)}_{best\ fit:\ 3+3} - 11 + \underbrace{31+2}_{\rm norm.}$$

FIT PROCEDURE

- Fit strategy (weigthed average): $\chi^2_{\rm total} = \sum_{i \text{ in data basis}} \frac{\chi^2_i}{\#\text{ data points}}$

- Two definitions of χ^2 [(un)correlated errors]:

$$\chi^2_1 = \sum_i \sum_k \frac{(n_k C_i - C(Q^2_i, \theta_i, \vec{p}\,))^2}{(\sigma_i + \nu_i)^2}$$

$$\chi^2_2 = \sum_{i,j} \sum_k (n_k C_i - C(Q^2_i, \theta_i, \vec{p}\,))[V^{-1}]_{ij} (n_k C_j - C(Q^2_j, \theta_j, \vec{p}\,))$$

$$V_{ij} = \sigma_i \sigma_j \delta_{ij} + \nu_i \nu_j$$

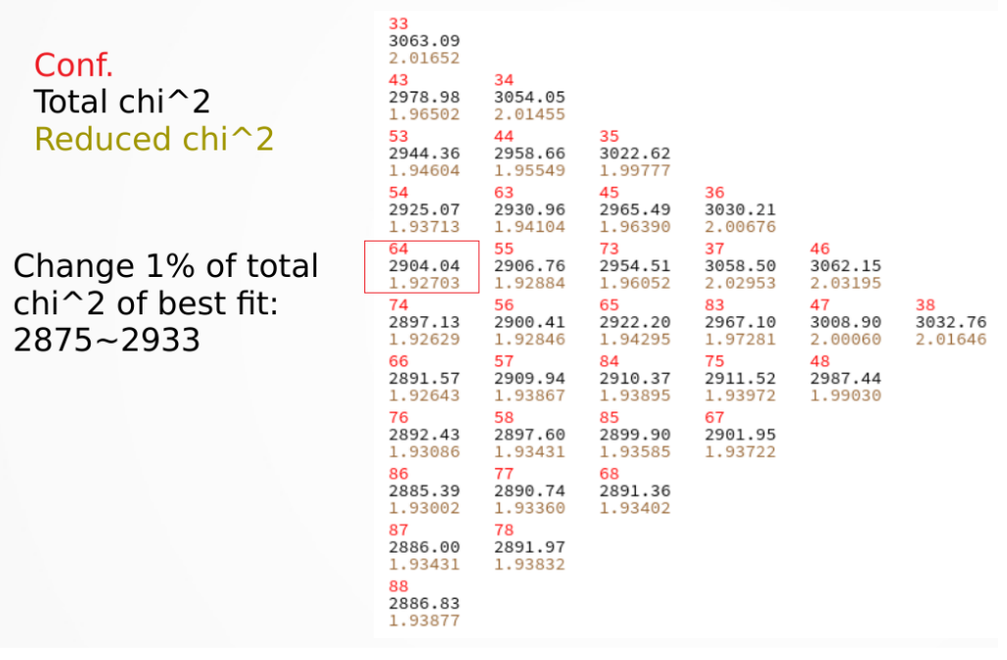
  - C_i = cross section data
  - $C(Q^2_j, \theta_j, \vec{p}\,)$ = XS for a given FF parametrization
  - $\vec{p}$ = parameter values for a given FF parametrization
  - n_k = normalization constants for various data sets k
  - V_{ij} = covariance matrix w/ stat. (σ_i) & syst. (ν_i) errors

TYPES of UNCERTAINTIES

- Systematic and statistical, only recently fully incorporated

- Systematics: Vary the number of poles
  around the best solution
  so that $\delta\chi^2_{\rm total} \leq 1\%$

  Höhler et al. (1976)



- Statistical errors, use
  - bootstrap method

    Efron, Tibshirani (1986)
  - Bayesian analysis

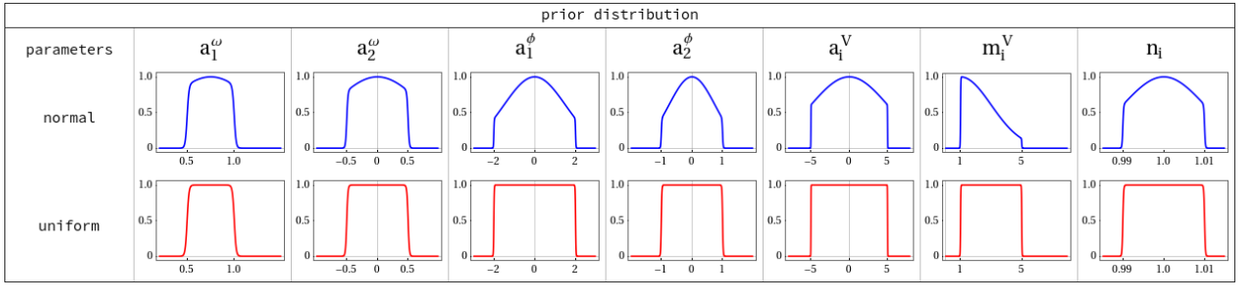
    Bayes (1763)

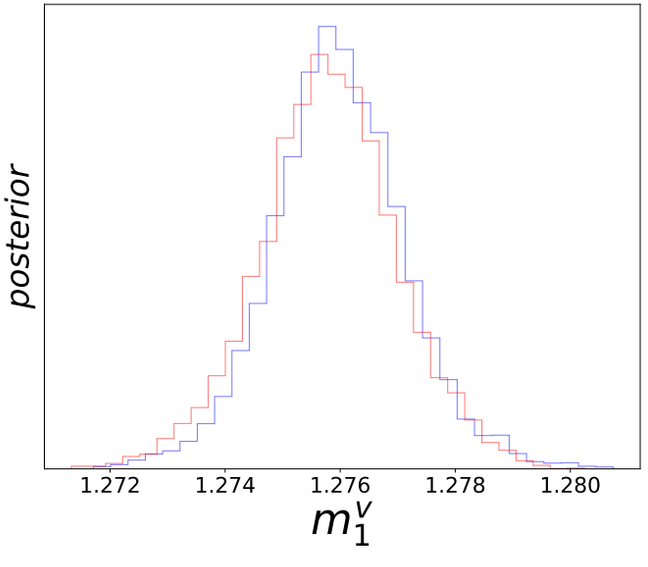
BAYESIAN ANALYSIS

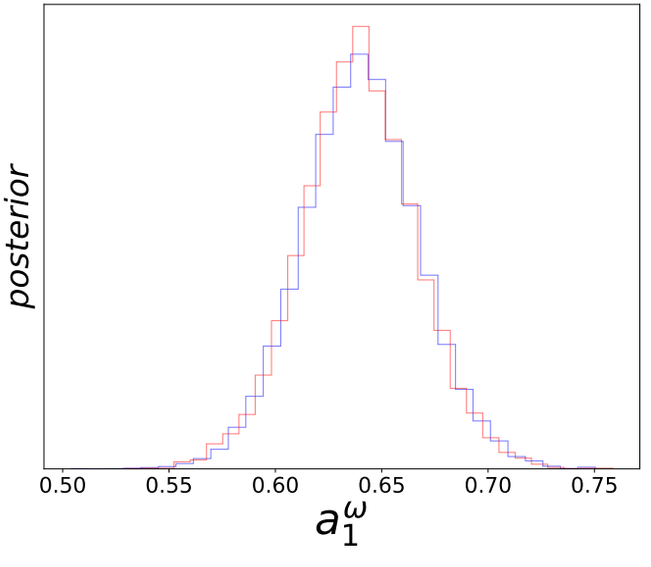
- Bayes theorem: $P(\text{parameters}|\text{data}) = \dfrac{P(\text{parameters})P(\text{data}|\text{parameters})}{P(\text{data})}$

  posterior $\sim$ prior $\times$ likelihood

- Bayesian analysis of the PRad data (71 data pts)



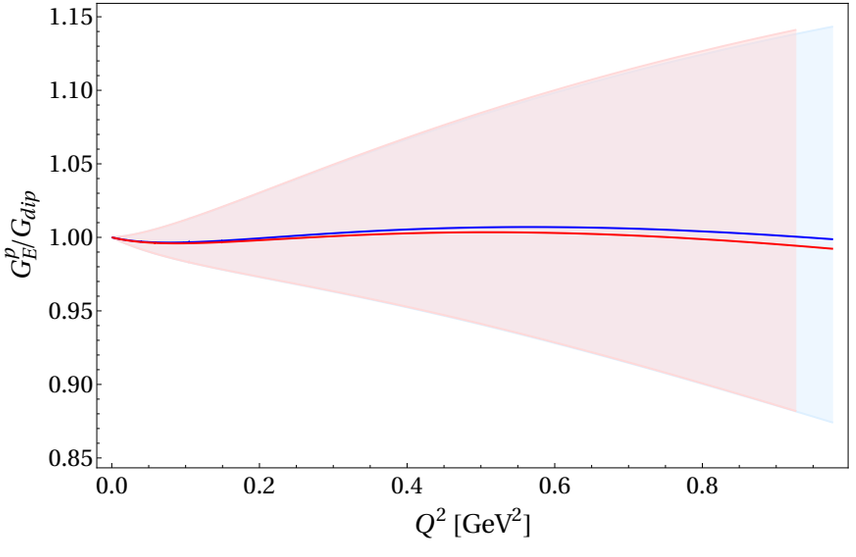


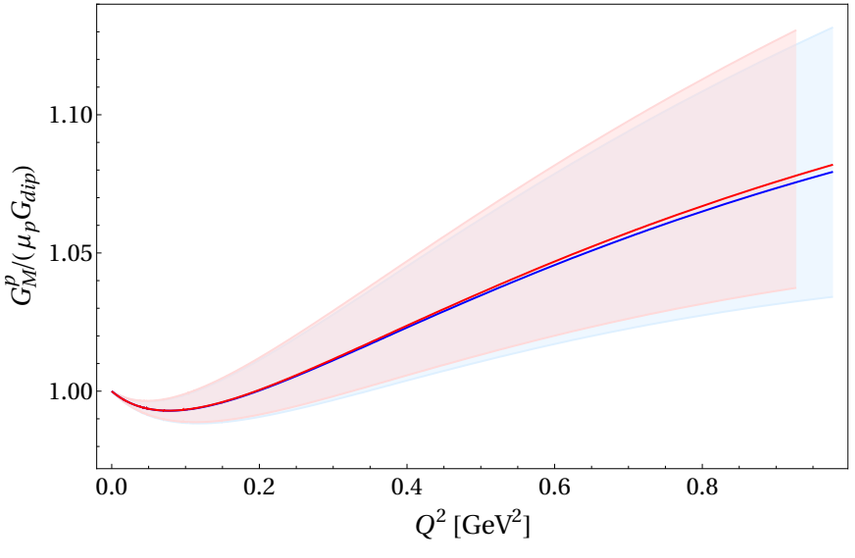


BAYESIAN versus BOOTSTRAP

- Bootstrap sampling in comparison to the Bayesian analysis [PRad data]

Method	r_E^p [fm]	r_M^p [fm]
Bayes normal	$\mathbf{0.828 \pm 0.011}$	$\mathbf{0.843 \pm 0.004}$
Bayes uniform	$\mathbf{0.828 \pm 0.011}$	$\mathbf{0.843 \pm 0.004}$
Bootstrap	$\mathbf{0.828 \pm 0.012}$	$\mathbf{0.843 \pm 0.005}$





↪ identical results, but bootstrap much faster

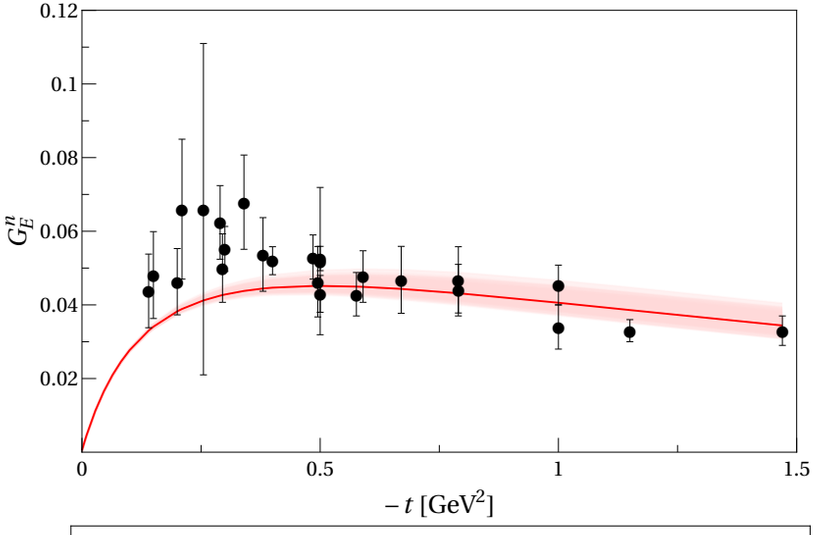
↪ will use the bootstrap method to determine the statistical error

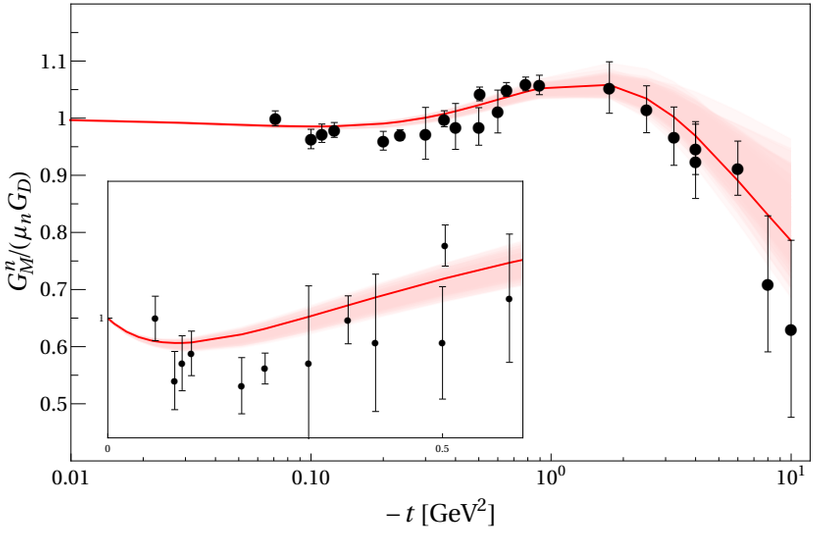
Results

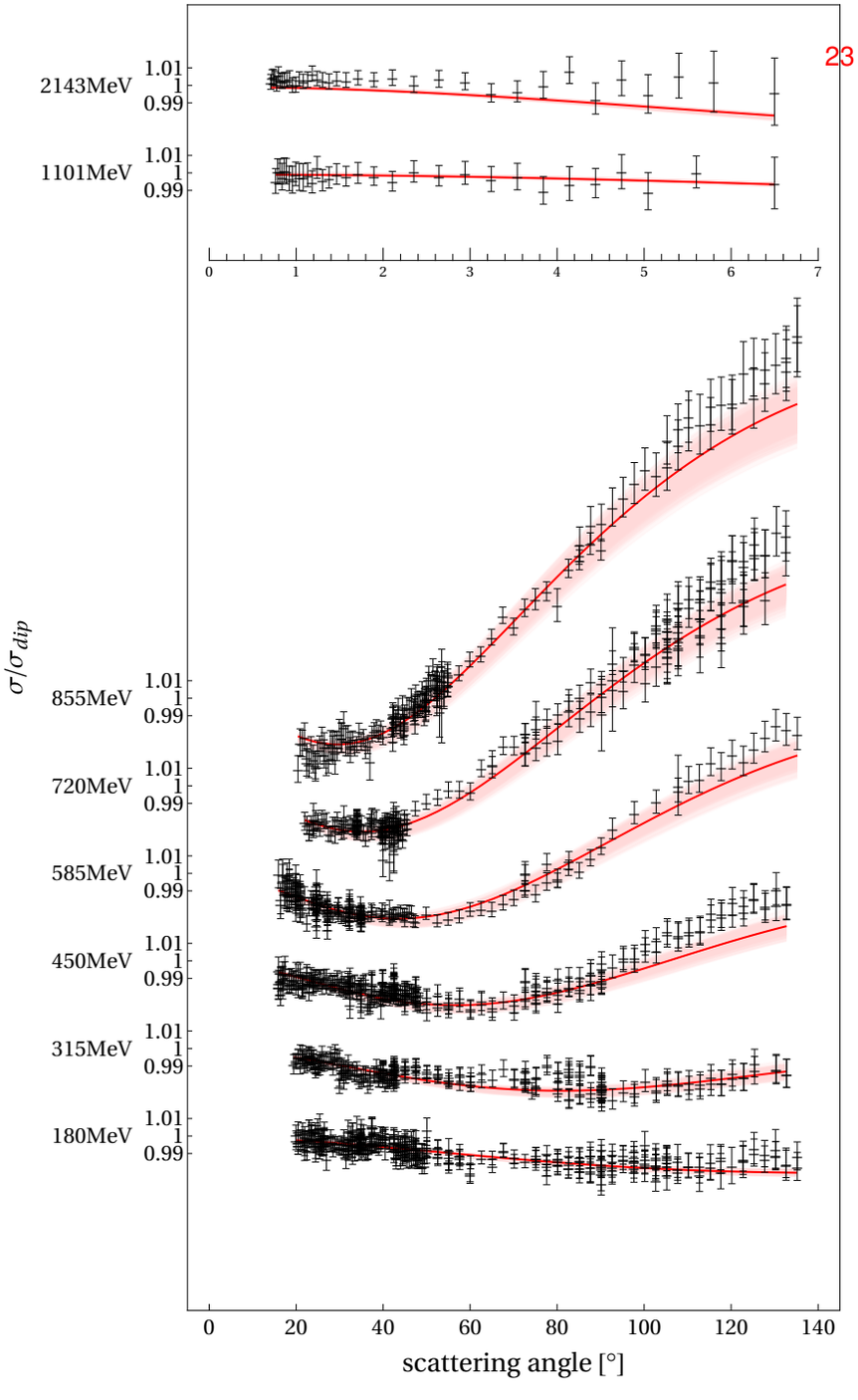
Lin, Hammer, UGM, Phys. Rev. Lett. **128** (2022) 5, 052002

SPACE-LIKE RESULTS I

- ep scattering data and neutron ffs:
  [error bands from bootstrap, χ^2/dof=1.223]

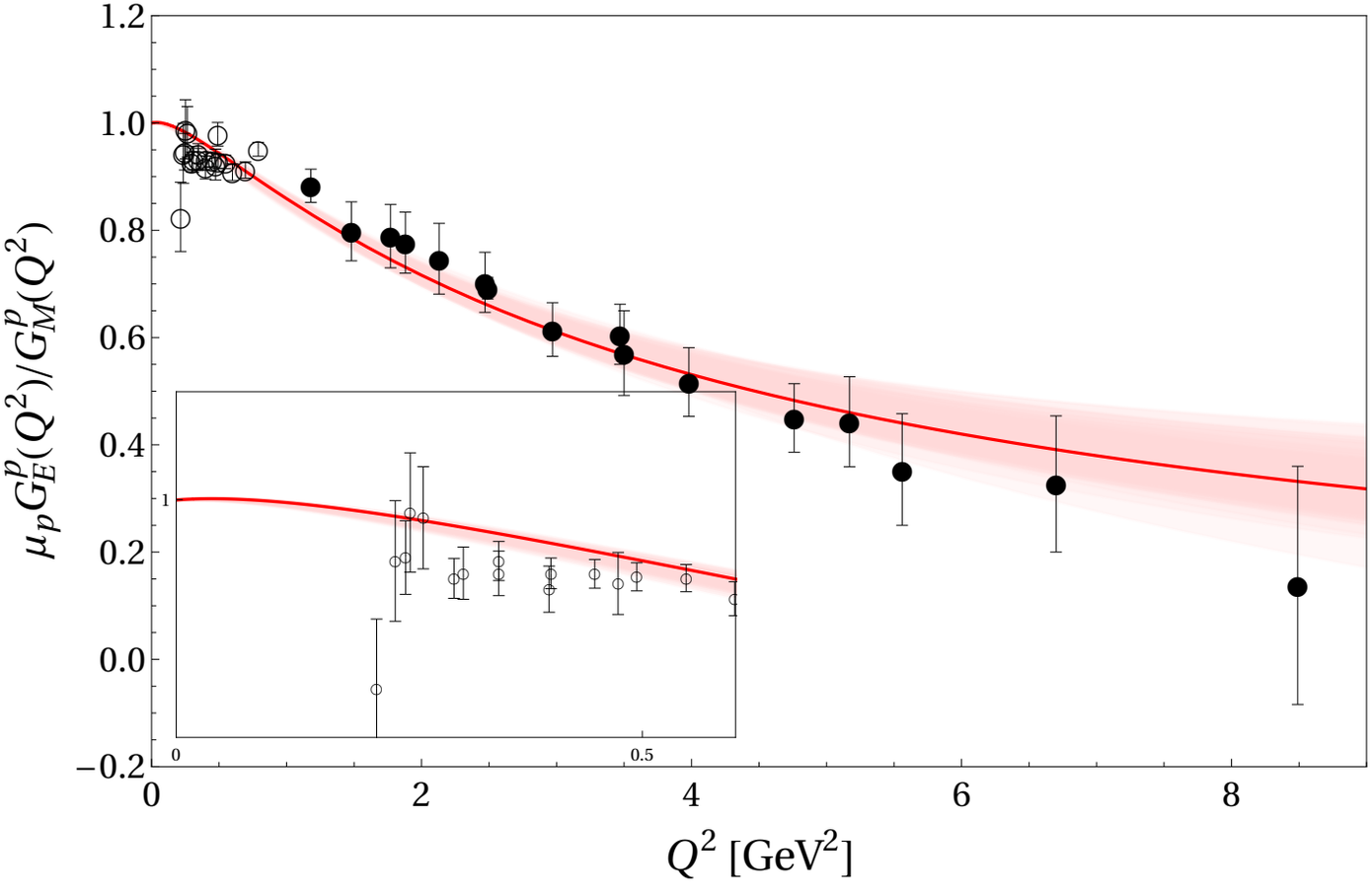






SPACE-LIKE RESULTS II

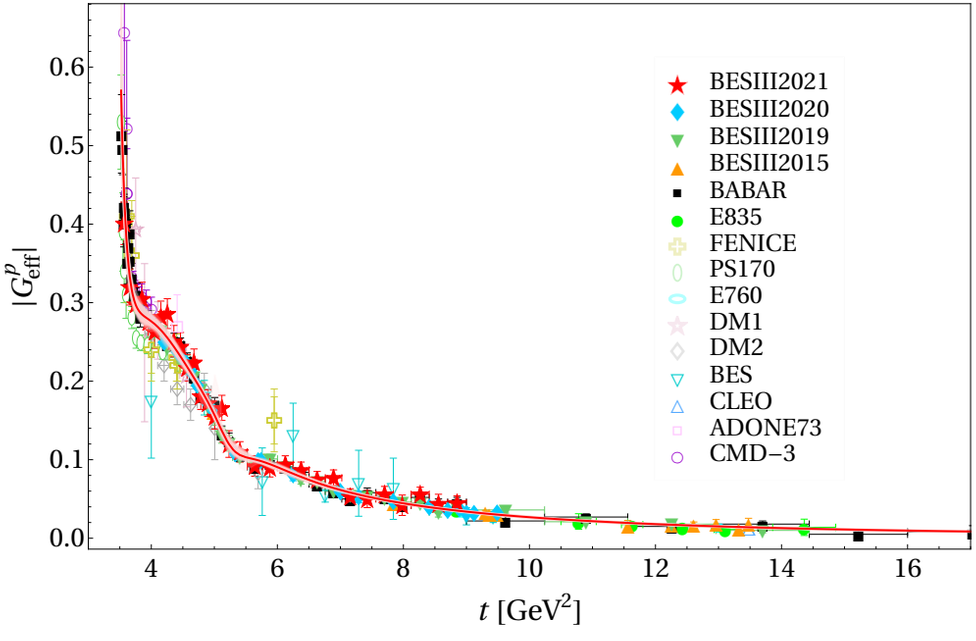
- Proton form factor ratio [Jlab data]:

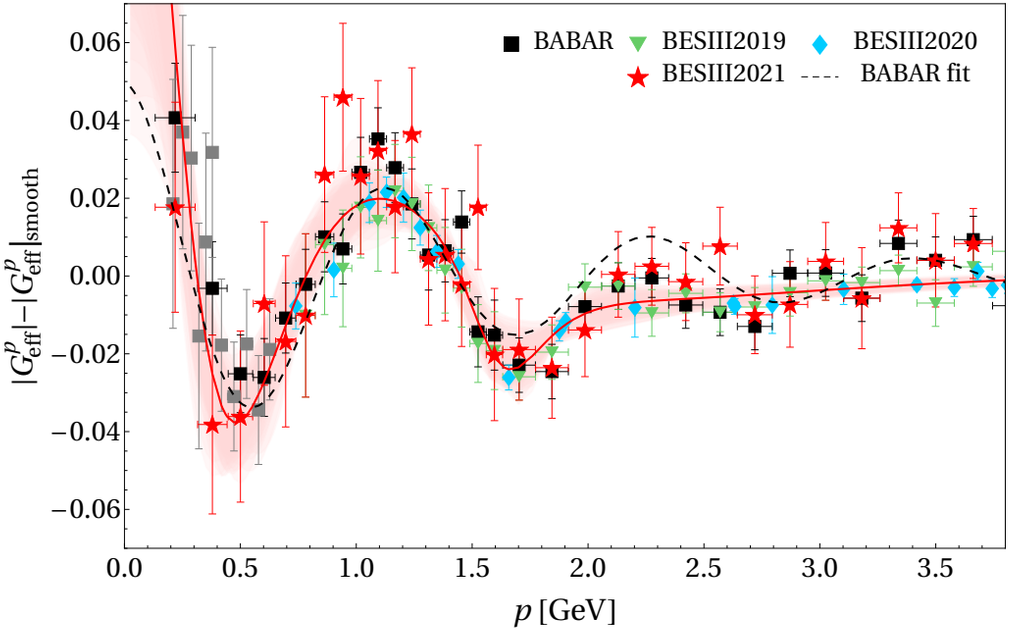


  - empty symbols: not fitted
  - zero crossing disfavored

TIME-LIKE RESULTS I

- Proton effective form factor:





open symbols not fitted

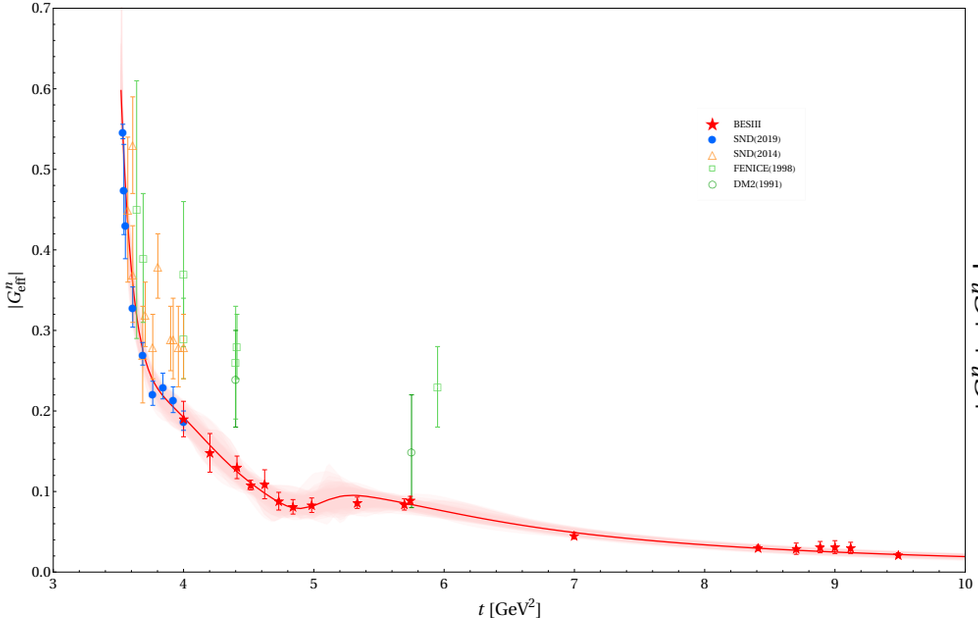
$$|G_{\rm eff}^p|_{\rm smooth} = \frac{7.7}{(1+t/14.8)(1-t/0.71)^2}$$

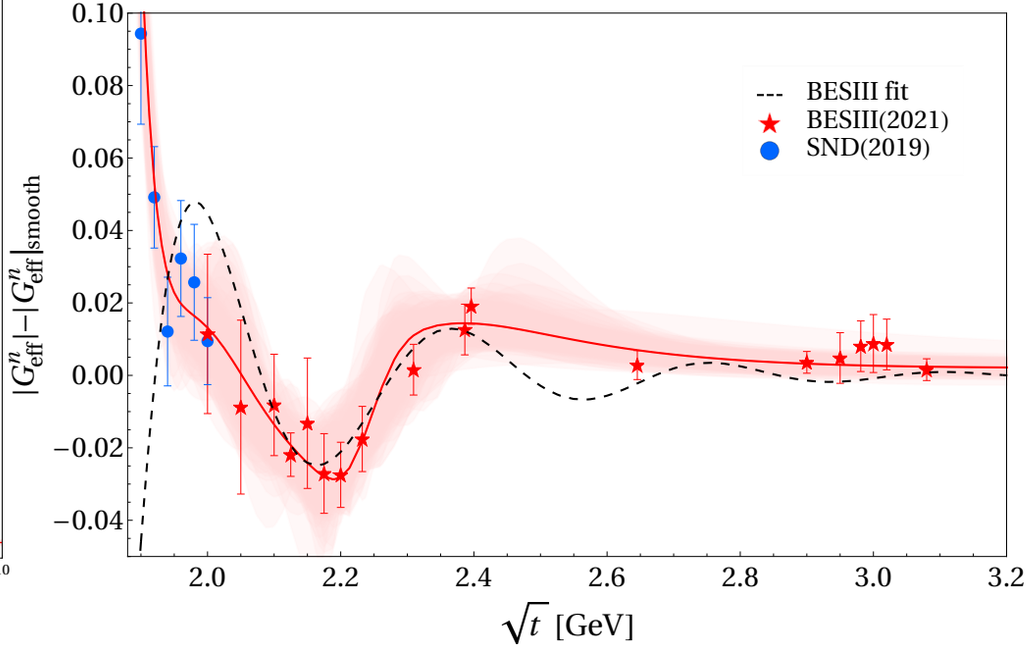
BaBar fit formula:

$$F_p = A^{\rm osc}\exp(-B^{\rm osc}p) \times \cos(C^{\rm osc}p + D^{\rm osc})$$

TIME-LIKE RESULTS II

- Neutron effective form factor:





open symbols not fitted

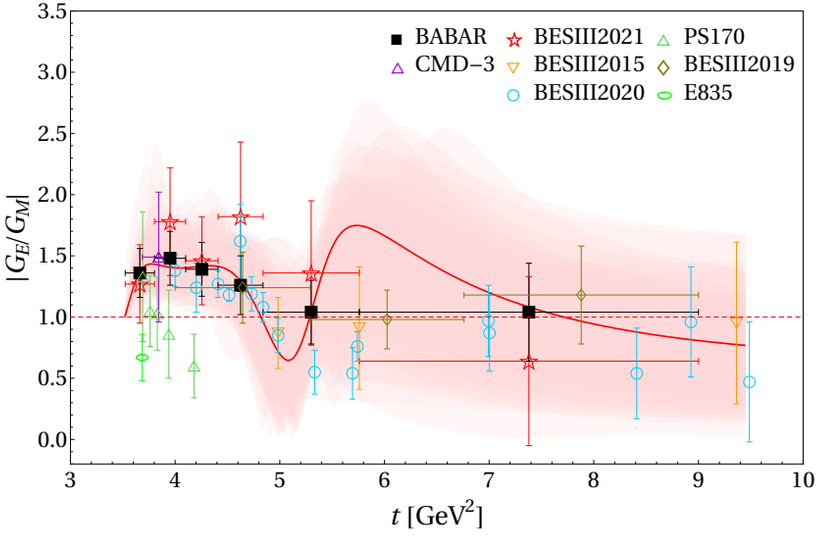
$$|G^n_{\rm eff}|_{\rm smooth} = \frac{4.87}{(1+t/14.8)(1-t/0.71)^2}$$

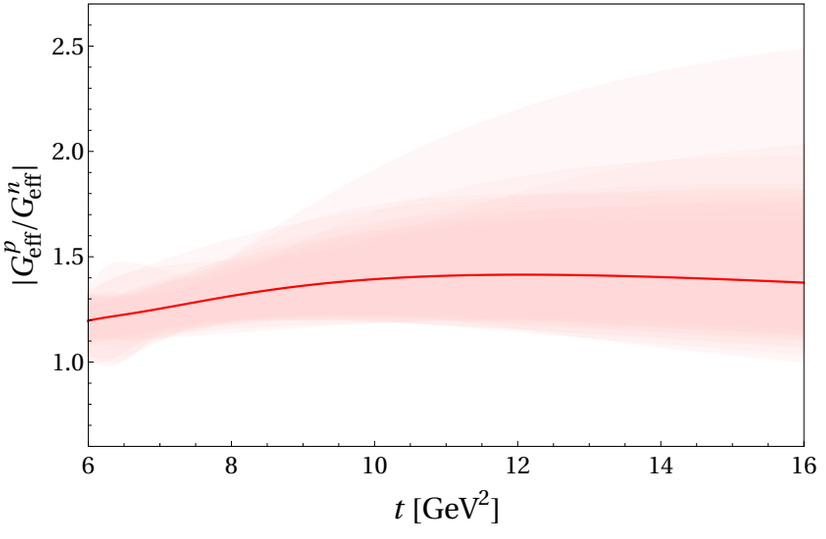
BESIII fit formula:

$$F_p = A^{\rm osc} \exp(-B^{\rm osc} p) \times \cos(C^{\rm osc} p + D^{\rm osc})$$

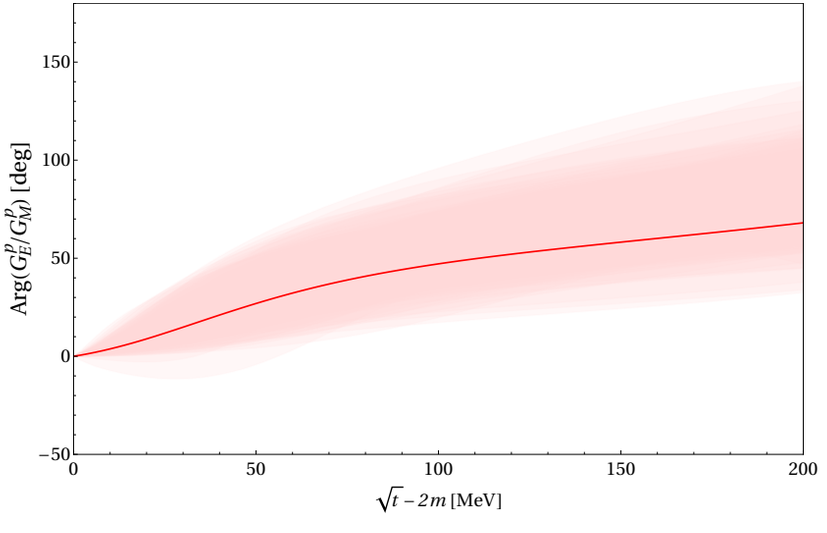
TIME-LIKE RESULTS III

- More fits and predictions [pQCD & phases]





open symbols not fitted



The proton radius

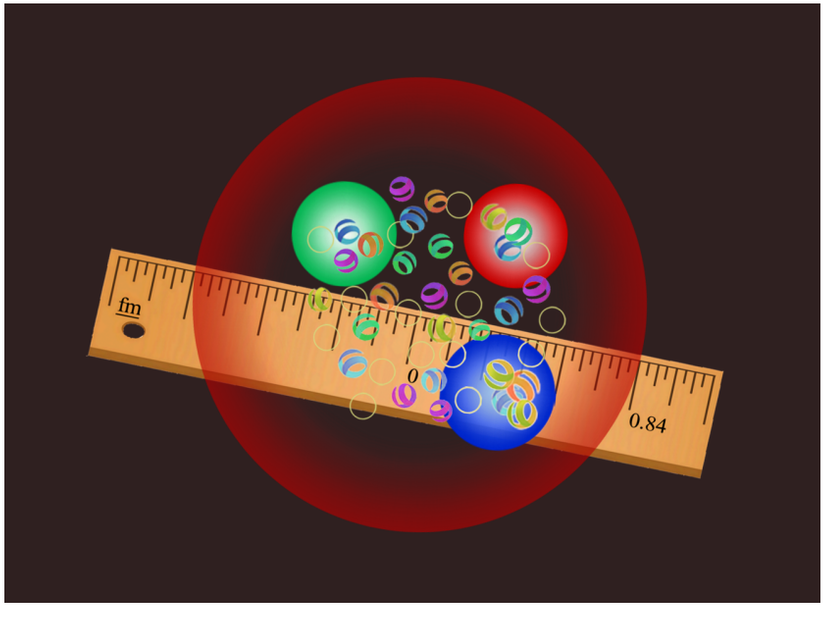


Fig. courtesy Yong-Hui Lin

PROTON CHARGE RADIUS

- Definition: $\boxed{r_p^2 \equiv -6\, G_E'(0)}$

- Measurements:

  – Leptonic hydrogen Lamb shift [in principle 2 numbers]

$$\Delta E_{LS} = \Delta E_1 + \Delta E_2 C(r_p^2) + \mathcal{O}(m_{\rm red}\alpha_{\rm EM}^2)$$

$$C(r_p^2) = c_1 + c_2 r_p^2 + \mathcal{O}(m_{\rm red}\alpha_{\rm EM}^2)$$

  – Lepton-proton scattering (Rosenbluth sep.)

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{\rm Mott}}{d\Omega}\frac{1}{1+\tau}\left(G_E^2 + \frac{\tau}{\varepsilon}G_M^2\right)(1+\delta_{\rm rad.}) + \mathcal{O}(m_{\rm red}\alpha_{\rm EM}^2)$$

- The neglected sibling, the proton magnetic radius:

$$(r_p^M)^2 \equiv -(6/\mu_p)\, G_M'(0)$$

PROTON CHARGE RADIUS - LEPTONIC LAMB SHIFT

- Energy levels in hydrogen:

$$E_{n\ell j} = R_\infty \left(-\frac{1}{n^2} + f_{n\ell j}\left(\alpha, \frac{m_e}{m_p}, \ldots\right) + \delta_{\ell 0} \frac{C_{\rm NS}}{n^3} r_p^2 \right)$$

$$f_{n\ell j}\left(\alpha, \tfrac{m_e}{m_p}, \ldots\right) = X_{20}\alpha^2 + X_{30}\alpha^3 + X_{31}\alpha^3 \ln\alpha + X_{40}\alpha^4 + \ldots$$

  - n, ℓ, j - principal, orbital, total ang. momentum quantum numbers
  - $f_{n\ell j}$ - relativistic corr's, vacuum effects, other QED corrections
  - m_e/m_p enters through the coefficients X_{20}, X_{30}, ... (recoil)
  - $C_{\rm NS}$ calculable leading order correction due to the finite r_p
  - higher order charge distributions are included in $f_{n\ell j}$

  $\Rightarrow$ must measure at least 2 transitions to pin down the two unknowns

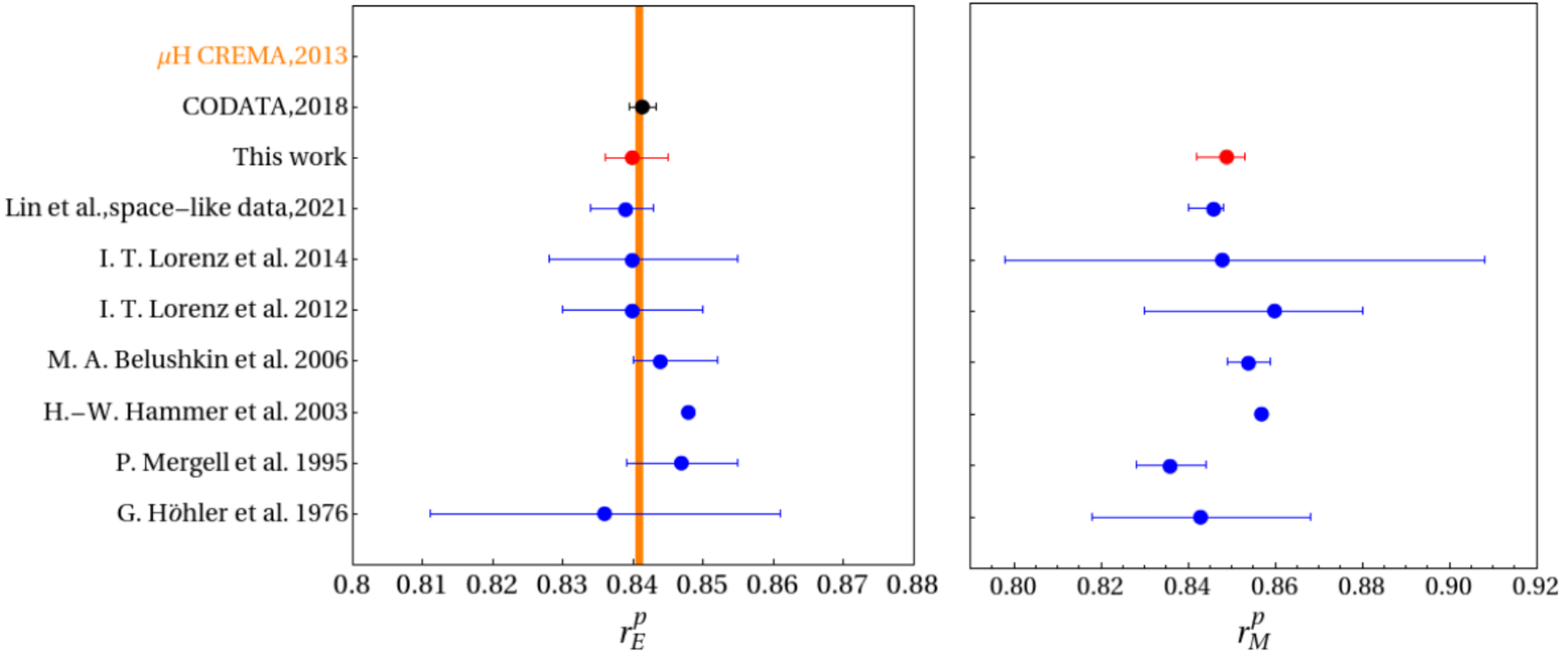
  $\Rightarrow$ this is done in recent measurements, but not before! [inconsistency]

PROTON CHARGE & MAGNETIC RADIUS

- Our determination incl. statistical and systematic errors:

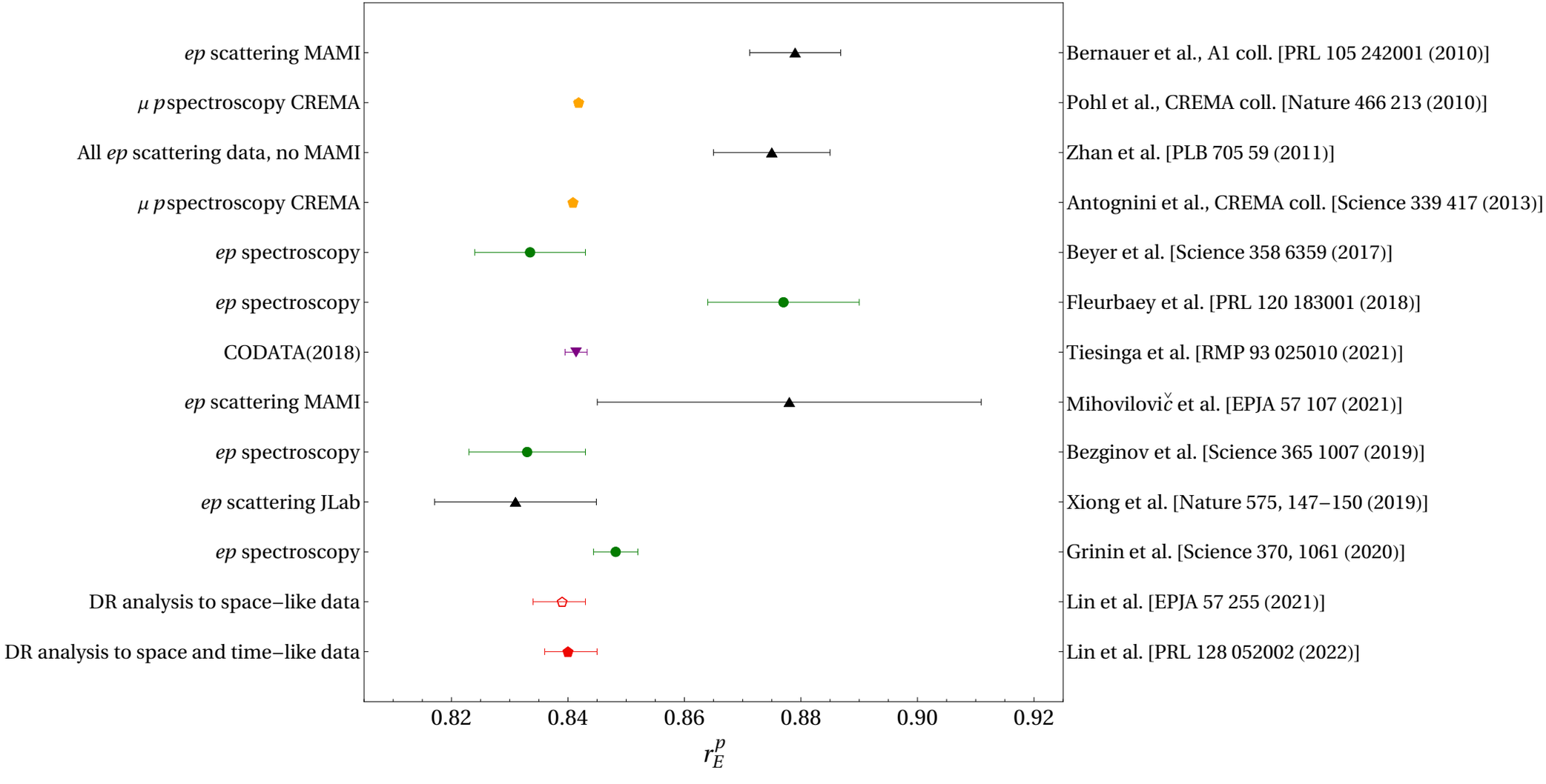
$$r_E^p = 0.840^{+0.003+0.002}_{-0.002-0.002}\ \text{fm}\ , \quad r_M^p = 0.849^{+0.003+0.001}_{-0.003-0.004}\ \text{fm}$$

- Comparison to earlier DR determinations (and some data)



PROTON CHARGE RADIUS

- Comparison to recent measurements:

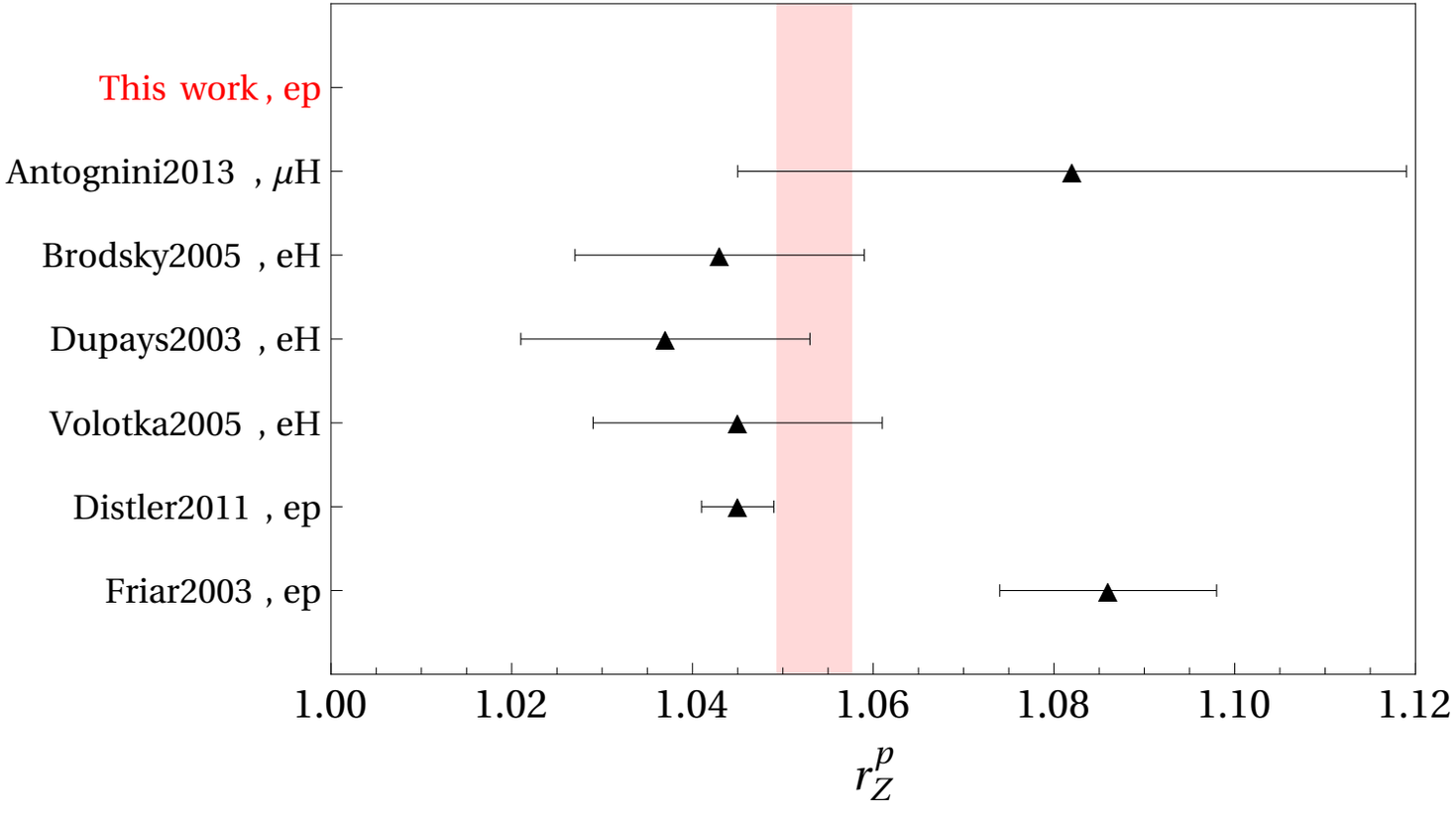


ZEMACH RADIUS & THIRD MOMENT

- Our determination incl. statistical and systematic errors:

$$r_Z = \frac{2}{\pi}\int_{-\infty}^{0}\frac{dt}{t\sqrt{-t}}\left(\frac{G_E(t)G_M(t)}{1+\kappa_p}-1\right) = 1.054^{+0.003+0.000}_{-0.002-0.001}\ \text{fm}$$

$$\langle r^3\rangle_{(2)} = \frac{24}{\pi}\int_{-\infty}^{0}\frac{dt}{t^2\sqrt{-t}}\left(G_E^2(t)-1-\frac{t}{3}\langle r^2\rangle_p\right) = 2.310^{+0.022+0.014}_{-0.018-0.015}\ \text{fm}^3$$



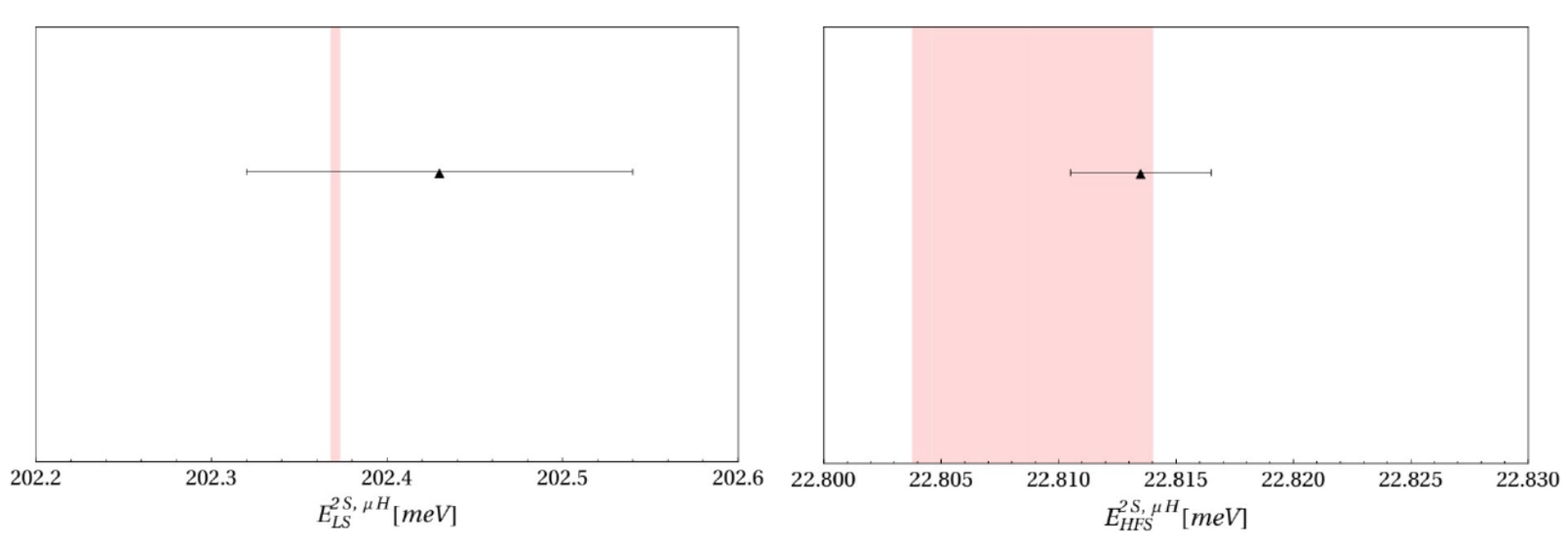
COMPARISON TO SPECTROSCOPY OBSERVABLES

- Relevant formulas:

$$\Delta E_{LS} = 206.0336(15) - 5.2275(10)\langle r_p^2\rangle + 0.0347\langle r^3\rangle_{(2)}$$

$$\Delta E_{HFS} = 22.9843(30) - 0.1621(10)\, r_Z$$

Antognini et al., Ann. Phys. **331** (2013) 127



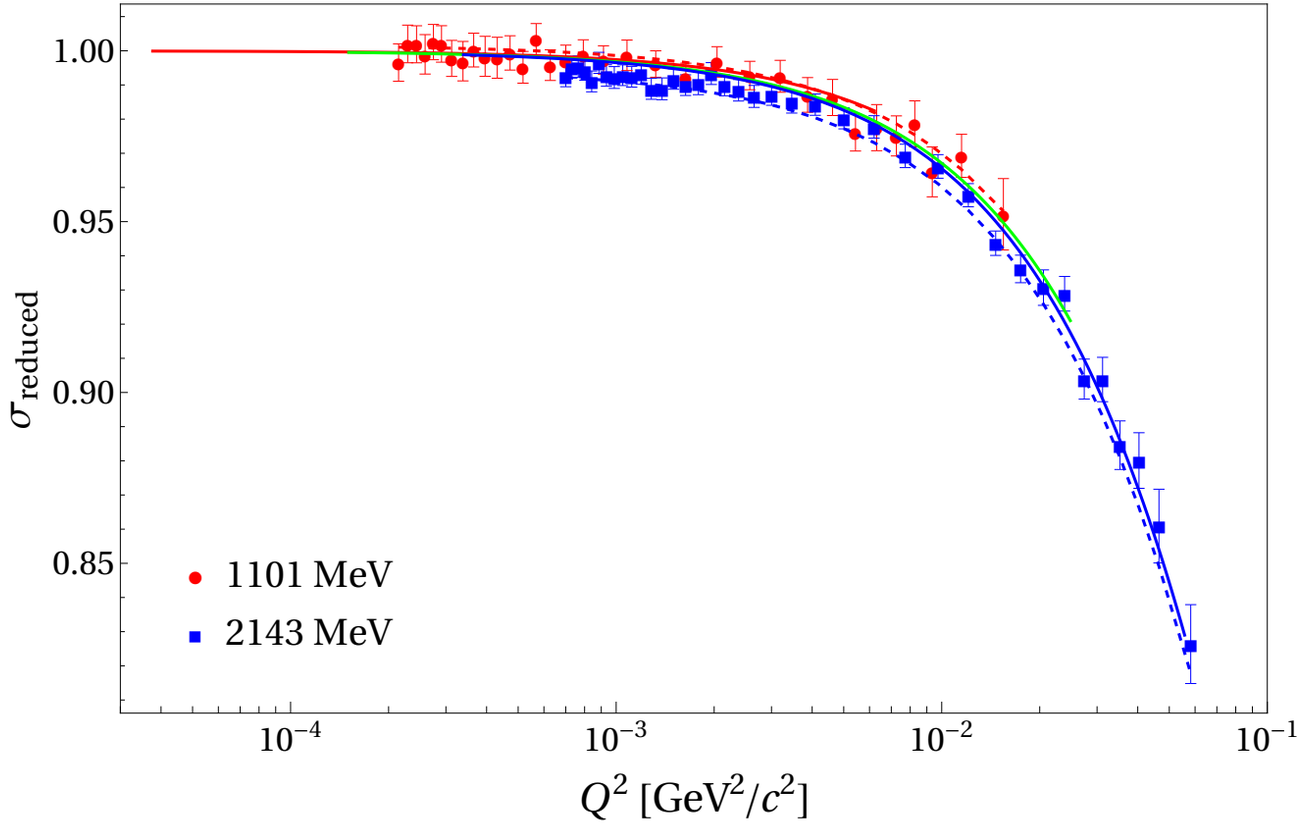
black triangles = our results

red bands from Antognini et al., Science **339** (2013) 417

PREDICTIONS for PRAD-II

Lin, Hammer, UGM, Phys. Lett. B 827 (2022) 136981 [2111.09619 [hep-ph]]

- Predictions for the upcoming PRad-II and e^+p scattering



$\hookrightarrow$ Predictions for $E_\gamma = \mathbf{0.7}, \mathbf{1.4}, \mathbf{2.1}$ GeV

NEUTRON RADII

- The charge squared neutron $(r_E^n)^2$ radius was mostly input in DR analyses, but not the magnetic one

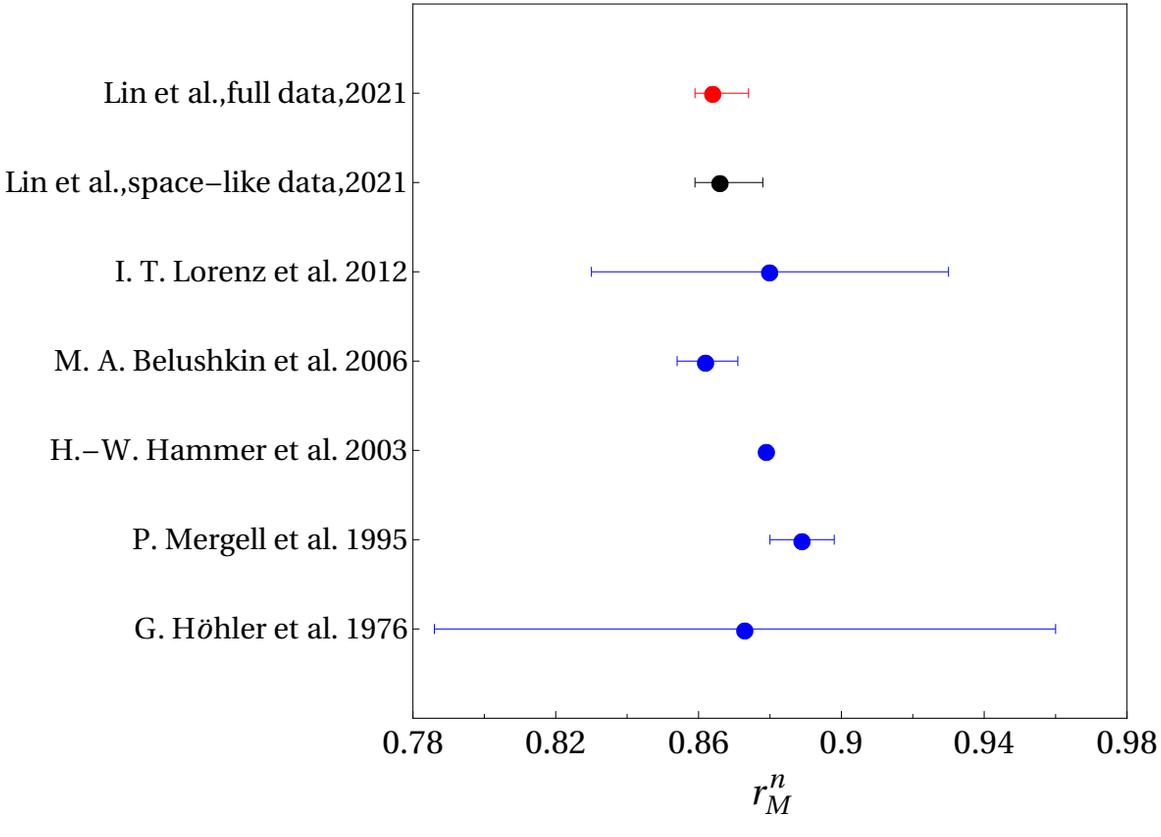
$$r_M^n = 0.864^{+0.004+0.006}_{-0.004-0.001}\ \text{fm}$$

↪ rather stable over time

↪ but larger variation

↪ always the largest em radius!

↪ lattice QCD gives rather comparable isovector radii (p& n)



SUMMARY & OUTLOOK

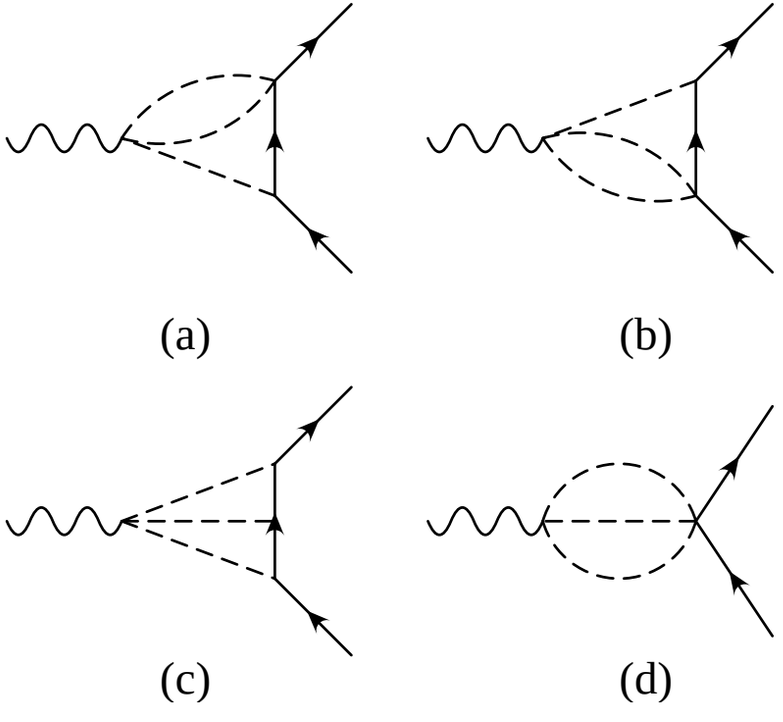
- Dispersion theory is the best tool to analyze the nucleon em FFs
- Description of all data, time- and space-like
- Always a small proton charge radius (0.84 fm), magnetic one bigger (0.85 fm)
- Most recent experiments tend to the small radius
- From a puzzle to precision Hammer, UGM, Sci.Bull. **65** (2020) 257
- Predictions for PRad-II & positron-proton scattering (also MESA/Mainz)
- Magnetic radius of the neutron is the largest one
- Theory challenge: better understanding of the oscillations in $|G_{\rm eff}^{p,n}|$
- Experimental challenges
  - $\hookrightarrow$ proton form factor ratio at $Q^2 \simeq 10\,\text{GeV}^2$
  - $\hookrightarrow$ more resolved form factor measurements in the time-like region
  - $\hookrightarrow$ μp scattering - testing lepton flavor universality (MUSE, AMBER)

Spares

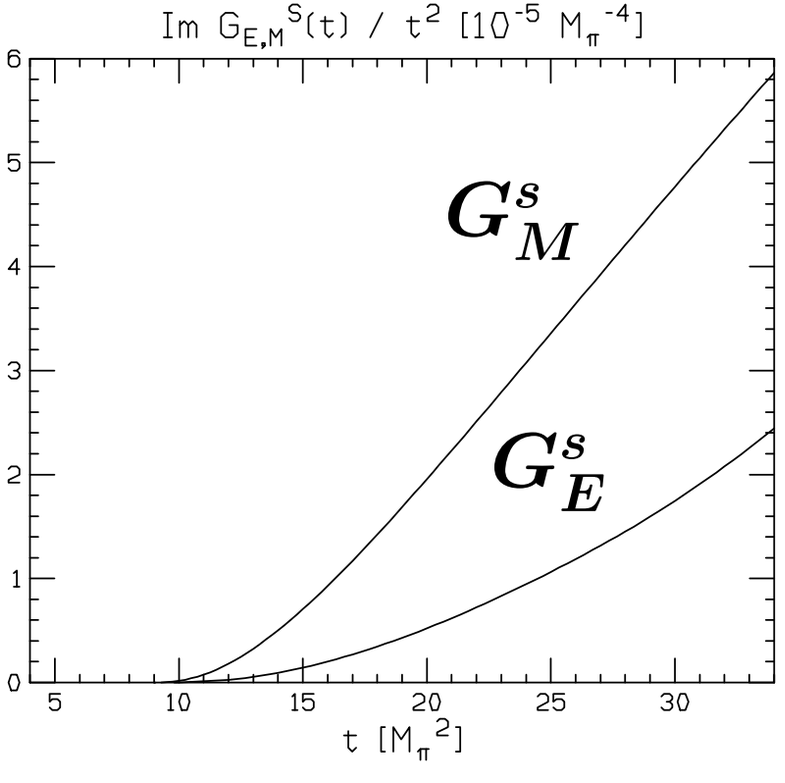
ISOSCALAR SPECTRAL FUNCTION

Bernard, Kaiser, UGM, Nucl. Phys. A **611** (1996) 429 [hep-ph/9607428]

- Two-loop CHPT calculation



- Electric/magnetic spectral fcts



⋆ **no** shoulder on the left wing

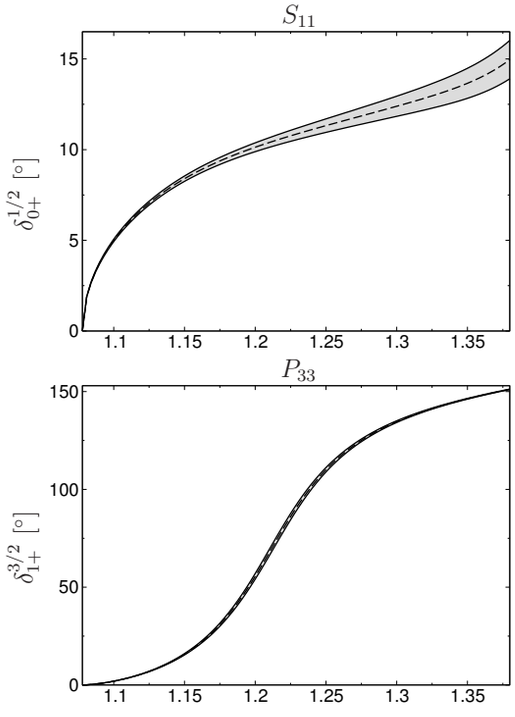
⋆ **clean** omega-pol dominance

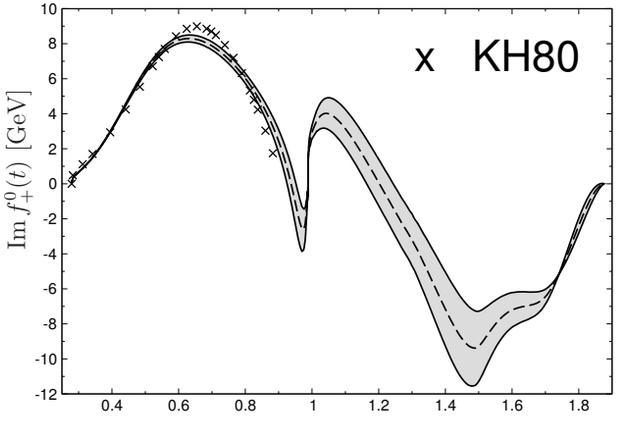
Once more on the isovector spectral functions

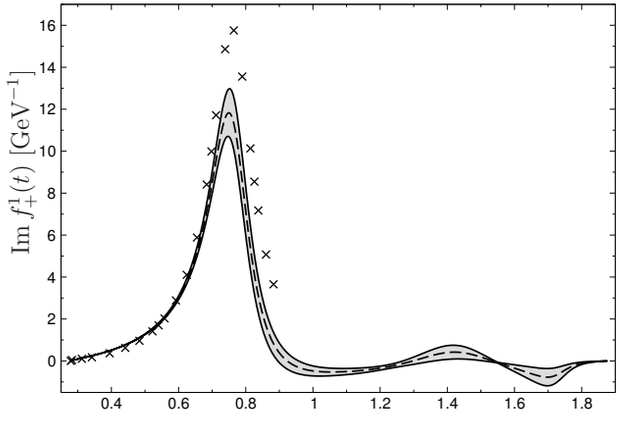
Hoferichter, Kubis, Ruiz de Elvira, Hammer, UGM, Eur. Phys. J. A **52** (2016)331
[arXiv:1609.06722 [nucl-th]]

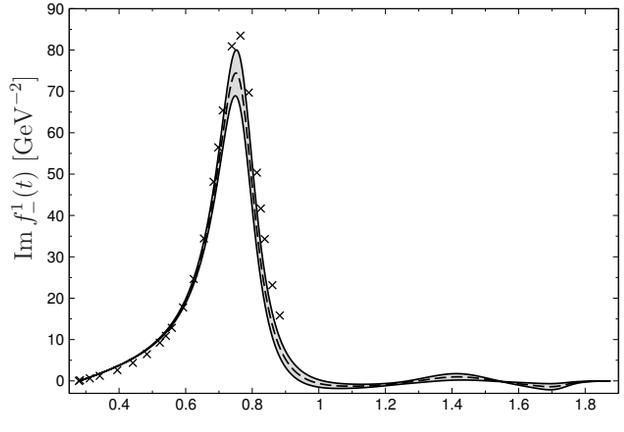
ROY-STEINER EQUATION ANALYSIS

- improve the isovector spectral functions by
  - ↪ updated πN amplitudes from Roy-Steiner equations
  - ↪ include modern data (esp. pionic hydrogen & deuterium)
  - ↪ better treatment of isospin-violating effects
  - ↪ construct the pion FF from precise knowledge of $\delta_1^1(s)$
  - ↪ perform systematic error analysis









Hoferichter, Ruiz de Elvira, Kubis, UGM, Phys. Rev. Lett. **115** (2015) 092301; Phys. Rev. Lett. **115** (2015) 192301; Phys. Rept. **625** (2016) 1; J.Phys. G**45** (2018) 024001

NEW ISOVECTOR SPECTRAL FUNCTIONS

- Precise determinations of the isovector spectral functions

