Karlsruher Institut für Technologie

The Drell-Yan process in the high invariant mass region: the impact of mixed QCD-electroweak corrections

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Madrid, 31/03/2022

In collaboration with: F. Buccioni, F. Caola, H. Chawdhry, F. Devoto, M. Heller, A. von Manteuffel, K. Melnikov, R. Röntsch
Based on: arXiv 2203.11237

Outlook:

Why

- are we interested in higher order corrections?
- are we focused on Drell-Yan?
- are mixed QCD-EW corrections are useful?
- is the computation complicated?

How

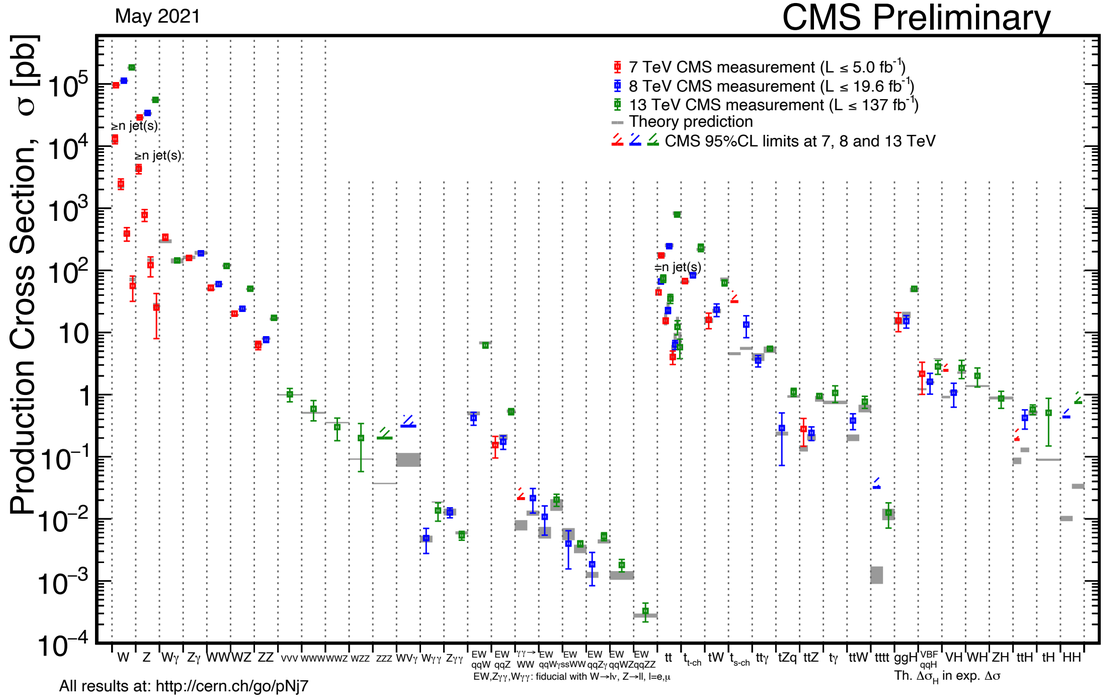
- can we overcome the intrinsic difficulties of the computation?
- can we exploit what was already known?

What

- can we learn from the results?

Motivations

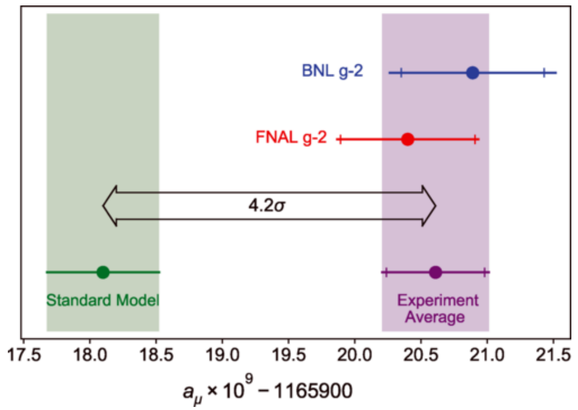
LHC continues to confirm the Standard Model

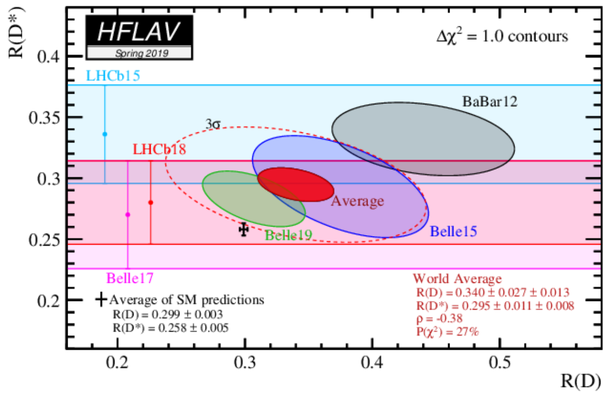


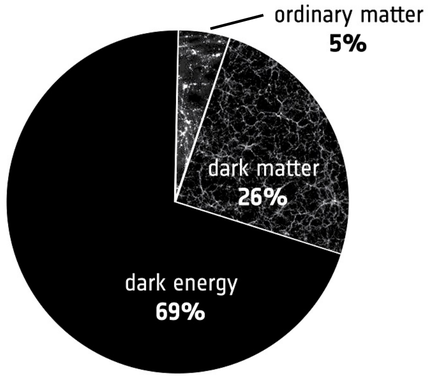
BUT

still several tensions with SM are present:

- **flavour anomalies**
- **g-2**
- **dark matter**







High precision theoretical predictions

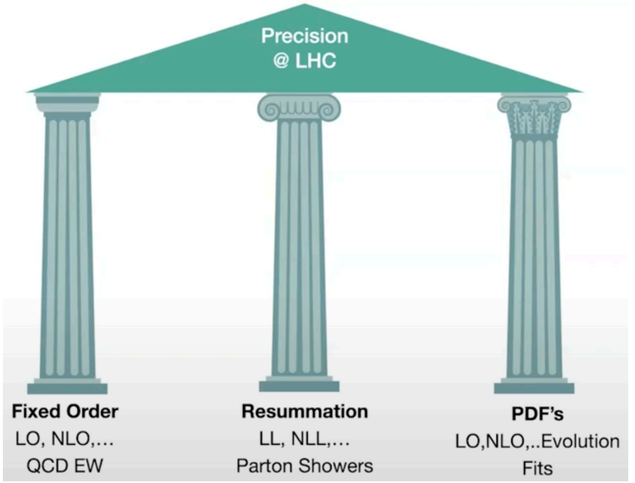
→ Constrain NP

→ Discover and characterise NP

Motivations

Requires advances in all aspects of collider physics:

- Parton distribution functions
- Fixed order calculations ← This talk
- Resummation
- Parton shower



Factorisation theorem

$$\mathrm{d}\sigma = \sum_{i,j} \int \mathrm{d}x_1\, \mathrm{d}x_2\, f_i(x_1) f_j(x_2)\, \mathrm{d}\hat{\sigma}_{ij}\big(\{p_i\}, \mu^2, x_1, x_2\big)$$

[Maltoni, GGI Tea Breaks '21]

Fixed-order predictions represent one of the pillars for precision physics at LHC

$$\mathrm{d}\hat{\sigma}_{ij} = \mathrm{d}\hat{\sigma}_{ij,\mathrm{LO}}\big(1 + \alpha_s\, \Delta^{QCD}_{ij,\mathrm{NLO}} + \alpha_{ew}\, \Delta^{EW}_{ij,\mathrm{NLO}} + \alpha_s^2\, \Delta^{QCD}_{ij,\mathrm{NNLO}} + \alpha_s\, \alpha_{ew}\, \Delta^{QCD\otimes EW}_{ij,\mathrm{NNLO}} + \ldots\big)$$

Parton distributions: $f_{i,j}$

Ren./fact. scale: μ

Partonic cross section: $\mathrm{d}\hat{\sigma}_{ij}$

Momentum fractions: x_1, x_2

External momenta: $\{p_i\}$

Motivations: why Drell-Yan?

Drell-Yan is one of the **“standard candles”** at the LHC:

- **Clear experimental signature**, extremely well-controlled:
  - > $Z \to \ell^+\ell^-$: detection of pair of leptons (high $p_\perp$)
  - > $W \to \ell\nu$: one lepton with high $p_\perp$ + missing $p_\perp$

- **Detector calibration** (e.g. detector response to lepton energy in NCDY events)

- **Large production rate:**
  - > relevant background for NP searches also at future colliders
  - > accurate theoretical predictions are necessary

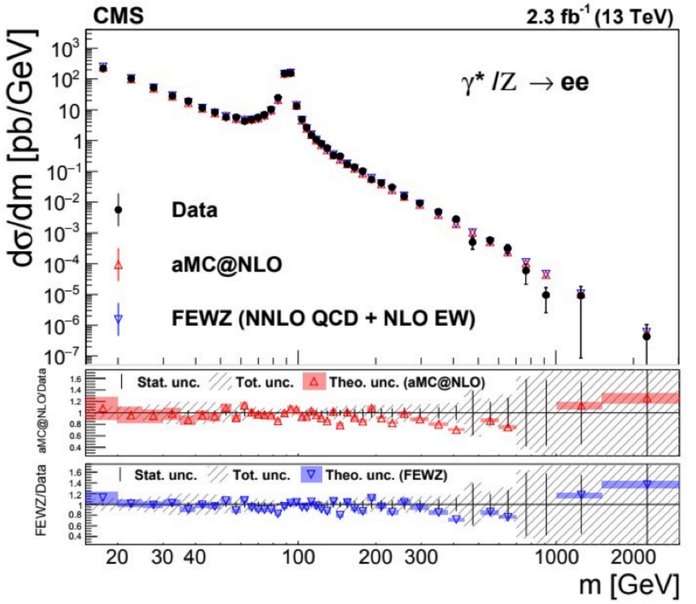
- Historically investigated as application of Parton Model and used to discover W and Z bosons

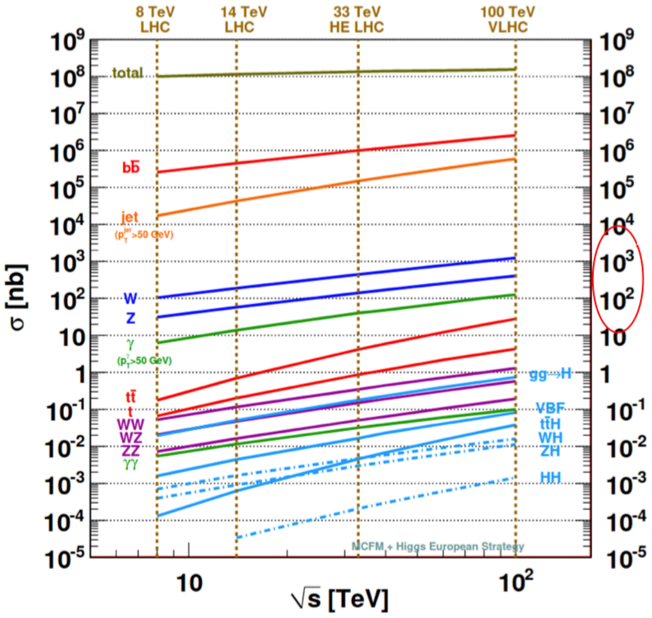
- **Information about PDFs**

> separation of valence quarks through W charge asymmetry

- Precise **determination of the SM parameters**

> CCDY at the W resonance: determination of the W-boson mass m_W

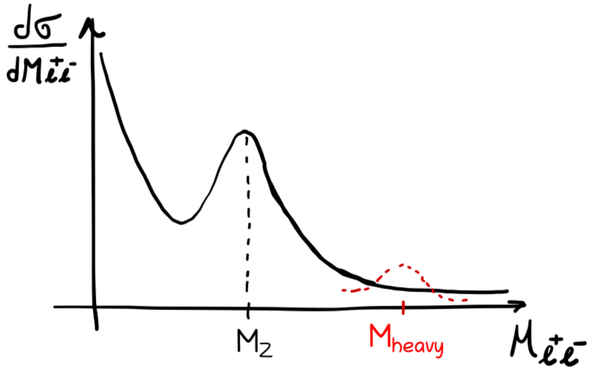




Motivations: why Drell-Yan is interesting?

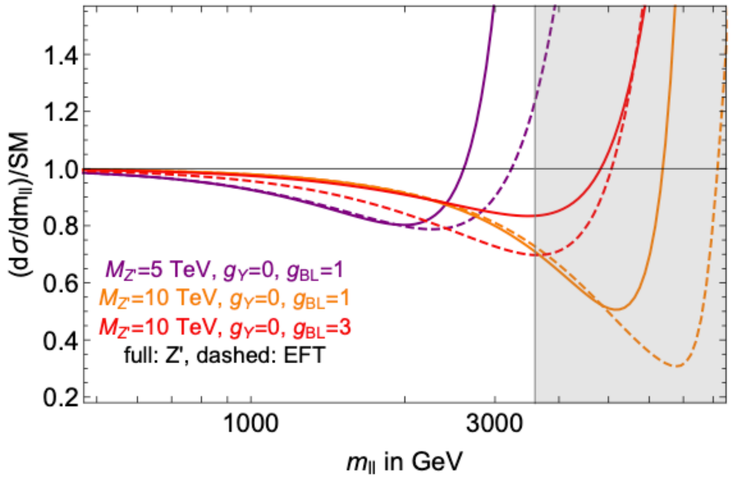
• Hunting New Physics (NP) at hight energies

- Many extensions to the SM contain weakly-coupled states which can decay into leptons



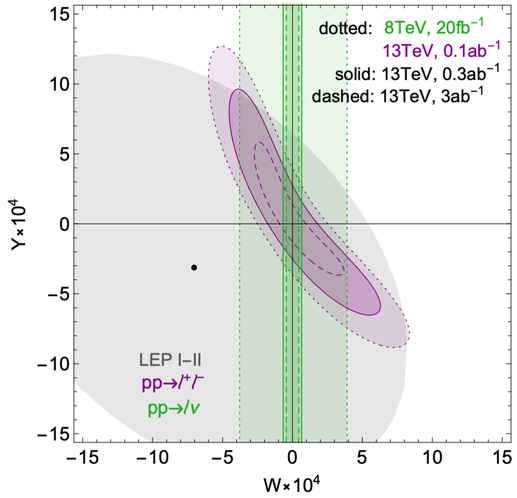
- Constrain heavy NP in a model-independent way using SMEFT

Impact of dimension-6 operators grows with energy
[Peskin, Takeuchi '90] [Barbieri, Pomarol, Rattazzi, Strumia '04]
[Alioli, Farina, Pappadopulo, Ruderman '17] [Farina et al.'17]



Projections demonstrate that hadron colliders can be used to perform electroweak precision test

High energy of the LHC can compensate for the limited precision



Motivations: high precision > QCD corrections

$$\mathrm{d}\sigma_{ij} = \mathrm{d}\sigma_{ij,\,\mathrm{LO}}\left(1 + \alpha_s\,\Delta^{QCD}_{ij,\,\mathrm{NLO}} + \alpha_s^2\,\Delta^{QCD}_{ij,\,\mathrm{NNLO}} + \alpha_s^3\,\Delta^{QCD}_{ij,\,\mathrm{N3LO}} + \ldots\right)$$

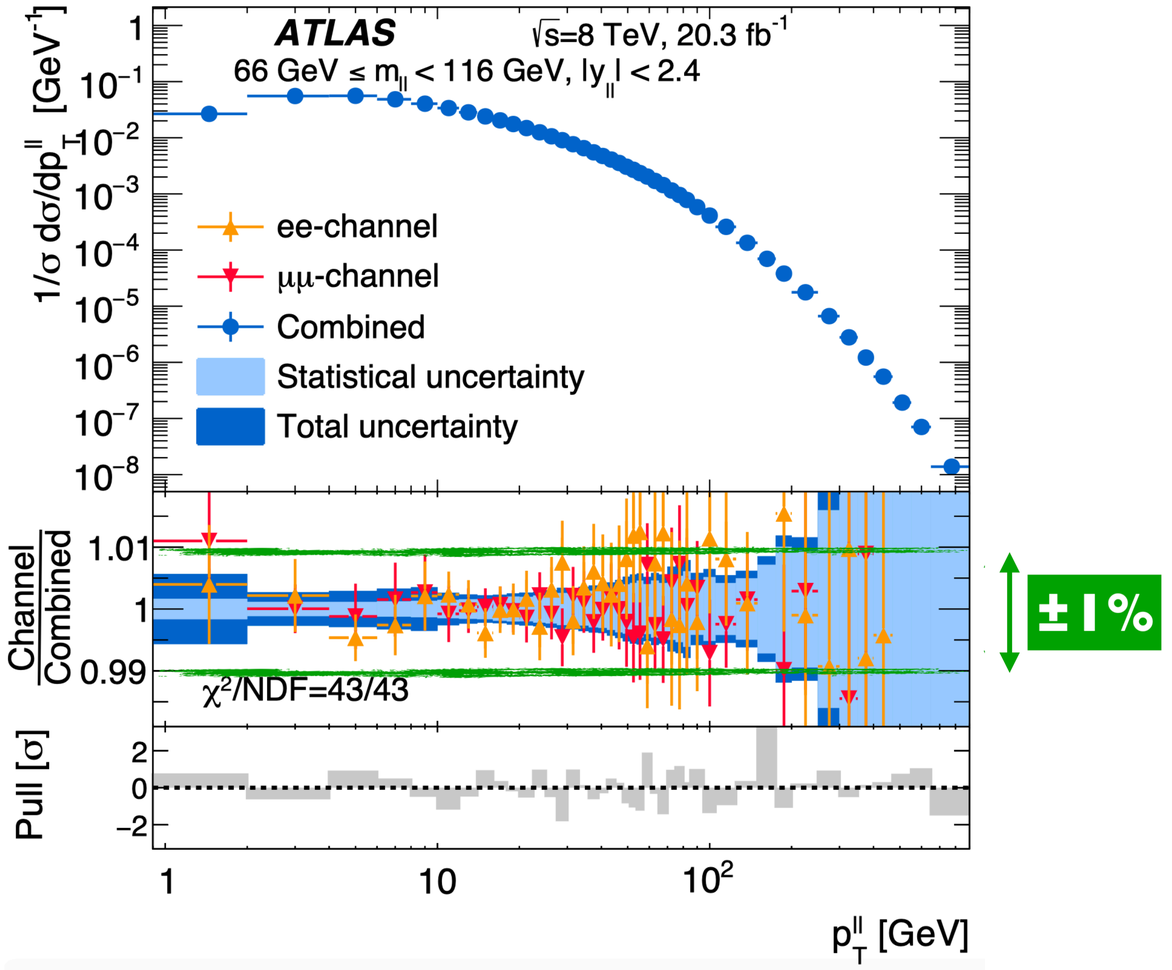
Couplings: $\alpha_s \sim 0.1$, $\alpha \sim 0.01$

Experimental precision usually $\sim$ few % level

NLO QCD $\sim \mathcal{O}(\alpha_s) \sim 10\,\%$

NNLO QCD $\sim \mathcal{O}(\alpha_s^2) \sim 1\,\%$

N3LO QCD $\sim \mathcal{O}(\alpha_s^3) \sim 0.1\,\%$



Brief historical recap

$\sigma^{\text{nlo}}_{\text{QCD}}$

[Altarelli, Ellis, Martinelli '79]

$\sigma^{\text{nnlo}}_{\text{QCD}}$

[Hamberg, van Neerven, Matsuura '91]

[Harlander, Kilgore '02]

$\mathrm{d}\sigma^{\text{nnlo}}_{\text{QCD}}$ (fully diff.+leptonic decay)

[Anastasiou, Dixon, Melnikov, Petriello '03, '04]

[Melnikov, Petriello '06]

[Catani, Cieri, Ferrera, de Florian, Grazzini '09]

[Catani, Ferrera, Grazzini '10]

$\sigma^{\text{n3lo}}_{\text{QCD}}$

[Duhr, Dulat, Mistlberger '20]

$\mathrm{d}\sigma^{\text{n3lo}}_{\text{QCD}}$

[Camarda, Cieri, Ferrera '21]

[Chen, Gehrmann, Glover, Huss, Yang '21]

[Chen, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli '22]

Motivations: high precision > QCD corrections

$$d\sigma_{ij} = d\sigma_{ij,\,\mathrm{LO}}\left(1 + \alpha_s\,\Delta^{QCD}_{ij,\,\mathrm{NLO}} + \alpha_s^2\,\Delta^{QCD}_{ij,\,\mathrm{NNLO}} + \alpha_s^3\,\Delta^{QCD}_{ij,\,\mathrm{N3LO}} + \ldots\right)$$

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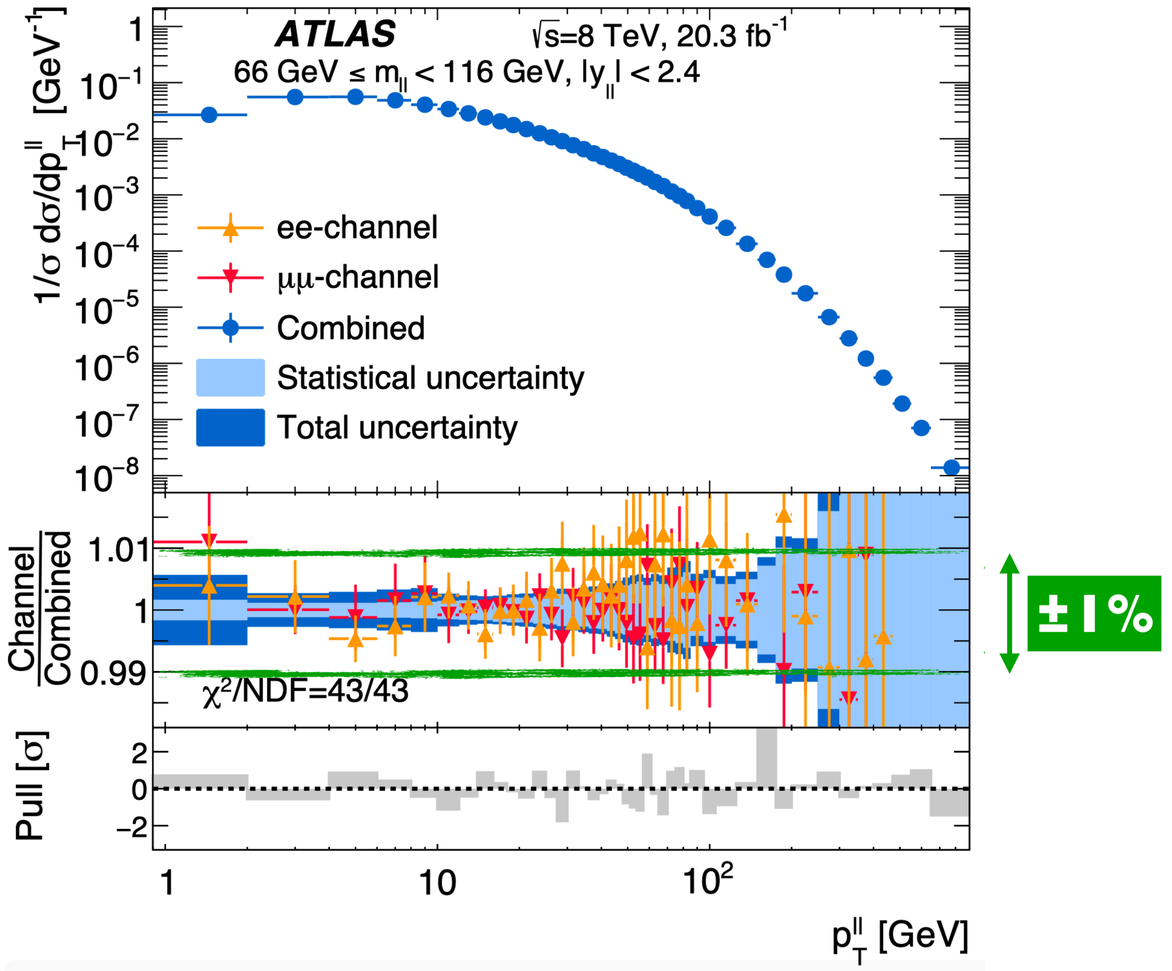
NNLO QCD $\sim \mathcal{O}(\alpha_s^2) \sim 1\,\%$

N3LO QCD $\sim \mathcal{O}(\alpha_s^3) \sim 0.1\,\%$

Is it enough?

NLO EW $\sim \mathcal{O}(\alpha) \sim 1\,\%$

Mixed QCD×EW corrections: $\sim \mathcal{O}(\alpha_s\,\alpha) \sim 0.1\,\%$



Motivations: why mixed QCD×EW corrections?

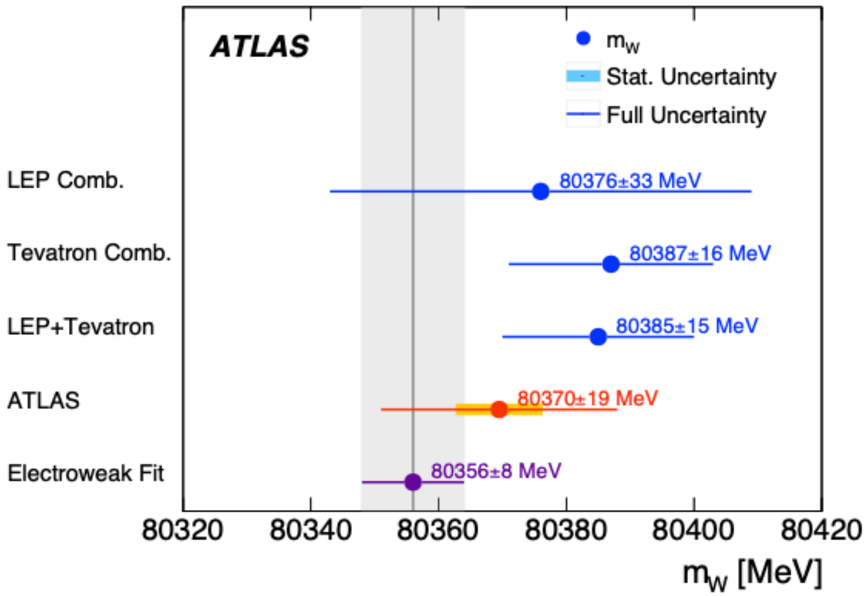
$$\mathrm{d}\sigma_{ij} = \mathrm{d}\sigma_{ij,\mathrm{LO}}\left(1 + \alpha_s\,\Delta^{QCD}_{ij,\mathrm{NLO}} + \alpha_{ew}\,\Delta^{EW}_{ij,\mathrm{NLO}} + \alpha_s^2\,\Delta^{QCD}_{ij,\mathrm{NNLO}} + \alpha_s\,\alpha_{ew}\,\Delta^{QCD\otimes EW}_{ij,\mathrm{NNLO}} + \alpha_s^3\,\Delta^{QCD}_{ij,\mathrm{N3LO}} + \ldots\right)$$

Couplings: $\alpha_s \sim 0.1\,,\ \alpha \sim 0.01$

Experimental precision usually $\sim$ few % level

NLO EW $\sim \mathcal{O}(\alpha) \sim 1\,\%$

Mixed QCD×EW corrections: $\sim \mathcal{O}(\alpha_s\,\alpha) \sim 0.1\,\%$



Why do we need this level of precision in theoretical prediction?

- **Target precision** sometimes is **lower than percent level**:
  - ATLAS has measured $m_W = (80\,370 \pm 19)\,\mathrm{MeV}$ *[ATLAS '17]*
  - ATLAS and CMS collaborations aim to reduce uncertainty to $\mathcal{O}(10\,\mathrm{MeV})$
    - > match precision electroweak fits
    - > would mean $\mathcal{O}(0.01\%)$ uncertainty

← On-shell vector boson production +decay

Motivations: why mixed QCD×EW corrections?

$$\mathrm{d}\sigma_{ij} = \mathrm{d}\sigma_{ij,\,\mathrm{LO}}\left(1 + \alpha_s\,\Delta^{QCD}_{ij,\,\mathrm{NLO}} + \alpha_{ew}\,\Delta^{EW}_{ij,\,\mathrm{NLO}} + \alpha_s^2\,\Delta^{QCD}_{ij,\,\mathrm{NNLO}} + \alpha_s\,\alpha_{ew}\,\Delta^{QCD\otimes EW}_{ij,\,\mathrm{NNLO}} + \alpha_s^3\,\Delta^{QCD}_{ij,\,\mathrm{N3LO}} + \dots\right)$$

Couplings: $\alpha_s \sim 0.1\,,\ \alpha \sim 0.01$

Experimental precision usually $\sim$ few % level

At high energy, this naive power counting is spoiled by Sudakov logs!

Why do we need this level of precision in theoretical prediction?

- **EW corrections enhanced at high energy** *[Denner, Pozzorini '01]*:

  Massless gauge theories: singular soft and collinear logs from virtual corrections have to be cancelled by real-gauge boson radiation.

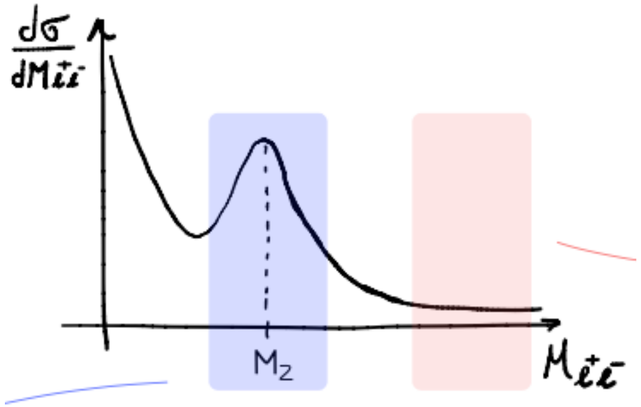
  In **EW theory** $m_{W/Z}$ provides a **physical cut-off**, real Z, W can be detected as distinguishable particles.

  Large logs from virtual corrections are of physical significance

$$\frac{\alpha}{4\pi\sin^2\theta_W}\log^2\left(\frac{s}{m_W^2}\right) \sim 6\,\%\,, \quad \frac{\alpha}{4\pi\sin^2\theta_W}\log\left(\frac{s}{m_W^2}\right) \sim 1\,\%\,, \quad \sqrt{s} = 1\mathrm{TeV}$$

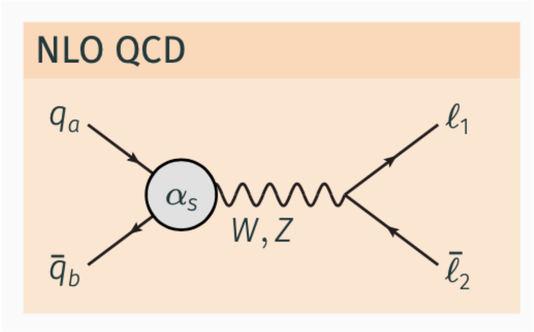
2 interesting regions to examine:

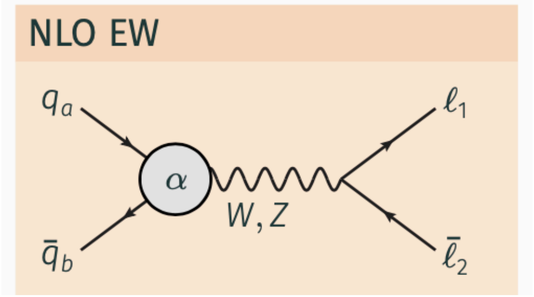
Resonance region: Z, W production
- Precise EW parameters determination

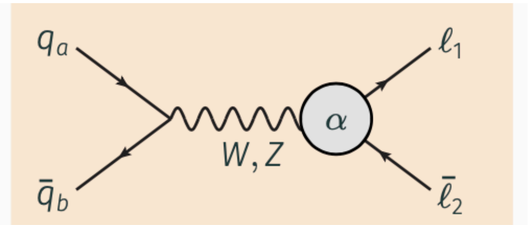


Off-shell region: l^+l^- production
- New Physics searches

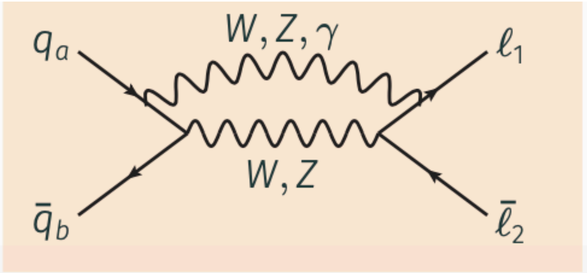
Analogies:







Crucial difference:



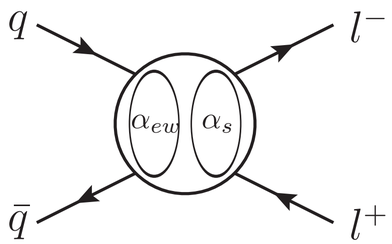
In the resonance region *[Dittmaier, Huss, Schwinn '14]*:

$$\mathcal{O}\left(\alpha_{ew}\frac{\Gamma_V}{M_V}\right) \sim \mathcal{O}(\alpha_{ew}^2) \rightarrow$$

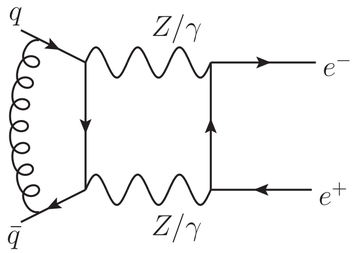
Suppressed way below percent level
Phenomenologically negligible

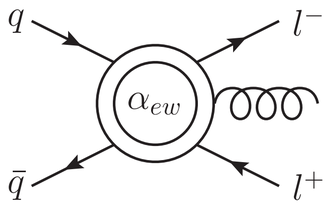
Ingredients for off-shell calculation at NNLO

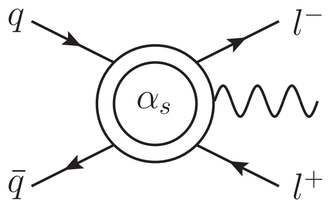
- Fully differential description of mixed QCD-EW effects is a complicated problem



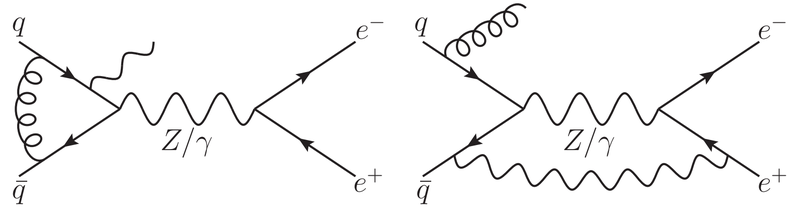
2-loop virtual + one-loop squared

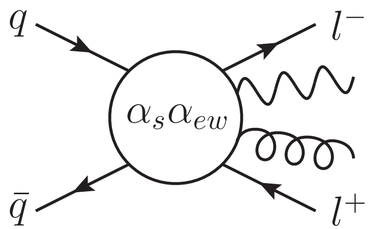




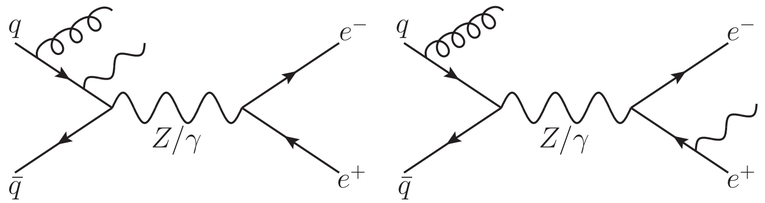


1-loop with one extra radiation



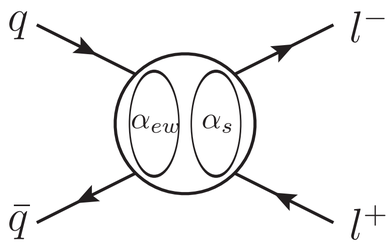


Tree-level double real

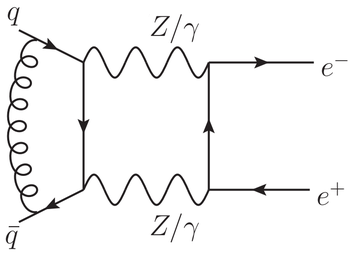


Ingredients for off-shell calculation at NNLO

❖ Fully differential description of mixed QCD-EW effects is a complicated problem



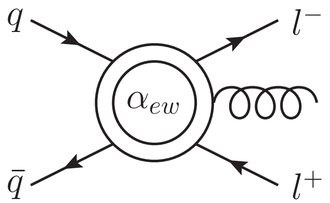
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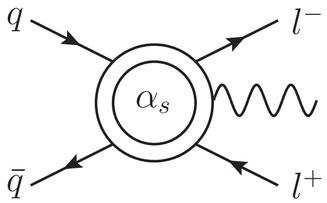


- **Several mass scales involved**
- **Fully analytic** calculation of two-loop integrals and helicity amplitudes *[Heller, von Manteuffel, Schabinger, Spiesberger '21]*
- Results expressed in terms of **well known functions** (ex: GPLs)
- Can be evaluated for **any phase-space point**

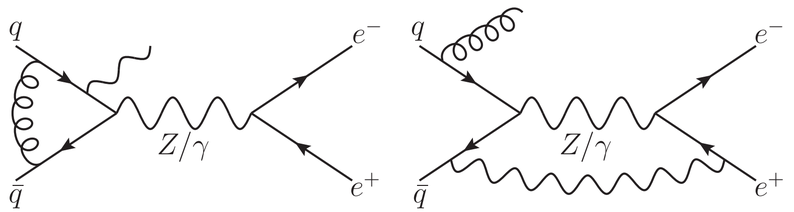
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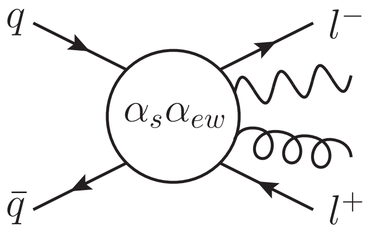




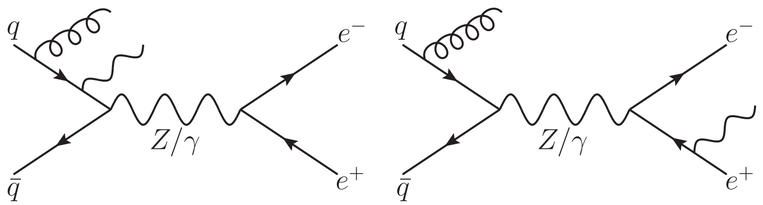
1-loop with one extra radiation



- **One-loop amplitude in degenerate kinematics**
- **OpenLoops** for numerical evaluation *[Cascioli, Maierhöfer, Pozzorini '12] [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller '19]*



Tree-level double real



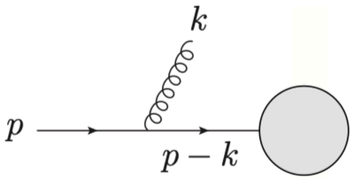
- Real emission contributions develop **soft and collinear singularities** and cannot be evaluated directly
- **Nested-soft-collinear subtraction** scheme *[Caola, Melnikov, Röntsch 1702.01352]*

IR singularities

Higher order corrections contain **infrared singularities from soft and/or collinear radiation**.

- Virtual corrections:
  - Explicit IR singularities from loop integrations
  - Poles in $1/\epsilon$

- Real corrections:
  - Singularities after integration over full phase space of radiated parton

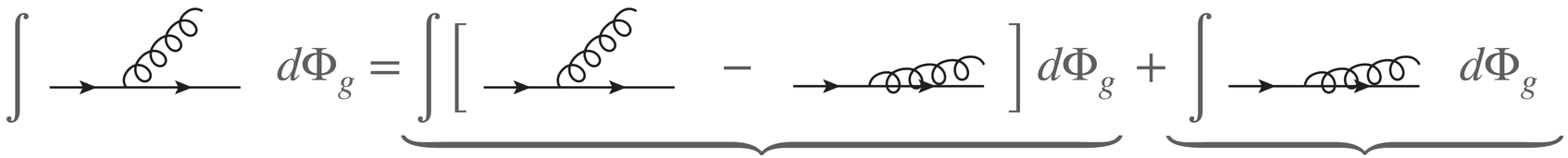


$$\sim \frac{1}{(p-k)^2} = \frac{1}{2E_pE_k(1-\cos\theta)} \underset{\substack{E_k \to 0 \\ \text{or} \\ \theta \to 0}}{\longrightarrow} \infty .$$

$$\int \frac{\mathrm{d}^{d-1}k}{(2\pi)^{d-1}2E_k} |M(\{p\},k)|^2 \underset{\substack{E_k \to 0 \\ \theta \to 0}}{\sim} \int \frac{\mathrm{d}E_k}{E_k^{1+2\epsilon}} \frac{\mathrm{d}\theta}{\theta^{1+2\epsilon}} \times |M(\{p\})|^2 \sim \frac{1}{4\epsilon^2} .$$

- **Integrating implies losing kinematic information** (needed for distributions, kinematic cuts, …)

→ **Subtraction scheme**: extract singularities without integrating over full phase space of radiated partons



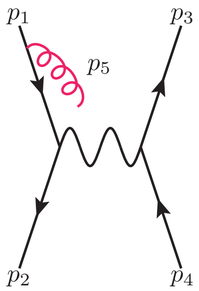
Finite in d=4, integrable numerically

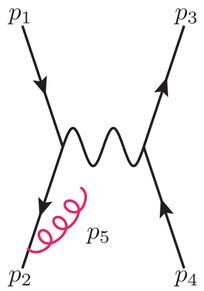
exposes the same $1/\epsilon$ poles as the virtual correction

Nested soft-collinear subtraction: NLO QCD

What do we need to know to define the counterterm?

- Which singular limits do contribute

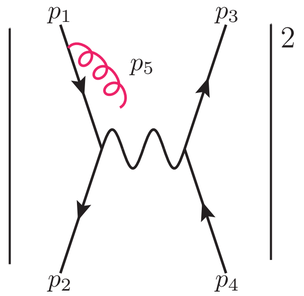




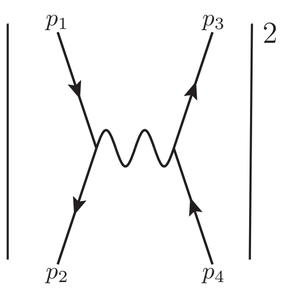
$$\mathscr{M} \sim \frac{1}{(k_1-k_5)^2} + \frac{1}{(k_2-k_5)^2} \sim \frac{1}{E_1 E_5 (1-\vec{n}_1\cdot\vec{n}_5)} + \frac{1}{E_2 E_5 (1-\vec{n}_2\cdot\vec{n}_5)}$$

$$\mathscr{M} \to \infty \quad \begin{cases} E_5 \to 0 & \to S_5 \\ \vec{n}_1 \parallel \vec{n}_5 & \to C_{15} \\ \vec{n}_2 \parallel \vec{n}_5 & \to C_{25} \end{cases}$$

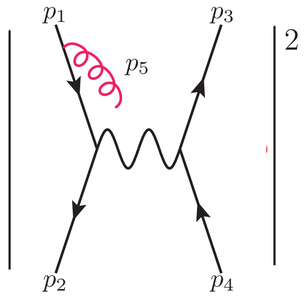
- How they act on the relevant contributions



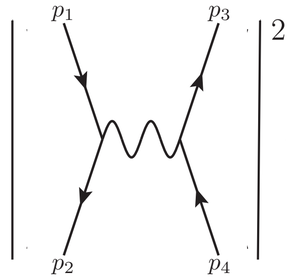
$$\underbrace{p_1 \| p_5}\ g_s^2\ \frac{1}{p_1\cdot p_5}\ \boxed{P_{qg}(z)}$$



Splitting kernel



$$\underbrace{E_3 \to 0}\ g_s^2\ \boxed{Eik(p_1,p_2;p_5)}$$



Eikonal kernel

$$P_{qg}(z) = \frac{1+z^2}{1-z} - \epsilon(1-z)\,, \qquad z = \frac{s_{12}}{s_{12}+s_{23}}$$

$$Eik(p_1,p_2;p_5) = \frac{p_1\cdot p_2}{(p_1\cdot p_5)\,(p_2\cdot p_5)}$$

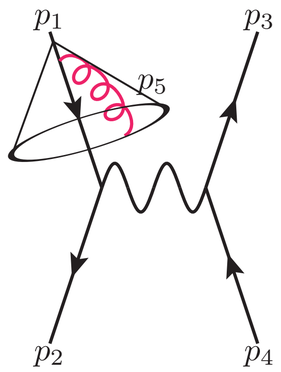
Nested soft-collinear subtraction: NLO EW

- How to simplify their treatment

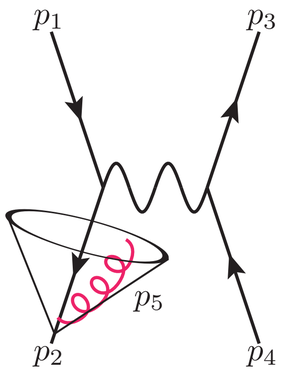
  - Phase space partition: useful to isolate collinear singularities (as in FKS), full freedom in their definition

$$\omega^{5j} \left| \mathscr{M}(1_q, 2_{\bar{q}}, 3_{l^-}, 4_{l^+}; 5_g) \right|^2$$

Partition function



ω^{51} Damp: $\vec{n}_2 \parallel \vec{n}_5$
Enhance: $\vec{n}_1 \parallel \vec{n}_5$

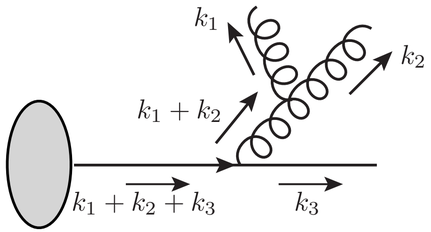


ω^{52} Damp: $\vec{n}_1 \parallel \vec{n}_5$
Enhance: $\vec{n}_2 \parallel \vec{n}_5$

  - Sector-by-sector subtraction:
    - Each sector contains a minimum number of singularities
    - Simple parametrisation

$$\omega^{5j} = \left[\left(I - C_{5j} \right) + C_{5j} \right] \omega^{5j} = \left(I - C_{5j} \right) \omega^{5j} + C_{5j}$$

Why is NNLO so difficult?



$$\sim \frac{1}{E_1 E_2 (1 - \vec{n}_1 \cdot \vec{n}_2)} \frac{1}{E_1 E_2 (1 - \vec{n}_1 \cdot \vec{n}_2) + E_1 E_3 (1 - \vec{n}_1 \cdot \vec{n}_3) + E_2 E_3 (1 - \vec{n}_2 \cdot \vec{n}_3)}$$

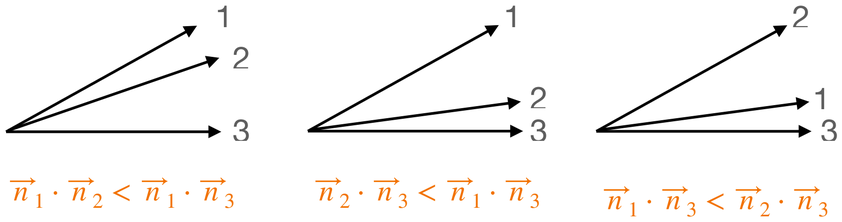
$E_1 \to 0 \quad E_2 \to 0 \quad E_1, E_2 \to 0$ - Either gluon or both gluons are soft

$\vec{n}_1 \parallel \vec{n}_2 \parallel \vec{n}_3$ - Both gluons collinear to the quark

$\vec{n}_1 \parallel \vec{n}_2$ - Gluons collinear to each other

Strongly ordered configurations have also to be included:

$$E_1 \ll E_2, \quad E_2 \ll E_1$$



All the contributing singular regimes have to be disentangle and regularised!

Many schemes are available:

Antenna *[0505111]*

ColorfulNNLO *[1603.08927]*

→ Nested-soft-collinear subtraction *[1702.01352]*

Residue subtraction *[1005.0274]*

New strategies have been explored:

Analytic Sector Subtraction *[1806.09570]*

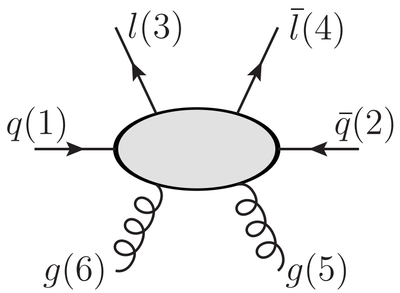
Geometric IR subtraction *[1804.07949]*

Unsubtraction *[1608.01584]*

FDR *[1208.5457]*

NNLO QCD: difficulties and solutions

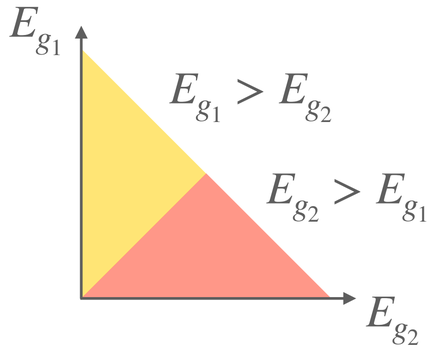
Examples: $q\bar{q} \to Z \to e^- e^+ g\, g$ *[Caola, Melnikov, Röntsch 1702.01352]*



Soft limits:

- Non-trivial structure of double soft eikonal
- Strongly-ordered limits to disentangle

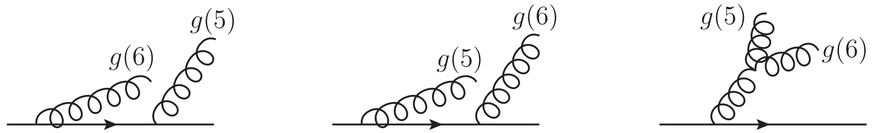
$$1 = \theta\left(E_{g_1} - E_{g_2}\right) + \theta\left(E_{g_2} - E_{g_1}\right)$$



Collinear limits:

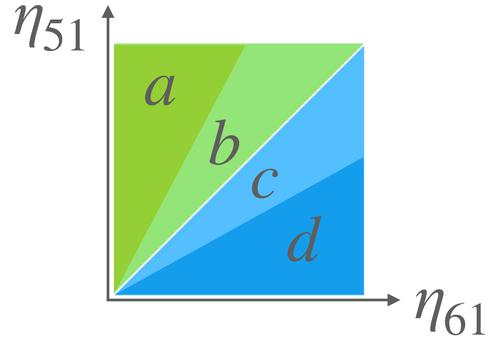
- Single, double and triple collinear limits to disentangle
- Non-trivial structure of triple collinear limits
- Strongly-ordered limits to disentangle in triple collinear sectors

$$1 = \sum_i \omega^i\,, \qquad i \in \left\{(51,61),(52,62),(51,62),(52,61)\right\}$$



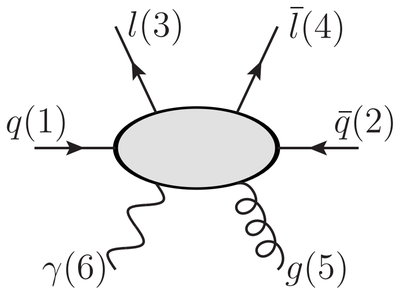
$$\eta_{ab} = \frac{1 - \cos\vartheta_{ab}}{2}$$

$$\omega^{51,61} = \omega^{51,61}\left(\theta_a + \theta_b + \theta_c + \theta_d\right)$$



Mixed QCD-EW: differences with respect to NNLO QCD

Examples: $q\bar{q} \to Z \to e^- e^+ g\,\gamma$ *[Buccioni, Caola, Chawdhry, Devoto, Heller, von Manteuffel, Melnikov, Röntsch, CSS]*



Soft limits:

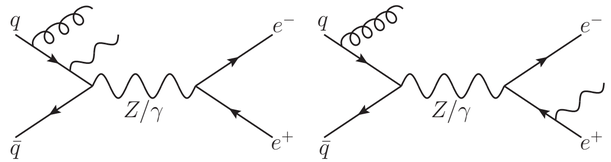
• Double soft limit factorises into NLO QCD x NLO EW → No need for energy ordering

$$\lim_{E_g, E_\gamma \to 0} |\mathscr{M}_{RR}|^2 = g_s^2\, \mathrm{Eik}(p_1, p_2; p_5)\; e^2 \sum_{i,j} Q_i\, Q_j\, \mathrm{Eik}(p_i, p_j; p_6)\, |\mathscr{M}_B|^2$$

Collinear limits:

• Single, double and triple collinear limits to disentangle

→ More **sectors** to account for final state radiation



$$1 = \sum_{i=1}^{4} \sum_{j=1}^{2} \omega^{\gamma i, g j}$$

• Non-trivial structure of triple collinear limits

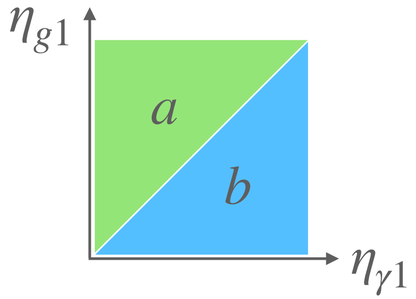
→ BUT **abelianization** of known results *[de Florian, Der, Fabre '18][Delto, Jaquier, Melnikov, Röntsch '19]*

• Strongly-ordered limits to disentangle in triple collinear sectors

→ BUT **no photon-gluon** collinearity



$$\omega^{51,61} = \omega^{51,61}\left(\theta_a + \theta_b\right)$$



Finite parts I

- Final result as combination of two- and one-loop corrections, double real radiation and pdf renormalisation:

✓ **Poles cancellation** proven independently on E_{max} and $\omega^{\gamma i,gj}$ functions

✓ **Fully differential**

✓ **Fully analytic**

✓ **Fully local > much more stable!**

✓ Cumbersome result for the finite parts

✓ In the **CoM** reference **frame**, $E_1 = E_2 = E_c$, **the result simplifies** remarkably

✓ Further **choice** $E_{max} = E_c$ reduces the number of terms that survive

✓ Simple structure arise, **compact expressions**

The considerations above hold for all the partonic channels, here we present explicit result for $q\bar{q} \to e^+e^-(g\gamma)$.
It is convenient to convenient to write the cross section in terms of contributions with different multiplicities and kinematic features

$$d\hat{\sigma}^{q\bar{q}}_{\text{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\text{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\text{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{reg},g\gamma}$$

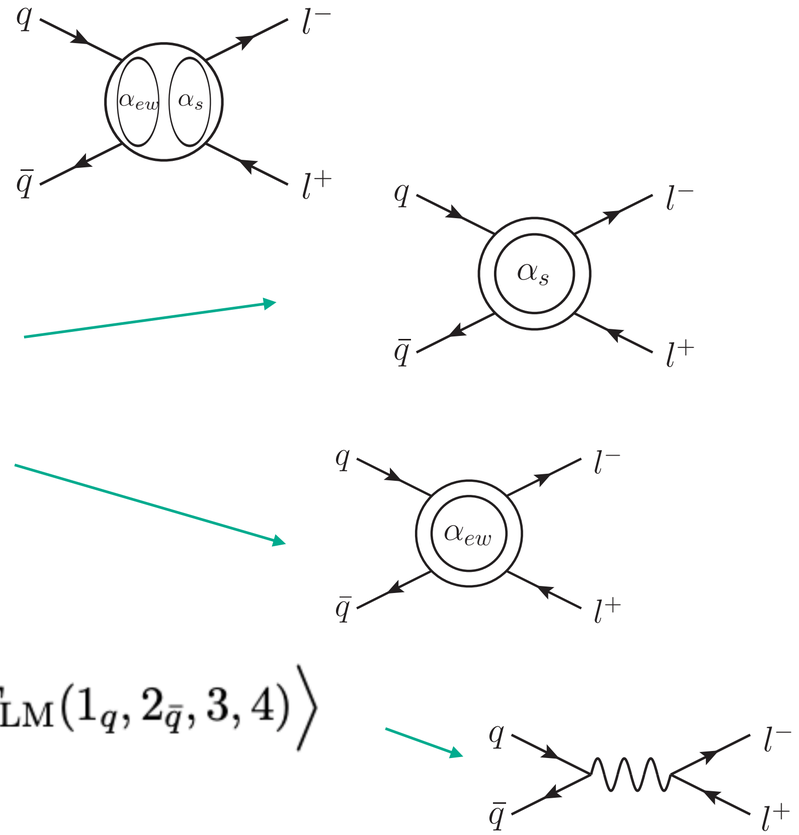
Finite parts III

$$d\hat{\sigma}^{q\bar{q}}_{\text{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\text{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\text{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{reg},g\gamma}$$

Elastic contributions arise from:

- double-unresolved real emissions of photons and gluons
- finite remainders of virtual corrections (EW and/or QCD virtual radiation)

$$\begin{aligned}
2s \cdot d\hat{\sigma}^{q\bar{q}}_{\text{el},g\gamma} = & \left\langle F^{(\text{QCD}\times\text{EW}),\,\text{fin}}_{\text{LVV+LV}^2}(1_q, 2_{\bar{q}}, 3, 4) \right\rangle \\
& + [\alpha] \left\langle \left[\mathcal{G}_{\text{EW}} + 3Q_q^2 \log\left(\frac{s}{\mu^2}\right) \right] F^{(\text{QCD}),\,\text{fin}}_{\text{LV}}(1_q, 2_{\bar{q}}, 3, 4) \right\rangle \\
& + [\alpha_s]\, C_F \left[\frac{2}{3}\pi^2 + 3\log\left(\frac{s}{\mu^2}\right) \right] \left\langle F^{(\text{EW}),\,\text{fin}}_{\text{LV}}(1_q, 2_{\bar{q}}, 3, 4) \right\rangle \\
& + [\alpha]\,[\alpha_s]\, C_F \left\langle \left\{ Q_q^2 \left[-\frac{4\pi^4}{45} + (2\pi^2 + 32\zeta_3) \log\left(\frac{s}{\mu^2}\right) \right.\right.\right. \\
& \left.\left.\left. + \left(9 - \frac{4\pi^2}{3}\right) \log^2\left(\frac{s}{\mu^2}\right) \right] + \mathcal{G}_{\text{EW}} \left(\frac{2\pi^2}{3} + 3\log\left(\frac{s}{\mu^2}\right) \right) \right\} F_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4) \right\rangle
\end{aligned}$$



Recurring structure

$$\mathcal{G}_{\text{EW}} = Q_q^2 \frac{2\pi^2}{3} + Q_e^2 \left(13 - \frac{2\pi^2}{3}\right) + 2\,Q_q Q_e \left[3\log\left(\frac{\eta_{13}}{\eta_{23}}\right) + 2\,\text{Li}_2(1-\eta_{13}) - 2\,\text{Li}_2(1-\eta_{23}) \right]$$

Finite parts IV

$$d\hat{\sigma}^{q\bar{q}}_{\text{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\text{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\text{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{reg},g\gamma}$$

Boosted contributions arise from final state where either a gluon and a photon are collinear to incoming partons, or one of them is collinear and the other is soft.

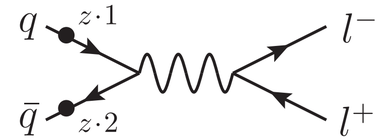
$$2s \cdot d\sigma^{q\bar{q}}_{\text{bt},g\gamma} = [\alpha]\,[\alpha_s]\,2C_F\,Q_q^2 \int_0^1 dz_1 dz_2\, \tilde{P}^{\text{NLO}}_{qq}(z_1, E_c) \left\langle \frac{F_{\text{LM}}(z_1 \cdot 1, z_2 \cdot 2, 3, 4)}{z_1\, z_2} \right\rangle \tilde{P}^{\text{NLO}}_{qq}(z_2, E_c)$$

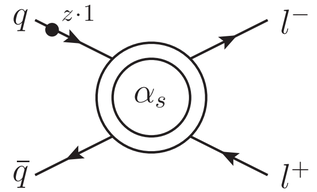
$$+ \sum_{i=1}^{2} \int_0^1 dz\, \tilde{P}^{\text{NLO}}_{qq}(z, E_c) \Big[[\alpha]\, Q_q^2 \left\langle F^{(i),(\text{QCD}),\,\text{fin}}_{\text{LV}}(1_q, 2_{\bar{q}}, 3, 4; z) \right\rangle$$

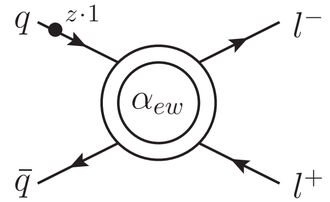
$$+ [\alpha_s]\, C_F \left\langle F^{(i),(\text{EW}),\,\text{fin}}_{\text{LV}}(1_q, 2_{\bar{q}}, 3, 4; z) \right\rangle \Big]$$

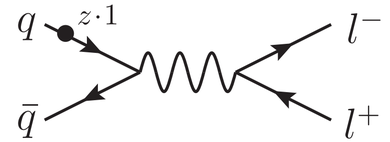
$$+ [\alpha]\,[\alpha_s]\,C_F \sum_{i=1}^{2} \int_0^1 dz \left\langle \left\{ Q_q^2\, P^{\text{NNLO}}_{qq}(z, E_c) + \tilde{P}^{\text{NLO}}_{qq}(z, E_c) \right.\right.$$

$$\left.\left. \times \left[Q_e^2\, G_{e^2} + 2Q_q Q_e \left(G^{(1,2)}_{eq} + (-1)^i \log\left(\frac{s_{i3}}{s_{i4}}\right) \log(z) \right) \right] \right\} F^{(i)}_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4; z) \right\rangle$$









Recurring structure

$$G^{(i,j)}_{eq} = \text{Li}_2(1-\eta_{i3}) - \text{Li}_2(1-\eta_{i4}) - \text{Li}_2(1-\eta_{j3}) + \text{Li}_2(1-\eta_{j4})$$
$$+ \left[\frac{3}{2} - \log\left(\frac{E_3}{E_c}\right)\right] \log\left(\frac{\eta_{i3}}{\eta_{j3}}\right) - \left[\frac{3}{2} - \log\left(\frac{E_4}{E_c}\right)\right] \log\left(\frac{\eta_{i4}}{\eta_{j4}}\right),$$

$$G_{e^2} = 13 - \frac{2}{3}\pi^2 + \log^2\left(\frac{E_3}{E_4}\right) + \left[3 - 2\log\left(\frac{E_3 E_4}{E_c^2}\right)\right] \log(\eta_{34}) + 2\text{Li}_2(1-\eta_{34})$$

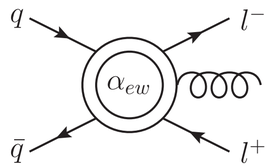
$$\tilde{P}^{\text{NLO}}_{qq}(z, E) = 4\mathcal{D}_1(z) - 2(1+z)\log(1-z) + (1-z) + 2\log\left(\frac{2E_c}{\mu}\right)\left(2\mathcal{D}_0(z) - (1+z)\right)$$

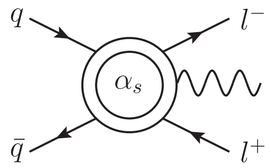
Finite parts V

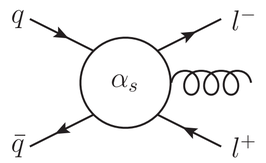
$$d\hat{\sigma}^{q\bar{q}}_{\text{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\text{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\text{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{reg},g\gamma}$$

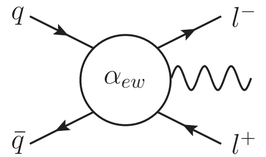
The $\mathcal{O}_{nlo}$ terms describes contributions of $l^-l^+\gamma$ and l^-l^+g. They arise from virtual corrections to these final states and from remnants of $l^-l^+g\gamma$ state in case either a gluon or a photon become unresolved.

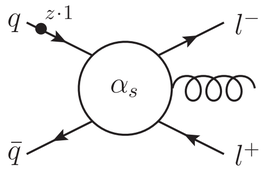
$$\begin{aligned}
2s \cdot d\sigma^{q\bar{q}}_{\mathcal{O}_{\text{nlo}},g\gamma} = &\left\langle \mathcal{O}^{g}_{\text{nlo}}\, F^{(\text{EW}),\,\text{fin}}_{\text{LV}}(1_q, 2_{\bar{q}}, 3, 4|5_g)\right\rangle \\
&+ \left\langle \mathcal{O}^{\gamma}_{\text{nlo}}\, F^{(\text{QCD}),\,\text{fin}}_{\text{LV}}(1_q, 2_{\bar{q}}, 3, 4|5_\gamma)\right\rangle \\
&+ [\alpha]\left\langle \mathcal{O}^{g}_{\text{nlo}}\left[Q_q^2\left(\frac{2}{3}\pi^2 + 3\log\left(\frac{s}{\mu^2}\right)\right) + 2Q_qQ_e\, G^{(1,2)}_{eq} + Q_e^2\, G_{e^2}\right] F_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_g)\right\rangle \\
&+ [\alpha_s]\, C_F\left[\frac{2}{3}\pi^2 + 3\log\left(\frac{s}{\mu^2}\right)\right]\left\langle \mathcal{O}^{\gamma}_{\text{nlo}}\, F_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_\gamma)\right\rangle \\
&+ [\alpha]\, Q_q^2 \sum_{i=1}^{2}\int_0^1 dz \left\langle \mathcal{O}^{g}_{\text{nlo}}\left[\tilde{P}^{\text{NLO}}_{qq}(z, E_c) + \tilde{\omega}^{\gamma i, gi}_{\gamma\|i}\, \log\eta_{i5}\, \bar{P}^{\text{AP},0}_{qq,R}(z)\right] F^{(i)}_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_g; z)\right\rangle \\
&+ [\alpha_s]\, C_F \sum_{i=1}^{2}\int_0^1 dz \left\langle \mathcal{O}^{\gamma}_{\text{nlo}}\left[\tilde{P}^{\text{NLO}}_{qq}(z, E_c) + \tilde{\omega}^{\gamma i, gi}_{g\|i}\, \log\eta_{i5}\, \bar{P}^{\text{AP},0}_{qq,R}(z)\right] F^{(i)}_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_\gamma; z)\right\rangle
\end{aligned}$$

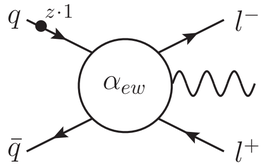












Recurring structure $\quad \bar{P}^{\text{AP},0}_{qq,R}(z) = 2\mathcal{D}_0(z) - (1+z)\,,$

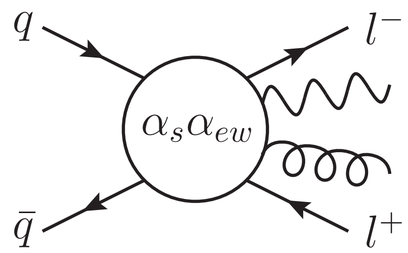
Finite parts VI

$$d\hat{\sigma}^{q\bar{q}}_{\text{mix},g\gamma} = d\hat{\sigma}^{q\bar{q}}_{\text{el},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{bt},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\mathcal{O}_{\text{nlo}},g\gamma} + d\hat{\sigma}^{q\bar{q}}_{\text{reg},g\gamma}$$

The regulated term is fully resolved and can be implemented numerically in $d = 4$.
Subtraction terms involve known eikonal contributions and splitting functions and are organised through partition functions

$$2s \cdot d\hat{\sigma}^{q\bar{q}}_{\text{reg},g\gamma} = \left\langle (I - S_g)(I - S_\gamma)\, \Omega_1^{q\bar{q}}\, F_{\text{LM}}(1_q, 2_{\bar{q}}, 3, 4|5_g, 6_\gamma) \right\rangle$$

$$\begin{aligned}
\Omega_1^{q\bar{q}} = {} & (1 - C_{g\gamma,1})(1 - C_{g1})\, \omega^{\gamma 1, g1}\, \theta_A^{(1)} + (1 - C_{g\gamma,1})(1 - C_{\gamma 1})\, \omega^{\gamma 1, g1}\, \theta_B^{(1)} \\
& + (1 - C_{g\gamma,2})(1 - C_{g2})\, \omega^{\gamma 2, g2}\, \theta_A^{(2)} + (1 - C_{g\gamma,2})(1 - C_{\gamma 2})\, \omega^{\gamma 2, g2}\, \theta_B^{(2)} \\
& + (1 - C_{g2})(1 - C_{\gamma 1})\, \omega^{\gamma 1, g2} + (1 - C_{g1})(1 - C_{\gamma 2})\, \omega^{\gamma 2, g1} \\
& + (1 - C_{g2})(1 - C_{\gamma 3})\, \omega^{\gamma 3, g2} + (1 - C_{g2})(1 - C_{\gamma 4})\, \omega^{\gamma 4, g2} \\
& + (1 - C_{g1})(1 - C_{\gamma 3})\, \omega^{\gamma 3, g1} + (1 - C_{g1})(1 - C_{\gamma 4})\, \omega^{\gamma 4, g1},
\end{aligned}$$



Finite parts II

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- ✓ **Compact results**
- ✓ **Recurring structures**
- ✓ **Fully differential in the final state kinematics**
- ✓ **Fast numerical implementation**
- ✓ **Other partonic channels present even simpler results**

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Full result fits one slide!!!

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- ✓ **Recurring structures**
- ✓ **Fully differential in the final state kinematics**
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Phenomenology: fiducial cross section

Set up for the results:

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strong qq and qg cancellation

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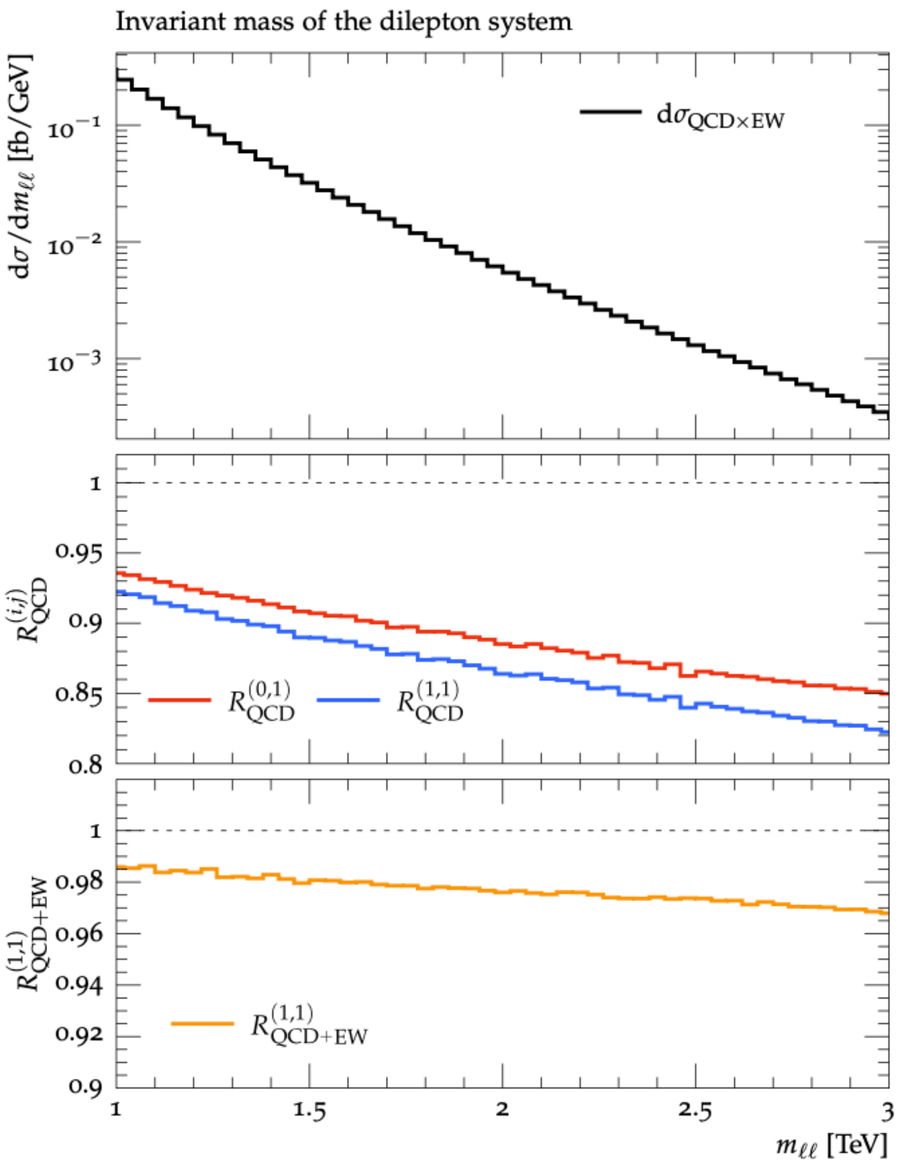
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→ QCDxEW corrections larger than NNLO QCD ones!

Kinematic distributions



$$d\sigma_{\rm QCD\times EW} = d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)} + d\sigma^{(2,0)} + d\sigma^{(1,1)}$$

What do we learn in the $m_{ll} \in [1, 3]\,\mathrm{TeV}$ region?

✓ The **cross section drops** of more than 3 orders of magnitude

✓ **Impact of NLO EW** corrections on NLO QCD results:

- Large corrections growing from $\mathcal{O}(-6\%)$ to $\mathcal{O}(-15\%)$

✓ **Impact of NLO EW+mixed QCD×EW** correction on NLO QCD results:

- Grow from $\mathcal{O}(-8\%)$ to $\mathcal{O}(-18\%)$

✓ **Impact of mixed QCD×EW** correction on NLO QCD+NLO EW results:

- Grow from $\mathcal{O}(-1.5\%)$ to $\mathcal{O}(-3\%)$

$$R^{(0,1)}_{\rm QCD} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}} \qquad R^{(1,1)}_{\rm QCD} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)} + d\sigma^{(1,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}} \qquad R^{(1,1)}_{\rm QCD+EW} = \frac{R^{(1,1)}_{\rm QCD}}{R^{(0,1)}_{\rm QCD}}$$

Conclusions

1. **Mixed QCDxEW** corrections to Drell-Yan are important to **search for NP in the high energy regime**

2. Calculating such correction is difficult

3. The **results** are now **available** and manifest a remarkably **simple structure**

4. **Mixed QCD-EW** amount to about **-1%** of the **LO** cross-section → **larger** than expected from **coupling magnitude**

5. **Good approximation** by the **product** of **QCD and EW** corrections in the **TeV region**

6. **Interesting impact on several observables**

Mixed QCD×EW corrections: state of the art

Theoretical developments

- Progress on two-loop master integrals *[Bonciani, Di Vita, Mastrolia, Schubert '16][Heller, von Manteuffel, Schabinger '19][Hasan, Schubert '20]*
- 2-loop amplitudes for $2 \to 2$ neutral current DY for massless leptons *[Heller, von Manteuffel, Schabinger, Spiesberger '20]*
- 2-loop amplitudes for $2 \to 2$ neutral current DY for massive leptons *[Armadillo, Bonciani, Devoto, Rana, Vicini '22]*

On-shell Z/W production ($2 \to 1$ process)

- Analytic mixed QCD-QED corrections to the inclusive production of an on-shell Z *[de Florian, Der, Fabre '18]*
- Fully differential mixed QCD-QED corrections to the production of an on-shell Z *[Delto, Jaquier, Melnikov, Röntsch '19]*
- Total on-shell Z production cross section in fully analytic with mixed QCD-EW corrections *[Bonciani, Buccioni, Rana, Vicini '20]*
- Fully differential on-shell Z and W production with mixed QCD-EW corrections *[Buccioni et al. '20] [Behring et al. '20]*

On-shell Z/W production ($2 \to 1$ process)

- Dominant mixed QCD-EW correction in the pole approximation NC and CC DY process *[Dittmaier, Huss, Schwinn '14, 15]*
- Approximate corrections available in parton showers based on factorised approach *[Balossini et at.'10][Bernaciak, Wackeroth '12] [Barzè et al. '12, '13] [Calame et al. '17]*
- Neutrino-pair production including mixed QCD-QED corrections *[Cieri, de Florian, Der, Mazzitelli '21]*
- Neutrino-pair production including mixed QCD-EW corrections *[Buonocore, Grazzini, Kallweit, Savoini, Tramontano '21]*
- Complete mixed QCD-EW corrections to NC and CC DY process *[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini '22]*

Thank you for your attention!

Backup

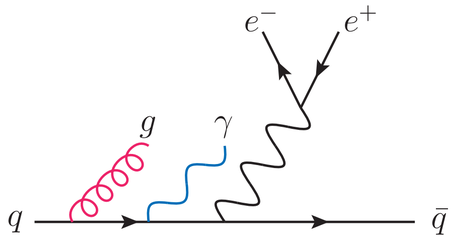
Phase space partitions

- **Unitary partition**
- Select a **minimum number of singularities** in each sector

$$1 = \underbrace{\omega^{\gamma 1,g1} + \omega^{\gamma 2,g2}}_{\text{Triple collinear partition}} + \underbrace{\omega^{\gamma 1,g2} + \omega^{\gamma 2,g1} + \omega^{\gamma 3,g2} + \omega^{\gamma 3,g1} + \omega^{\gamma 4,g2} + \omega^{\gamma 4,g1}}_{\text{Single collinear partition}}$$

$\omega^{\gamma 1,g1}$ contains $C_{\gamma 1}$, C_{g1}

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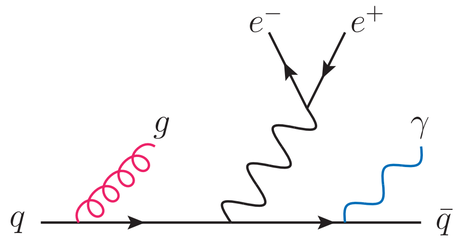


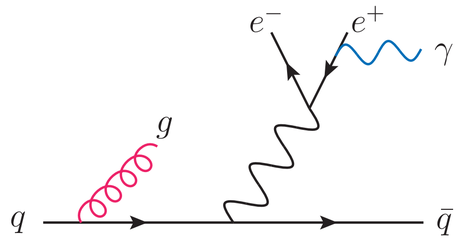
$\sim$ NNLO $\rightarrow$ **difficult!**

Overlapping singularities remain

$\omega^{\gamma i,gj}$ contains $C_{\gamma i}$, C_{gj}

Large rapidity difference in each sector





$\sim$ NLO $\times$ NLO $\rightarrow$ **simple!**

$$\omega^{\gamma i,gj} = \frac{\eta_{gj}^{-1}}{\sum_{k=1}^{2} \eta_{gk}^{-1}} \times \frac{\eta_{\gamma i}^{-1}}{\sum_{k=1}^{4} \eta_{\gamma k}^{-1}}$$

- Explicit **form is arbitrary**
- **Tuning** to simplify the analytic integration of the counterterm and the numerical stability of the fully-regulated term

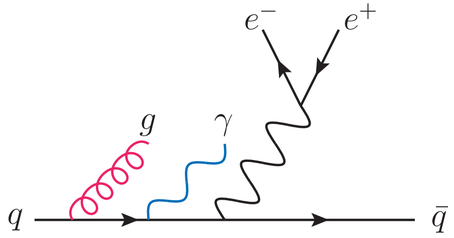
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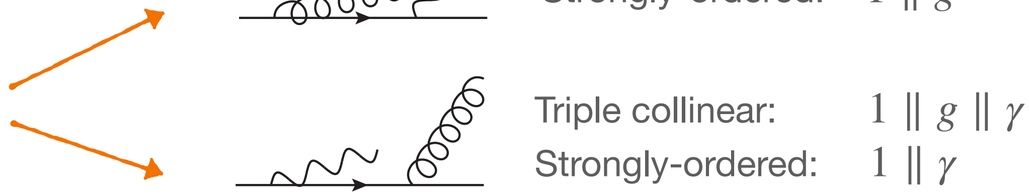
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Triple collinear: $1 \parallel g \parallel \gamma$

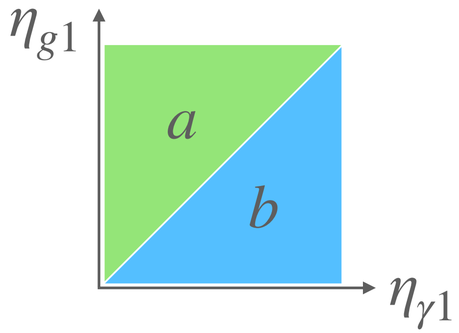
Strongly-ordered: $1 \parallel g$

Triple collinear: $1 \parallel g \parallel \gamma$

Strongly-ordered: $1 \parallel \gamma$

Define **angular ordering** to separate singularities

$$1 = \theta_a + \theta_b = \theta(\eta_{g1} - \eta_{\gamma 1}) + \theta(\eta_{\gamma 1} - \eta_{g1})$$



Phenomenology: m_{ll} windows

$$\sigma = \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} + \dots$$

LO · NLO QCD · NLO EW · NNLO QCD · QCDxEW

Question: can we capture NNLO QCDxEW by only computing (NLO QCD)·(NLO EW) $\rightarrow \delta\sigma^{(1,1)}_{\text{fact}}$?

$$\begin{aligned}
\Phi^{(1)} &: \ 200\ \text{GeV} < m_{\ell\ell} < 300\ \text{GeV}, \\
\Phi^{(2)} &: \ 300\ \text{GeV} < m_{\ell\ell} < 500\ \text{GeV}, \\
\Phi^{(3)} &: \ 500\ \text{GeV} < m_{\ell\ell} < 1.5\ \text{TeV}, \\
\Phi^{(4)} &: \ 1.5\ \text{TeV} < m_{\ell\ell} < \infty.
\end{aligned}$$

σ [fb]	$\sigma^{(0,0)}$	$\delta\sigma^{(1,0)}$	$\delta\sigma^{(0,1)}$	$\delta\sigma^{(2,0)}$	$\delta\sigma^{(1,1)}$	$\delta\sigma^{(1,1)}_{\text{fact.}}$
$\Phi^{(1)}$	1169.8	254.3	−30.98	10.18	−10.74	−6.734
$\Phi^{(2)}$	368.29	71.91	−11.891	2.85	−4.05	−2.321
$\Phi^{(3)}$	82.08	14.31	−4.094	0.691	−1.01	−0.7137
$\Phi^{(4)} \times 10$	9.107	1.577	−1.124	0.146	−0.206	−0.1946

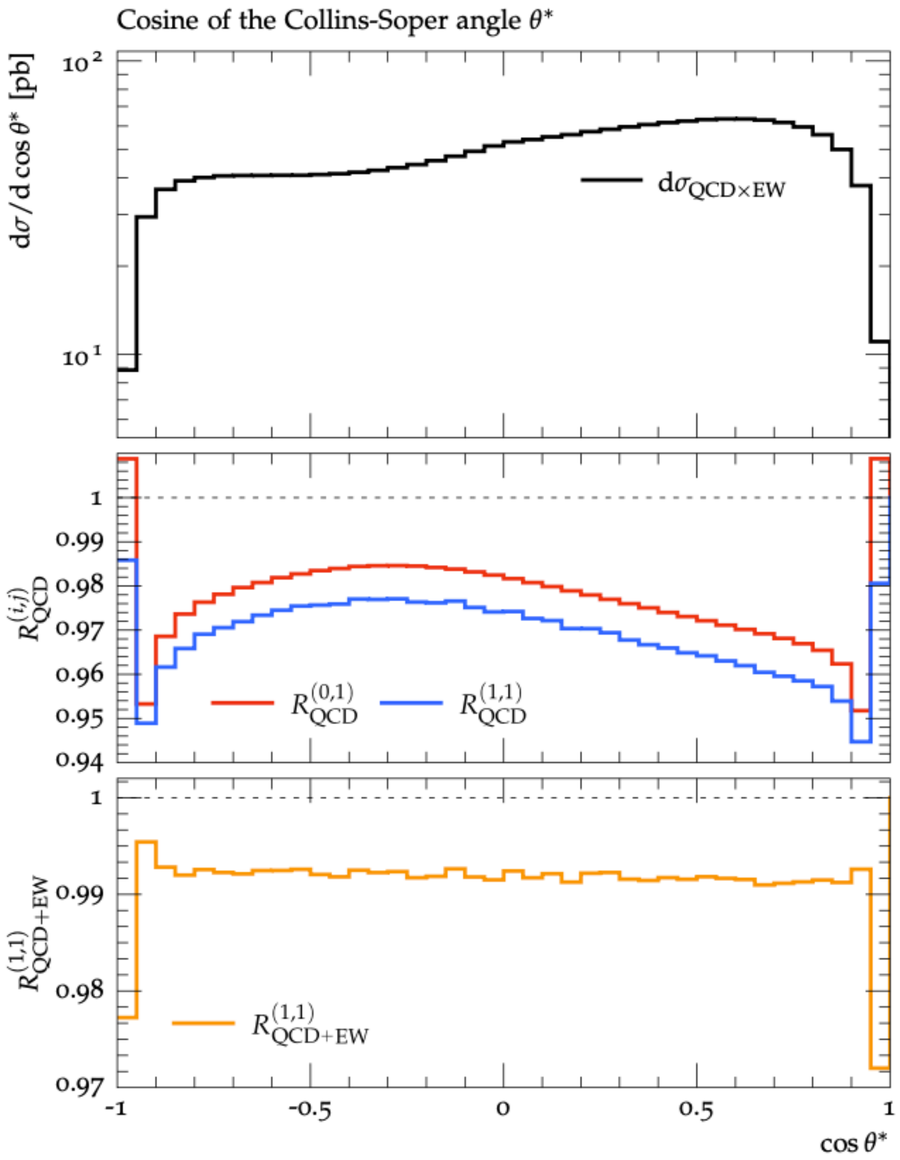
What do we learn?

✓ **At high invariant mass** ($m_{ll} > 1.5\,\text{TeV}$) **factorised approx.** captures more than **90%** of the **exact result**

$$\frac{\delta\sigma^{(1,1)}_{\text{fact.}}}{\sigma^{(0,0)}} = \left[\frac{\delta\sigma^{(1,0)}}{\sigma^{(0,0)}} \sim 0.17\right] \cdot \left[\frac{\delta\sigma^{(0,1)}}{\sigma^{(0,0)}} \sim -0.12\right] \sim -0.021 \qquad \frac{\delta\sigma^{(1,1)}}{\sigma^{(0,0)}} \sim -0.023$$

⟶ Expected: factorised approx. correctly reproduces the leading Sudakov logs, which dominate at high invariant mass

Kinematic distributions



Angular distributions can test quarks to leptons interactions → current structure

Collins-Soper angle: $$\cos\theta^* = \frac{p_{l^-}^+ p_{l^+}^- - p_{l^-}^- p_{l^+}^+}{m_{ll}\sqrt{m_{ll}^2 + p_{\ell\ell,\perp}^2}} \times \mathrm{sgn}(p_{ll,z})$$

$$\sigma_F = \int_0^1 d\cos\theta^* \frac{d\sigma(pp \to l^- l^+)}{d\cos\theta^*}, \qquad \sigma_B = \int_{-1}^0 d\cos\theta^* \frac{d\sigma(pp \to l^- l^+)}{d\cos\theta^*}$$

Forward-backward asymmetry:

$$A_{\mathrm{FB}} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

	$\tilde{A}_{\mathrm{FB}}$	A_{FB}
$\Phi^{(1)}$	$0.1442^{+0.05\%}_{-0.31\%}$	$0.1440^{+0.11\%}_{-0.09\%}$
$\Phi^{(2)}$	$0.1852^{+0.08\%}_{-0.40\%}$	$0.1847^{+0.10\%}_{-0.19\%}$
$\Phi^{(3)}$	$0.2401^{+0.13\%}_{-0.64\%}$	$0.2388^{+0.06\%}_{-0.47\%}$
$\Phi^{(4)}$	$0.3070^{+0.49\%}_{-1.5\%}$	$0.3031^{+0.19\%}_{-1.2\%}$

$$R_{\mathrm{QCD}}^{(0,1)} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}} \qquad R_{\mathrm{QCD}}^{(1,1)} = \frac{d\sigma^{(0,0)} + d\sigma^{(1,0)} + d\sigma^{(0,1)} + d\sigma^{(1,1)}}{d\sigma^{(0,0)} + d\sigma^{(1,0)}} \qquad R_{\mathrm{QCD+EW}}^{(1,1)} = \frac{R_{\mathrm{QCD}}^{(1,1)}}{R_{\mathrm{QCD}}^{(0,1)}}$$

Theoretical uncertainties

Theoretical uncertainties can be estimated by varying the central scale by a factor of 2, and changing the input scheme for the electroweak parameters

$$\sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} = 1928.3^{+1.8\%}_{-0.15\%}\ \text{fb}.$$

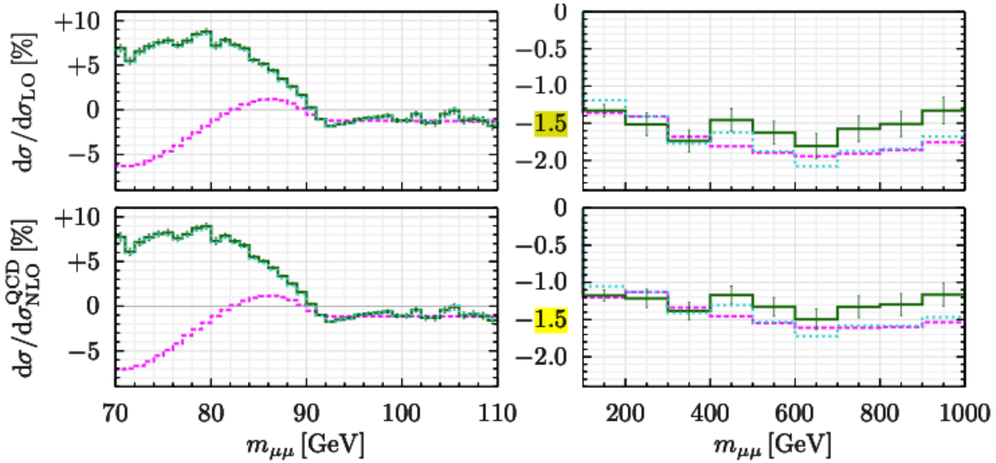
The mixed QCD-EW corrections are about -1% and therefore they are comparable in size to the theoretical uncertainties.

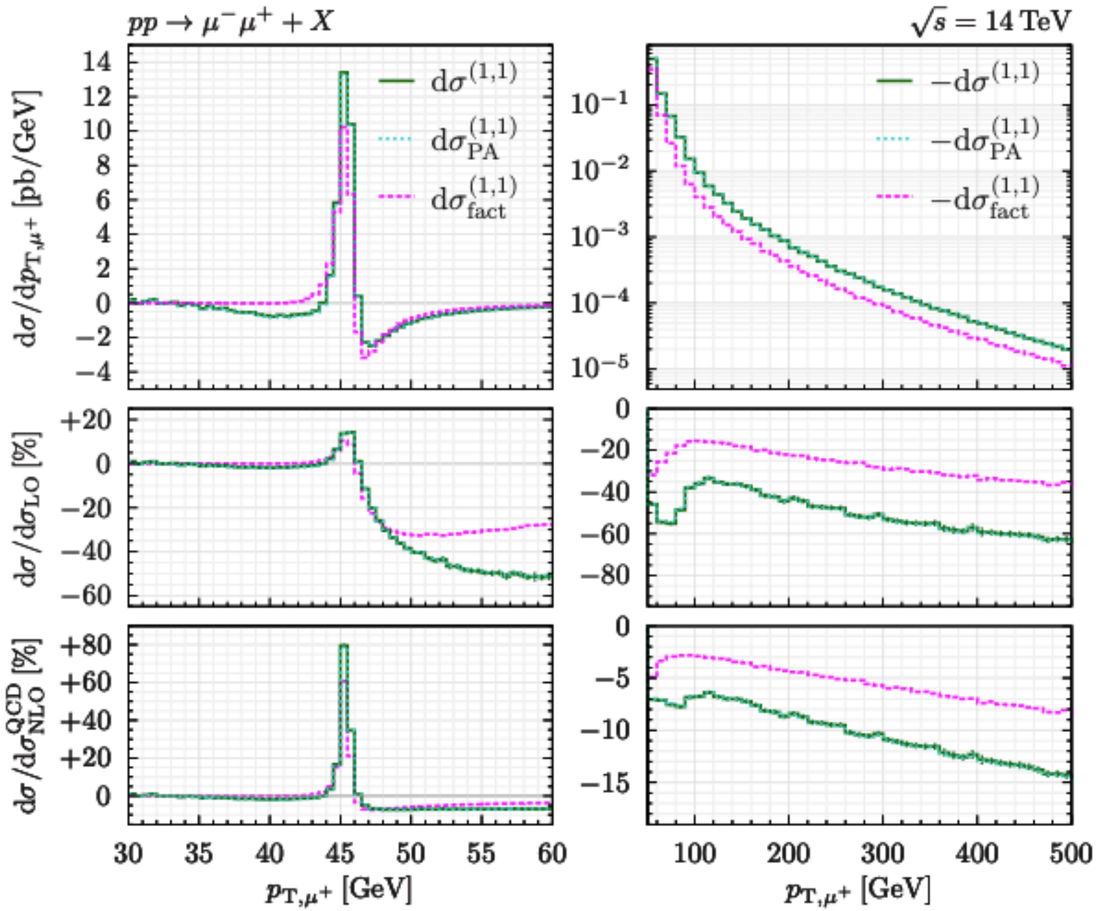
$$\sigma_{\text{QCD}\times\text{EW}} \equiv \sigma^{(0,0)} + \delta\sigma^{(1,0)} + \delta\sigma^{(0,1)} + \delta\sigma^{(2,0)} + \delta\sigma^{(1,1)} = 1912.6^{+0.65\%}_{-0\%}\ \text{fb}.$$

The mixed QCD-EW corrections remove a large source of input-scheme dependence coming from NLO QCD contribution.
Pure EW scheme uncertainty is reduced from about 1% to about 0.5% after the inclusion of mixed corrections

Distributions for QCDxEW corrections to dilepton production

- First complete QCDxEW corrections presented by *[Bonciani, Buonocore, Grazzini, Kallweit, Rana, Tramontano, Vicini '21]*
- **Two-loop amplitudes** evaluated with help of **semi-analytic method**
- **IR singularities** regulated by $q_\perp$ **subtraction** as implemented in MATRIX.
- Results for **massive leptons** (mass as IR regulator)
  - **Fiducial cross section** increased by $\sim 0.5\,\%$ relative to LO
  - Larger impact at **high-pT**: -60% correction





- High **invariant mass**: correction $\sim -1.5\,\%$ w.r.t. NLO QCD.
- Factorised approximation works well at the Jacobian peak, fails at higher pT.

IR regularisation: subtraction vs slicing

$F(x)$ arbitrary complicated function

$$I = \lim_{\epsilon \to 0} \left[\int_0^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right]$$

Goal: compute *I* without relying on the analytic evaluation of the integral

Slicing

$$I \sim \lim_{\epsilon \to 0} \left[F(0) \int_0^\delta \frac{dx}{x} x^\epsilon + \int_\delta^1 \frac{dx}{x} x^\epsilon F(x) - \frac{1}{\epsilon} F(0) \right] = F(0) \log \delta + \int_\delta^1 \frac{dx}{x} x^\epsilon F(x)$$

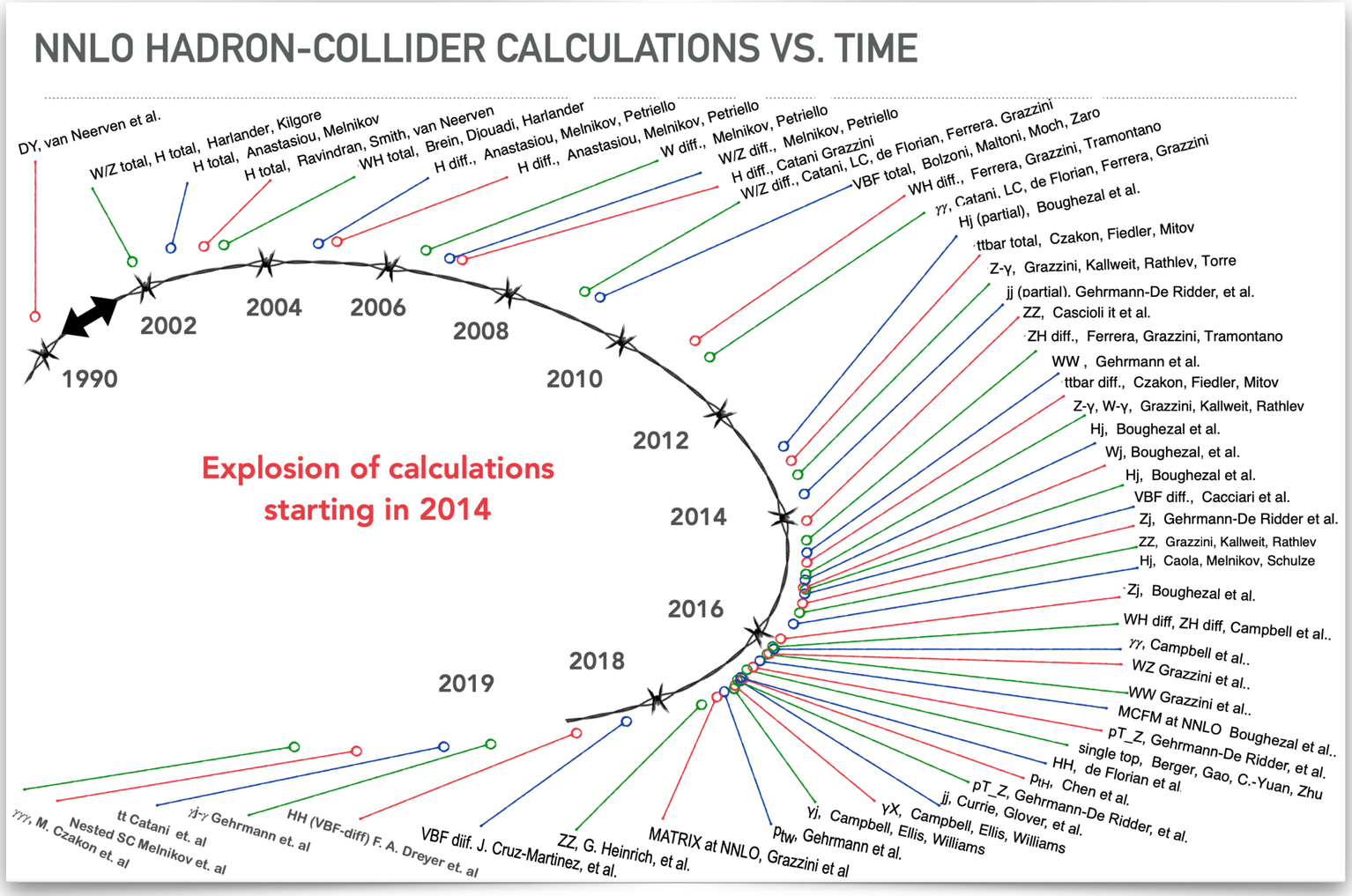
Slicing parameter $\delta \ll 1$ $\rightarrow$ power dependence on the slicing parameter in the result

Subtraction

$$I = \lim_{\epsilon \to 0} \left[\underbrace{\int_0^1 \frac{dx}{x} x^\epsilon \Big(F(x) - F(0) \Big)}_{\text{Regulated, finite for } \epsilon \to 0} + \underbrace{\int_0^1 \frac{dx}{x} x^\epsilon F(0)}_{\text{Extract } 1/\epsilon \text{ pole}} - \frac{1}{\epsilon} F(0) \right]$$

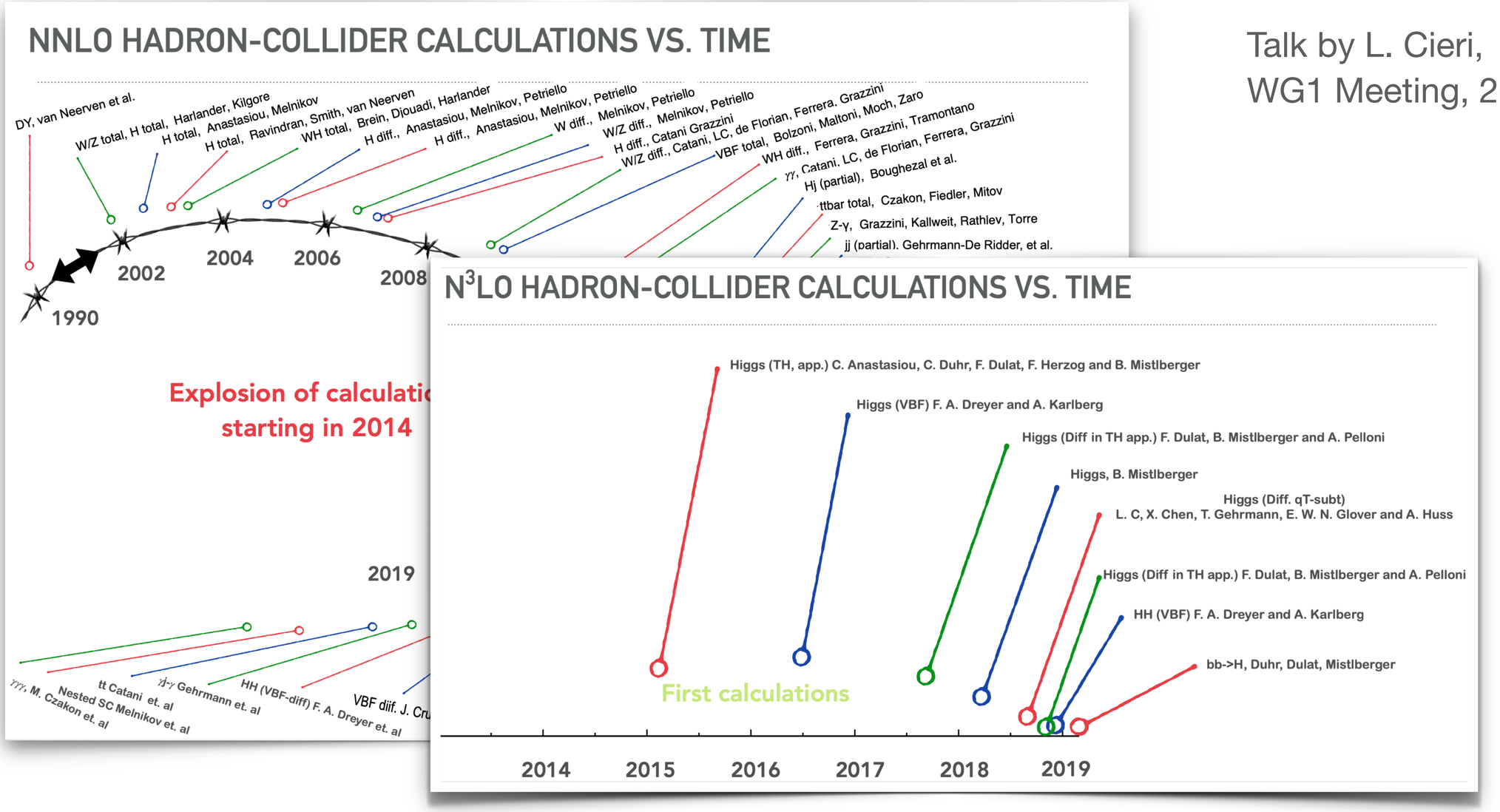
Counterterm: the definition may be involved!

Motivations



Talk by L. Cieri,
WG1 Meeting, 2019

Motivations



Talk by L. Cieri,
WG1 Meeting, 2019

Why is NNLO so difficult?

At NLO two main strategies have been implemented

Catani Seymour:

- Counterterm contribution: reproduces the **IR singularities** related to a dipole in **all of the phase space** [complicated structure]
- Full counterterm: sum of **contributions**, each **parametrised differently**
- **Analytic integration** of each term [non trivial, complicated structure of the counterterm]

FKS:

- **Partition** of the radiative phase space with sector functions
- **Different parametrisation** for each sector
- **Analytic integration**, after getting rid of sector functions [non trivial, non optimised parametrisation]

Detail informations of NNLO kernels also available ~ 20 years ago

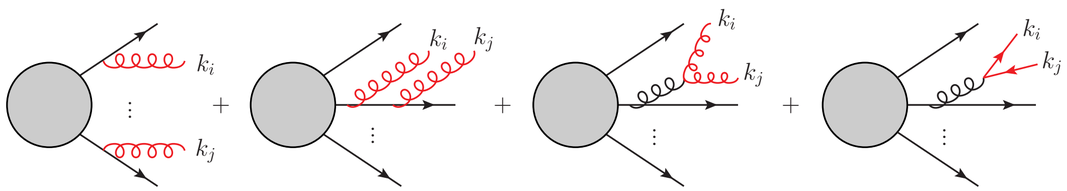
(N3LO kernels partially available *[Catani, Colferai, Torrini 1908.01616, Del Duca, Duhr, Haindl, Lazopoulos Michel 1912.06425, Dixon, Herrmann, Kai Yan, Hua Xing Zhu 1912.09370Yu Jiao Zhu 2009.08919]*)

Why is NNLO so difficult?

Under IR singular limits, the RR factorise into: (universal kernel) x (lower multiplicity matrix elements)

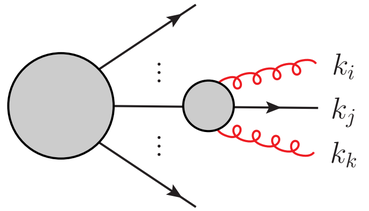
Double soft limit *[Catani, Grazzini 9903516,9810389]*

$$\lim_{k_i,\,k_j\to 0} RR_{n+2}\left(\{k\}_n, k_i, k_j\right) \sim \mathrm{Eik}\left(\{k\}_n, k_i, k_j\right) \otimes B_n\left(\{k\}_n\right)$$



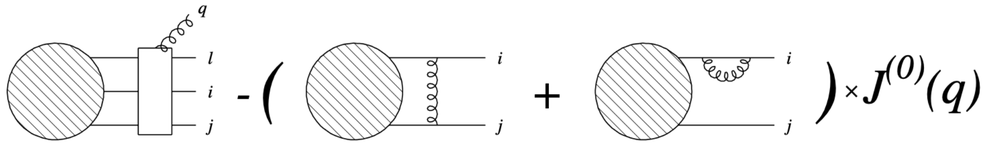
Triple collinear limit *[Catani, Grazzini 9903516,9810389]*

$$\lim_{k_i \| k_j \| k_k} RR_{n+2}\left(\{k\}_{n-1}, k_i, k_j, k_k\right) \sim \frac{1}{s_{ijk}^2} P\left(k_i, k_j, k_k\right) \otimes B_n\left(\{k\}_{n-1}, k_{ijk}\right)$$



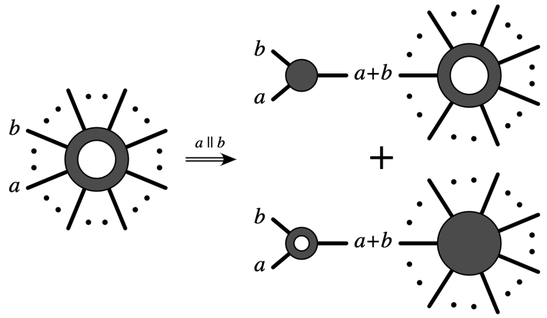
One loop single soft limit *[Catani, Grazzini 0007142]*

$$\lim_{k_i\to 0} RV_{n+1}\left(\{k\}_n, k_i\right) \sim \mathrm{Eik}\left(\{k\}_n, k_i\right) \otimes V_n\left(\{k\}_n\right) + \widetilde{\mathrm{Eik}}\left(\{k\}_n, k_i\right) \otimes B_n\left(\{k\}_n\right)$$



One loop single collinear limit *[Kosower 9901201, Bern, Del Duca, Kilgore, Schmidt 9903516]*

$$\lim_{k_i \| k_\to 0} RV_{n+1}\left(\{k\}_n, k_i\right) \sim \frac{1}{s_{ij}}\left[P\left(k_i, k_j\right) \otimes V_n\left(\{k\}_n\right) + \widetilde{P}\left(k_i, k_j\right) \otimes B_n\left(\{k\}_n\right)\right]$$



$$\mathbf{S}_{ij}\,RR(\{k\}) \propto \sum_{c,d\neq i,j}\left[\sum_{e,f\neq i,j} I^{(i)}_{cd}\,I^{(j)}_{ef}\,B_{cdef}\big(\{k\}_{\not ij}\big)+I^{(ij)}_{cd}\,B_{cd}\big(\{k\}_{\not ij}\big)\right]$$

$$I^{(i)}_{cd}=\frac{s_{cd}}{s_{ic}\,s_{id}} \qquad I^{(ij)}_{cd}=2\,T_R\,I^{(q\bar q)(ij)}_{cd}-2\,C_A\,I^{(gg)(ij)}_{cd} \qquad s_{ab}=2p_a\cdot p_b$$

$$I^{(q\bar q)(ij)}_{cd}=\frac{s_{ic}s_{jd}+s_{id}s_{jc}-s_{ij}s_{cd}}{s_{ij}^2(s_{ic}+s_{jc})(s_{id}+s_{jd})} \qquad I^{(gg)(ij)}_{cd}=\frac{(1-\epsilon)(s_{ic}s_{jd}+s_{id}s_{jc})-2s_{ij}s_{cd}}{s_{ij}^2(s_{ic}+s_{jc})(s_{id}+s_{jd})}+s_{cd}\,\frac{s_{ic}s_{jd}+s_{id}s_{jc}-s_{ij}s_{cd}}{s_{ij}s_{ic}s_{jd}s_{id}s_{jc}}\left[1-\frac{1}{2}\,\frac{s_{ic}s_{jd}+s_{id}s_{jc}}{(s_{ic}+s_{jc})(s_{id}+s_{jd})}\right]$$

$$\mathbf{C}_{ijk}\,RR(\{k\}) \propto \frac{1}{s_{ijk}^2}\,P^{\mu\nu}_{ijk}\big(s_{ir},s_{jr},s_{kr}\big)\,B_{\mu\nu}\big(\{k\}_{\not ijk},k_{ijk}\big) \qquad P^{\mu\nu}_{ijk}B_{\mu\nu}=P_{ijk}\,B+Q^{\mu\nu}_{ijk}\,B_{\mu\nu}$$

$$P^{(3g)}_{ijk}=C_A^2\left\{\frac{(1-\epsilon)s_{ijk}^2}{4s_{ij}^2}\left(\frac{s_{jk}}{s_{ijk}}-\frac{s_{ik}}{s_{ijk}}+\frac{z_i-z_j}{z_{ij}}\right)^2+\frac{s_{ijk}}{s_{ij}}\left[4\,\frac{z_iz_j-1}{z_{ij}}+\frac{z_iz_j-2}{z_k}+\frac{(1-z_kz_{ij})^2}{z_iz_kz_{jk}}+\frac{5}{2}\,z_k+\frac{3}{2}\right]\right.$$

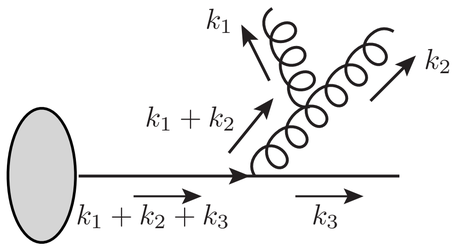
$$\left.+\frac{s_{ijk}^2}{2s_{ij}s_{ik}}\left[\frac{2z_iz_jz_{ik}(1-2z_k)}{z_kz_{ij}}+\frac{1+2z_i(1+z_i)}{z_{ik}z_{ij}}+\frac{1-2z_iz_{jk}}{z_jz_k}+2z_jz_k+z_i(1+2z_i)-4\right]+\frac{3(1-\epsilon)}{4}\right\}+perm\,.$$

$$z_a=\frac{s_{ar}}{s_{ir}+s_{jr}+s_{kr}}\,,\ z_{ab}=z_a+z_b$$

$$Q^{(3g)\mu\nu}_{ijk}=C_A^2\,\frac{s_{ijk}}{s_{ij}}\left\{\left[\frac{2z_j}{z_k}\frac{1}{s_{ij}}+\left(\frac{z_jz_{ik}}{z_kz_{ij}}-\frac{3}{2}\right)\frac{1}{s_{ik}}\right]\tilde k_i^2\,q_i^{\mu\nu}+\left[\frac{2z_i}{z_k}\frac{1}{s_{ij}}-\left(\frac{z_jz_{ik}}{z_kz_{ij}}-\frac{3}{2}-\frac{z_i}{z_k}+\frac{z_i}{z_{ij}}\right)\frac{1}{s_{ik}}\right]\tilde k_j^2\,q_j^{\mu\nu}-\left[\frac{2z_iz_j}{z_{ij}z_k}\frac{1}{s_{ij}}+\left(\frac{z_jz_{ik}}{z_kz_{ij}}-\frac{3}{2}-\frac{z_i}{z_j}+\frac{z_i}{z_{ik}}\right)\frac{1}{s_{ik}}\right]\tilde k_k^2\,q_k^{\mu\nu}\right\}+perm\,.$$

Key problem: several **different invariants** combined into **non-trivial** and various **structures**, to be integrated over a **6-dim PS.**

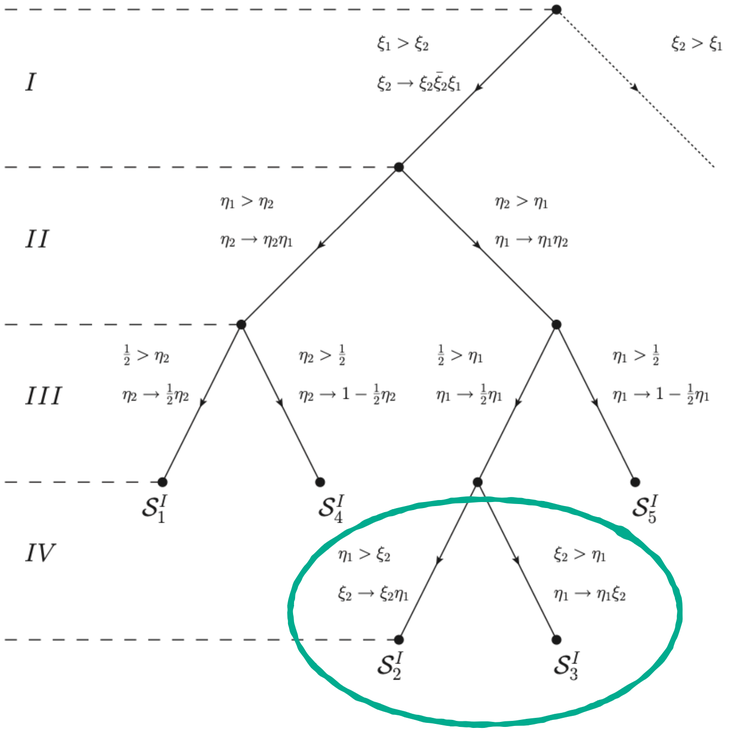
1. Clear understanding of which singular configurations do actually contribute



$$\sim \frac{1}{(k_1+k_2)^2}\frac{1}{(k_1+k_2+k_3)^2} = \frac{1}{2k_1\cdot k_2}\frac{1}{2k_1\cdot k_2 + 2k_1\cdot k_3 + 2k_2\cdot k_3} \iff k_1 \to 0 \text{ and } k_2 \parallel k_3$$

Entangled soft-collinear limits of diagrams can not be treated in a process-independent way.
Do non-commutative limits actually contribute?

STRIPPER was implemented taking into account all the possible choices of soft and collinear limits order -> redundant configurations were included

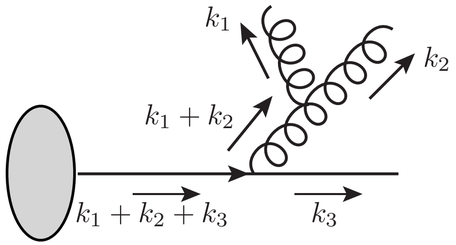


Gauge invariant amplitudes are free of entangled singularities
thanks to color coherence: soft parton does not resolve angles of the collinear partons

Soft-collinear limits can be described by taking the known soft and collinear limits sequentially

2. Get to the point where the problem is well defined

a) Identify the overlapping singularities
b) Regulate them



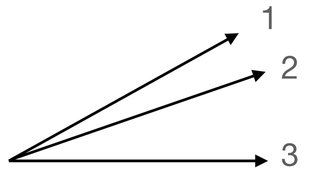
$$\sim \frac{1}{E_1 E_2 (1 - \vec{n}_1 \cdot \vec{n}_2)} \frac{1}{E_1 E_2 (1 - \vec{n}_1 \cdot \vec{n}_2) + E_1 E_3 (1 - \vec{n}_1 \cdot \vec{n}_3) + E_2 E_3 (1 - \vec{n}_2 \cdot \vec{n}_3)}$$

Soft origin: $E_1 \to 0 \quad E_2 \to 0 \quad E_1, E_2 \to 0$

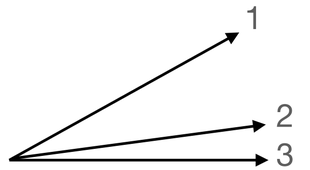
$E_1 \ll E_2\,, \quad E_2 \ll E_1$

Collinear origin: $\vec{n}_1 \parallel \vec{n}_2 \quad \vec{n}_1 \parallel \vec{n}_2 \parallel \vec{n}_3$

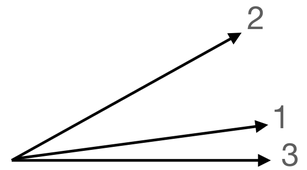
Includes **strongly ordered** configurations



$\vec{n}_1 \cdot \vec{n}_2 < \vec{n}_1 \cdot \vec{n}_3$



$\vec{n}_2 \cdot \vec{n}_3 < \vec{n}_1 \cdot \vec{n}_3$



$\vec{n}_1 \cdot \vec{n}_3 < \vec{n}_2 \cdot \vec{n}_3$

Soft and collinear modes do not intertwine: soft subtraction can be done globally. Collinear singularities have still to be regulated.
Strongly ordered configurations have to be properly taken into account.

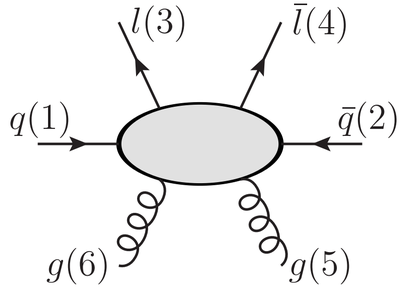
Phase space partitions

Efficient way to simplify the problem: introduce **partition functions** (following FKS philosophy):

- **Unitary partition**
- Select a **minimum number of singularities** in each sector
- Do **not affect** the **analytic integration** of the counterterms

Definition of partition functions benefits from remarkable degree of **freedom**: different approaches can be implemented

Examples: Nested soft-collinear subtraction $q\bar{q} \to Z \to e^- e^+ \, g \, g$ *[Caola, Melnikov, Röntsch 1702.01352]*



$$1 = \omega^{51,61} + \omega^{52,62} + \omega^{51,62} + \omega^{52,61}$$

$$\omega^{51,61} = \frac{\rho_{25}\,\rho_{26}}{d_5\,d_6}\left(1 + \frac{\rho_{15}}{d_{5621}} + \frac{\rho_{16}}{d_{5612}}\right) \qquad \omega^{51,62} = \frac{\rho_{25}\,\rho_{16}\,\rho_{56}}{d_5\,d_6\,d_{5612}}$$

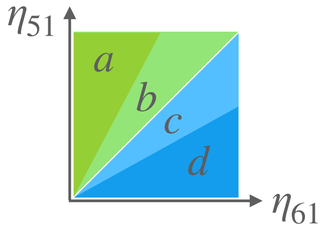
$$\omega^{52,62} = \frac{\rho_{15}\,\rho_{16}}{d_5\,d_6}\left(1 + \frac{\rho_{25}}{d_{5621}} + \frac{\rho_{26}}{d_{5612}}\right) \qquad \omega^{52,61} = \frac{\rho_{15}\,\rho_{26}\,\rho_{56}}{d_5\,d_6\,d_{5621}}$$

$$\rho_{ab} = 1 - \cos\vartheta_{ab}\,, \; \eta_{ab} = \rho_{ab}/2$$

$$d_{i=5,6} = \rho_{1i} + \rho_{2i} = 2$$

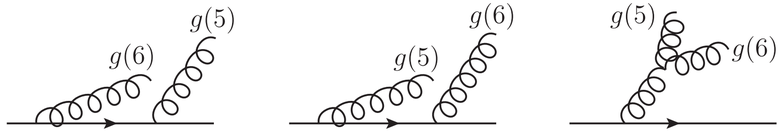
$$d_{5621} = \rho_{56} + \rho_{52} + \rho_{61}$$

$$d_{5612} = \rho_{56} + \rho_{51} + \rho_{62}$$



$$1 = \theta\left(\eta_{61} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{61} < \eta_{51}\right) + \theta\left(\eta_{51} < \frac{\eta_{61}}{2}\right) + \theta\left(\frac{\eta_{61}}{2} < \eta_{51} < \eta_{61}\right)$$

$$= \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}$$



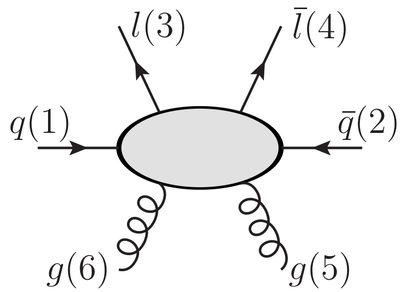
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Advantages:

1. Simple definition
2. Structure of collinear singularities fully defined
3. Same strategy holds for NNLO mixed QCDxEW processes
4. **Minimum number of sector**

Disadvantages:

1. Partition based on angular ordering -> Lorentz invariance not preserved
   -> angles defined in a given reference frame
2. Theta function

3. Solve the PS integrals

The problem is now well defined:

A. **Singular kernels** and their nested limits have to be **subtracted from the double real correction** to get integrable object

$$\int d\Phi_{n+2}\, RR_{n+2} = \int d\Phi_{n+2} \underbrace{\left[RR_{n+2} - K_{n+2}\right]}_{\text{Fully regulated real emission contribution}} + \int d\Phi_{n+2}\, K_{n+2} \qquad K_{n+2} \supset C_{ij},\, C_{kl},\, S_i,\, S_{ij},\, C_{ijk}$$

$\longrightarrow$ Numerical evaluation

Examples: Nested soft-collinear subtraction $q\bar{q} \to Z \to e^- e^+ g\, g$ *[Caola, Melnikov, Röntsch]*

$$[\,\mathrm{d}f_i\,] = \frac{\mathrm{d}^d k_i}{(2\pi)^d}\,(2\pi)\,\delta_+(k_i^2)$$

$$\begin{aligned}
\mathrm{d}\hat{\sigma}^{NNLO}_{\text{resolv.}} = \int \theta(E_5 - E_6)\,\theta(E_{\max} - E_5) \Bigg\{ & \sum_{i,j\in\{1,2\},i\neq j} \left(1 - C_{5i}\right)\left(1 - C_{6j}\right)\left(1 - S_{56}\right)\left(1 - S_6\right)[\mathrm{d}k_5]\,[\mathrm{d}k_6]\,\omega^{5i,6j}\,B\left(\{k\}_{1\ldots6}\right) \\
& + \sum_{i\in\{1,2\}} \Big[\theta^{(a)}\left(1 - C_{i56}\right)\left(1 - C_{6i}\right) + \theta^{(b)}\left(1 - C_{i56}\right)\left(1 - C_{56}\right) \\
& \quad + \theta^{(c)}\left(1 - C_{i56}\right)\left(1 - C_{5i}\right) + \theta^{(d)}\left(1 - C_{i56}\right)\left(1 - C_{56}\right)\Big][\mathrm{d}k_5]\,[\mathrm{d}k_6]\,\omega^{5i,6i}\,B\left(\{k\}_{1\ldots6}\right)\Bigg\}
\end{aligned}$$

Explicit expression depends on the scheme

3. Solve the PS integrals

The problem is now well defined:

A. **Singular kernels** and their nested limits have to be **subtracted from the double real correction** to get integrable object

$$\int d\Phi_{n+2}\, RR_{n+2} = \int d\Phi_{n+2} \left[RR_{n+2} - K_{n+2} \right] + \int d\Phi_{n+2}\, K_{n+2} \qquad K_{n+2} \supset C_{ij},\, C_{kl},\, S_i,\, S_{ij},\, C_{ijk}$$

B. **Counterterms** have to be **integrated over the unresolved phase space**

$$I = \int \mathrm{PS}_{\mathrm{unres.}} \otimes \mathrm{Limit} \otimes \mathrm{Constraints}$$

The ‘Limit’ component is universal and known. The phase space is well defined. Constraints may vary depending on the scheme.

Several kinematic structures have to be integrated **analytically** over a 6-dim PS.

Different approximations and techniques can be applied: the results assume different form depending on the adopted strategy

Two main structure are the most complicated ones and affect most of the physical processes:

- **Double soft**
- **Triple collinear**

Kernels integration

Examples: Nested soft-collinear subtraction $q\bar{q} \to Z \to e^- e^+ \, g \, g$ *[Caola, Delto, Frellesvig, Melnikov 1807.05835, Delto, Melnikov 1901.05213]*

Two soft parton (5,6) and two hard massless radiator (1,2): arbitrary relative angle between the three-momenta of the radiators

$$I_{12}^{(gg)(56)} = \frac{(1-\epsilon)(s_{51}s_{62} + s_{52}s_{61}) - 2s_{56}s_{12}}{s_{56}^2(s_{51}+s_{61})(s_{52}+s_{62})} + s_{12}\,\frac{s_{51}s_{62} + s_{52}s_{61} - s_{56}s_{12}}{s_{56}s_{51}s_{62}s_{52}s_{61}}\left[1 - \frac{1}{2}\,\frac{s_{51}s_{62} + s_{52}s_{61}}{(s_{51}+s_{61})(s_{52}+s_{62})}\right]$$

$$I_{S_{56}}^{(gg)} = \int [\mathrm{d}k_5]\,[\mathrm{d}k_6]\,\theta(E_{\max} - E_5)\,\theta(E_5 - E_6)\,I_{12}^{(gg)(56)}\left(k_1, k_2, k_5, k_6\right) \qquad [\,\mathrm{d}f_i\,] = \frac{\mathrm{d}^d k_i}{(2\pi)^d}\,(2\pi)\,\delta_+(k_i^2)$$

$$E_5 = E_{\max}\,\xi \qquad E_6 = E_{\max}\,\xi\,z \qquad 0 < \xi < 1\,,\, 0 < z < 1$$

Reverse unitarity: **map phase space integrals onto loop integrals** *[Anastasiou, Melnikov 0207004]*
after defining integral families, integration-by-part identities. Differential equations w.r.t. the ratio of energies of emitted gluons at fixed angle.
Boundary conditions for z=0, and arbitrary angle

Kernels integration

Examples: Nested soft-collinear subtraction $q\bar{q} \to Z \to e^- e^+ g\, g$ *[Caola, Delto, Frellesvig, Melnikov 1807.05835, Delto, Melnikov 1901.05213]*

$$
\begin{aligned}
I^{(gg)}_{S_{56}} &= (2E_{\max})^{-4\epsilon} \left[\frac{1}{8\pi^2}\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\right]^2 \left\{\frac{1}{2\epsilon^4} + \frac{1}{\epsilon^3}\left[\frac{11}{12} - \ln(s^2)\right]\right. \\
&+ \frac{1}{\epsilon^2}\left[2\mathrm{Li}_2(c^2) + \ln^2(s^2) - \frac{11}{6}\ln(s^2) + \frac{11}{3}\ln 2 - \frac{\pi^2}{4} - \frac{16}{9}\right] \\
&+ \frac{1}{\epsilon}\left[6\mathrm{Li}_3(s^2) + 2\mathrm{Li}_3(c^2) + \left(2\ln(s^2) + \frac{11}{3}\right)\mathrm{Li}_2(c^2) - \frac{2}{3}\ln^3(s^2)\right. \\
&\quad + \left(3\ln(c^2) + \frac{11}{6}\right)\ln^2(s^2) - \left(\frac{22}{3}\ln 2 + \frac{\pi^2}{2} - \frac{32}{9}\right)\ln(s^2) \\
&\quad \left. - \frac{45}{4}\zeta_3 - \frac{11}{3}\ln^2 2 - \frac{11}{36}\pi^2 - \frac{137}{18}\ln 2 + \frac{217}{54}\right] \\
&+ 4\mathrm{G}_{-1,0,0,1}(s^2) - 7\mathrm{G}_{0,1,0,1}(s^2) + \frac{22}{3}\mathrm{Ci}_3(2\delta) + \frac{1}{3\tan(\delta)}\mathrm{Si}_2(2\delta) \\
&+ 2\mathrm{Li}_4(c^2) - 14\mathrm{Li}_4(s^2) + 4\mathrm{Li}_4\left(\frac{1}{1+s^2}\right) - 2\mathrm{Li}_4\left(\frac{1-s^2}{1+s^2}\right) \\
&+ 2\mathrm{Li}_4\left(\frac{s^2-1}{1+s^2}\right) + \mathrm{Li}_4(1-s^4) + \left[10\ln(s^2) - 4\ln\left(1+s^2\right)\right. \\
&\left. + \frac{11}{3}\right]\mathrm{Li}_3(c^2) + \left[14\ln(c^2) + 2\ln(s^2) + 4\ln\left(1+s^2\right) + \frac{22}{3}\right]\mathrm{Li}_3(s^2) \\
&+ 4\ln(c^2)\mathrm{Li}_3(-s^2) + \frac{9}{2}\mathrm{Li}_2^2(c^2) - 4\mathrm{Li}_2(c^2)\mathrm{Li}_2(-s^2) + \left[7\ln(c^2)\ln(s^2)\right. \\
&\left. - \ln^2(s^2) - \frac{5}{2}\pi^2 + \frac{22}{3}\ln 2 - \frac{131}{18}\right]\mathrm{Li}_2(c^2) + \left[\frac{2}{3}\pi^2 - 4\ln(c^2)\ln(s^2)\right]\times \\
&\mathrm{Li}_2(-s^2) + \frac{\ln^4(s^2)}{3} + \frac{\ln^4\left(1+s^2\right)}{6} - \ln^3(s^2)\left[\frac{4}{3}\ln(c^2) + \frac{11}{9}\right] \\
&+ \ln^2(s^2)\left[7\ln^2(c^2) + \frac{11}{3}\ln(c^2) + \frac{\pi^2}{3} + \frac{22}{3}\ln 2 - \frac{32}{9}\right] - \frac{\pi^2}{6}\ln^2\left(1+s^2\right) \\
&+ \zeta_3\left[\frac{17}{2}\ln(s^2) - 11\ln(c^2) + \frac{7}{2}\ln\left(1+s^2\right) - \frac{21}{2}\ln 2 - \frac{99}{4}\right] + \ln(s^2)\times \\
&\left[-\frac{7\pi^2}{2}\ln(c^2) + \frac{22}{3}\ln^2 2 - \frac{11}{18}\pi^2 + \frac{137}{9}\ln 2 - \frac{208}{27}\right] - 12\mathrm{Li}_4\left(\frac{1}{2}\right) \\
&+ \frac{143}{720}\pi^4 - \frac{\ln^4 2}{2} + \frac{\pi^2}{2}\ln^2 2 - \frac{11}{6}\pi^2\ln 2 + \frac{125}{216}\pi^2 + \frac{22}{9}\ln^3 2 \\
&\left. + \frac{137}{18}\ln^2 2 + \frac{434}{27}\ln 2 - \frac{649}{81} + \mathcal{O}(\epsilon)\right\},
\end{aligned}
$$

$$
\delta = \frac{\delta_{12}}{2},\ s = \sin\frac{\delta_{12}}{2},\ c = \cos\frac{\delta_{12}}{2}
$$

$$
\mathrm{Ci}_n(z) = \frac{\mathrm{Li}_n(e^{iz}) + \mathrm{Li}_n(e^{-iz})}{2},\ \mathrm{Si}_n(z) = \frac{\mathrm{Li}_n(e^{iz}) - \mathrm{Li}_n(e^{-iz})}{2i}
$$

Singular structure of the RR

Under fundamental limits, the RR factorise into: (universal kernel) x (lower multiplicity matrix elements)

$$\mathbf{S}_{ij}\, RR(\{k\}) \propto \sum_{c,d\neq i,j} \left[\sum_{e,f\neq i,j} I^{(i)}_{cd}\, I^{(j)}_{ef}\, B_{cdef}\big(\{k\}_{\not{i}\not{j}}\big) + I^{(ij)}_{cd}\, B_{cd}\big(\{k\}_{\not{i}\not{j}}\big) \right]$$

$$\mathbf{C}_{ijk}\, RR(\{k\}) \propto \frac{1}{s^2_{ijk}}\, P^{\mu\nu}_{ijk}\big(s_{ir}, s_{jr}, s_{kr}\big)\, B_{\mu\nu}\big(\{k\}_{\not{i}\not{j}\not{k}}, k_{ijk}\big)$$

$$\mathbf{C}_{ijkl}\, RR(\{k\}) \propto \frac{1}{s_{ij}\, s_{kl}}\, P^{\mu\nu}_{ij}\big(s_{ir}, s_{jr}\big)\, P^{\rho\sigma}_{kl}\big(s_{kr'}, s_{lr'}\big)\, B_{\mu\nu\rho\sigma}\big(\{k\}_{\not{i}\not{j}\not{k}\not{l}}, k_{ij}, k_{kl}\big)$$

$$\mathbf{SC}_{ijk}\, RR(\{k\}) = \mathbf{CS}_{jki} RR(\{k\}) \propto \frac{1}{s_{jk}} \sum_{c,d\neq i} P^{\mu\nu}_{jk}\; I^{(i)}_{cd}\; B^{cd}_{\mu\nu}\big(\{k\}_{\not{i}\not{j}\not{k}}, k_{jk}\big)$$

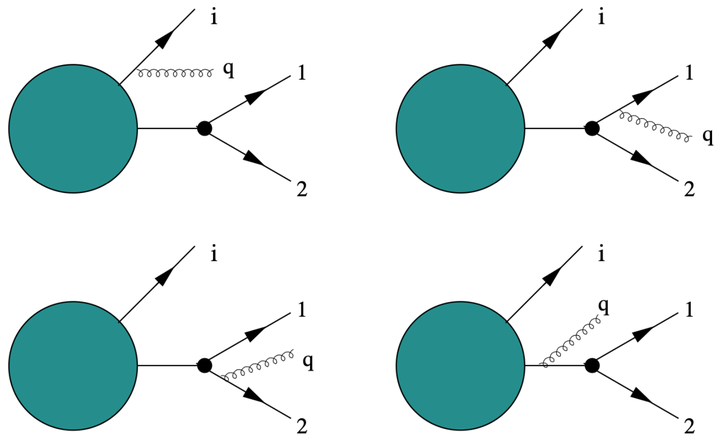
Born-level kinematics does not satisfy the mass-shell condition and momentum conservation

→ Momentum mapping needed!

Colour coherence and disentangled soft-collinear singularities

Parton q is soft and partons $1,2$ are collinear *[Catani, Grazzini 9908523]*

$$\left|\mathscr{M}_{g,a_1,a_2,\ldots,a_n}(q,p_1,p_2,\ldots,p_n)\right|^2 \simeq -\frac{2}{s_{12}}(4\pi\mu^{2\epsilon}\alpha_s)^2 \left\langle \mathscr{M}_{a,a_3,\ldots,a_n}(p,p_3,\ldots,p_n)\left|\hat{\mathbf{P}}_{a_1a_2}\left[\mathbf{J}^{\dagger}_{(12)\mu}(q)\,\mathbf{J}^{\mu}_{(12)}(q)\right]\right|\mathscr{M}_{a,a_3,\ldots,a_n}(p,p_3,\ldots,p_n)\right\rangle$$



Mother parton: $a \to a_1 + a_2$

Altarelli-Parisi splitting functions: spin correlations

Soft current: colour correlations

$$\mathbf{J}^{\dagger}_{(12)\mu}(q)\,\mathbf{J}^{\mu}_{(12)}(q) \simeq \sum_{i,j=3}^{n} \mathbf{T}_i \cdot \mathbf{T}_j\, \mathscr{S}_{ij}(q) + 2\sum_{i=3}^{n} \mathbf{T}_i \cdot \mathbf{T}_{(12)}\, \mathscr{S}_{i\,(12)}(q)$$

$$\mathbf{T}_{(12)} = \mathbf{T}_1 + \mathbf{T}_2 \qquad \mathscr{S}_{ij}(q) = \frac{2\,s_{ij}}{s_{iq}\,s_{jq}} \qquad \mathscr{S}_{i\,(12)}(q) = \frac{2\,(s_{i1}+s_{i2})}{s_{iq}\,(s_{1q}+s_{2q})}$$

The soft-collinear limit at $\mathcal{O}\left(\alpha_s^2\right)$ is fully described in a factorised way, where the factors are the soft eikonal function and the Altarelli-Parisi splitting functions that control IR limits at $\mathcal{O}\left(\alpha_s\right)$.

This simplification, which is due to colour coherence, was not performed in FKS.

Mixed EWxQCD corrections to on shell Z production+decay

13TeV LHC

G_μ scheme

$p_\perp^{l_1} > 24\,\text{GeV}$ $\quad p_\perp^{l_2} > 16\,\text{GeV}$

$-2.4 < y_l < 2.4$

$R = 0.1$

$\mu_F = \mu_R = m_Z/2$

$m_{ll} > 50\,\text{GeV}$ (dressed leptons)

Type	Inclusive	Cuts	Cuts (production)
$\Delta^{\text{QED}}_{\text{NLO}}$	$+2.3 \times 10^{-3}$	-5.3×10^{-3}	$+2.2 \times 10^{-3}$
$\Delta^{\text{weak}}_{\text{NLO}}$	-5.5×10^{-3}	-5.0×10^{-3}	-5.0×10^{-3}
$\Delta^{\text{EW}}_{\text{NLO}}$	-3.2×10^{-3}	-1.0×10^{-2}	-2.8×10^{-3}
$\Delta^{\text{QCD}-\text{QCD}}_{\text{NNLO}}$	$+1.3 \times 10^{-2}$	$+5.8 \times 10^{-3}$	$+5.8 \times 10^{-3}$
$\Delta^{\text{QCD}-\text{QED}}_{\text{NNLO}}$	$+5.5 \times 10^{-4}$	-5.9×10^{-3}	$+1.4 \times 10^{-4}$
$\Delta^{\text{QCD}-\text{weak}}_{\text{NNLO}}$	-1.6×10^{-3}	-2.1×10^{-3}	-2.1×10^{-3}
$\Delta^{\text{QCD}-\text{EW}}_{\text{NNLO}}$	-1.1×10^{-3}	-8.0×10^{-3}	-2.0×10^{-3}

tiny NLO EW corrections

[Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]

(QCDx)QED corrections strongly sensitive to selection cuts on the leptons

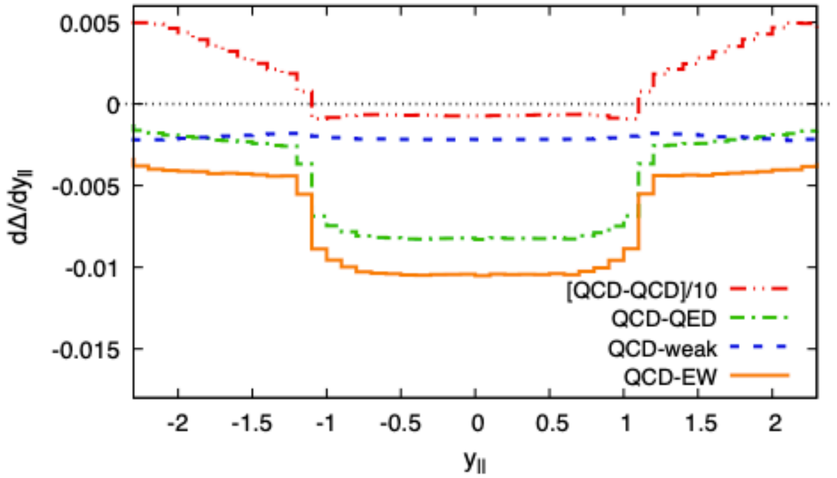
Production process. It removes (strong) dependence on lepton cuts: weak corrections dominant over QED

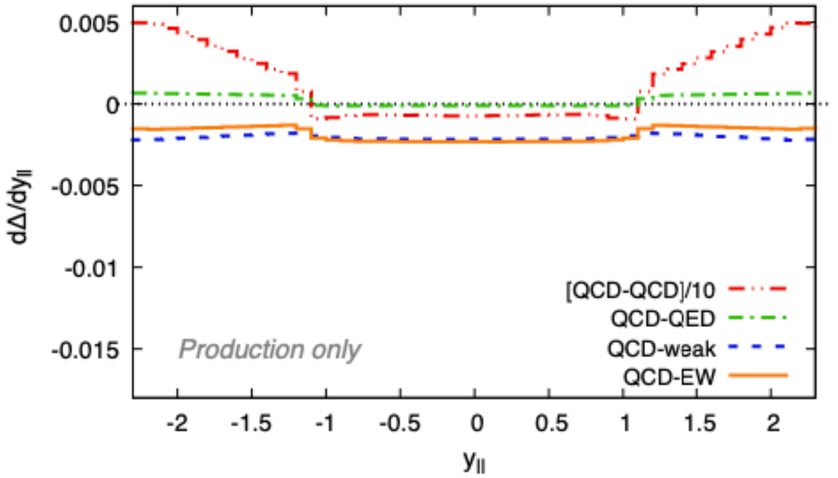
$$d\Delta^i = \frac{d\sigma^i}{d\sigma_{\text{LO}} + d\sigma^{\text{QCD}}_{\text{nlo}}}$$

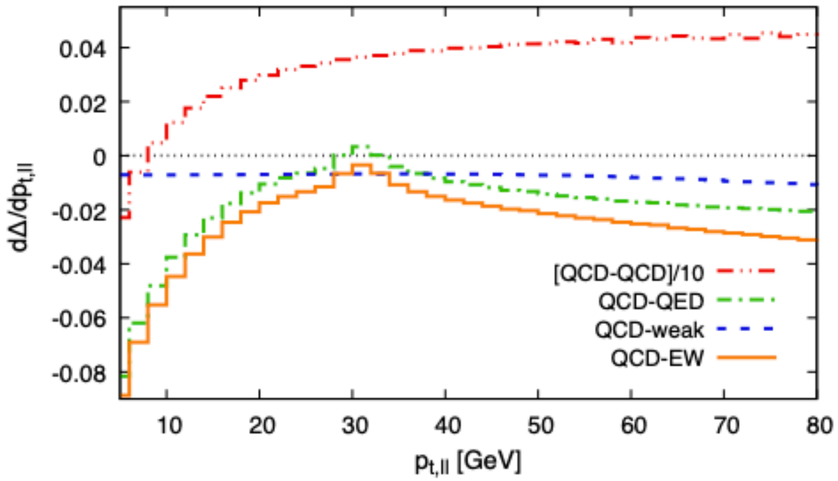
$$i \in \{\text{EW}, \text{QCD} - \text{EW}, \text{QCD} - \text{QCD}\}$$

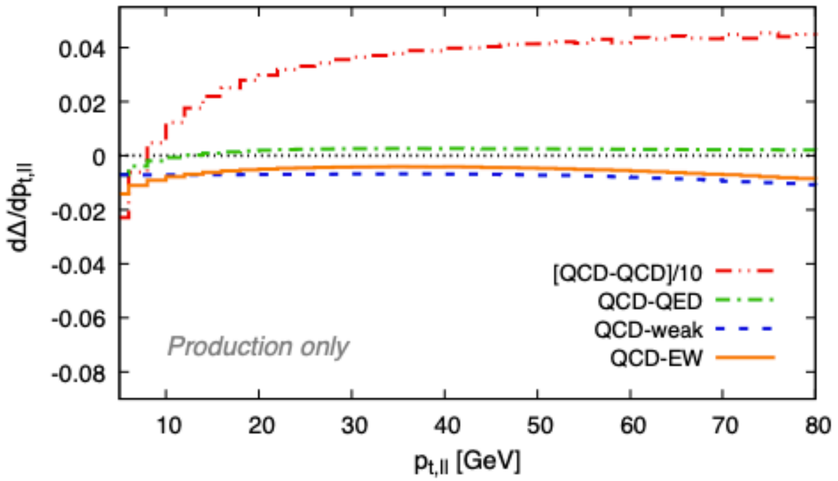
$$\text{EW} = \text{QED} + \text{weak}$$

Mixed EWxQCD corrections to on shell Z production+decay







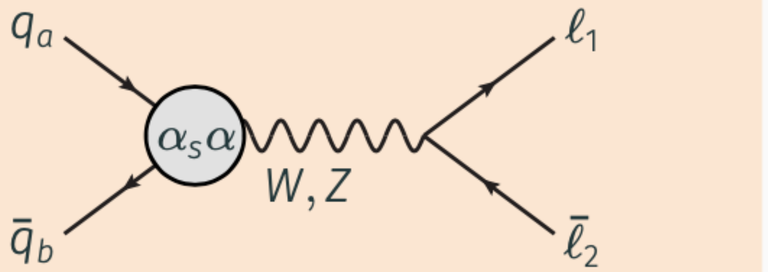


[Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]

- QCDxWeak corrections dominate over QCDxQED in the production mechanism
- In some kinematics regions QCDxEW effects comparable to NNLO QCD: central y_{ll} and $p_\perp^{ll}$
- The impact of mixed QCDxEW, as well as the QED-Weak interplay, is observable dependent

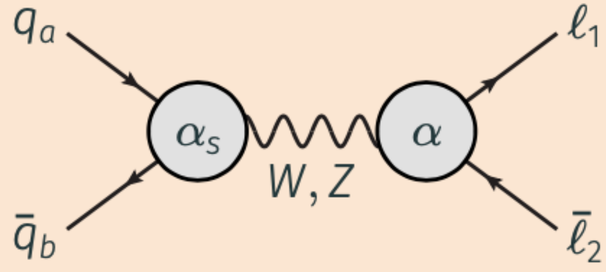
EW and QCD corrections to W and Z production

Mixed QCD-EW: Initial-Initial



- Correction of NLO×NLO type
- Already investigated *[Dittmaier, Huss, Schwinn '15] [Carloni, Calame et al. '16]*

Mixed QCD-EW: Initial-Final



- Correction of NNLO type
- Active research activity

[De Florian, Der, Fabre '18]
[Cieri, de Florian, Der, Mazzitelli '20]
[Bonciani, Buccioni, Rana, Triscari, Vicini '19]
[Bonciani, Buccioni, Rana, Vicini '20]
[Dittmaier, Schmidt, Schwarz '20]
[Heller, von Manteuffel, Schabinger, Spiesberger '20]
[Buonocore, Grazzini, Kallweit, Savoini, Tramontano '20]
[Delto, Jaquier, Melnikov, Röntsch '19]
[Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]
[Behring, Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '21]

Mixed QCDxEW corrections: the real-virtual component

1-loop correction to $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)\gamma(p_5) + e^+(p_3)\,e^-(p_4)g(p_5)$

IR subtraction proceeds as for NLO:

- soft singularities are extracted first
- Collinear singularities are extracted from the the soft-regulated term

$$
\begin{aligned}
2s \cdot d\sigma^{\mathrm{RV}} = {} & \left\langle S_g\, F^{\mathrm{EW}}_{LRV}(1,2,3,4\,|\,5_g)\right\rangle + \left\langle S_\gamma\, F^{\mathrm{QCD}}_{LRV}(1,2,3,4\,|\,5_\gamma)\right\rangle \\
& + \left\langle (I - S_g)\,(C_{g1} + C_{g2})\, F^{\mathrm{EW}}_{LRV}(1,2,3,4\,|\,5_g)\right\rangle + \sum_{k=1}^{4} \left\langle (I - S_\gamma)\, C_{\gamma k}\, F^{\mathrm{QCD}}_{LRV}(1,2,3,4\,|\,5_\gamma)\right\rangle \\
& + \left\langle \mathcal{O}_{\mathrm{nlo}}\, F^{\mathrm{EW}}_{LRV}(1,2,3,4\,|\,5_g)\right\rangle + \left\langle \mathcal{O}_{\mathrm{nlo}}\, F^{\mathrm{QCD}}_{LRV}(1,2,3,4\,|\,5_\gamma)\right\rangle
\end{aligned}
$$

Several simplifications occur with respect to the pure QCD case:

- soft limit simplify into abelian-like contributions

$$
S_g\, F^{\mathrm{EW}}_{LRV}(1,2,3,4\,|\,5_g) = 2C_F g^2_{s,b}\, \frac{\rho_{12}}{E_5^2\,\rho_{15}\,\rho_{25}}\, F^{EW}_{LV}(1,2,3,4)
$$

- One-loop collinear kernel can be obtain via abelianization > fewer terms

$$
C_{g1}\, F^{\mathrm{EW}}_{LRV}(1,2,3,4\,|\,5_g) = \frac{g^2_{s,b}}{E_5^2\,\rho_{51}} \left[(1-z)\, C_F\, P_{qq}(z)\, \frac{F^{\mathrm{EW}}_{LV}(z\cdot 1,2,3,4)}{z} + [\alpha]\, C_F\, Q_q^2\, \frac{\Gamma^3(1-\epsilon)\,\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)}\, \frac{(2E_1^2)^{-\epsilon}}{\rho_{15}^{\epsilon}}\, P^{\mathrm{loop},1}_{qq}(z)\, \frac{F_{LM}(z\cdot 1,2,3,4)}{z} \right]
$$

Mixed QCDxEW corrections: VV and RV component

2-loop correction to $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

$$\left\langle F^{\text{QCD}\times\text{EW}}_{LVV+LV^2}(1,2,3,4)\right\rangle = \frac{\alpha_s(\mu)}{2\pi}\frac{\alpha(\mu)}{2\pi}\left[I^{\text{QCD}}\cdot I^{\text{QED}} + \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\frac{1}{\epsilon}H_{QCD\times EW}\right]\left\langle F_{LM}(1,2,3,4)\right\rangle$$

$$+\frac{\alpha_s(\mu)}{2\pi}I^{\text{QCD}}\left\langle F^{\text{EW, fin}}_{LV}(1,2,3,4)\right\rangle + \frac{\alpha(\mu)}{2\pi}I^{\text{QED}}\left\langle F^{\text{QCD, fin}}_{LV}(1,2,3,4)\right\rangle + \left\langle F^{\text{QCD}\times\text{EW, fin}}_{LVV+LV^2}(1,2,3,4)\right\rangle$$

Genuine NNLO hard-triple collinear contribution

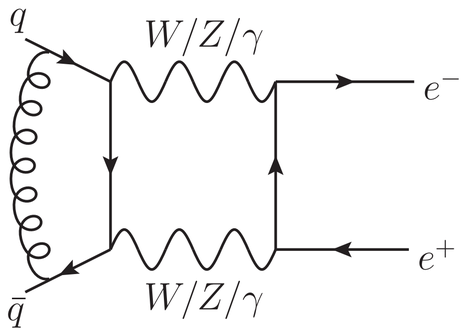
$$H_{QCD\times EW} = 2C_F Q_q^2\left(\frac{\pi^2}{2} - 6\zeta_3 - \frac{3}{8}\right)$$

Universal IR operators
[Catani '98]

Finite part one-loop EW/QCD correction

Finite part of the two-loop mixed QCDxEW correction

- Handle chiral couplings in dimensional regularisation
  - > γ_5 really a 4-dimentional object > need for a prescription in d-dimension
- Solving master integrals: non-ratitonalabile roots
- Adapt the result for a fast Monte-Carlo integration



Mixed QCDxEW corrections: VV and RV component

2-loop correction to $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

$$\left\langle F^{\mathrm{QCD\times EW}}_{LVV+LV^2}(1,2,3,4)\right\rangle = \frac{\alpha_s(\mu)}{2\pi}\frac{\alpha(\mu)}{2\pi}\left[I^{\mathrm{QCD}}\cdot I^{\mathrm{QED}} + \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\frac{1}{\epsilon}H_{QCD\times EW}\right]\left\langle F_{LM}(1,2,3,4)\right\rangle$$

$$+\frac{\alpha_s(\mu)}{2\pi}\,I^{\mathrm{QCD}}\left\langle F^{\mathrm{EW,\,fin}}_{LV}(1,2,3,4)\right\rangle + \frac{\alpha(\mu)}{2\pi}\,I^{\mathrm{QED}}\left\langle F^{\mathrm{QCD,\,fin}}_{LV}(1,2,3,4)\right\rangle + \left\langle F^{\mathrm{QCD\times EW,\,fin}}_{LVV+LV^2}(1,2,3,4)\right\rangle$$

Genuine NNLO hard-triple collinear contribution

$$H_{QCD\times EW} = 2C_F\,Q_q^2\left(\frac{\pi^2}{2} - 6\zeta_3 - \frac{3}{8}\right)$$

Universal IR operators
[Catani '98]

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Finite part of the two-loop mixed QCDxEW correction

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IR subtraction proceeds as for **NLO**:

- soft singularities are extracted first
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$$2s\cdot d\sigma^{\mathrm{RV}} = \left\langle S_g\,F^{\mathrm{EW}}_{LRV}(1,2,3,4\,|\,5_g)\right\rangle + \left\langle S_\gamma\,F^{\mathrm{QCD}}_{LRV}(1,2,3,4\,|\,5_\gamma)\right\rangle$$

$$+\left\langle (I-S_g)\,(C_{g1}+C_{g2})\,F^{\mathrm{EW}}_{LRV}(1,2,3,4\,|\,5_g)\right\rangle + \sum_{k=1}^{4}\left\langle (I-S_\gamma)\,C_{\gamma k}\,F^{\mathrm{QCD}}_{LRV}(1,2,3,4\,|\,5_\gamma)\right\rangle$$

$$+\left\langle \mathcal{O}_{\mathrm{nlo}}\,F^{\mathrm{EW}}_{LRV}(1,2,3,4\,|\,5_g)\right\rangle + \left\langle \mathcal{O}_{\mathrm{nlo}}\,F^{\mathrm{QCD}}_{LRV}(1,2,3,4\,|\,5_\gamma)\right\rangle \longrightarrow \text{Integrable in the PS, explicit poles}$$

Brief historical recap

$\sigma^{\text{nlo}}_{\text{QCD}}$

[Altarelli, Ellis, Martinelli '79]

$\sigma^{\text{nnlo}}_{\text{QCD}}$

[Hamberg, van Neerven, Matsuura '91]

[Harlander, Kilgore '02]

$\mathrm{d}\sigma^{\text{nnlo}}_{\text{QCD}}$ (fully diff.+leptonic decay)

[Anastasiou, Dixon, Melnikov, Petriello '03, '04]

[Melnikov, Petriello '06]

[Catani, Cieri, Ferrera, de Florian, Grazzini '09]

[Catani, Ferrera, Grazzini '10]

Complete EW corr. to W prod.

[Dittmaier, Krämer '02]

[Bauer, Wackerste '04]

[Zykunov '06]

[Arbuzov, Bardin, Bondarenko, Christina, Kalinovskaya, Nanava, Sadykov '06]

[Carloni, Calame, Montagna, Nicrosini, Vicini '16]

Complete EW corr. to Z prod.

[Bauer, Brein, Hollik, Schappacher, Wackeroth '02]

[Zykunov '07]

[Carloni, Calame, Montagna, Nicrosini, Vicini '07]

[Arbuzov, Bardin, Bondarenko, Christina, Kalinovskaya, Nanava, Sadykov '08]

[Dittmaier, Huber '09]

$\sigma^{\text{n3lo}}_{\text{QCD}}$

[Duhr, Dulat, Mistlberger '20]

$\mathrm{d}\sigma^{\text{n3lo}}_{\text{QCD}}$

[Camarda, Cieri, Ferrera '21]

[Chen, Gehrmann, Glover, Huss, Yang '21]

Mixed QCDxEW corrections: the double real component

Mixed QCD×EW correction to neutral-current $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

$$\left\langle (I-S_g)(I-S_\gamma)\,F_{LM}(1,2,3,4\,|\,5,6)\right\rangle = \left\langle F_{LM}^{s_r\,c_s}(1,2,3,4\,|\,5,6)\right\rangle + \left\langle F_{LM}^{s_r\,c_t}(1,2,3,4\,|\,5,6)\right\rangle + \left\langle F_{LM}^{s_r\,c_r}(1,2,3,4\,|\,5,6)\right\rangle$$

Contains **soft-regulated single collinear singularities**
> matrix elements of lower multiplicity

Contains **soft-regulated triple collinear singularities**
> matrix elements of lower multiplicity
> partition functions select 1 strongly-ordered limit each

All singularities removed through nested subtractions:
> evaluable in 4-dimensions
> only terms involving fully-resolved real-real matrix element

Analytic results for non trivial integrals from triple collinear (and for double soft) limit calculated in
[Caola, Delto, Frellesvig, Melnikov '18, Delto, Melnikov '19]

After integration subtraction terms return lower multiplicity terms multiplied by explicit poles

$\left\langle F_{LM}(1,2,3,4)\right\rangle$

Elastic LO-like

$\left\langle F_{LM}(z_1\cdot 1, z_2\cdot 2,3,4)\right\rangle$
$\left\langle F_{LM}(z\cdot 1,2,3,4)\right\rangle$
$\left\langle F_{LM}(1,z\cdot 2,3,4)\right\rangle$

Boosted LO-like

$\left\langle \mathcal{O}^g_{\rm nlo}\,F_{LM}(1,2,3,4\,|\,5_g)\right\rangle$
$\left\langle \mathcal{O}^\gamma_{\rm nlo}\,F_{LM}(1,2,3,4\,|\,5_\gamma)\right\rangle$

Elastic NLO-like

$\left\langle \mathcal{O}^g_{\rm nlo}F_{LM}(z\cdot 1,2,3,4\,|\,5_g)\right\rangle$ $\left\langle \mathcal{O}^\gamma_{\rm nlo}F_{LM}(1,z\cdot 2,3,4\,|\,5_\gamma)\right\rangle$
$\left\langle \mathcal{O}^g_{\rm nlo}F_{LM}(1,z\cdot 2,3,4\,|\,5_g)\right\rangle$ $\left\langle \mathcal{O}^\gamma_{\rm nlo}F_{LM}(z\cdot 1,2,3,4\,|\,5_\gamma)\right\rangle$

Boosted NLO-like

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Mixed QCD×EW correction to neutral-current $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

$$\left\langle (I-S_g)(I-S_\gamma)\,F_{LM}(1,2,3,4\,|\,5,6)\right\rangle = \sum_{i=1}^{4}\left\langle (I-S_g)(I-S_\gamma)\,\Omega_i^{q\bar{q}}\,F_{LM}(1,2,3,4\,|\,5,6)\right\rangle$$

$$\Omega_1^{q\bar{q}} = (1-C_{g\gamma,1})(1-C_{g1})\,\omega^{\gamma 1,g1}\,\theta_a + (1-C_{g\gamma,1})(1-C_{\gamma 1})\,\omega^{\gamma 1,g1}\,\theta_b + \dots$$

All singularities removed via nested subtractions:
> evaluable in 4-dimensions
> only terms involving fully-resolved RR matrix element

Triple-collinear counterterm:

$$\Omega_2^{q\bar{q}} = C_{g\gamma,1}(1-C_{g1})\,\omega^{\gamma 1,g1}\,\theta_A + C_{g\gamma,1}(1-C_{\gamma 1})\,\omega^{\gamma 1,g1}\,\theta_B + (1 \leftrightarrow 2)$$

Analytic results for non trivial integrals from triple collinear limit calculated in *[Caola, Delto, Frellesvig, Melnikov '18, Delto, Melnikov '19]*

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Boosted NLO-like

Nested soft-collinear subtraction: generalities

Extension of FKS subtraction to NNLO

- Fully local and fully analytic

- Transparent treatment of IR singularities

  Independent subtraction of **soft and collinear divergences** > colour coherence

  Partition by means of sector functions

- Flexibility

  Core structure depends only on the partons contributing to the process

  Modular building blocks

→ Color coherence:

- on-shell, gauge invariant QCD scattering amplitudes
- soft gluons do not resolve details of collinear splittings > only sensitivity to total color charge

→ **Soft and collinear limits treated independently** > regularise soft singularities first, then collinear

Nested soft-collinear subtraction: NLO EW

NLO EW correction to neutral-current $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

- Basic ingredients

$$\mathrm{d}\sigma^{\mathrm{nlo}} = \mathrm{d}\sigma^{\mathrm{R}} + \mathrm{d}\sigma^{\mathrm{V}} + \mathrm{d}\sigma^{\mathrm{PDF}}$$

Real contribution | Virtual correction | PDF renormalisation

- Real radiation

$$\mathrm{d}\sigma^{\mathrm{R}} = \frac{1}{2s}\int [\mathrm{d}p_\gamma]\, F_{LM}(1,2,3,4;5) \equiv \left\langle F_{LM}(1,2,3,4;5)\right\rangle$$

$$F_{LM}(1,2,3,4;5) = \mathrm{d}\,\mathrm{PS}_{3,4}\left|\mathscr{M}(1,2,3,4;5)\right|^2 \mathscr{O}_{\mathrm{kin}}(1,2,3,4;5)$$

Lorentz-invariant phase space for leptons (incl. conservation δ)

Matrix element squared

IR-safe observable

$$[\,\mathrm{d}f_i\,] = \frac{\mathrm{d}^{d-1}k_i}{(2\pi)^{d-1}\,2E_k}\,\theta(E_{\max} - E_k)$$

Unresolved phase space (partonic CoM)

Arbitrary parameter

- Soft and collinear operators

$$S_i\,A = \lim_{E_i\to 0} A\,, \qquad C_{ij}\,A = \lim_{\rho_{ij}\to 0} A\,.$$

$$\rho_{ij} = 1 - \cos\vartheta_{ij}\,, \quad \eta_{ij} = \rho_{ij}/2$$

Nested soft-collinear subtraction: NLO EW

NLO EW correction to neutral-current $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

- Complete IR subtraction

$$\langle F_{LM}(1,2,3,4\,|\,5)\rangle = \langle S_5\,F_{LM}(1,2,3,4\,|\,5)\rangle + \langle (I-S_5)\,F_{LM}(1,2,3,4\,|\,5)\rangle$$

$$= \langle S_5\,F_{LM}(1,2,3,4\,|\,5)\rangle + \sum_{i=1}^{4} \langle C_{5i}\,(I-S_5)\,F_{LM}(1,2,3,4\,|\,5)\rangle + \sum_{i=1}^{4} \langle \mathcal{O}^{(i)}_{\text{nlo}}\,\omega^{5i}\,F_{LM}(1,2,3,4\,|\,5)\rangle$$

Soft subtraction term:
complete decoupling of the unresolved photon

Hard-collinear subtraction term:
decoupling of the unresolved photon

Fully subtracted term is finite:
numerical integration in 4-dimensions

$$\mathcal{O}_{\text{nlo}} \equiv \sum_{i=1}^{4} \mathcal{O}^{(i)}_{\text{nlo}}\,\omega^{5i} \equiv \sum_{i=1}^{4} \left(I-C_{5i}\right)\left(I-S_5\right)\omega^{5i}$$

Phase space singularities are make explicit by analytically integrating the subtraction contributions

Partition functions do not affect the integration of the singular kernels!

Nested soft-collinear subtraction: NLO EW

NLO EW correction to neutral-current $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

- Final result as combination of virtual corrections and pdf renormalisation:
  - ✓ Poles cancellation proven independently on E_{max} and ω^{5i} functions
  - ✓ Simple structure arise, compact expressions
  - ✓ Fully differential

$$2s \cdot \mathrm{d}\sigma^{\mathrm{nlo}} = \langle \mathcal{O}_{\mathrm{nlo}}\, F_{LM}(1,2,3,4\,|\,5)\rangle + \langle F_{LV}^{\mathrm{fin}}(1,2,3,4)\rangle$$

→ Finite remainder of virtual corrections

$$+\frac{\alpha(\mu)}{2\pi}\left\{ Q_q^2 \int_0^1 dz \left(P_{qq}^{(0)}(z)\,\log\left(\frac{s}{\mu^2}\right) + P_{qq}^{'}(z)\right)\left[\left\langle \frac{F_{LM}(z\cdot 1,2,3,4)}{z}\right\rangle + \left\langle \frac{F_{LM}(1,z\cdot 2,3,4)}{z}\right\rangle\right]\right.$$

→ Boosted LO-like terms, convoluted with splitting functions

Elastic LO-like terms

$$+\left[Q_q^2 \frac{2\pi^2}{3} + Q_e^2\left(13 - \frac{2\pi^2}{3}\right)\right.$$

$$+2\,Q_q\,Q_e\left[\left(\log\left(\frac{E_{max}^2}{E_4\,E_1}\right)+\frac{3}{2}\right)\log\left(\frac{\eta_{13}\,\eta_{24}}{\eta_{23}\,\eta_{14}}\right)\right.$$

$$\left.\left.\left.+\mathrm{Li}_2(1-\eta_{13}) + \mathrm{Li}_2(1-\eta_{24}) - \mathrm{Li}_2(1-\eta_{14}) - \mathrm{Li}_2(1-\eta_{23})\right]\left\langle F_{LM}(1,2,3,4)\right\rangle\right\}$$

$$P_{qq}^{(0)}(z) = 2\,D_0(z) - (1+z) + \frac{3}{2}\,\delta(1-z)\,,$$

$$P_{qq}^{'}(z) = 4\,D_1(z) + (1-z) - 2\,(1+z)\,\log(1-z)\,.$$

What about NNLO?

Detailed informations of NNLO kernels also available ~ 20 years ago (N3LO kernels are now partially available)

Extraction of real-emission singularities was the main bottleneck for NNLO predictions

Example: dijet two-loop amplitudes ~ 20 years ago *[Glover, Oleari, Tejeda-Yeomans '01]*,

dijet production at NNLO ~ 4 ago *[Currie, De Ridder, Gehrmann, Glover, Huss, Pires '17]*

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Why is NNLO so difficult?

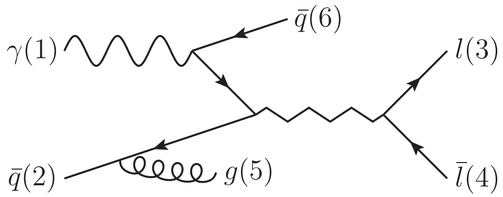
1. Clear understanding of which singular configurations do actually contribute
2. Get to the point where the problem is well defined
3. Solve the phase space integrals of the relevant limits

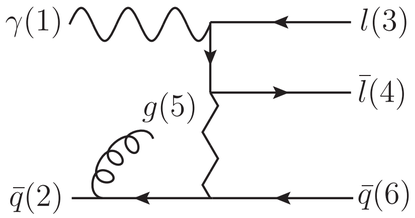
The RR component: generalities

The differences between the on-shell and the off-shell calculation

In the **off-shell case** the **simplifications** mentioned for the on-shell computation are valid, but they are **not enough**:

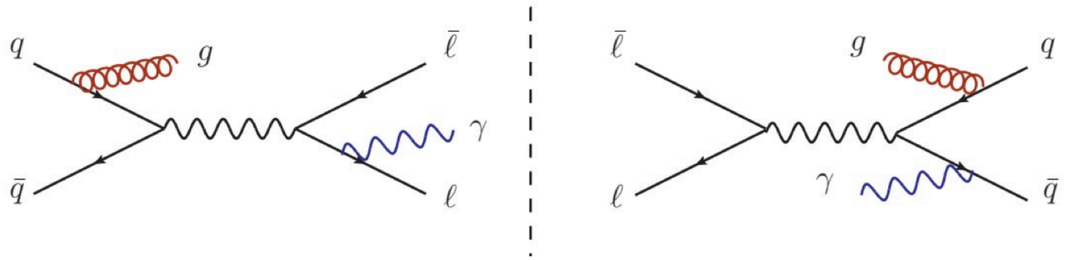
- new diagrams from initial state photons





Extend the partition to isolate singularities from configurations with collinear photons and leptons

- interference between “initial-initial” and “initial-final” corrections



- Only singular for soft photons
- Soft singularities can be treated as iterated NLO-like singularities

The RR component: generalities

From NNLO QCD to mixed EWxQCD: the on-shell case as intermediate step

> **Final state radiation** introduces additional soft and collinear singularities leading to new kinematic structure

> **The phase space partition** has to be **extended**

BUT

Absence of gluon-gluon interactions

> no true double-soft singularities

> no spin correlations

> pure abelian contributions

> **abelianization** of known results

- Adapting the NNLO QCD computation is practically all you need to obtain the mixed QCDxEW correction in the resonance region.

[de Florian, Der, Fabre '18][Delto, Jaquier, Melnikov, Röntsch '19] [Buccioni, Caola, Delto, Jaquier, Melnikov, Röntsch '20]

- In the **high energy region**, **full control of the subtraction procedure** is crucial to be able to adapt the results from NNLO QCD. A straightforward abelianization is not sufficient.

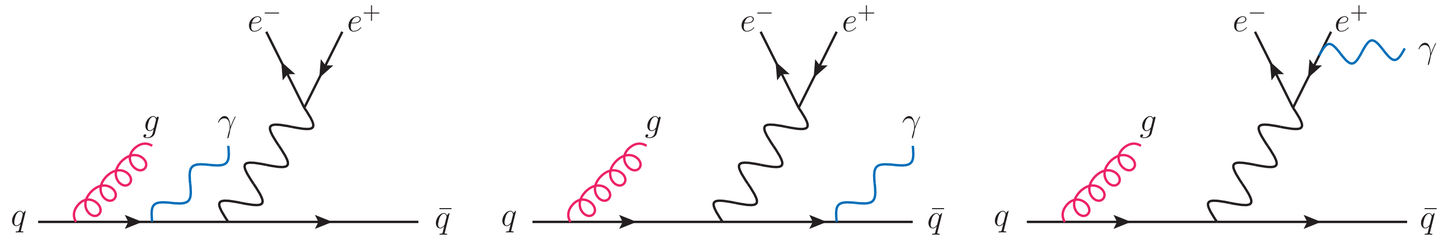
Mixed QCDxEW corrections: the double real component

Mixed QCD×EW correction to neutral-current $q(p_1)\,\bar{q}(p_2) \to e^+(p_3)\,e^-(p_4)$

- Several singular limits:

$$S_i\,A = \lim_{E_i \to 0} A\,, \qquad C_{ij}\,A = \lim_{\rho_{ij} \to 0} A\,, \qquad C_{ijk}\,A = \lim_{\rho_{ij},\rho_{jk},\rho_{ik} \to 0} A$$

- Difficulty: numerous collinear singularities to regulate



- Simplification: trivial soft singularities structure: no strongly-ordered soft limits,
  double soft limit completely factorised into abelian-like terms

$$\left\langle F_{LM}(1,2,3,4\,|\,5,6)\right\rangle = \left\langle S_g\,S_\gamma\,F_{LM}(\ldots|\,5,6)\right\rangle + \left\langle \left[(I-S_g)\,S_\gamma + (I-S_\gamma)\,S_g\right]F_{LM}(\ldots|\,5,6)\right\rangle + \left\langle (I-S_g)\,(I-S_\gamma)\,F_{LM}(\ldots|\,5,6)\right\rangle$$

Double soft subtraction term — Single soft subtraction term — Soft-finite term

Second and third term contain (potentially **overlapping**) **collinear singularities**

> as for NLO we introduce **phase space partitioning**