DEPARTAMENTO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

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Instituto de Matemática Interdisciplinar

SEMINARIO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

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An Operator-Valued Version of V.P. Potapov's Matrix-valued Factorization Result

In joint work with In Sung Hwang (Sungkyungkwan University, Republic of Korea) and Woo Young Lee (Seoul National University, Republic of Korea), we have recently considered several questions emerging from the Beurling-Lax-Halmos Theorem, which characterizes the shift-invariant subspaces of vector-valued Hardy spaces. That is, a backward shift-invariant subspace is a model space $H(\Delta) \equiv H^2_E \ominus \Delta H^2_E$, for some operator-valued inner function Δ. Of special interest for us is the notion of "Beurling degree" for an inner function, which we recently introduced. We first establish a connection between the spectral multiplicity of the model operator (the truncated backward shift) and the Beurling degree of the corresponding characteristic function. Next, we consider the case of multiplicity-free: more precisely, for which characteristic function Δ of the model operator T does it follow that T is multiplicity-free, i.e., T has multiplicity 1? We prove that if Δ has a meromorphic pseudo-continuation of bounded type in the complement of the closed unit disk and the adjoint of the flip of Δ is an outer function, then T is multiplicity-free. Finally, we focus on rational symbols and study V.P. Potapov's celebrated theorem, that an $n \times n$ matrix function is rational and inner if and only if it can be represented as a finite Blaschke-Potapov product. We extend this result to the operator-valued case. As a corollary, we prove that when $\Delta \in H^\infty(\mathbb{T}, B(E))$ is a left inner divisor of $z \cdot I_E$, then Δ is a Blaschke-Potapov factor. This requires a new notion of operator-valued rational function in the infinite multiplicity case; that is, $\Phi \in H^\infty(\mathbb{T}, B(D, E))$ is said to be rational if $\theta H^2(\mathbb{T}, E) \subseteq \ker H_{\Phi^*}$, where θ is a finite Blaschke product and H_{Φ^*} denotes the Hankel operator with symbol Φ^*.

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