DEPARTAMENTO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

Facultad de Ciencias MATEMÁTICAS

Instituto de Matemática Interdisciplinar

SEMINARIO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

Jan Cholewa
Universidty of Silesia in Katowice

On homogeneous operators and semigroups in homogeneous spaces

Homogeneous spaces are function spaces in R N which include, among others, Lebesgue and Morrey ones. Homogeneous operators in such spaces form a class that includes, for example, the Laplacian and associated fractional powers. Homogeneous semigroups, in turn, are the ones generated by homogeneous operators. Analysis of homogeneous operators and semigroups in homogeneous spaces leads to more precise results than in the general case. As we show in this talk spectrum of a homogeneous operator is made of half-lines originating at zero in the complex plane. Thus both spectrum and resolvent set are cones. Also, given any ray in the resolvent set a sharp estimate of the resolvent operator along the ray is derived. Concerning homogeneous semigroups we give several results characterising their generators and indicating that with homogeneity the conditions in Hille-Yosida and LumerPhillips theorems for generation of semigroups, or sectoriality condition for generation of analytic semigroups, are easier to check. We moreover show that homogeneous semigroups have to satisfy sharp estimates, which are determined by the homogeneity alone. Perturbations of homogeneous operators and semigroups are also discussed. We give, for example, a result which proves from homogeneity the relative boundedness. In particular, we show that any lower degree perturbation of a sectorial operator is still sectorial. Regarding the perturbed semigroup we then obtain smoothing estimates. For perturbation of the same degree involving selfadjoint operators in homogeneous Hilbert spaces we exhibit, for example, a threshold phenomena for the spectrum. Finally we apply the results to some linear diffusion problems, including fractional diffusion and Hardy type potentials considered in the joint work [1].

[1] J.W. Cholewa and A. Rodriguez-Bernal, On some PDEs involving homogeneous operators. Spectral analysis, semigroups and Hardy inequalities, J. Differential Equations 315 (2022), 1–56.

Organizado por el Departamento de Análisis Matemático y Matemática Aplicada y el Instituto de Matemática Interdisciplinar

Fecha: Jueves 17 de noviembre de 2022
Hora: 13:00 h.
Lugar: Sala 209 (Alberto Dou)
Facultad de CC. Matemáticas