DEPARTAMENTO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

Facultad de Ciencias MATEMÁTICAS

Instituto de Matemática Interdisciplinar

SEMINARIO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

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An introduction to the big and little lip functions

Given a continuous function $f: R \to R$ with $M_f(x,r) = \sup_{|x-y| \le r} |f(x) - f(y)|$, the so-called``Big Lip'' and ``Little Lip'' functions are defined as follows: $Lip\ f(x) = lim\ sup_{r \to 0^+} \frac{M_f(x,r)}{r}$ $lip f(x) = lim\ inf_{r \to 0^+} \frac{M_f(x,r)}{r}$. The behavior of these functions is intimately related to the differentiability of f. The Rademacher-Stepanov Theorem tells us that f is differentiable almost everywhere on the set $L_f = \{x: Lip\ f(x) < \infty\}$. On the other hand, as Balogh and Csörnyei showed, this theorem no longer holds if we replace L_f with $l_f = \{x: lip\ f(x) < \infty\}$. They give an example where $lip\ f(x) = 0$ a.e. but $Lip\ f(x) = \infty$ for all $x \in R$ so L_f is the empty set and f is nowhere differentiable. However, they also show that if $l_f = R$ then f is differentiable on a set of positive measure and thus L_f has positive measure as well. In this talk, I explore the relationship between $lip\ f$ and $Lip\ f$ as well as between L_f and l_f . I will also pose a number of open problems.

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