DEPARTAMENTO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

Facultad de Ciencias MATEMÁTICAS

Instituto de Matemática Interdisciplinar

SEMINARIO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

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On the gradient problem of C. E. Weil

It is well-known that if $f : (a, b) \to \mathbb{R}$ is everywhere differentiable on the open interval (a, b) then its derivative, f' is Darboux and Baire one. It is less widely known that derivatives have one more property, the so called Denjoy–Clarkson property. This property states that if (α, β) is any open interval then its inverse image by the derivative, that is, $(f')^{-1}(\alpha, \beta)$ is either empty, or of positive one dimensional Lebesgue measure. The gradient problem of C. E. Weil is about the multidimensional version of the Denjoy–Clarkson property.

Assume $n \geq 2$, $G \subset \mathbb{R}^n$ is an open set and $f : G \to \mathbb{R}$ is an everywhere differentiable function. Then $\nabla f = (\partial_1 f, \ldots, \partial_n f)$ maps G into $\mathbb{R}^n$. Assume $\Omega \subset \mathbb{R}^n$ is open. Is it true that $(\nabla f)^{-1}(\Omega) = \{p \in G : \nabla f(p) \in \Omega\}$ is either empty, or of positive n-dimensional Lebesgue measure? During the talk I will discuss ideas (coming from geometric measure theory and dynamical systems) behind my solution, some later progress in the area done by other mathematicians, and some related results on the way to the solution.

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