



DEPARTAMENTO DE
ANÁLISIS MATEMÁTICO Y
MATEMÁTICA APLICADA



SEMINARIO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

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On the stability problem in linear dynamics

Let us recall the following classical result from the 1960s in the area of linear dynamics:

Hartman's theorem. *Every invertible hyperbolic operator on a Banach space is structurally stable.*

The name *Hartman's theorem* for the above result was coined by Charles Pugh. Actually, it was originally established by Philip Hartman in 1960 for operators on finite-dimensional euclidean spaces. The extension to arbitrary Banach spaces was independently obtained by Jacob Palis and Charles Pugh in the late 1960s, motivated by an argument due to Jürgen Moser. The above theorem is a major tool for proving the celebrated *Grobman-Hartman theorem* on the local linearization of diffeomorphisms around hyperbolic fixed points.

The basic question related to *Hartman's theorem* is whether or not its converse is always true. In other words, we have the following natural question: *If an invertible operator T on a Banach space X is structurally stable, is it necessarily hyperbolic?*

It was soon realized that the answer is positive in the finite-dimensional setting (Joel Robbin, 1972), but the full question remained open for more than 50 years. This problem was finally settled in a 2021 joint paper of the speaker with Ali Messaoudi.

In our talk we will give an overview of the solution to this problem and its relationship with the notion of *generalized hyperbolicity* and the *generalized Grobman-Hartman theorem*. We will also present a recent result due to the speaker asserting that *every invertible generalized hyperbolic operator on a Banach space is time-dependent stable*.

The following basic open problem will be proposed:

The Stability Problem in Linear Dynamics: *Characterize the notion of structural stability for invertible operators on Banach (or Hilbert) spaces.*

References

- [1] N. C. Bernardes Jr. and A. Messaoudi, *A generalized Grobman-Hartman theorem*, Proc. Amer. Math. Soc. **148** (2020), 4351-4360.
- [2] N. C. Bernardes Jr. and A. Messaoudi, *Shadowing and structural stability for operators*, Ergodic Theory Dynam. Systems **41** (2021), 961-980.
- [3] N. C. Bernardes Jr., *Time-dependent stable operators*, to appear.
- [4] J. M. Franks, *Time dependent stable diffeomorphisms*, Invent. Math. **24** (1974), 163-172.

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Facultad de CC. Matemáticas, UCM