



DEPARTAMENTO DE
ANÁLISIS MATEMÁTICO Y
MATEMÁTICA APLICADA



SEMINARIO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

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The double commutant property for composition operators on Hardy space

Abstract

The commutant of a family \mathcal{F} of operators on a Hilbert space is defined as the family \mathcal{F}' of all the operators that commute with every element of \mathcal{F} . It turns out that \mathcal{F}' is a unital algebra, closed in the weak operator topology. The double commutant of the family \mathcal{F} is defined as $\mathcal{F}'' = (\mathcal{F}')'$. It follows immediately from this definition that $\mathcal{F} \subseteq \mathcal{F}''$.

John von Neumann proved in 1930 that the double commutant of a unital, selfadjoint algebra \mathcal{A} of operators on a Hilbert space coincides with the closure of \mathcal{A} in the weak operator topology.

A non-selfadjoint algebra \mathcal{A} of operators on a Hilbert space is said to enjoy the double commutant property provided that \mathcal{A}'' coincides with the closure of \mathcal{A} in the weak operator topology. We are interested in the double commutant property for singly generated algebras, that is, operator algebras of the form $\mathcal{A} = \{p(A) : p \text{ is a polynomial}\}$, where A is a non-selfadjoint operator.

We consider composition operators C_φ defined on the Hardy space $H^2(\mathbb{D})$ by the formula $C_\varphi f = f \circ \varphi$, $f \in H^2(\mathbb{D})$, where the symbol φ is an analytic selfmap of the open unit disc \mathbb{D} .

A typical source of symbols is the family of linear fractional transformations φ of the Riemann sphere that satisfy $\varphi(\mathbb{D}) \subseteq \mathbb{D}$. Such symbols may have either one or two fixed points. There is a classification of the composition operators C_φ according to the fixed point configuration of the symbol φ .

Those with only one fixed point (that must lie on $\partial\mathbb{D}$) are called parabolic. It turns out that a composition operator corresponding to a parabolic symbol φ is similar to a multiplication operator on the Sobolev algebra $W^{1,2}[0, +\infty)$. This representation allows us to prove that the algebra generated by C_φ has the double commutant property if and only if φ fails to be automorphic.

Joint work with Fernando León-Saavedra (UCA), John S. Petrovic (WMU) and Luis Rodríguez-Piazza (US).

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