



DEPARTAMENTO DE  
ANÁLISIS MATEMÁTICO Y  
MATEMÁTICA APLICADA



# SEMINARIO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

**Miguel Lacruz**  
**Universidad de Sevilla**

## Composition operators and Sobolev algebras

### Abstract

Our interest in Sobolev algebras stems from our work on the commutant of a composition operator  $C_\varphi$  induced on the Hardy space  $H^\infty(\mathbb{D})$  by an analytic self-map  $\varphi$  of the open unit disc. We focus on linear fractional transformations  $\varphi$  that take the unit disc into itself. They can be classified, according to their fixed point configuration, as elliptic, hyperbolic, loxodromic or parabolic.

As it turns out, all parabolic composition operators  $C_\varphi$  share their family of eigenfunctions, and this allows us to construct a Banach space isomorphism between  $H^2(\mathbb{D})$  and the Sobolev algebra  $W^{1,2}[0, +\infty)$ , in such a way that  $C_\varphi$  is similar to a multiplication operator by a cyclic function on  $W^{1,2}[0, +\infty)$ .

The aim of this talk is to present a complete characterization of those linear fractional composition operators  $C_\varphi$  that have a minimal commutant, or enjoy von Neumann's double commutant property. We shall pay very close attention to the parabolic case, explaining how the immersion lemma turns  $W^{1,2}[0, +\infty)$  into a Banach algebra under pointwise multiplication, describing the periodic Sobolev multipliers, and leaving a few open problems in this line of research.

*Joint work with Fernando León-Saavedra (UCA), John S. Petrovic (WMU) and Luis Rodríguez-Piazza (US).*

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**Date: Tuesday, may 28, 2024, 13:00h**  
**Place: Seminario Alberto Dou (Room 209)**  
**Facultad de CC. Matemáticas, UCM**