DEPARTAMENTO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

Facultad de Ciencias MATEMÁTICAS

Instituto de Matemática Interdisciplinar

SEMINARIO DE ANÁLISIS MATEMÁTICO Y MATEMÁTICA APLICADA

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Self-similar sets: measurement, irregularity and complexity. A computational approach.

Self-similar sets, such as the Sierpinski gasket, are nowadays the playing field of many mathematical developments. In contrast to this fact, some elementary questions about them remain unknown: What are the Hausdorff and packing measure of the Sierpinski gasket? Is there a method for numerical calculation of these measures in self-similar spaces? How are the tangent measures to a self-similar set? The n-dimensional unit cube is an example of self-similar set generated by a special system of similarities. Is the high regularity of Euclidean spaces obtained in this way completely lost if the similarity system changes? Or should some kind of regularity be preserved for any self-similar set? In this talk we discuss the above topics through the study of the geometry of spherical neighborhoods of well separated self-similar sets in Euclidean spaces. We demonstrate that the packing and the centred Hausdorff measures can be used as a dual pair that serve both, computational purposes and also to understand the local structure of irregular sets, by providing orderly information on the pointwise asymptotic behavior of their spherical densities. We calculate, with error bounds, the centred-Hausdorff and the packing measures of the Sierpinski gasket and of the penta-Sierpinski gasket, and we give the complete spectra of asymptotic spherical densities of these fractals. As an example of the theoretical applications of these computations, we propose a unique index that captures the irregularity of self-similar sets by the oscillation range of the spherical density at almost every point. We also discuss the complexity of self-similar sets in terms of the vastness of their spherical neighborhood sets, a method potentially applicable to general metric spaces. This is joint work with M. Llorente and M. E. Mera.

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