

Aislantes topológicos de bandas planas y su uso en información cuántica

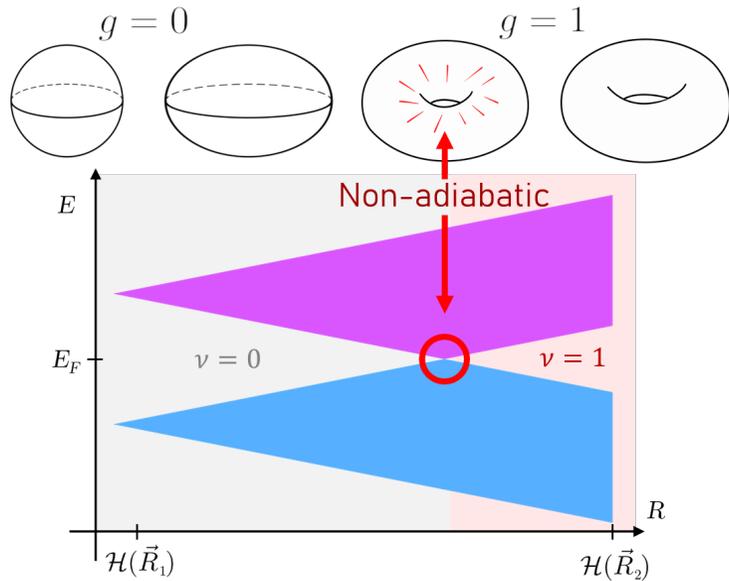
Juan Zurita, Charles Creffield and Gloria Platero



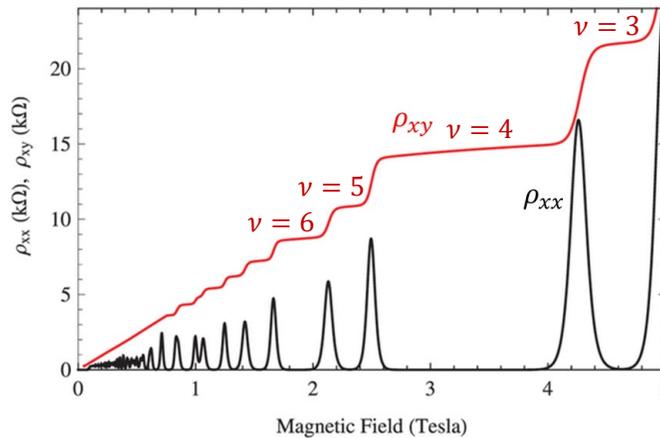
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- I. Introducción
- II. La escalera de Creutz
- III. Modelos relacionados
- IV. Aplicaciones en información cuántica

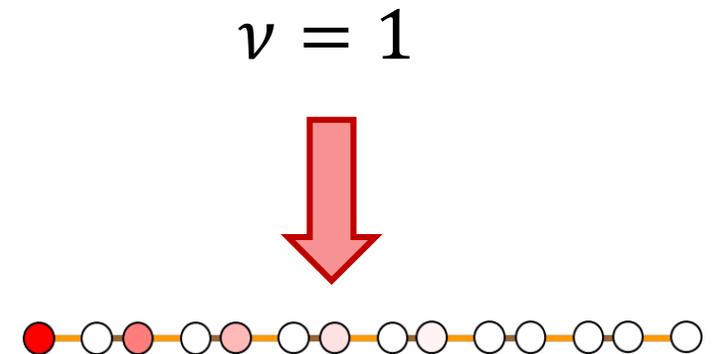
Aislantes topológicos



Fases caracterizadas por invariantes topológicos



Propiedades cuantizadas



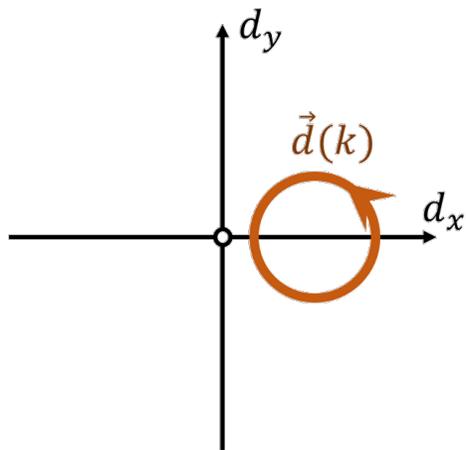
Correspondencia entre el bulk y la frontera

Invariantes topológicos (1D)

Como índice de $\vec{d}(k)$ alrededor de 0:

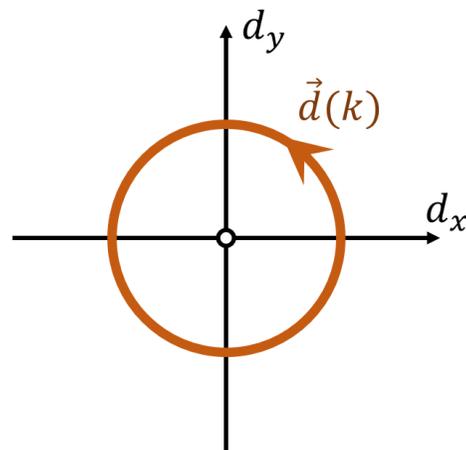
$$\mathcal{H}(k) = d_x(k)\sigma_x + d_y(k)\sigma_y = \vec{d}(k) \cdot \vec{\sigma}$$

$\nu = 0$



Trivial

$\nu = 1$



Topológico

Como integral de una cantidad local:

$$\nu \cdot \pi = \mathcal{Z} \quad \text{mód } 2\pi \quad \text{Fase de Zak}$$

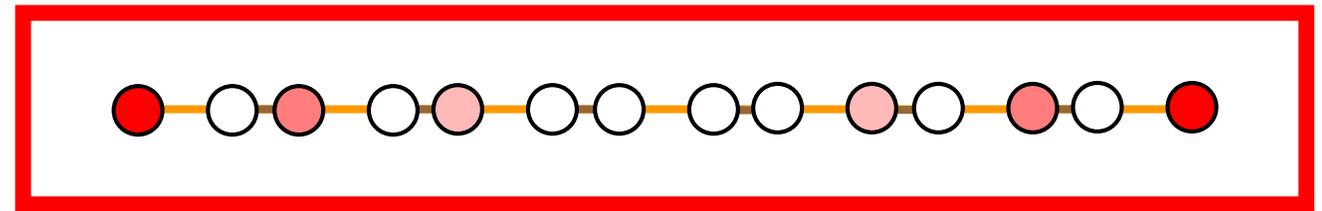
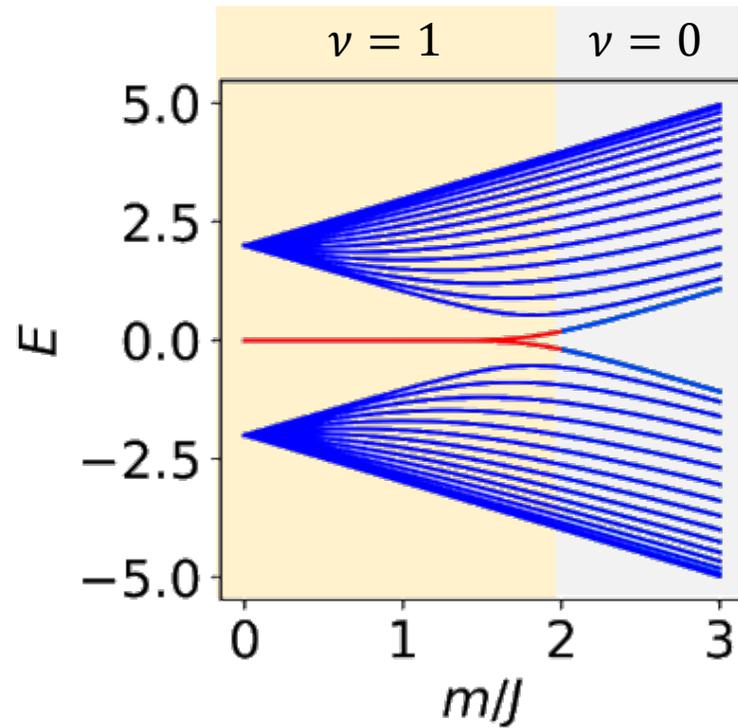
$$\mathcal{Z} = \oint_{\text{ZB}} \mathcal{A} dk$$

Conexión de Berry:

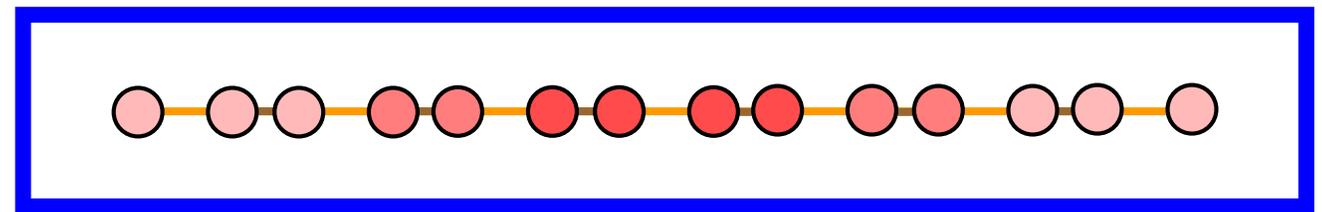
$$\mathcal{A} = i \langle u(k) | \partial_k | u(k) \rangle$$

Estados de borde topológicos

Condiciones de contorno abiertas



Estados de borde

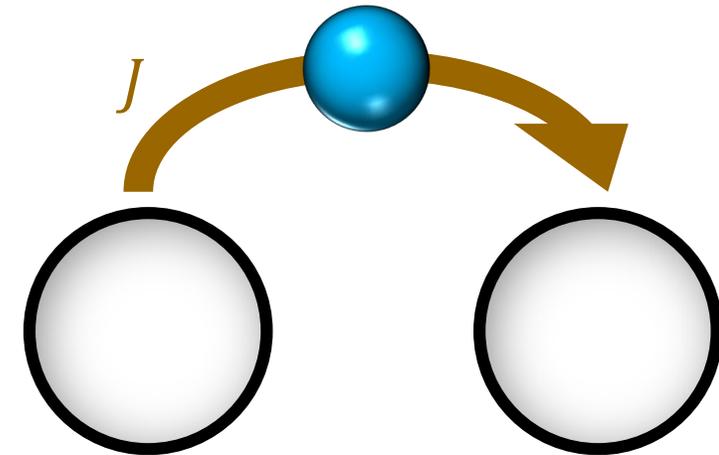
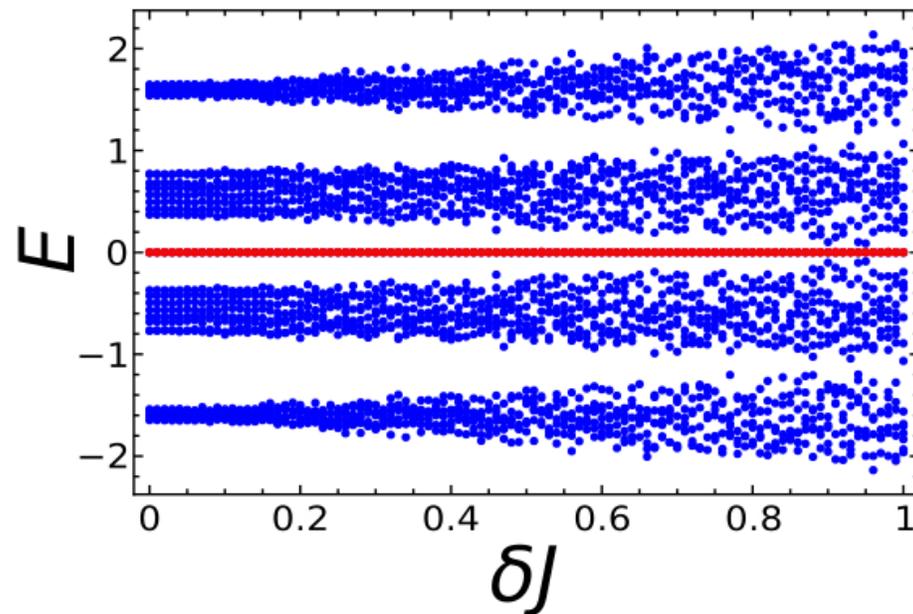


Estados del bulk

Resistencia al desorden

Condiciones de contorno abiertas

Estados de borde protegidos



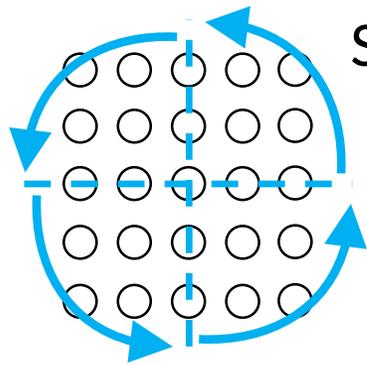
$$J = J_{\text{clean}} + R\delta J$$

Amplitud de salto

$$-1/2 < R < 1/2$$

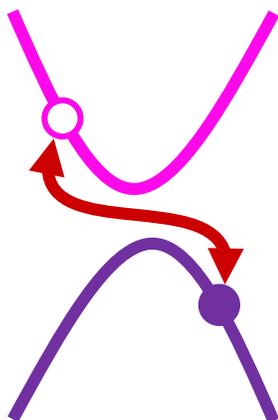
Variable aleatoria

Simetrías discretas



Simetrías cristalinas
(unitarias)

$$[\mathcal{H}, U] = 0$$



Conjugación
partícula-hueco
(antiunitaria)

$$\mathcal{C} = \vartheta K$$

$$\{\mathcal{H}, \mathcal{C}\} = 0$$

Inversión temporal
(antiunitaria)

$$\mathcal{T} = \tau K$$

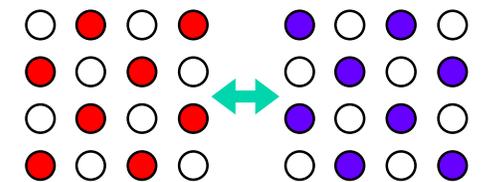
$$[\mathcal{H}, \mathcal{T}] = 0$$



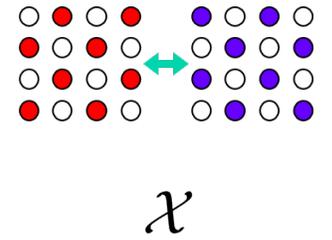
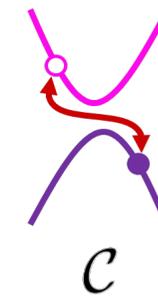
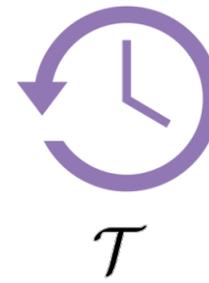
Simetría quiral
o de subred
(unitaria, hermítica)

$$\mathcal{X} = \mathcal{C}\mathcal{T}$$

$$\{\mathcal{H}, \mathcal{X}\} = 0$$

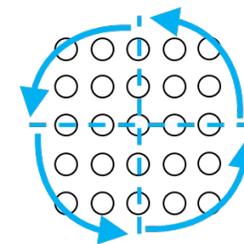


Clases de simetría



Clase	Simetrías			Dimensión espacial d								
	\mathcal{T}	\mathcal{C}	\mathcal{X}	1	2	3	4	5	6	7	8	...
A	0	0	0	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	...
AIII	0	0	1	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	\mathbb{Z}	0	...
AI	1	0	0	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	...
BDI	1	1	1	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	...
D	0	1	0	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0	\mathbb{Z}_2	...
DIII	-1	1	1	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	0	...
AII	-1	0	0	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	\mathbb{Z}	...
CII	-1	-1	1	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	0	...
C	0	-1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	0	...
CI	1	-1	1	0	0	\mathbb{Z}	0	\mathbb{Z}_2	\mathbb{Z}_2	\mathbb{Z}	0	...

Aislantes topológicos cristalinos

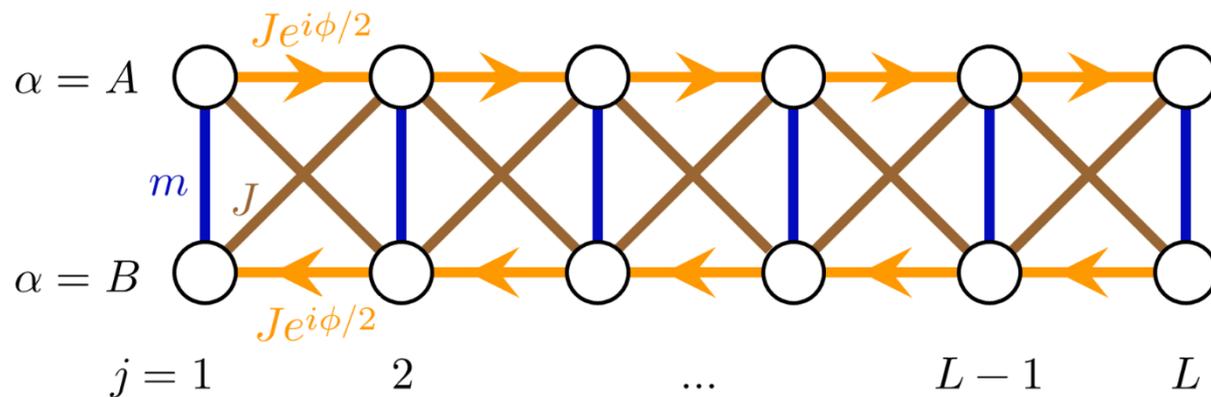


Simetrías cristalinas
(unitarias)

$$[\mathcal{H}, U] = 0$$

Schön.	C_1	C_4	S_4	C_{4h}	D_4	C_{4v}	D_{2d}	D_{4h}
HM	1	4	$\bar{4}$	4/m	422	4mm	$\bar{4}2m$	4/mmm
	C_{q+1}	C_{q+1}^4	C_q^2	C_q^4	C_{q+1}^5	$C_q^2 \times C_{q+1}$	$C_q^2 \times C_{q+1}$	C_q^5
A	0	0	\mathbb{Z}^2	\mathbb{Z}^4	0	\mathbb{Z}^2	\mathbb{Z}^2	\mathbb{Z}^5
AIII	\mathbb{Z}	\mathbb{Z}^4	0	0	\mathbb{Z}^5	\mathbb{Z}	\mathbb{Z}	0
	\mathcal{R}_{q-3}	$\mathcal{R}_{q-3}^2 \times C_{q+1}$	$\mathcal{R}_{q-4} \times \mathcal{R}_{q-2}$	$\mathcal{R}_{q-4}^2 \times C_q$	\mathcal{R}_{q-3}^5	$\mathcal{R}_{q-4}^2 \times \mathcal{R}_{q-5}$	$\mathcal{R}_{q-4}^2 \times \mathcal{R}_{q-3}$	\mathcal{R}_{q-4}^5
AI	0	0	\mathbb{Z}	\mathbb{Z}^3	0	\mathbb{Z}^2	\mathbb{Z}^2	\mathbb{Z}^5
BDI	0	\mathbb{Z}	0	0	0	\mathbb{Z}	0	0
D	0	0	\mathbb{Z}	\mathbb{Z}	0	0	0	0
DIII	\mathbb{Z}	\mathbb{Z}^3	\mathbb{Z}_2	0	\mathbb{Z}^5	0	\mathbb{Z}	0
AII	\mathbb{Z}_2	\mathbb{Z}_2^2	$\mathbb{Z} \times \mathbb{Z}_2$	\mathbb{Z}^3	\mathbb{Z}_2^5	\mathbb{Z}^2	$\mathbb{Z}^2 \times \mathbb{Z}_2$	\mathbb{Z}^5
CII	\mathbb{Z}_2	$\mathbb{Z}_2^2 \times \mathbb{Z}$	\mathbb{Z}_2	\mathbb{Z}_2^2	\mathbb{Z}_2^5	$\mathbb{Z}_2^2 \times \mathbb{Z}$	\mathbb{Z}_2^3	\mathbb{Z}_2^5
C	0	0	$\mathbb{Z}_2 \times \mathbb{Z}$	$\mathbb{Z}_2^2 \times \mathbb{Z}$	0	\mathbb{Z}_2^3	\mathbb{Z}_2^2	\mathbb{Z}_2^5
CI	\mathbb{Z}	\mathbb{Z}^3	0	0	\mathbb{Z}^5	\mathbb{Z}_2	\mathbb{Z}	0

La escalera de Creutz



$$\mathcal{H}_C = - \sum_{R=1,A}^{L-1,B} \left\{ J_\rho c_{R+1}^\dagger c_R + J c_{R+1}^\dagger c_{\bar{R}} + h.c. \right\} - m \sum_{R=1,A}^{L,B} c_R^\dagger c_{\bar{R}}$$

Flujo magnético

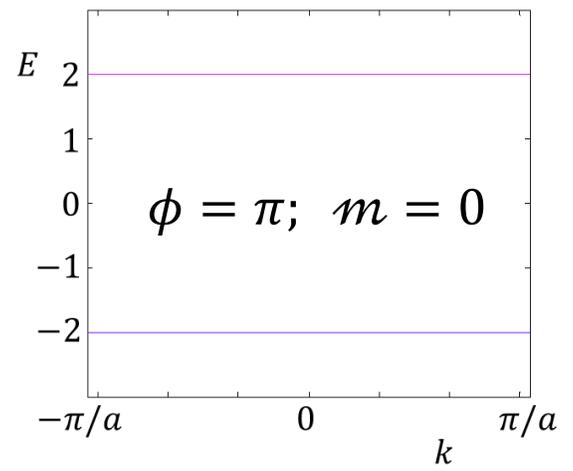
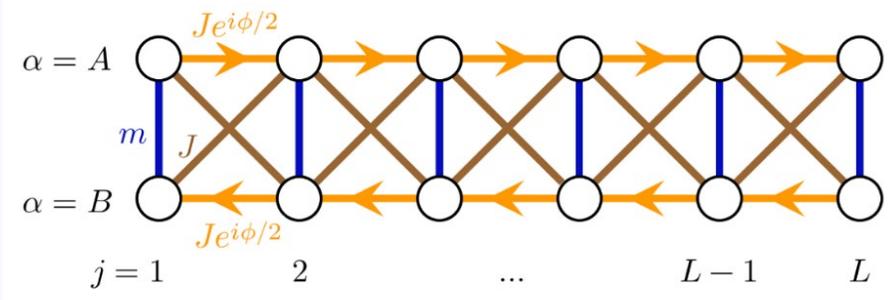
$$\phi = \frac{q}{\hbar} \Phi \equiv 2\pi \frac{\Phi}{\Phi_0}$$

$$J_\rho = J e^{is_\rho \phi/2}$$

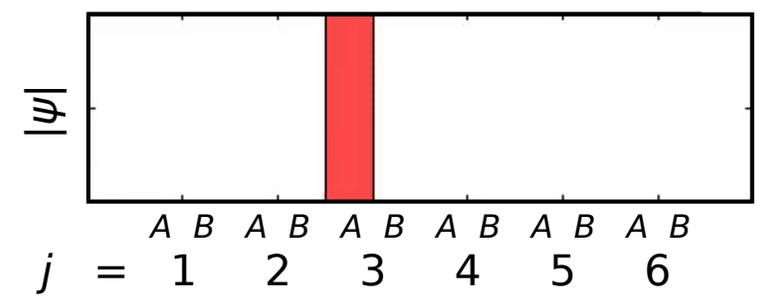
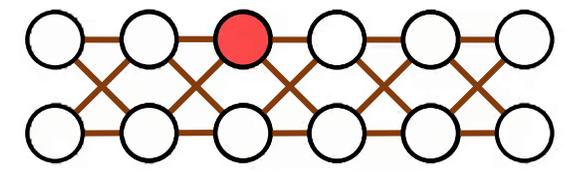
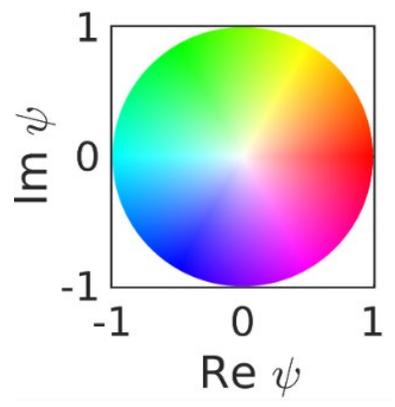
$$s_{A/B} = \pm 1$$

II. La escalera de Creutz

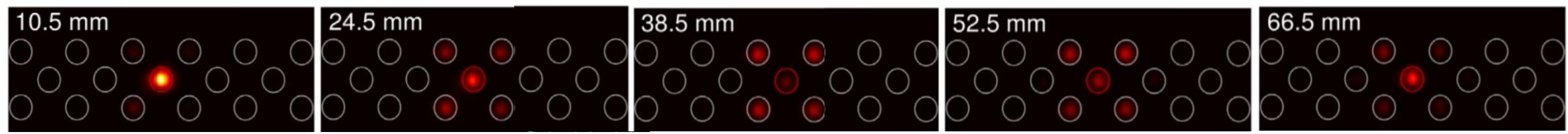
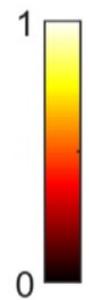
Bandas planas



Caging de Aharonov-Bohm en la escalera de Creutz



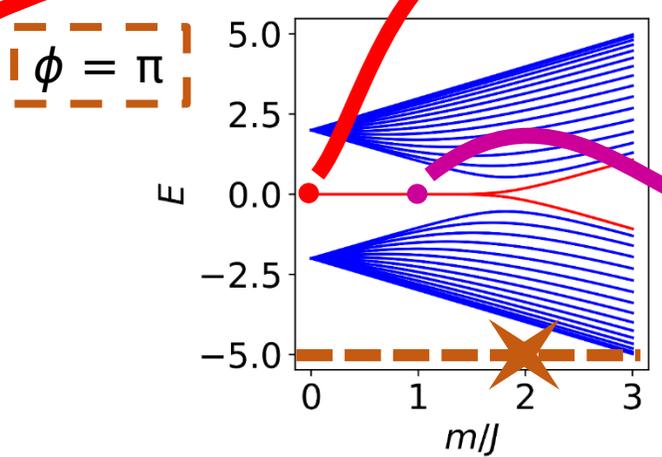
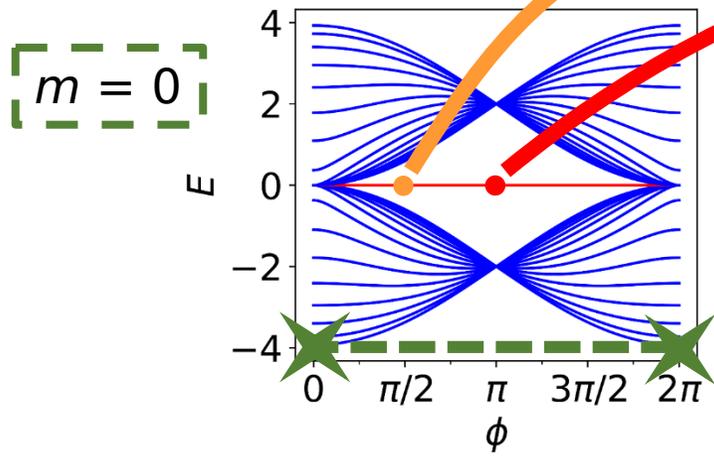
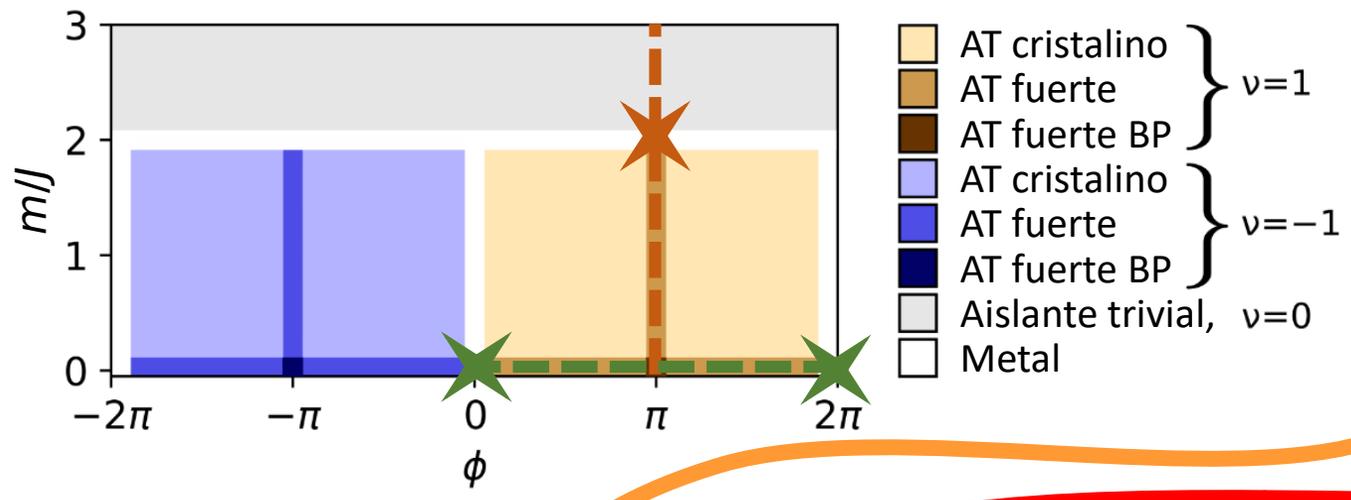
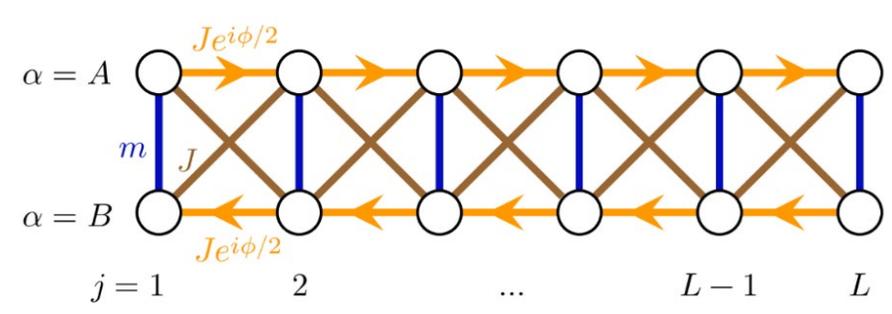
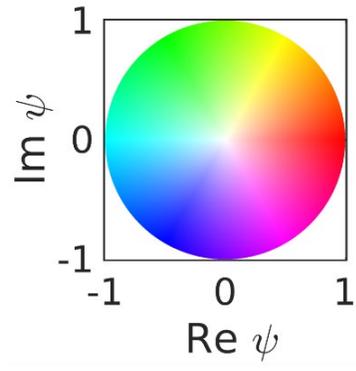
Observación del caging de AB en una red fotónica (cadena de rombos):



[4] S. Mukherjee et al. Physical Review Letters (2018)

II. La escalera de Creutz

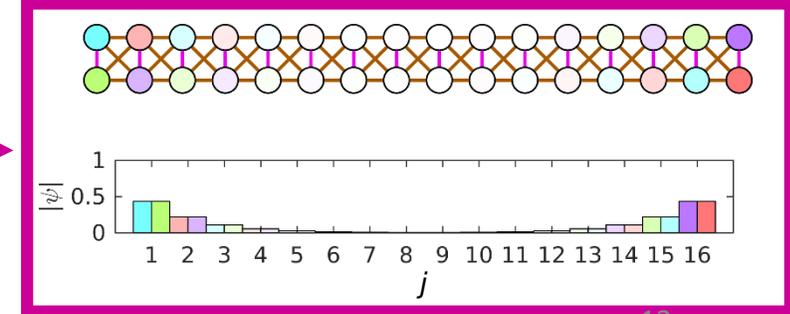
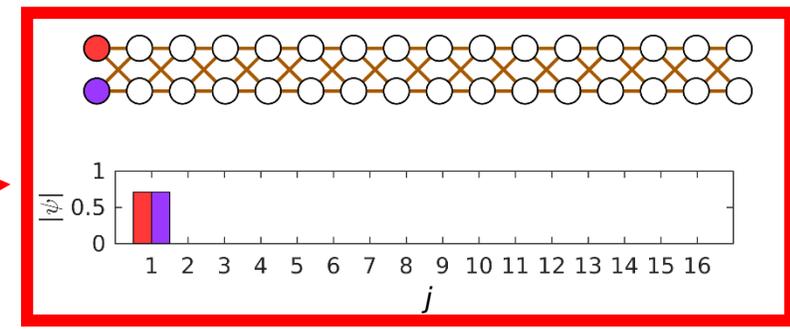
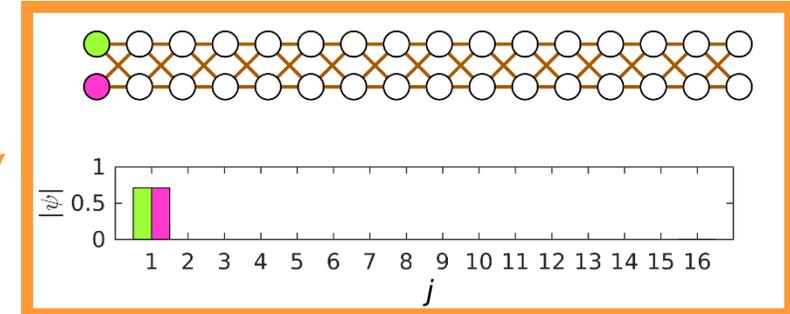
Fases topológicas



$\phi = \pi/2$
 $m = 0$

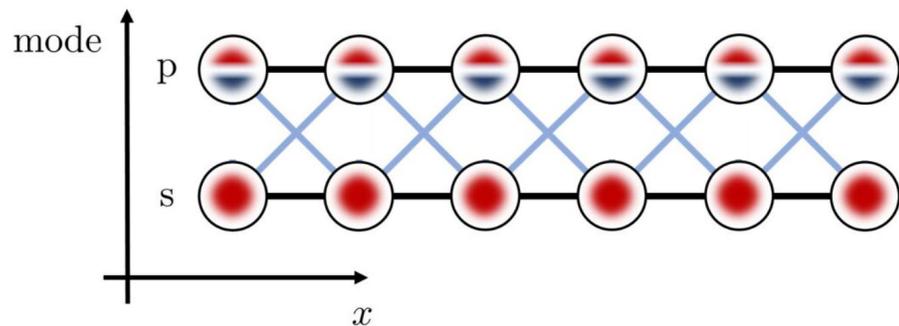
$\phi = \pi$
 $m = 0$

$\phi = \pi$
 $m = J$

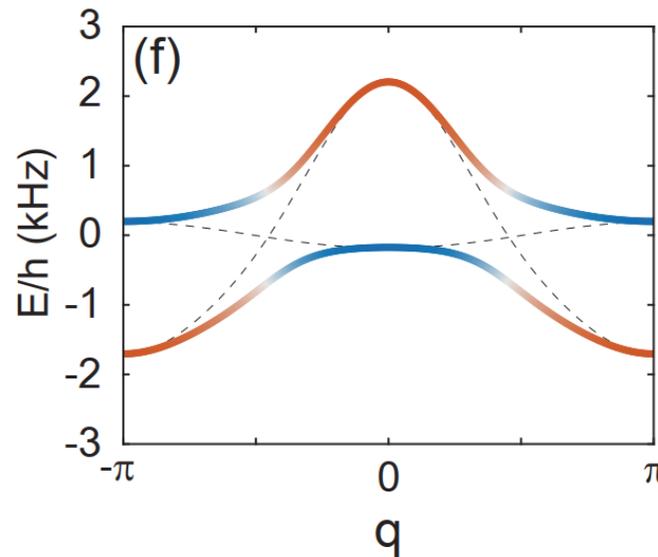


Implementación experimental

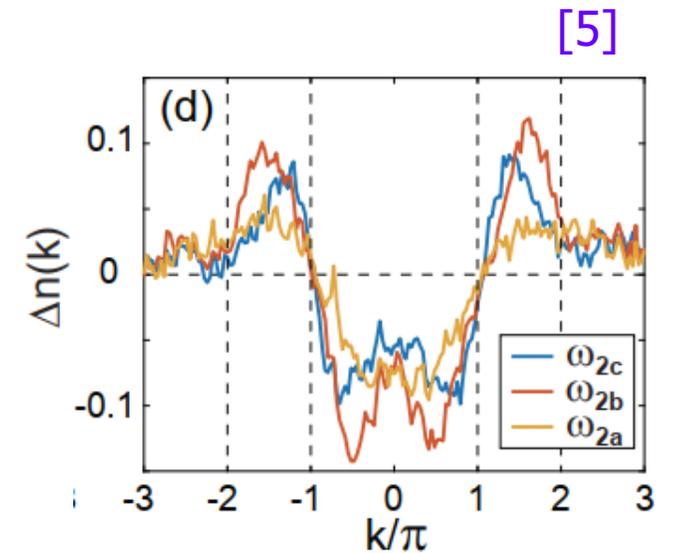
Átomos fríos en una red óptica:



Esquema



Bandas de energía (teoría)

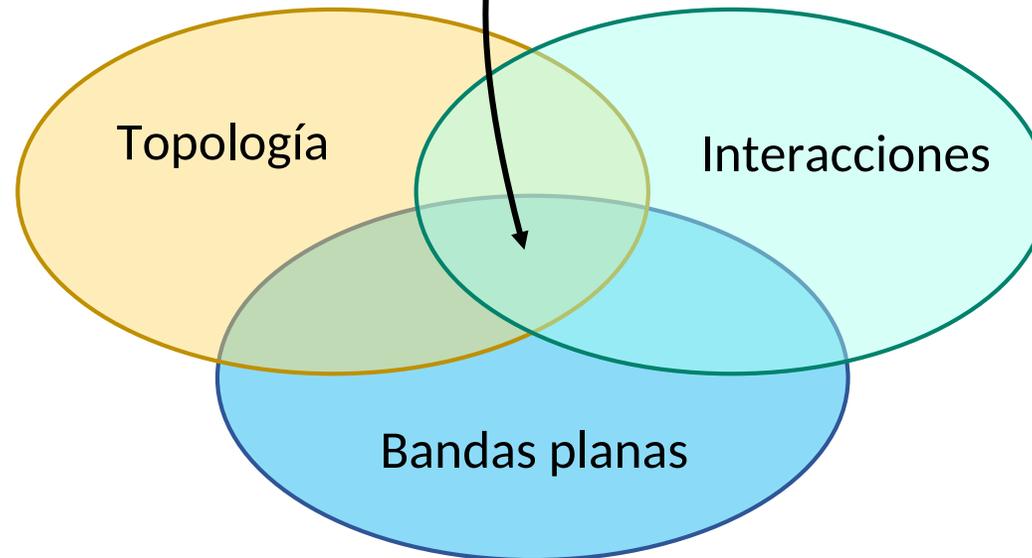
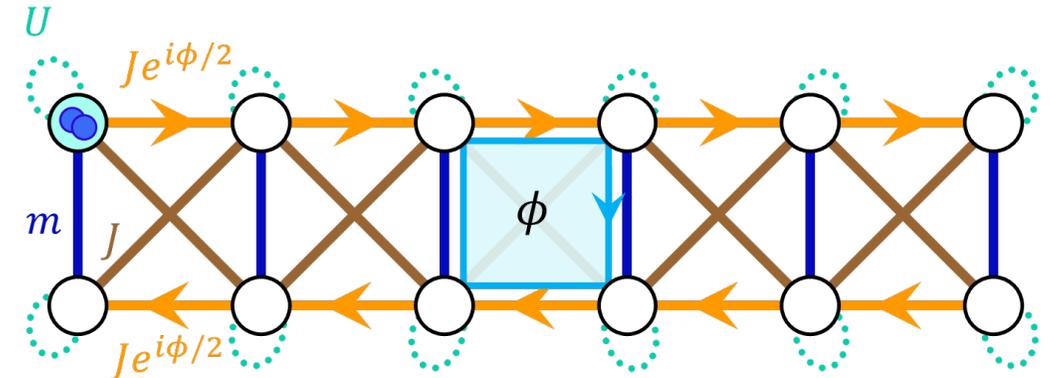


Distribución de momentos (medido)

[5] J. H. Kang, J. H. Han and Y. Shin. New Journal of Physics (2020)

Escalera de Creutz-Hubbard

$$\mathcal{H}_{CH} = \mathcal{H}_C + \frac{U}{2} \sum_{j,\alpha} c_{j\alpha}^\dagger c_{j\alpha}^\dagger c_{j\alpha} c_{j\alpha}$$

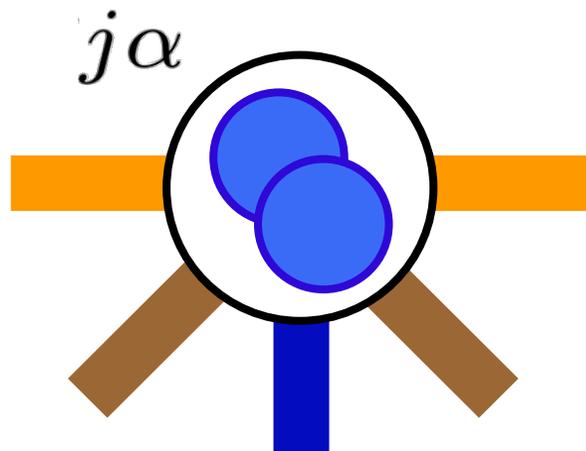


Escalera de Creutz-Hubbard: dos bosones

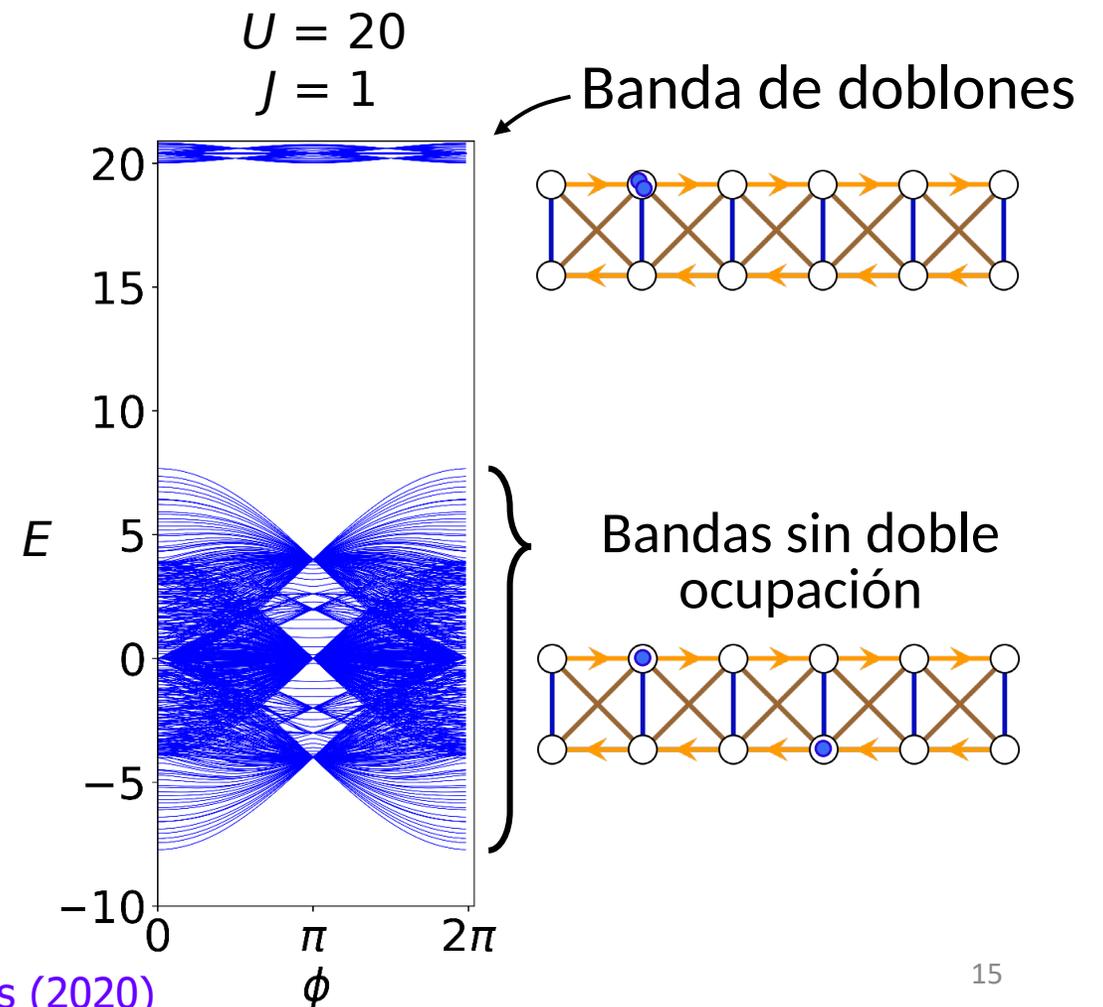
$U \gg J$ Régimen de interacción repulsiva fuerte

$$d_{j\alpha}^\dagger = \frac{1}{\sqrt{2}} c_{j\alpha}^\dagger c_{j\alpha}^\dagger \quad d_{j\alpha} = \frac{1}{\sqrt{2}} c_{j\alpha} c_{j\alpha}$$

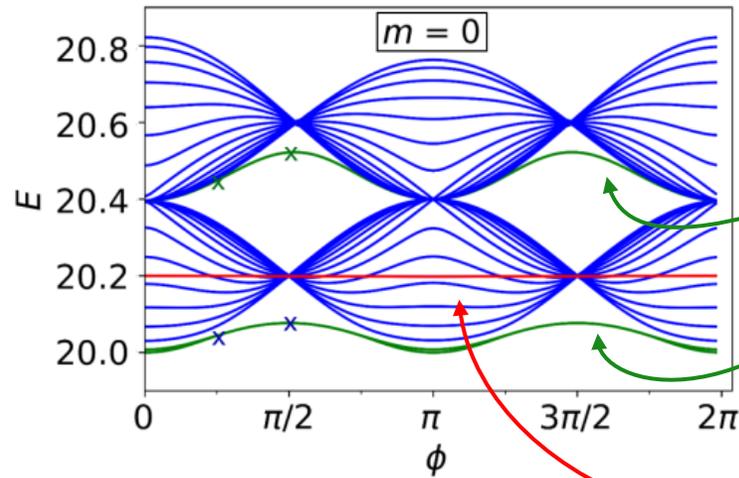
Operadores de doblones



$$d_{j\alpha}^\dagger |0\rangle = |2j\alpha\rangle$$

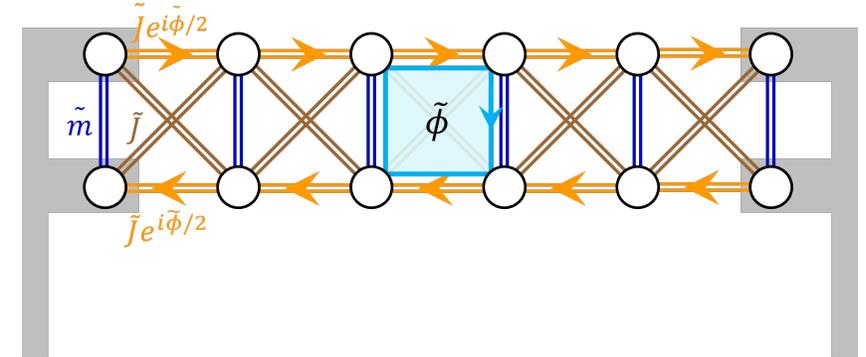
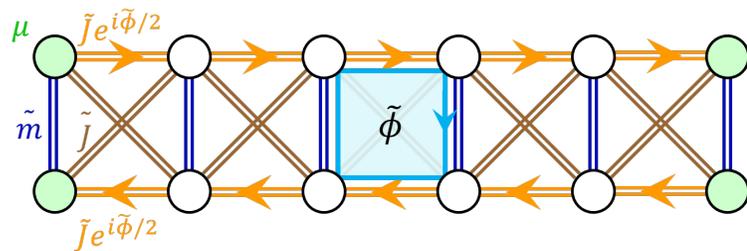
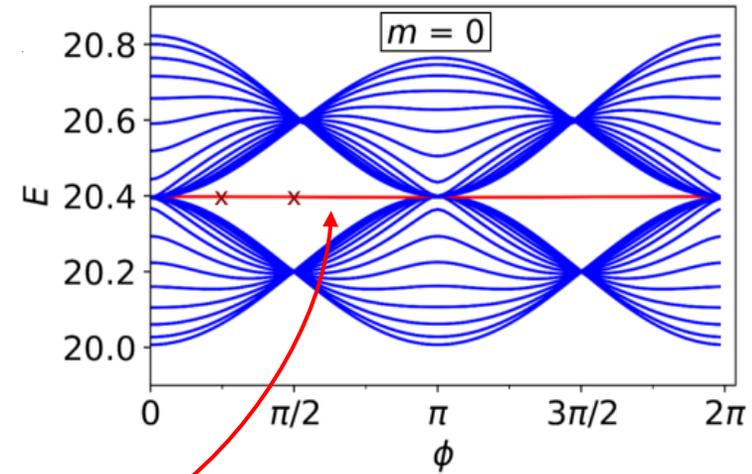


Escalera de Creutz-Hubbard: estados de borde

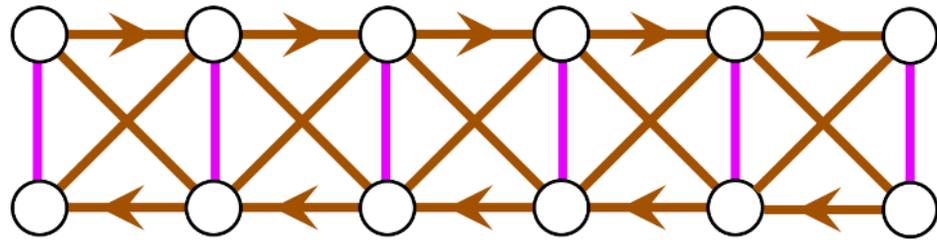


Estados de borde de Tamm

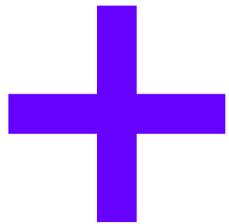
Estados de borde topológicos



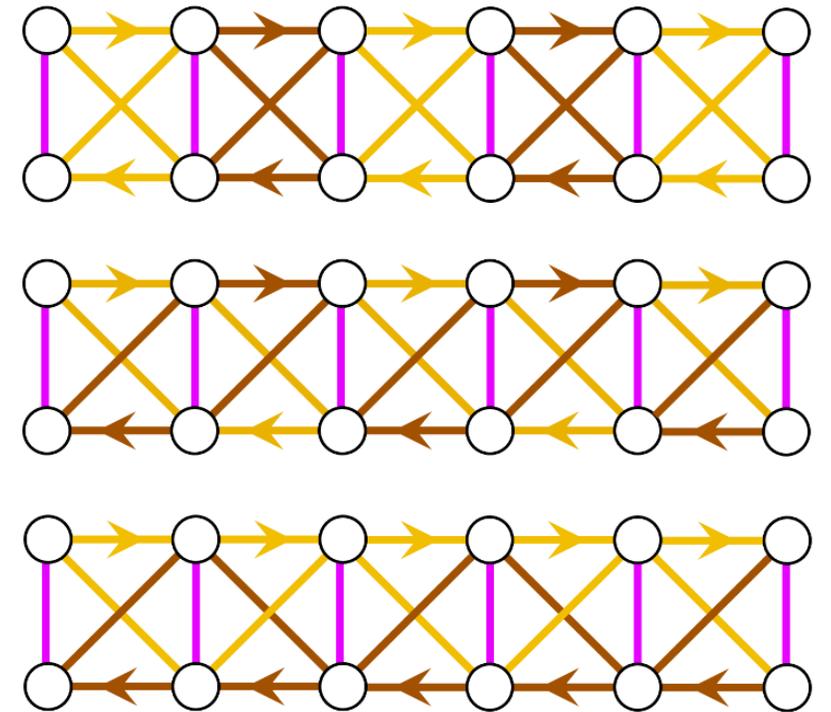
Escaleras CSSH



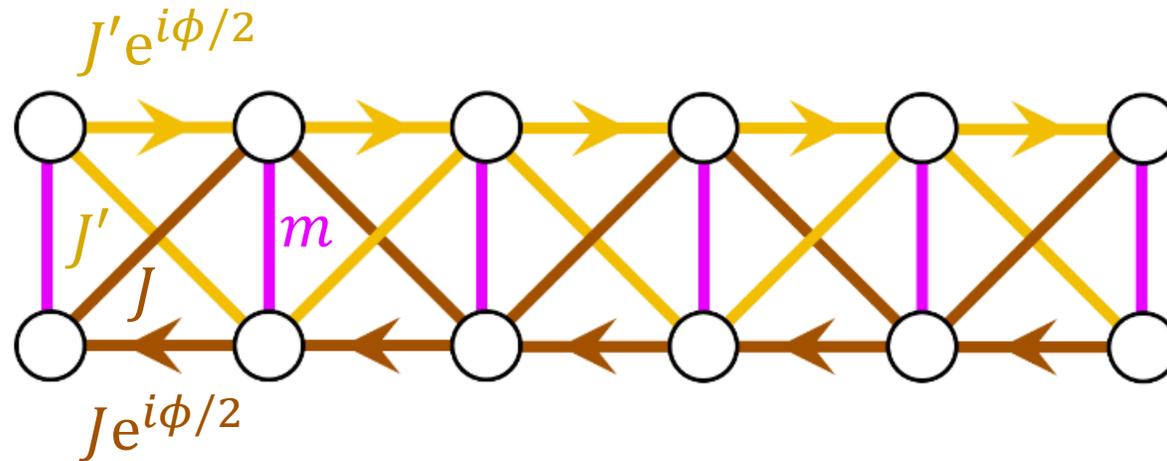
Escalera de Creutz



Cadena SSH



Escaleras CSSH

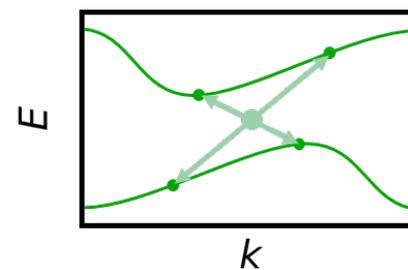
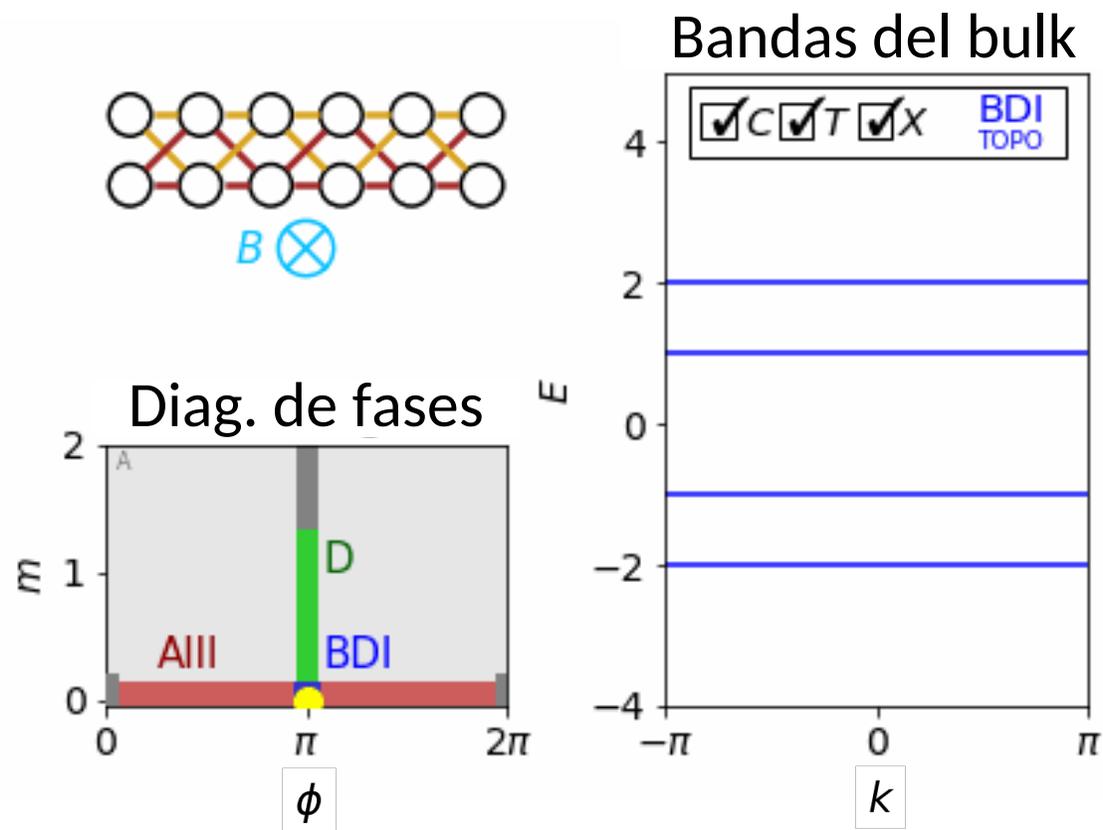


$$J = \xi \sin^2 \theta$$

$$J' = \xi \cos^2 \theta$$

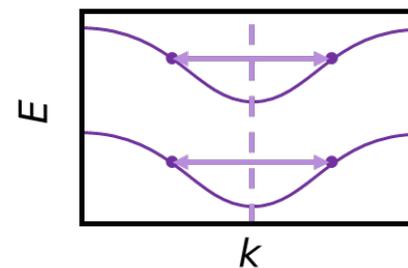
$$\left\{ \begin{array}{l} \theta \text{ Relacionado con } J/J' \\ \phi \text{ Flujo magnético} \\ m \text{ Amplitud de salto vertical} \end{array} \right.$$

Escaleras CSSH: Clases de simetría



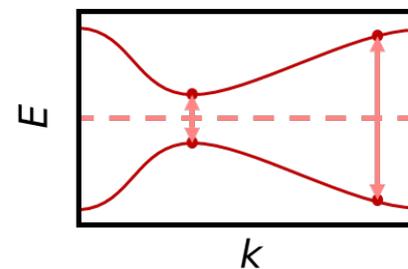
Simetría
partícula-hueco (C)

$$E_i(k) = -E_j(-k)$$



Simetría de
inversión temporal (T)

$$E_i(k) = E_i(-k)$$



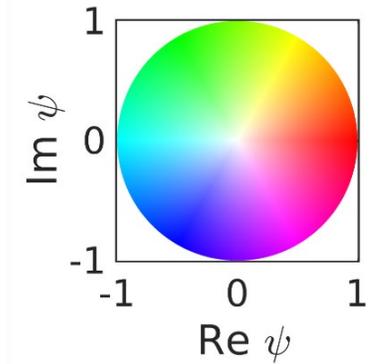
Simetría
quiral (X)

$$E_i(k) = -E_j(k)$$

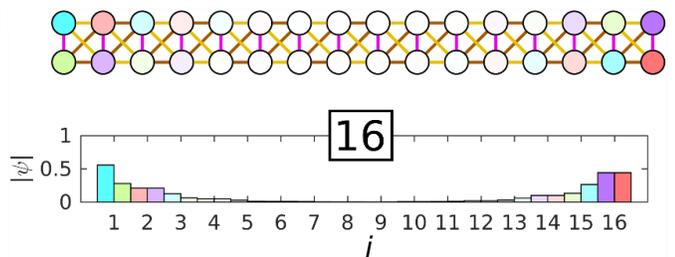
Class	A	AIII	AI	BDI	D	DIII	AII	CII	C	CI
1D T. Inv.		\mathbb{Z}		\mathbb{Z}	\mathbb{Z}_2	\mathbb{Z}_2		$2\mathbb{Z}$		

[1] J. Zurita, C. Creffield, G. Platero. Quantum (2021)

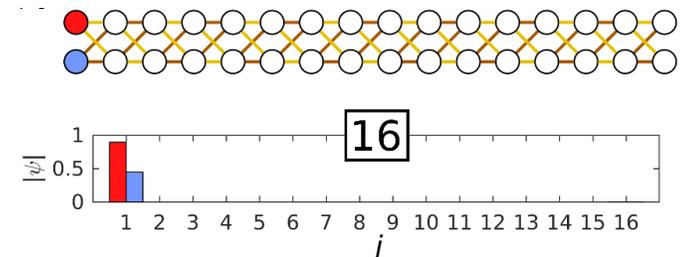
Escaleras CSSH: Estados de borde



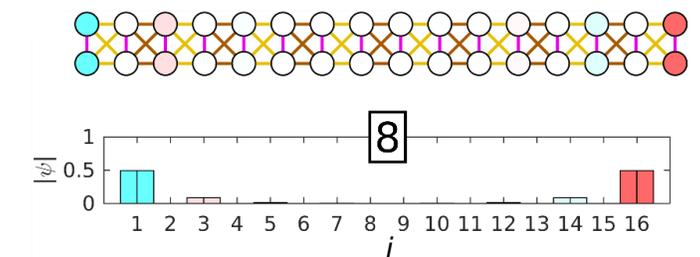
Estados protegidos



De tipo Creutz

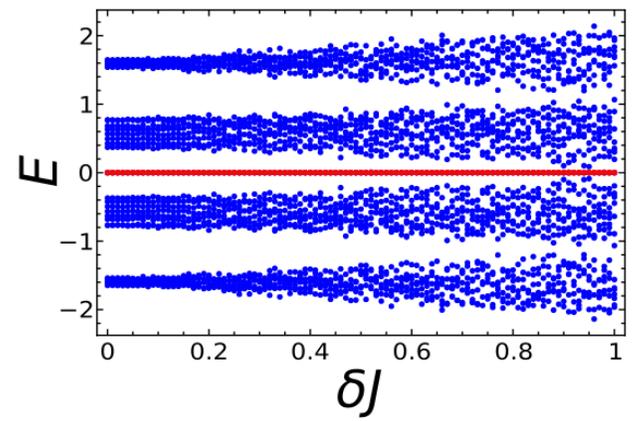


De tipo Creutz, compactos



De tipo SSH

Comportamiento frente al desorden:

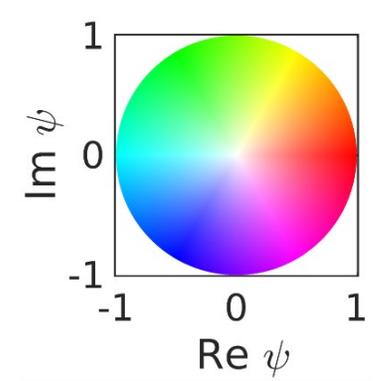


$$J = J_{\text{clean}} + R\delta J$$

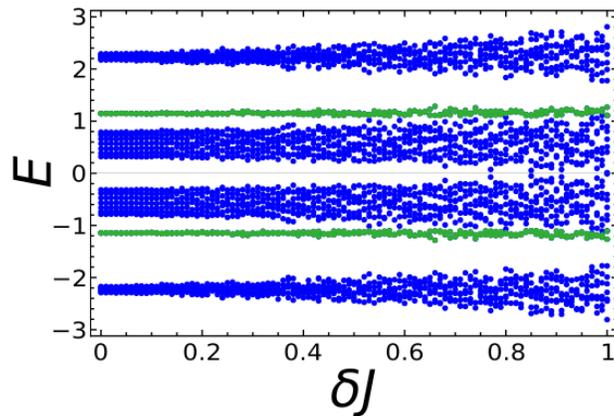
$$-1/2 < R < 1/2$$

[1] J. Zurita, C. Creffield, G. Platero. Quantum (2021)

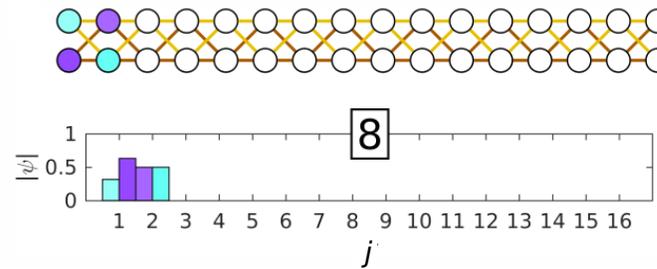
Escaleras CSSH: Estados de borde



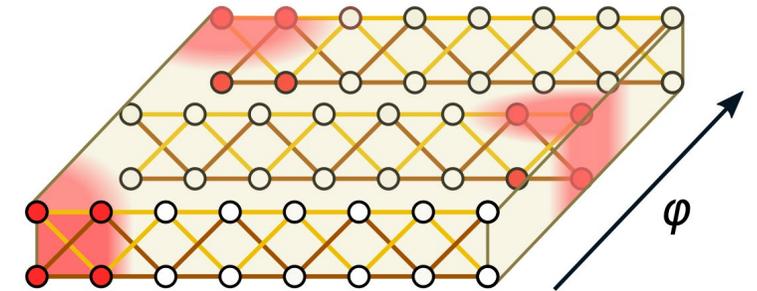
Estados no protegidos: número de Chern



Comportamiento frente al desorden

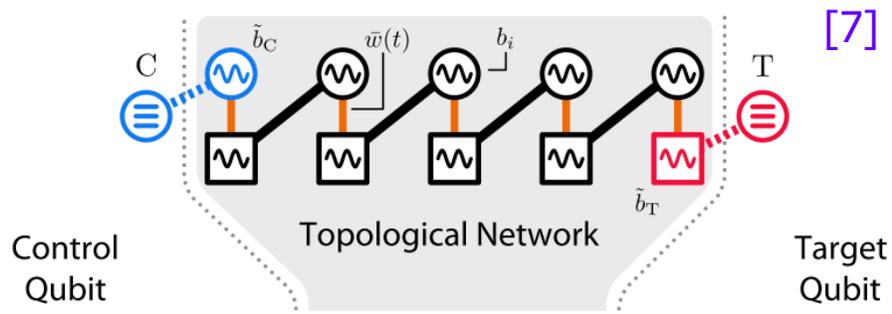


Estado no protegido, 1^{er} gap



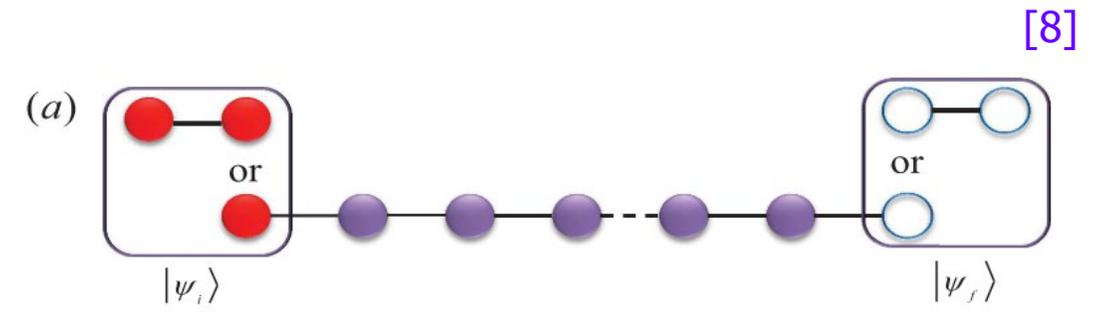
Material efectivo 2D

Aplicaciones en información cuántica



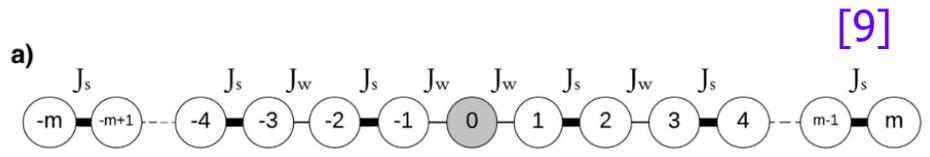
Transferencia de estados usando una red fotónica

[7]



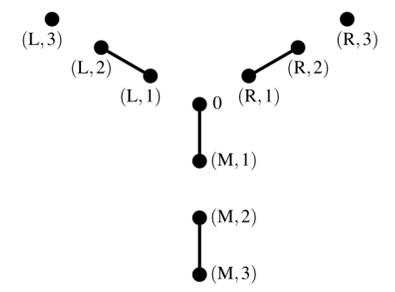
Transferencia de estados usando una red de qubits

[8]



Memoria cuántica y transferencia en cadenas de espines

[9]



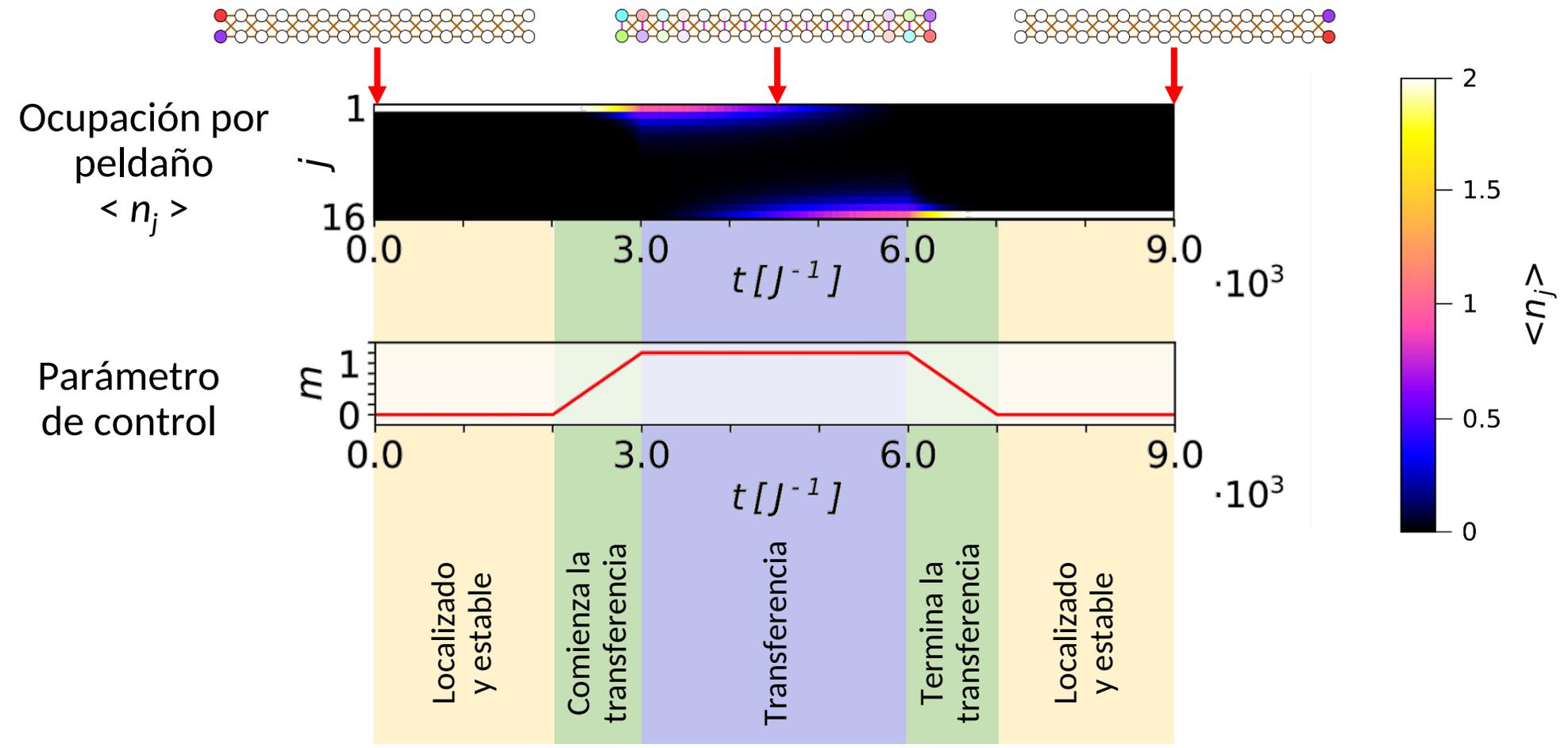
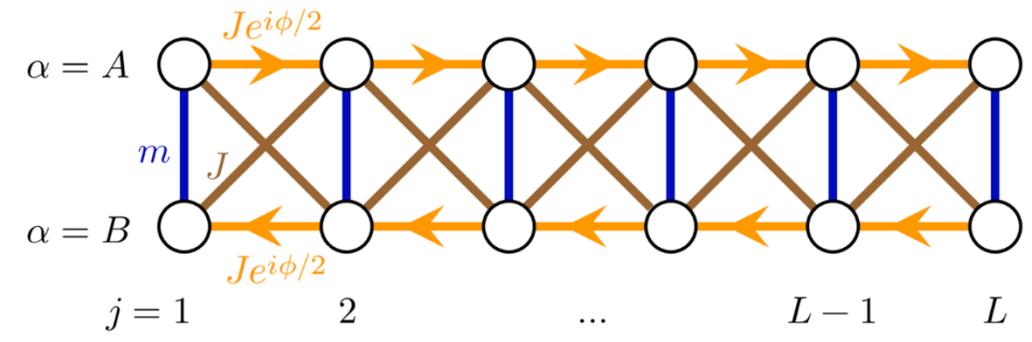
Puerta lógica cuántica

[10]

[7] N. Lang, H. Büchler. npj Quantum Information (2017)
 [9] M. P. Estarellas et al. Scientific Reports (2017)

[8] F. Mei et al. Physical Review A (2018)
 [10] P. Boross et al. Physical Review B (2019)

Transferencia controlada

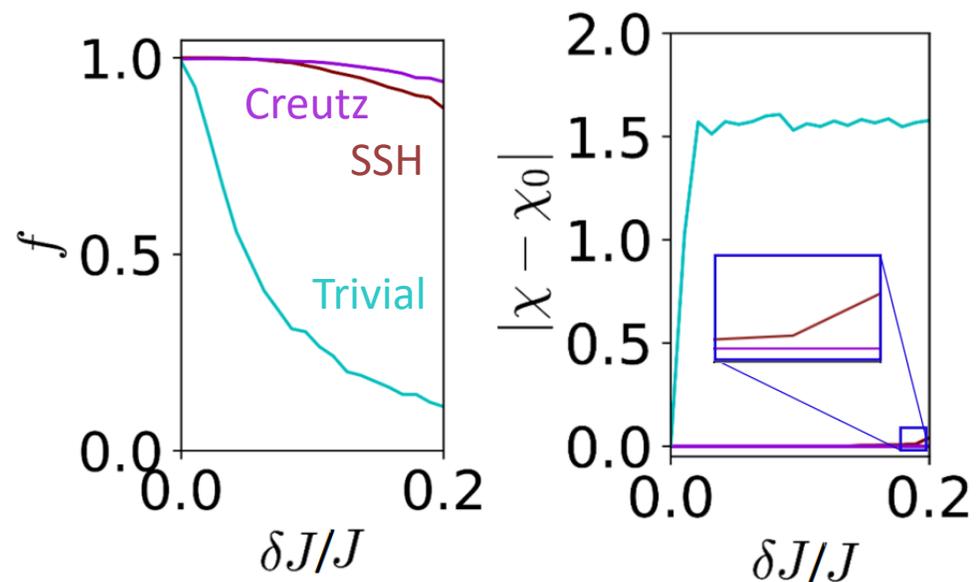


[11] J. Zurita, C. Creffield, G. Platero, *artículo en preparación* (2021)

Transferencia controlada con desorden

Desorden en amplitudes de salto

Preserva la simetría



$$f = |\langle \mathcal{R} | \psi(T) \rangle|^2$$

Fidelidad

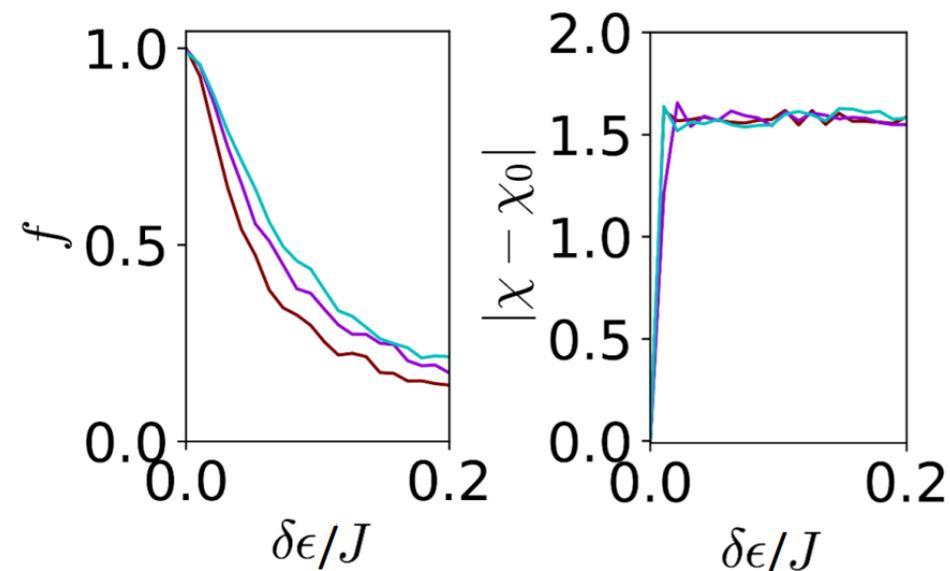
$$\chi = \text{Arg}(\langle \mathcal{R} | \psi(T) \rangle)$$

Fase

Media sobre
1000 realizaciones

Desorden en energías

Rompe la simetría



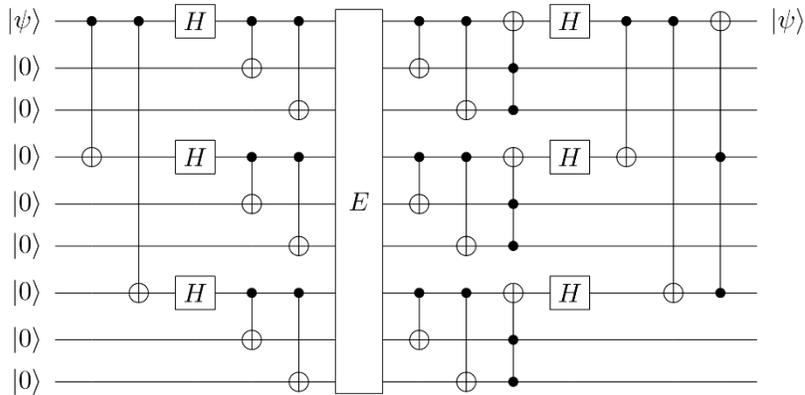
$$J = J_{\text{clean}} + R\delta J$$

$$\epsilon = \epsilon_{\text{clean}} + R\delta \epsilon$$

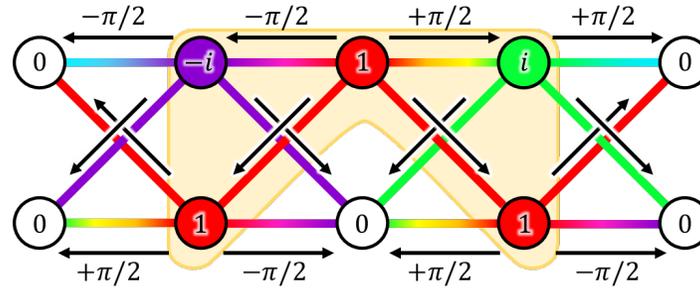
$$-1/2 < R < 1/2$$

[11] J. Zurita, C. Creffield, G. Platero, *artículo en preparación* (2021)

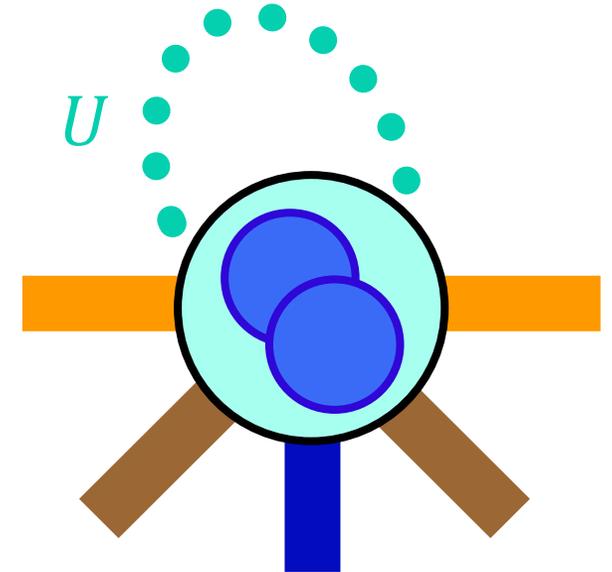
Ideas en desarrollo



Topología
+
Corrección de
errores



Aplicaciones del AB
caging en protocolos
cuánticos



Topología con
interacciones débiles

¡Gracias por vuestra atención!

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