The bond term premium in an estimated DSGE model with real-time learning*

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ABSTRACT: Adaptive learning (AL) takes over other features needed to generate a sizable term premium under rational expectations. Indeed, a time-varying term premium emerges in a first-order approximation of a DSGE model under AL. The estimated AL term premium shares important features with the corresponding measures obtained in the literature using no-arbitrage affine term structure models. Thus, the correlation between the AL term premium and that estimated by the New York Fed corresponding to the 10-year zero coupon yield is 0.96. Moreover, the two term premium measures are highly persistent and they are countercyclical, but the GDP growth rate and the federal funds rate lead the two term premium measures for about 2 years. Monetary policy shocks explain a sizable share (20%) of AL term premium fluctuations.

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1 Introduction

Term premia on medium- and long-term nominal bonds are viewed as compensations for inflation and consumption risks faced by investors over the lifespan of bonds. There is a large body of literature that finds that estimated term premium measures obtained from no-arbitrage affine term structure models are sizable, persistent and fluctuate significantly over time (e.g. Gürkaynak and Wright, 2012 and references therein). However, standard macroeconomic DSGE models have difficulty in explaining term premium dynamics. Rudebusch and Swanson (2012) have made progress by combining several features. They show that a combination of (i) Epstein-Zin preferences (Epstein and Zin, 1989), so risk aversion can be modeled independently from the intertemporal elasticity of substitution; (ii) a third-order approximation;[1] (iii) both long-run real, as in Bansal and Yaron (2004), and nominal risks; and (iv) a huge risk aversion is needed to explain term premium dynamics. Using similar RE-DSGE models, Dew-Becker (2014) and Kliem and Meyer-Gohde (2017) estimate two term premium measures with rather distinctive empirical features.[2] In spite of the progress achieved in this literature, all these (estimated/calibrated) DSGE models rely on an unpleasantly huge risk aversion parameter, which is needed to overcome the lack of economic uncertainty implied by the RE hypothesis in the characterization of term premium dynamics.

This paper departs from the RE assumption by considering adaptive learning (AL), which puts uncertainty about the economic environment at the central stage in the characterization of consumption and inflation risks defining the term premium. Under AL, agents do not know the structure of the economy, so they face a first-order uncertainty. That is, while RE agents are able to identify all sources of uncertainty, AL agents have to learn—in general—about

[1] These DSGE models rely on the rational expectations (RE) assumption. In this class of models, a first-order approximation eliminates the term premium entirely due to the well-known property of certainty equivalence in linearized RE models. Indeed, a third-order approximation is needed to obtain a (model-based) time-varying term premium (Rudebusch and Swanson, 2008).

[2] Thus, the latter reproduces the downward trend displayed by the estimated term premium measures obtained from no-arbitrage affine term structure models since the early 1980's (for instance, Adrian, Crump and Moench, 2013) quite well. However, the term premium estimated by Dew-Becker (2014) shows an upward trend, at least at the start of the Great Moderation period, which is in contrast to other term premium measures.

how the economy behaves, and in particular about the alternative sources of fundamental uncertainty from the time series they observe at the time when they are forming their expectations in an imperfect information setup. I call this a first-order uncertainty because a time-varying term premium emerges in a first-order approximation of an AL-DSGE model. Moreover, I stress the importance of AL by assuming a logarithmic utility function when defining household preferences (i.e. a small risk aversion parameter). Of course, the risks associated with both long-run growth and long-run inflation considered in the RE literature are also present in an AL context due to the learning process about these long-run parameters with no need to assume that they are time varying.[3] In addition, the extended model distinguishes the expectations hypothesis (EH) of the term structure from the assumption made about how agents form their expectations (AL or RE), which enables us to assess the empirical importance of the EH independently of the AL and RE hypotheses.

I deviate from the RE assumption by assuming AL expectations to be based on small forecasting models as in Slobodyan and Wouters (2012a,b), where the expectation of a forward-looking variable is described as a least-squares projection on a small information set.[4] However, this AL-DSGE model differs from the AL model of Slobodyan and Wouters (2012a,b) in two important aspects. First, the term structure of interest rates is introduced. Moreover, I consider two alternative hypotheses for the term structure—which are embedded in the model—: (i) the set of optimality conditions characterizing household demand for government bonds with different maturities; and (ii) the EH of the term structure. The AL term premium is defined as the difference between the two yields implied by these two alternative

[3]Kozicki and Tinsley (2005) is an early paper considering adaptive expectations (instead of AL used here) to characterize term structure dynamics. More specific, they consider a model where the expectations hypothesis of the term structure is imposed (i.e. long-term interest rates are given by agents' beliefs about expected average future short-term rates), but in which their adaptive beliefs are determined by their perceptions of the central bank's long-run inflation target. As discussed in Gürkaynak and Wright (2012), there is a related literature attempting to explain term structure anomalies by shifting perceptions of the central bank's long-run inflation target. More recently, Sinha (2015, 2016) also investigates the implications of AL in the yield curve. However, her approach to AL is rather different to the one followed in this paper. Moreover, she does not address the implications of AL in the estimated bond term premium in her articles.

[4]This approach falls under the broad class of restricted perceptions equilibria, where agents use a misspecified model but form their beliefs optimally given the misspecification (Sargent, 1991; Hommes and Sorger, 1998; Evans and Honkapohja, 2012).

hypotheses.[5]

Second, agents form their expectations using both term structure information and inflation data observed in real time. As emphasized in Aguilar and Vázquez (2017, 2018), most estimated AL models typically consider forecasting models based on variables whose observable counterparts are only final revised data, which ignores an important informational issue for the characterization of both learning and term premium dynamics.[6] Indeed, actual learning dynamics are driven by data available to agents when they form their expectations in real time. I overcome this limitation associated with AL models by restricting small forecasting models to include only lagged term structure and inflation information, which is observed at the time expectations are formed in real time.

I consider an AL approach based on direct multi-step forecasting (Bhansali, 2002; Jordà, 2005) to characterize the multi-period-ahead expectations of forward-looking variables that show up when the DSGE model is extended with the term structure of interest rates. This is in contrast to the approach followed in the related AL literature using iterated forecasts, which are built in general on a misspecified model. As discussed in Jordà (2005), among others, direct forecasts associated with long-term horizons outperform iterated forecasts when dealing with misspecified models since misspecification errors are compounded with the forecast horizon. Direct multi-step forecasting introduces additional flexibility, which is ignored when iterated forecasts are considered, and this extra flexibility results in a better model fit (Aguilar and Vázquez, 2018). More importantly for the purpose of this paper, AL based on direct multi-step forecasting does not (by definition) impose the law of iterated expectations assumed when AL is based on iterated forecasts. This potential failure of the law of iterated

[5]As explained below, under RE the two yields coincide and there is no term premium under a first-order approximation. However, under AL there is a well defined term premium under a first-order approximation because the optimal yield is determined by consumption and inflation expectations whereas the EH yield is fully characterized by short-term rate expectations and these expectations may not be consistent with consumption and inflation expectations in an AL setting.

[6]Another exception is Milani (2011). He focuses on real-time data on output and inflation and their forecasts from the Survey of Professional Forecasters recorded in real time when estimating a small-scale DSGE model, but he ignores revised data on macroeconomic variables, which more accurately describe the actual economy in order to estimate and assess model's fit.

expectations allows for the existence of a well-defined term premium even under a first-order approximation of the AL model solution since the expectations of future consumption and inflation are not consistent with the expectations of the short-term interest rate over the lifespan of bonds under this type of AL. Moreover, the estimated AL expectations of inflation, the short-term interest rate and consumption growth are disciplined in order to fit well the corresponding forecasts reported in the Survey of Professional Forecasters. This strategy of disciplining AL expectations with survey data proves to be very useful for the estimation of an AL term premium that closely resembles the one estimated using no-arbitrage affine term structure models.

As pointed out by Rudebusch and Swanson (2012), RE-DSGE models exhibit a sort of dichotomy between macroeconomic and term premium dynamics. Thus, the macroeconomic dynamics of the model are not very sensitive to the additional second- and higher-order approximation terms introduced by Epstein-Zin preferences, while term premiums are fully determined by those second- and higher-order terms. In contrast, the AL-DSGE model introduced here displays no such dichotomy because macroeconomic and term structure dynamics fully interact under a first-order approximation for two main reasons. First, term structure information observed in real-time plays an important role in characterizing the (adaptive) learning process of the aggregate economy in the model and then learning dynamics determine both the macroeconomy and the term structure of interest rates. Second, the learning process based on direct multi-step forecasting results in a major source of non-linearity, which largely takes over the non-linear features introduced by a third-order approximation of the RE-DSGE model with Epstein-Zin preferences.

The AL and RE versions of the model are estimated using US quarterly data for 1983:3-2014:3. The estimation results show that the estimated term premium under AL shares important features with those estimated in the related literature. Thus, the correlation between the AL term premium and that estimated by the New York Fed—based on the five-factor, no-arbitrage term structure model suggested by Adrian, Crump and Moench (2013)—

associated with the 10-year zero coupon yield is almost perfect over the sample period (the contemporaneous correlation coefficient is 0.96). These two term premia are further highly persistent. Moreover, a comovement analysis shows that the AL term premium is countercyclical in line with the findings in the related literature (e.g. Campbell and Cochrane, 1999; and Cochrane and Piazzesi, 2005). Furthermore, both higher GDP growth rate and federal funds rate anticipate higher term premium for about 6-10 quarters. These comovement results are also in line with those implied by the estimated term premium of Adrian, Crump and Moench (2013). In sum, these findings suggest that AL takes over other factors—such as consumer preferences featuring huge risk aversion— needed to generate sizable term premium fluctuations. In particular, these findings provide empirical support for the hypothesis put forward in the literature (Barillas, Hansen and Sargent, 2009; Rudebusch and Swanson, 2012; among others) that model uncertainty, illustrated in this paper by the presence of a representative AL agent, is an alternative to the unpleasantly huge risk aversion parameters of 50 or 100 needed in RE-DSGE models to fit the data criticized in Lucas (2003).

The variance decomposition analysis shows that monetary policy shocks explain around 10% of the short-run fluctuations in the estimated AL term premium and that proportion increases to roughly 20% in the long-run variance decomposition. This finding suggests that AL induces a sizable propagation mechanism of monetary policy shocks to the term premium. The estimation results also show that the significance of the EH of the term structure is rather low across maturities.

The rest of the paper is organized as follows. Section 2 introduces a DSGE model with real-time AL. Section 3 shows the estimation results and discusses their implications. Section 4 analyzes the dynamic features of the estimated AL term premium in comparison with those estimated in the literature. Section 5 studies the comovement between the estimated term premium and the business cycle. Finally, Section 6 concludes.

2 A DSGE model with real-time adaptive learning

The model builds on the Smets and Wouters (2007) (henceforth called SW) model and its AL extension studied by Slobodyan and Wouters (2012a). This standard medium-scale estimated DSGE model contains both nominal and real frictions affecting the choices of households and firms. I extend the AL medium-scale DSGE model in three directions. First, I extend the model to account for the term structure of interest rates. I combine the two approaches followed separately in Aguilar and Vázquez (2017, 2018), by both (i) considering the standard consumption-based asset pricing equation associated with each maturity; and (ii) imposing the EH of the term structure, which is obtained by imposing the law of iterated expectations. This combination enables a term premium for each bond maturity to be defined and its dynamic features studied throughout the term structure of government bond yields. Second, only lagged term structure information and inflation data observed in real time are used in the small forecasting models for all forward-looking variables of the medium-scale DSGE model. Finally, AL expectations of consumption growth, inflation and short-term interest rate are disciplined by requiring that the deviations of estimated AL model expectations from the corresponding forecasts reported in the Survey of Professional Forecasters (SPF) be stationary. Disciplining expectations in these three dimensions proves to be important for the purpose of estimating an AL term premium measure because, first, the term premium is understood as a compensation for consumption and inflation uncertainty, so a sound characterization of inflation and consumption expectations becomes crucial. Second, a term premium measure takes the yield implied by the expectations hypothesis as a reference, so a sound characterization of short-term rate expectations is again important.

In contrast to Ormeño and Molnár (2015), I do not impose the more restrictive assumption that agents' expectations must match SPF forecasts up to a white noise error. In short, I allow for persistent deviations between model-based AL expectations and those reported in the SPF. The main reason is that term structure information is also disciplining agents' learning process. As a consequence, if I required the deviations of model expectations from the

SPF forecast to be white noise, the approach would force model-based expectations to mimic SPF forecasts, but would also somewhat overlook term structure information on forecasting whenever the two sources of information are not in line. It is important to recognize this, since there is evidence that term structure information is not used by professional forecasters on a consistent basis (Rudebusch and Williams, 2009).

I present these extensions of the model next. The remaining log-linearized equations of the model are presented in Appendix 1.

2.1 The DSGE model

The AL model builds on the standard SW model. Households maximize their utility that depends on their levels of consumption relative to an external habit component and leisure. Labor supplied by households is differentiated by a union with monopoly power setting sticky nominal wages à la Calvo (1983). Households rent capital to firms and decide how much capital to accumulate depending on the capital adjustment costs that they face, and how much capital is used in the production process depending on the capital utilization adjustment costs. Intermediate firms decide how much differentiated labor they hire to produce differentiated goods and set their prices à la Calvo. In addition, wages and prices are both partially indexed to lagged inflation when they are not re-optimized, introducing another source of nominal rigidity. Consequently, current prices depend on current and expected marginal costs and past inflation whereas current wages are linked to past and expected future inflation and wages. I deviate from the monetary policy rule in the SW model by assuming that the monetary authorities follow a Taylor-type rule reacting to expected inflation and a term spread as defined below. The main findings of the paper are not affected by considering a standard Taylor rule, but I want to characterize the central banker and private agents as sharing a similar degree of uncertainty about the aggregate economy in general, and inflation in particular.

The model contains 26 shocks: seven structural disturbances associated with technology, demand-side, monetary policy, and price and wage mark-up shocks as in the SW model; seven term structure shocks; eleven shocks describing the deviations of consumption growth, inflation and short-term interest rate model expectations from their respective observable counterparts reported in the SPF and one shock associated with the inflation revision process.

2.2 The term structure extension

This section introduces the term structure of interest rates in the SW model. From the first-order conditions characterizing the optimal decisions of the representative consumer, it is possible to obtain the standard consumption-based asset pricing equation associated with each maturity:

$$E_t\left[\beta^j \frac{U_C(C_{t+j}, L_{t+j})\left(exp(\varepsilon_t^{\{j\}})(1+R_t^{\{j\}})\right)^j}{U_C(C_t, L_t)\prod_{k=1}^{j}(1+\pi_{t+k})}\right] = 1,\ for\ j =\ 1,\ 2,\ ...,\ n,$$

where E_t stands for the RE or the AL operator depending on the scenarios analyzed below, β is the discount factor, U_C denotes the marginal utility consumption, C_t, L_t, π_t, $R_t^{\{j\}}$ and $\varepsilon_t^{\{j\}}$ denote consumption, labor, the rate of inflation, the nominal yield and the risk premium shock associated with the j-period maturity bond, respectively. The inclusion of a risk premium shock for each maturity is in line with the view of many authors of interpreting the gap between the pure-expectations-hypothesis-implied yield, $R_t^{\{j\}}$, and the observed yield as a measure of fluctuations in the risk premium (e.g. De Graeve, Emiris and Wouters, 2009). Moreover, since I focus on government bonds in the empirical analysis, $\varepsilon_t^{\{j\}}$ can be understood as a *convenience yield term* (see, among others, Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood et al., 2015; Del Negro et al., 2017) defined as a risk premium associated with the safety and liquidity features of government bonds relative to assets with the same payoff, but without such exceptional properties.

Considering the multiplicative-utility function assumed in the SW model, and after some algebra, the (linearized) consumption-based asset pricing equations can be written as

$$x_t = E_t x_{t+j} - \left(\frac{1-x_1}{\sigma_c}\right)\left[j r_t^{\{j\}} - \sum_{k=1}^{j} E_t \pi_{t+k} + j\varepsilon_t^{\{j\}}\right] + x_2\left(l_t - E_t l_{t+j}\right), \; for\, j = \; 1,\, 2,\, ...,\, n, \tag{1}$$

where lower case variables denote the log-deviation of consumption (and hours worked) from its balanced-growth (steady-state) value or, alternatively, the deviations of the nominal interest rate, nominal yields and the rate of inflation from their respective steady-state values. The following notation is used: $x_t = c_t - x_1 c_{t-1}$, $x_1 = \frac{h}{\gamma}$, $x_2 = \frac{(\sigma_c - 1)}{\sigma_c}\frac{WL}{C}$, where h and σ_c denote the habit formation and risk aversion parameters, and γ, W, L and C denote the balanced-growth rate and the steady-state values of the real wage, hours worked and consumption, respectively.

Under the law of iterated expectations, the former optimality condition (1) for $j = 1$ can be (iteratively) solved forward j-periods ahead to obtain:

$$x_t = E_t x_{t+j} - \left(\frac{1-x_1}{\sigma_c}\right)\sum_{k=1}^{j} E_t\left[r_{t+k-1}^{\{1\}} - E_t \pi_{t+k} + \varepsilon_{t+k-1}^{\{1\}}\right] + x_2\left(l_t - E_t l_{t+j}\right). \tag{2}$$

Since equations (1) and (2) must hold in equilibrium when the law of iterated expectations holds, they imply the expectations hypothesis (EH) of the term structure of interest rates:

$$\hat{r}_t^{\{j\}} = \frac{1}{j}\sum_{k=1}^{j} E_t r_{t+k-1}^{\{1\}} - \left(\varepsilon_t^{\{j\}} - \frac{1}{j}\sum_{k=1}^{j} E_t \varepsilon_{t+k-1}^{\{1\}}\right),$$

or

$$\hat{r}_t^{\{j\}} = \frac{1}{j}\sum_{k=0}^{j-1} E_t r_{t+k} + \xi_t^{\{j\}} = r_t^{EH\{j\}} + \xi_t^{\{j\}}, \tag{3}$$

where $\hat{r}_t^{\{j\}}$ is the yield consistent with (2) and the supraindex $\{1\}$ on r_{t+k} has been removed for sake of simplicity. Equation (3) states the EH of the term structure. That is, the nominal yield of the j-period maturity bond under the EH, $\hat{r}_t^{\{j\}}$, is equal to the average of the expectations of the short-term (1-period) nominal interest rate between periods t and

$t+j-1$, denoted by $r_t^{EH\{j\}}$, plus a term premium $\xi_t^{\{j\}} = \left(\frac{1}{j}\sum_{k=1}^{j} E_t \varepsilon_{t+k-1}^{\{1\}}\right) - \varepsilon_t^{\{j\}}$.

While standard AL approaches assume that the law of iterated expectations—implied by the RE hypothesis and used to derive (2) and (3)—holds for the subjective expectations of the representative agent (Honkapojha, Mitra and Evans, 2003), that law may in reality fail to hold for market expectations for several reasons. Thus, the aggregate forecasts used to discipline markets' expectations in the model might not satisfy the law of iterated expectations. More importantly, AL agents who acknowledge their limited knowledge of the true data generating process (i.e. they are aware that they are dealing with a misspecified model) may find it useful to follow a flexible forecasting approach by relying on direct multi-step forecasting instead of using iterated forecasts built on a misspecified model. As discussed in Bhansali (2002), Jordà (2005) and references therein, direct forecasts outperform iterated forecasts based on a misspecified model since misspecification errors are compounded with the forecast horizon. Moreover, the AL approach based on multi-step forecasting is in line with the way in which panelists from the Survey of Professional Forecasters behave. As emphasized below, Stark (2013) reports that these forecasters are quite flexible in their approach to forecasting and vary their methods with the forecast horizon. These features suggest that the law of iterated expectations does not hold in reality. The AL approach discussed below considers direct multi-step forecasting to describe the expectations of any forward-looking variable k-periods ahead, for $k = 1, 2, ..., j$.

The potential failure of the law of iterated expectations may have important implications on both macroeconomic and term structure dynamics. Thus, household decisions under AL based on (1) depend upon expectations of consumption, inflation and hours worked while (3) depends only on the short-term interest rate expectations. In order to analyze the relative importance of these two approaches, I define the following linear combination of the two nominal yields implied by (1) and (3) under AL (i.e. $r_t^{\{j\}}$ is the yield implied by the consumption based model and $r_t^{EH\{j\}}$ is the yield implied by the EH):

$$\bar{r}_t^{\{j\}} = \lambda_j r_t^{\{j\}} + (1 - \lambda_j) r_t^{EH\{j\}}, \tag{4}$$

where $0 \leq \lambda_j \leq 1$ for all j. Clearly, equation (4) embeds the two approaches, enabling the EH of the term structure to be distinguished from the expectational assumption considered. A value of λ_j close to zero indicates that the EH holds for the j-period maturity bond. Moreover, it can be estimated across the term structure of interest rates, which can be used to assess the importance of the EH along the yield curve.

The term structure premium associated with the j-period maturity bond is defined as the excess return that a consumer earns in expectation by buying this bond and holding it to maturity instead of buying short-term bonds and rolling them over for j periods (see, for instance, Rudebusch and Swanson, 2008, 2012; and Dew-Becker, 2014). Hence, the term premium can be expressed as the wedge between the (optimal) consumption-model based yield, $r_t^{\{j\}}$, and that implied by the EH, $r_t^{EH\{j\}}$:

$$tp_t^{\{j\}} = r_t^{\{j\}} - r_t^{EH\{j\}}. \tag{5}$$

Notice that this wedge vanishes under RE when there is no convenience yield term (i.e. $\varepsilon_t^{\{j\}} = 0$ for all j) since $r_t^{\{j\}} = r_t^{EH\{j\}}$ under RE. This is an implication of the well-known property of certainty equivalence in linearized models. However, there is a wedge due to the potential failure of the law of iterated expectations under AL approach based on direct multi-step forecasting.

In line with the SW model, I assume that the risk premium shock follows an AR(1):

$$\varepsilon_t^{\{1\}} = \rho^{\{1\}} \varepsilon_{t-1}^{\{1\}} + \eta_t^{\{1\}}, \tag{6}$$

whereas the remaining risk premium shocks $\varepsilon_t^{\{j\}}$, for $j > 1$, follow AR(1) processes augmented with an additional term that captures a potential interaction with the short-term risk premium shock:

$$\varepsilon_t^{\{j\}} = \rho^{\{j\}} \varepsilon_{t-1}^{\{j\}} + \rho_{\varepsilon}^{\{j\}} \eta_t^{\{1\}} + \eta_t^{\{j\}}. \tag{7}$$

That is, $\rho_{\varepsilon}^{\{j\}}$ captures the interaction of the risk premium innovation, $\eta_t^{\{1\}}$, with the risk premium shock, $\varepsilon_t^{\{j\}}$, associated with the j-period maturity government bond.

2.3 Real-time adaptive learning

By considering small forecasting models as in Adam (2005), Branch and Evans (2006), Slobodyan and Wouters (2012a,b) and Rychalovska, Slobodyan and Wouters (2016), I deviate from the minimum state variable (MSV) AL approach followed by Eusepi and Preston (2011) and others (Orphanides and Williams, 2005; Milani, 2007, 2008, 2011; Sinha, 2015, 2016), where agents' expectations are assumed to be based on a (linear) function of the state variables of the model. In contrast, small forecasting models assume that agents form their expectations based on the information provided by endogenous variables, such as those appearing in the optimality conditions of a DSGE model.

The consideration of small forecasting models based only on information *observed* when agents are forming their expectations in real time is arguably a more appealing approach to AL than the MSV approach on several grounds. Small forecasting models are robust to alternative models characterized by different MSV sets. This is an important feature because one of the main motivations for moving from the RE assumption to some sort of AL is that in reality agents do not know what the true model is. Consequently, they cannot know the actual MSV set and, in addition, they may have trouble in observing their values in real time. In sum, the small forecasting model approach followed in this paper recognizes that agents might be endowed in reality with much less information regarding the structure of the actual economy than the MSV approach to AL postulates. Taking into account the limited information scenario faced by agents in reality seems to be crucial in the estimation of the term premium since this is characterized by consumption and inflation risks as emphasized above.

Most papers in the AL literature consider only revised aggregate data when characterizing the linear forecasting models that agents follow to update their expectations, the so-called "perceived law of motion" (PLM). This assumption is problematic because revised aggregate data are not available to economic agents when they are forming their expectations in real time.[7] Aguilar and Vázquez (2017, 2018) introduce term spread information, which is observed in real time, in their AL-DSGE models. In this paper, I follow the second of their papers by considering real-time inflation data in addition to term structure information in the PLM.

Appendix 2 outlines how AL expectation formation works and how AL interacts with the rest of the economy. Here, I describe the small forecasting models that agents use to forecast the forward-looking variables of the DSGE model.

A PLM with information observed in real time

As emphasized in Aguilar and Vázquez (2018), agents must form their PLM using only information actually available at the time when they are forming their expectations. In this DSGE model augmented with term structure, I consider that agents combine three alternative forecasting models at the same time, track their forecasting performance, and use a variant of the Bayesian model averaging method to generate an aggregate forecast from the alternative forecasting models that is used to characterize their decisions.[8]

The first two specifications rely on the lagged 2-quarter and 1-year term spreads—$sp_{t-1}^{\{2\}} =$

[7] See Croushore (2011) and references therein for an analysis of aggregate data revisions and their consequences in several contexts.

[8] More precisely, for each forecasting model m_i, the agents track the value of

$$B_{i,t} = t \cdot log \left(det \left(\frac{1}{t} \sum_{i=1}^{t} u_i u_i^T \right) \right) + \kappa_i \cdot log(t),$$

where κ_i is the number of degrees of freedom in the forecasting model m_i, and u_i is the i-th model forecasting error. As pointed out in Slobodyan and Wouters (2008), this expression is a generalization of the sum of squared errors adjusted for degrees of freedom using the Bayesian information criterion penalty. Thus, given values of $B_{i,t}$, the weight of a model i at time t is proportional to $exp\left(-\frac{1}{2}B_{i,t}\right)$.

$r_{t-1}^{\{2\}} - r_{t-1}$ and $sp_{t-1}^{\{4\}} = r_{t-1}^{\{4\}} - r_{t-1}$, respectively. From a theoretical perspective, a PLM based on term structure information is rationalized by the interaction between term spreads and the expectations of both consumption and inflation implicitly implied by the set of optimality conditions (1). From an empirical perspective, the use of term structure information in the PLM is further motivated by the ability of term spreads to predict inflation (Mishkin, 1990) and real economic activity (Estrella and Hardouvelis, 1991, Estrella and Mishkin, 1997). Beyond term structure information, agents also have access to real-time data on the macroeconomic outlook. Hence, the third formulation relies on lagged real-time inflation. Formally, the three alternative small forecasting models are described as follows

$$\begin{cases} m_1: & E_t y_{t+j} = \theta_{1,y,t-1}^{\{j\}} + \beta_{1,y,t-1}^{\{j\}} sp_{t-1}^{\{2\}}, \\ m_2: & E_t y_{t+j} = \theta_{2,y,t-1}^{\{j\}} + \beta_{2,y,t-1}^{\{j\}} sp_{t-1}^{\{4\}}, \\ m_3: & E_t y_{t+j} = \theta_{3,y,t-1}^{\{j\}} + \beta_{3,y,t-1}^{\{j\}} \pi_{t-1,t}^{r}, \end{cases} \tag{8}$$

where $\pi_{t-1,t}^{r}$ is the first release of inflation associated with time $t-1$, which is released at time t.

Although the PLM (8) are determined by actual information available to agents at the time they formed their expectations, it is important to remark that the process of updating AL coefficients through the Kalman filter still depends on a few revised variables such as aggregate consumption and inflation. A possibility for overcoming this issue would be the use of additional vintage data (i.e. partially revised data), but this would inevitably increase the set of observable variables, which is already quite large as emphasized below. [9]

The inclusion of real-time inflation in the PLM requires a characterization of that variable. Following Casares and Vázquez (2016), I consider the following identity relating revised inflation, π_t, to both the initial announcement of inflation (i.e. real time inflation), $\pi_{t,t+1}^{r}$,

[9] Moreover, if the bulk of aggregate data revisions is mostly accomplished in the first revision, final revised data would be a good proxy of vintage revised data used in the updating procedure of AL coefficients.

and the final revisions, $rev^{\pi}_{t,t+S}$:

$$\pi_t = \pi^r_{t,t+1} + rev^{\pi}_{t,t+S},$$

where S denotes the number of periods (quarters) of delay for the final release. Many papers (e.g. Aruoba, 2008) have shown that US data revisions of many aggregate time series (e.g. inflation) are not rational forecast errors. More precisely, revisions are correlated to their initial (real-time) announcements and show persistence. Thus, I assume that

$$rev^{\pi}_{t,t+S} = b^r_{\pi}\pi^r_{t,t+1} + \epsilon^{\pi}_{t,t+S},$$

$$\epsilon^{\pi}_{t,t+S} = \rho^r_{\pi}\epsilon^{\pi}_{t-1,t+S-1} + \eta^{\pi r}_{t,t+S},$$

where $\eta^{\pi r}_{t,t+S}$ is a white noise innovation.

According to the PLM (8), direct multi-step forecasting is considered: each expectational horizon is estimated separately and thus they do not have to be consistent with each other. As discussed above, this AL approach based on direct multi-step forecasting overcomes the potential weakness of alternative AL models relying on iterated forecasts obtained from a misspecified forecasting model because misspecification errors are compounded with the forecast horizon.[10]

Through the time-varying learning parameters, the AL approach introduces a few non-linear features that help somewhat to overcome the log-linear approximation typically used in DSGE models. This feature improves model fit by capturing low frequency patterns in the data captured by the time-varying intercepts, $\theta^{\{j\}}_{i,y,t-1}$, in (8) (e.g. the downtrend of inflation in the last three decades). Moreover, the combination of different small forecasting models in the AL model helps, on the one hand, to address the multicollinearity issue arising when a single forecasting model including a number of highly correlated regressors is used. On the

[10] This approach is in clear contrast to the maintained beliefs hypothesis suggested in Preston (2005)—an approach also followed in Eusepi and Preston (2011) and Sinha (2015, 2016)— which imposes not only an infinite forecast horizon, but also considers iterated forecasts used under the MSV approach.

other hand, it adds flexibility, which is in line with how SPF panelists forecast. As pointed out by Stark (2013), *"SPF panelists are quite flexible in their approach to forecasting... They use a combination of models in forming their expectations, rather than just one model. And, they vary their methods with the forecast horizon ...*(i.e. they do not use iterated forecasts) *the panelists update their projections frequently, suggesting that their projections incorporate the most recent information available on the economy around the survey's deadline.*"

PLM disciplined by the Survey of Professional Forecasters (SPF)

AL can be criticized because it introduces additional degrees of freedom resulting in an arbitrary improvement in model fit—see, for instance, Adam and Marcet (2011). This criticism does not downplay the AL approach followed in this paper because, apart from relying on small forecasting models, the information set of agents is further restricted by the fact that it contains only information observed in real time. As a way of disciplining expectations even further, I assume that the deviations of agents' expectations of inflation, consumption growth and short term interest rate from the (observed) forecasts reported in the SPF follow stationary processes. I focus on short-term forecasting horizons due to SPF data availability. Moreover, short-term forecasting horizons arguably have more informational content than longer forecasts horizons. Formally, I assume

$$\epsilon_{\pi,t}^{\{j\}} = \rho_{\pi}^{\{j\}} \epsilon_{\pi,t-1}^{\{j\}} + \eta_{\pi,t}^{\{j\}}, \quad for\, j = \ 1,\ 2,\ 3,\ 4, \tag{9}$$

where $\epsilon_{\pi,t}^{\{j\}}$, denoting the deviation of the j-period-ahead inflation model expectations ($E_t\pi_{t+j}$) from their observable counterpart reported in the SPF, is assumed to follow an AR(1). Similarly, the deviation of the j-period-ahead consumption growth model expectations, $E_t\left(\Delta c_{t+j}\right)$, from their observable counterpart reported in the SPF, denoted by$\epsilon_{\Delta c,t}^{\{j\}}$, is also assumed to follow an AR(1):

$$\epsilon_{\Delta c,t}^{\{j\}} = \rho_{\Delta c}^{\{j\}} \epsilon_{\Delta c,t-1}^{\{j\}} + \eta_{\Delta c,t}^{\{j\}}, \quad for\, j = \ 1,\ 2,\ 3,\ 4. \tag{10}$$

Finally, the deviation of the j-period-ahead short-term interest rate model expectations ($E_t r_{t+j}$) from their observable counterpart reported in the SPF, denoted by$\epsilon_{r,t}^{\{j\}}$, is also assumed to follow an AR(1):

$$\epsilon_{r,t}^{\{j\}} = \rho_r^{\{j\}} \epsilon_{r,t-1}^{\{j\}} + \eta_{r,t}^{\{j\}}, \quad for\ j = 1,\ 2,\ 3. \tag{11}$$

As shown below, this way of disciplining the expectations of the short-term interest rate becomes crucial for estimating a term premium under AL that resembles the estimated term premium obtained from the no-arbitrage affine term structure model suggested by Adrian, Crump and Moench (2013).

2.4 Real-time monetary policy rule

In line with the limited information assumption imposed in the characterization of agents' expectations, the monetary policy rule is assumed to be determined only by inflation expectations and the lagged values of the 1-year term spread, which are actually available to the policymaker at the time of implementing monetary policy. Formally,

$$r_t = \rho_r r_{t-1} + (1 - \rho_r) r_\pi E_t \pi_{t+1} + r_{sp} sp_{t-1}^{\{4\}} + \varepsilon_t^r, \tag{12}$$

where ε_t^r , as in the SW model, is assumed to follow an AR(1) process with persistence parameter denoted by ρ_R. In contrast to the standard policy rule in the SW model, I assume, first, that the policy rate reacts to expected inflation instead of current inflation. This enables an informational symmetry to be maintained between the private sector and the central bank (i.e. the inflation expectations of the two types of agents coincide). Second, term spread appears as a direct determinant in the policy rule. That is, the term spread may play a role beyond the indirect role of being a determinant of inflation expectations as described in equation (8). Finally, the policy rate does not react to any measure of output gap (i.e. its level or growth rate) as assumed in the SW model. This last feature can be rationalized in the AL framework as follows. Output gap is hard to measure and its inclusion presumes that the central banker has a much richer information set than the private sector. Moreover, introducing the output gap would require a characterization of the fully flexible economy which would enlarge the set of forward-looking variables whose PLM must be estimated. Furthermore, as mentioned above, the term spread can be viewed as a proxy for expected changes in the output gap or other measures of economic activity (Estrella and Hardouvelis, 1991, Estrella and Mishkin, 1997).[11]

3 Estimation results

This section starts with a description of the data and the estimation approach, then goes on to discuss the model fit and estimation results.

3.1 Data and estimation approach

The set of observable variables is identical to the one considered by Slobodyan and Wouters (2012a) (i.e. the quarterly series of the inflation rate, the federal funds rate, the log of hours worked and the quarterly log differences of real consumption, real investment, real wages and real GDP) with the addition of the 1-year, 3-year, 5-year, 7-year and 10-year zero-coupon Treasury yields, the SPF forecasts about inflation and consumption growth rates from 1- to 4-quarter horizons, the SPF forecasts of the three-month TB rate from 1- to 3-quarter horizons and the first-release (real-time) inflation. GDP, consumption, investment and hours worked are measured in per-working age population terms. The set of measurement equations

[11] McCallum (1994) is an early paper emphasizing the role of term spreads as simple predictors regarding future macroeconomic conditions in the characterization of monetary policy. More recently, Vázquez, María-Dolores and Londoño (2013) investigated this role in the context of a small-scale New Keynesian monetary model augmented with term structure using both revised and real-time data.

is

$$
X_t = \begin{bmatrix} dlGDP_t \\ dlCONS_t \\ dlINV_t \\ dlWAG_t \\ dlP_t \\ lHours_t \\ FEDFUNDS_t \\ k-year\;TB\;yield_t \\ dlP^r_{t,t+1} \\ dlCONS_t^{e\{j\}} \\ dlP_t^{e\{j\}} \\ r_t^{e\{j\}} \end{bmatrix} = \begin{bmatrix} \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\gamma} \\ \overline{\pi} \\ \overline{l} \\ \overline{r} \\ \overline{r} + \overline{tp}^{\{4k\}} \\ \overline{\pi} \\ \overline{\gamma} \\ \overline{\pi} \\ \overline{r} \end{bmatrix} + \begin{bmatrix} y_t - y_{t-1} \\ c_t - c_{t-1} \\ i_t - i_{t-1} \\ w_t - w_{t-1} \\ \pi_t \\ l_t \\ r_t \\ \bar{r}_t^{\{4k\}} \\ \pi^r_{t,t+1} \\ E_t\left(c_{t+j} - c_{t+j-1}\right) + \epsilon_{c,t}^{\{j\}} \\ E_t \pi_{t+j} + \epsilon_{\pi,t}^{\{j\}} \\ E_t r_{t+j} + \epsilon_{r,t}^{\{j\}} \end{bmatrix}, \qquad (13)
$$

where l and dl denote the log and the log difference, respectively. $\overline{\gamma} = 100(\gamma - 1)$ is the common quarterly trend growth rate for real GDP, real consumption, real investment and real wages, which are the variables featuring a long-run trend. $\bar{l}$, $\overline{\pi}$, $\overline{r}$ and $\overline{tp}^{\{j\}}$ are the steady-state levels of hours worked, inflation, the federal funds rate and the term premium associated with the j-quarter zero-coupon Treasury yield, respectively. The superscripts e and $\{j\}$ in the last three rows of the measurement equation denote actual forecasts from the SPF and the corresponding forecast horizon, respectively. I consider $j = 1, 2, 3, 4$ for disciplining inflation and consumption growth expectations but $j = 1, 2, 3$ when disciplining short term interest rate expectations. Finally, k denotes the bond maturity expressed in years (i.e. $k = 1, 3, 5, 7, 10$); hence $4k$ denotes the maturity in quarters. Hence, the measurement equation involves 24 observable variables, which is much larger than the set of observable variables used in the estimation of standard medium-scale DSGE models (e.g. Smets and Wouters, 2007).

The AL model is estimated using US quarterly data for 1983:3-2014:3. As noted above, I consider consumption growth and short-term interest rate forecasts from the SPF in addition to the inflation forecasts used by Ormeño and Molnár (2015). Inflation forecasts have been reported since as far back as the late 1960's, but the consumption growth forecast time series start at 1981:3. I decided to start the sample period in 1983:3, around the start of the Great Moderation.[12]

Moreover, our model is also estimated for the Great Moderation period running from 1983:3 until 2007:4 to compare our estimated AL term premium measures with those estimated in RE-DSGE models by Dew-Becker (2014) and Kliem and Meyer-Gohde (2017). This exercise further enables me to assess the robustness of the estimated AL term premium when the Great Recession period is considered.

The estimation approach follows a two-step Bayesian estimation procedure. First, the log posterior function is maximized by mixing prior information on the parameters with the likelihood of the data. The prior assumptions are exactly the same as in Slobodyan and Wouters (2012a). Moreover, I consider rather loose priors for the parameters characterizing both bond term premium dynamics and the stationary processes characterizing the deviations of inflation, consumption growth and the short-term interest rate model expectations from the corresponding forecasts reported in the SPF. The second step implements the Metropolis-Hastings algorithm, which runs a very large sequence of draws of all the possible realizations for each parameter in order to obtain its posterior distribution.[13]

In addition to the calibrated parameters considered in Slobodyan and Wouters (2012a), I fix the relative risk aversion parameter at $\sigma_c = 1$ (that is, I assume a log-utility function

[12] In regards to the end period of analysis, although data that is called revised data was available up to 2017:3 when I started to carry out the empirical analysis, the earlier end date for the long sample was chosen so as to be consistent with the timing of the last revision for the data (ignoring any comprehensive or benchmark revisions that may be carried out in the future). In particular, there is a three-year lag before the GDP deflator used to define the rate of inflation is revised for the last time. This lag means that only the data up to 2014:3 can be considered as truly revised data.

[13] The DSGE models are estimated using Dynare codes gently provided by Sergey Slobodyan and Raf Wouters with a number of modifications to accommodate for the presence of the term structure of interest rates in both the structural model and the small forecasting models as described in equation (8).

on consumption). This restriction is imposed for several reasons. By imposing $\sigma_c = 1$, I do not have to deal with hours-worked expectations, which greatly reduces the number of PLM to be estimated. Moreover, while inflation and consumption expectations can be disciplined with the SPF reported at least since 1981, labor forecasts in the SPF started to be released much more recently in the fourth quarter of 2003. Another problem of considering SPF labor forecasts is that they are based on total payroll employment (extensive margin of labor) whereas the model's counterpart is total hours worked. Furthermore, higher values of the risk aversion parameter are needed to solve the bond term premium puzzle—Rudebusch and Swanson (2008, 2012)—under RE. By setting $\sigma_c = 1$, I impose a parameter value much closer to a standard parameterization of the utility function, which helps to highlight the contribution of AL to generating a sizable term premium.

Considering a long term maturity yield, such as the 10-year yield, means that the expectations of consumption, inflation and the short-term interest rate up to a 40-quarter horizon must be characterized. This results in a curse of dimensionality problem. To address this issue, I estimate the forecasting rules described in equation (8), using the SPF forecasts to discipline them as discussed above, and then I impose the following simple recursive structure for consumption, inflation and interest rate expectations on forecast horizons beyond those considered from the SPF:

$$\begin{cases} E_t c_{t+j} = \mu_c E_t c_{t+j-1}, & \\ E_t \pi_{t+j} = \mu_\pi E_t \pi_{t+j-1}, & for \quad j > 4 \\ E_t r_{t+j-1} = \mu_r E_t r_{t+j-2}, & \end{cases} \tag{14}$$

where the parameters μ_c, μ_π and μ_r are estimated jointly with the rest of model parameters. Since consumption and nominal interest rates are expected to be more persistent than inflation, the prior distribution assumed for these three parameters is a Beta-distribution with mean 0.9, 0.8 and 0.9, respectively, and standard deviation 0.15.

3.2 Model fit

I estimate the AL and the RE versions of the model. The posterior log data densities of the AL and RE models are 815.09 and 569.95, respectively. The difference between their log data densities is 245.14 points, which results in a huge posterior odd of 2.9e+106. This difference in favor of the AL learning specification suggests that a real-time learning approach based on term structure information greatly improves the joint fit of macroeconomic times series and the yield curve as well as the inflation, consumption growth and the short-term interest rate forecasts reported in the SPF. In particular, as shown in Figure 1, the AL model fits the zero-coupon yields for alternative maturities well. Moreover, Figure 2 shows that AL model's expectations resemble the SPF forecasts of inflation, consumption growth and the short-term interest rate, which are crucial to account effectively for the consumption and inflation risks characterizing the term premium.

Beyond the overall model fit based on the posterior log data density, I also analyze the performance of the AL model to reproduce selected second-moment statistics obtained from actual data as shown in Table 1. I focus on two types of moment: standard deviations and first-order autocorrelations. For standard deviations, I observe that the estimated AL model is able to match the volatility of all variables reasonably well, at least qualitatively. For first-order autocorrelation, the AL model reproduces the persistence of the growth rates of most real variables rather well, but overestimates inflation and real wage persistence.

Table 1. Actual and simulated second moments (1983:3-2014:3)

Actual data	Δc	Δinv	Δw	Δy	π
Standard deviation	0.61	1.93	0.87	0.63	0.24
Autocorrelation	0.35	0.67	-0.19	0.38	0.62
Simulated data					
Standard deviation	0.73	1.64	1.09	0.82	0.37
Autocorrelation	0.20	0.59	0.48	0.26	0.93

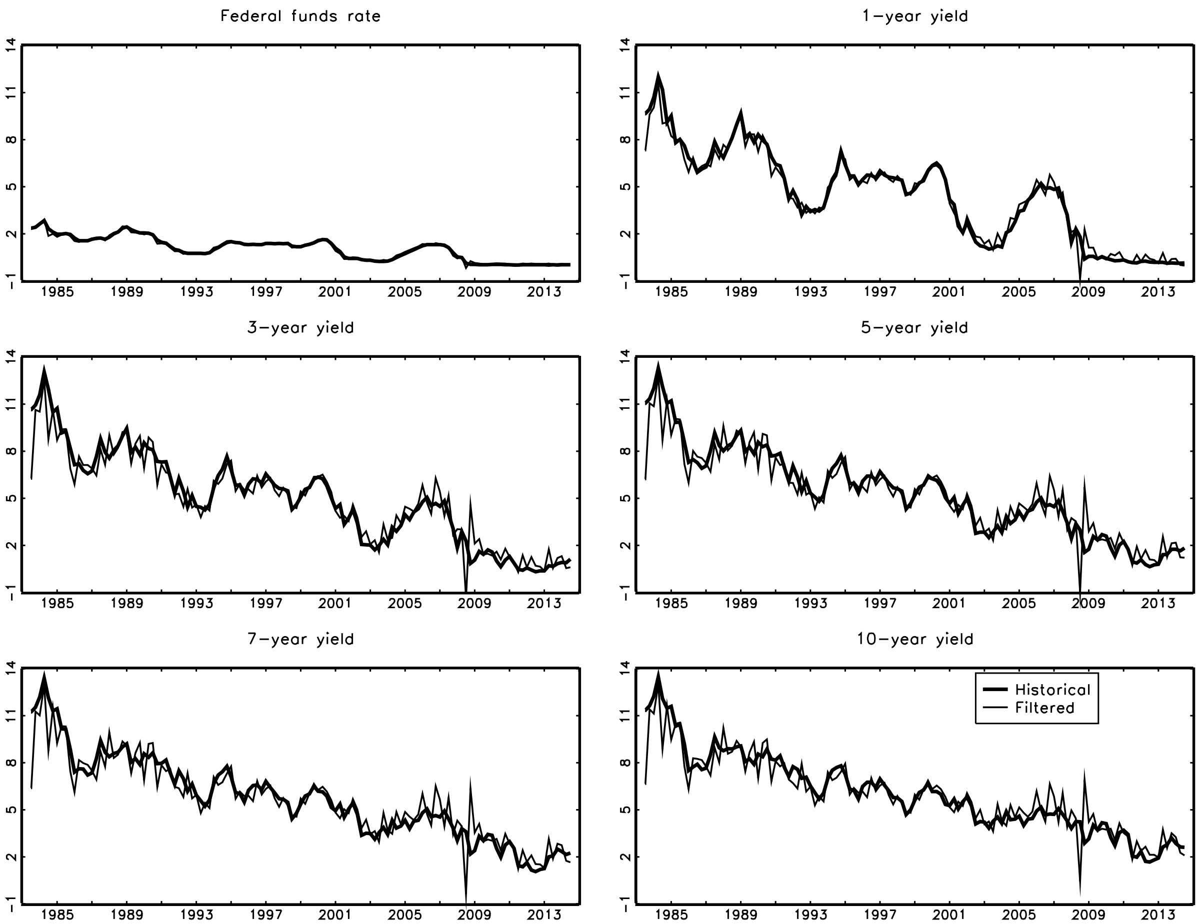

Figure 1. Historical and filtered annualized yields

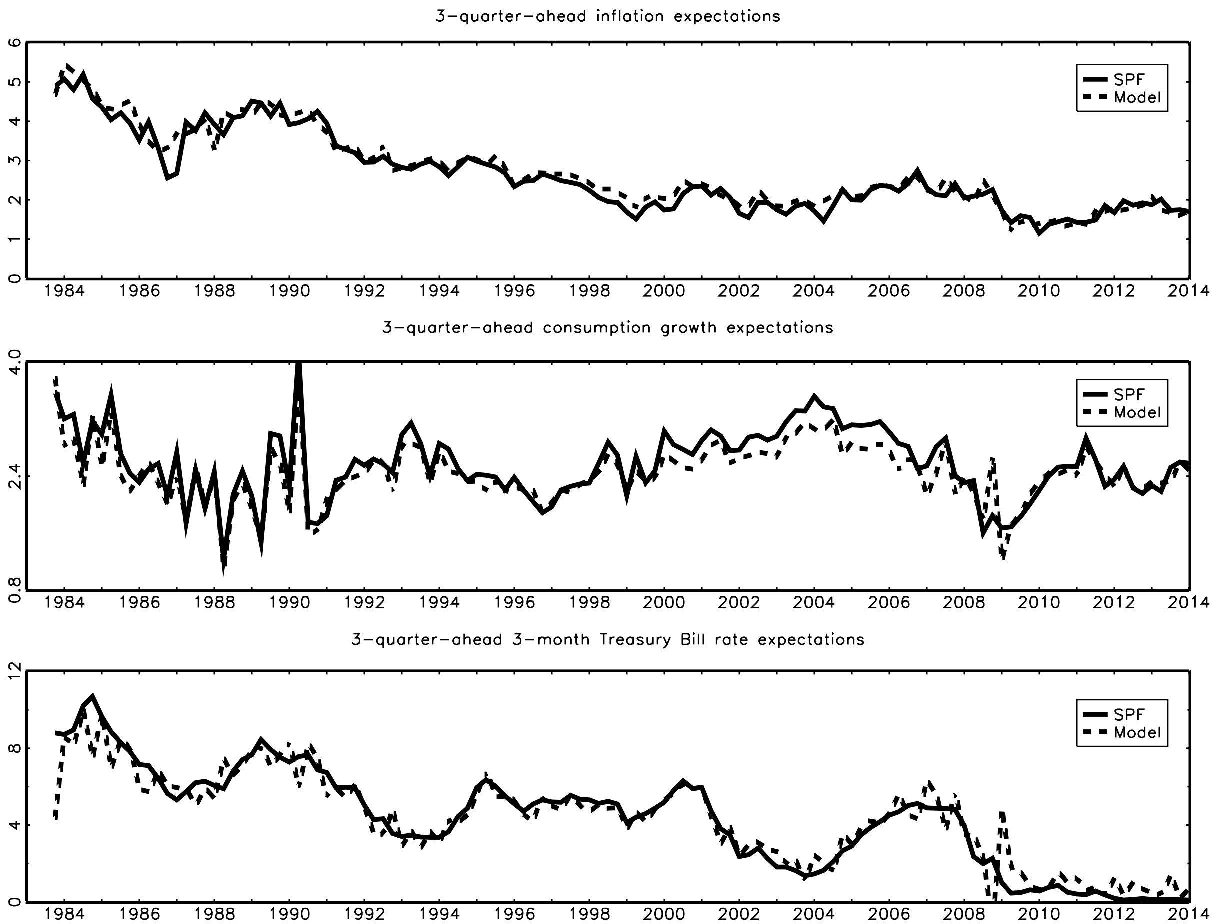

Figure 2. Expectations comparison

3.3 Posterior estimates

Table 2 shows the estimation results for a selected group of parameters featuring both endogenous and exogenous persistence for the AL and the RE (linearized) versions of the model.[14] Confirming results in Aguilar and Vázquez (2017, 2018), some sources of endogenous persistence lose a great deal of their importance. Thus, the estimates of the elasticity of the cost of adjusting capital, φ, the elasticity of the capital utilization adjusting cost, ψ, and Calvo's

[14] Appendix 3 displays the full set of parameter estimates.

price probability are much smaller under AL (1.47, 0.51, and 0.69) than under RE (7.26, 0.78 and 0.91). Regarding the exogenous sources of price and wage markup persistence, I find that the autoregressive coefficients of price and wage markup shocks are similar in the AL and RE models. However, moving average coefficients are lower in the AL model than in the RE model, which implies that these shocks are somewhat less persistent in the AL model than in the RE model. Regarding policy rule parameters, the AL model exhibits lower persistence than the RE model whereas the remaining coefficients are rather similar in the two models. In particular, the term spread coefficient, r_{sp}, takes values in the two models similar to those estimated by Smets and Wouters (2007) and Slobodyan and Wouters (2012a) for the output gap and the first difference of the output gap coefficients showing up in a standard Taylor rule, which supports the view that the term spread can be viewed as a sound proxy for the cyclical component of output.

The estimates of the steady-state term premium for the alternative maturities are slightly smaller under AL than under RE. The weight of the EH, measured by the weight $(1 - \lambda_j)$, is observed to show a U-shaped relationship with bond maturity for both AL and RE.[15]

The last column of Table 2 shows the estimates of the selected group of parameters obtained from the AL model for the Great Moderation period. In general, we observe that the parameter estimates are fairly robust. In particular, the confidence intervals of the policy rule parameter estimates associated with the whole sample (1983:3-2014:3) and the Great Moderation subsample (1983:3-2007:4) largely overlap, with the exception of the inflation coefficient which is slightly smaller during the Great Moderation. Similarly, the increasing pattern of the estimates of the steady-state term premium, $\bar{tp}^{\{j\}}$, as j increases and the hump-shaped pattern of weights, λ_j, are robust across sample periods. The confidence intervals

[15] As discussed above, the parameter λ_j typically cannot be identified under RE due to the certainty equivalence property associated with a first-order approximation of a DSGE model. However, λ_j can be identified in the present framework under RE because RE are also disciplined in the estimation exercise by using the forecasts reported in the SPF. Thus, the specification of the yield based on the consumption-based approach, $r_t^{\{j\}}$, relies on consumption and inflation expectations, which are disciplined by SPF data, whereas the yield based on the EH of the term structure, $r_t^{EH\{j\}}$, depends on interest rate expectations, which are also disciplined using SPF data.

associated with the estimate of the elasticity of the cost adjusting capital, φ, do not overlap, but they are closer than the one estimated from the RE version of the model. In spite of these differences in parameter estimates, the (implied) estimated AL term premium is fairly robust across the two sample periods as shown below.

Table 2. Selected parameter estimates

	1983:3-2014:3		1983:3-2007:4
	AL	RE	AL
log data density	815.09	569.95	652.96
Parameters associated with real rigidities			
habit formation (h)	0.49	0.33	0.48
	(0.46,0.52)	(0.30,0.36)	(0.44,0.52)
cost of adjusting capital (φ)	1.47	7.26	1.94
	(1.41,1.54)	(6.07,8.54)	(1.83,2.02)
capital utilization adjusting cost (ψ)	0.51	0.78	0.44
	(0.47,0.56)	(0.74,0.83)	(0.38,0.50)
Parameters associated with nominal rigidities			
Calvo price probability (ξ_p)	0.69	0.911	0.64
	(0.66,0.72)	(0.906,0.917)	(0.61,0.67)
Calvo wage probability (ξ_w)	0.38	0.34	0.35
	(0.33,0.42)	(0.24,0.42)	(0.31,0.39)
price indexation (ι_p)	0.19	0.12	0.18
	(0.13,0.25)	(0.08,0.17)	(0.15,0.22)
wage indexation (ι_w)	0.64	0.35	0.62
	(0.58,0.70)	(0.17,0.53)	(0.55,0.68)

Table 2. (*Continued*)

	1983:3-2014:3		1983:3-2007:4
	AL	RE	AL
Parameters associated with price and wage markups			
mark-up price AR coef. (ρ_p)	0.94	0.87	0.93
	(0.91,0.98)	(0.82,0.93)	(0.89,0.96)
mark-up wage AR coef. (ρ_w)	0.95	0.968	0.93
	(0.93,0.99)	(0.965,0.971)	(0.89,0.96)
mark-up price MA coef. (μ_p)	0.63	0.86	0.67
	(0.58,0.67)	(0.81,0.94)	(0.62,0.72)
mark-up wage MA coef. (μ_w)	0.51	0.81	0.30
	(0.47,0.56)	(0.76,0.87)	(0.25,0.35)
Policy rule parameters			
inertia (ρ_r)	0.57	0.84	0.59
	(0.50,0.63)	(0.82,0.85)	(0.52,0.68)
inflation (r_π)	2.14	2.15	1.84
	(2.10,2.22)	(2.02,2.28)	(1.78,1.91)
term spread (r_{sp})	0.08	0.05	0.09
	(0.04, 0.11)	(0.04,0.06)	(0.07,0.13)

Aguilar and Vázquez (2018) discuss at length the transmission mechanism of shocks and the PLM associated with a rather similar version of this AL model. Consequently, we move on in the next section to the main goal of this paper: the analysis of term premium dynamics under AL.

Table 2. (*Continued*)

	1983:3-2014:3		1983:3-2007:4
	AL	RE	AL
Steady-state term premium for alternative maturities			
1-year ($\bar{tp}^{\{4\}}$)	-0.16	0.06	-0.03
	(-0.20,-0.12)	(0.04,0.08)	(-0.10,0.02)
3-year ($\bar{tp}^{\{12\}}$)	0.18	0.20	0.20
	(0.12,0.25)	(0.15,0.26)	(0.16,0.24)
5-year ($\bar{tp}^{\{20\}}$)	0.35	0.31	0.31
	(0.32,0.39)	(0.17,0.44)	(0.27,0.34)
7-year ($\bar{tp}^{\{28\}}$)	0.43	0.66	0.36
	(0.39,0.46)	(0.56,0.74)	(0.33,0.38)
10-year ($\bar{tp}^{\{40\}}$)	0.50	0.66	0.42
	(0.47,0.54)	(0.10,0.35)	(0.40,0.45)
The importance of the EH weight: $(1-\lambda_j)$			
1-year (λ_1)	0.63	0.09	0.62
	(0.57,0.69)	(0.08,0.11)	(0.56,0.70)
3-year (λ_3)	0.72	0.30	0.93
	(0.64,0.81)	(0.24,0.36)	(0.89,0.98)
5-year (λ_5)	0.88	0.73	0.98
	(0.83,0.95)	(0.68,0.83)	(0.95,1.0)
7-year (λ_7)	0.82	0.996	0.86
	(0.78,0.88)	(0.993,1.0)	(0.83,0.89)
10-year (λ_{10})	0.77	0.91	0.78
	(0.70, 0.83)	(0.88,0.94)	(0.73,0.81)

Notes: Parameter notation and standard deviation in parentheses

4 AL term premium features

This section analyzes the term premium features across medium- and long-term maturities. To that end, I compare the estimated AL term premia with those estimated from the non-arbitrage affine model of Adrian, Crump and Moench (2013).[16]

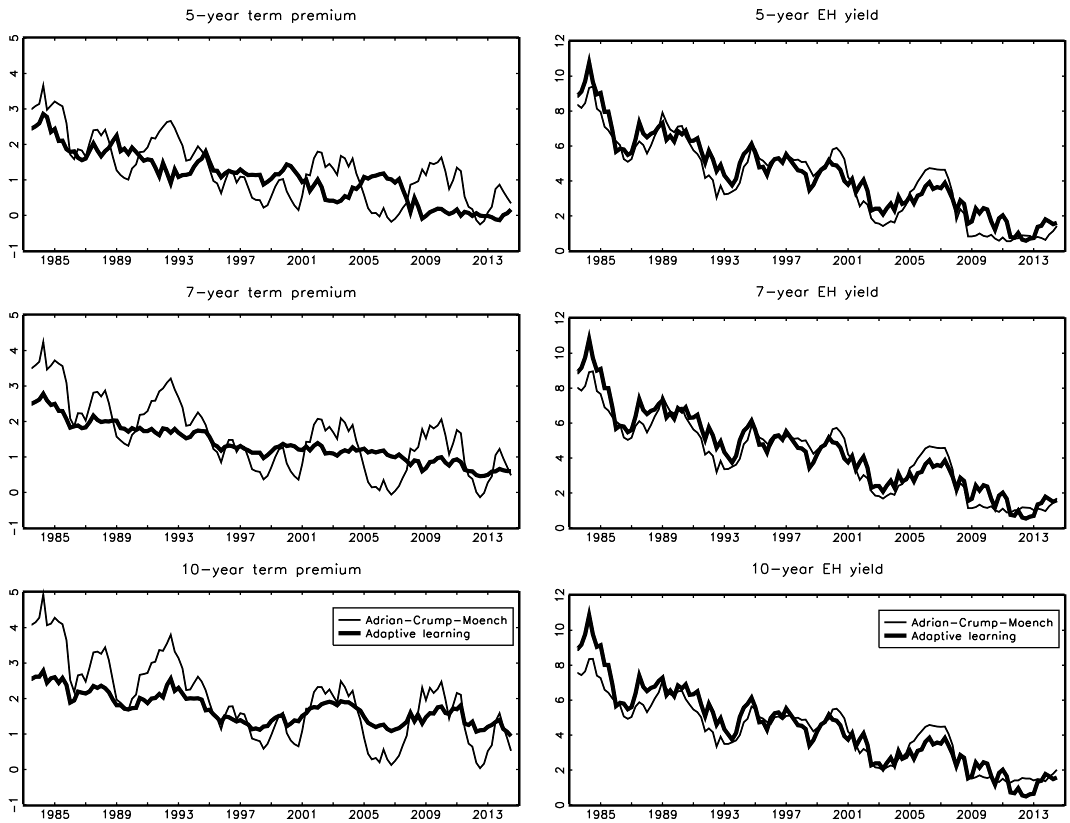

Figure 3. Annualized term premia and EH yields

Note: The AL term premia reported in this figure are computed as $4\times\left[\left(r_t^{\{j\}}-r_t^{EH\{j\}}\right)+\overline{tp}^{\{j\}}\right]$ and the yields implied by EH are calculated as $4\times\left(\overline{r}+r_t^{EH\{j\}}\right)$ where $\overline{r}$ is the steady-state short-term nominal interest rate.

[16] As shown in Kliem and Meyer-Gohde (2017), alternative non-arbitrage affine models, such as those estimated by Kim and Wright (2008) and Adrian, Crump and Moench (2013), result in comparable term premium time series. I focus on the latter because it is published and regularly updated by the New York Federal Reserve.

The left column graphs of Figure 3 shows the estimated term premia from the AL model studied in this paper (thick line), together with that based on the non-arbitrage affine model of Adrian, Crump and Moench (2013) (thin line) —henceforth the ACM term premium— for the sample period 1983:3-2014:3. Two main conclusions emerge from these graphs. First, the term premia associated with the alternative maturities exhibit a downward trend consistent with that observed for inflation during this sample period, but the downward trend looks milder for the AL term premia. Second, although they are smoother than the ACM term premia, the AL term premia display sizable fluctuations. As shown in the right column graphs of Figure 3, these two conclusions are largely explained by the different features displayed by the EH yields (i.e. risk neutral yields) estimated by Adrian, Crump and Moench (2013) and those estimated under AL. Thus, the EH yields under AL exhibit a more pronounced downward trend and they are more volatile, which result in smoother and flatter term premia time series. These findings are line with those found in Bauer, Rudebusch and Wu (2014) for the US when correcting for estimation small-sample bias in dynamic term structure models.

Beyond the visual inspection provided by Figure 3, Table 3 shows statistics (mean, standard deviation, first-order autocorrelation and contemporaneous cross-correlations) that point to further similarities between these two alternative term premia across maturities. Thus, the estimated AL term premium is sizable, highly persistent and closely correlated with the ACM term premium. This is particularly striking for the term premium associated with the 10-year yield, for which the contemporaneous correlation between the two term premia is a strikingly high figure of 0.96. This is a quite remarkable finding because the AL model, a fully-fledged DSGE model, is very different from the non-arbitrage affine model of the term structure (i.e. a reduced-form model) used to estimate the ACM term premium.

Table 3. Term premium statistics (1983:3-2014:3)

	Mean		Standard deviation	
	ACM	AL	ACM	AL
5-year	1.29	1.08	0.89	0.72
7-year	1.59	1.37	0.99	0.53
10-year	1.95	1.71	1.11	0.44
	Auto correlation		Cross correlation	
	ACM	AL	ACM,AL	
5-year	0.95	0.97	0.61	
7-year	0.95	0.98	0.80	
10-year	0.95	0.95	0.96	

Table 4 shows the variance decomposition of two important nominal variables (inflation and the policy interest rate) and the term premium associated with the 5- and 10-year maturity yields ($tp^{\{20\}}$ and $tp^{\{40\}}$, respectively). Focusing on the term premia, it is interesting to observe that, apart from risk premium and (short- and long-term) term premium shocks, a sizable share of the variance decomposition of the AL term premium associated with medium- and long-term maturity yields is explained by monetary policy shocks. For instance, the share of monetary policy shocks in the short-term variance decomposition of the 10-year term premium is 8.5, whereas the long-term variance decomposition increases to 18%. This finding shows a sizable propagation mechanism of monetary policy shocks to the term premium.

Table 4. Variance decomposition

	π	r	$tp^{\{20\}}$	$tp^{\{40\}}$
Productivity	1.5/1.5	0/0	0.1/0.1	0.1/0.1
Risk premium	1.4/2.8	2.3/4.1	9.2/12.8	11.3/18.3
Exogenous spending	0.1/0.4	0/0	0/0	0/0
Invest. specific tech.	0/1.1	0/0.1	0/0.1	0/0.1
Monetary policy	6.2/15.3	75.5/63.9	13.7/23.3	8.5/18
Price mark-up	62.7/42.6	0.8/1.0	4.9/3.3	3.2/2.6
Wage mark-up	3.7/4.3	0/0.1	0.2/0.3	0.1/0.3
Inflation revision	1.1/0.5	0.8/0.3	6.2/2.1	4/1.6
Short-term premium	20.6/28.2	19.9/29.5	59.1/52.5	38.9/42.3
Long-term premium	2.7/3.4	0.7/1.1	6.4/5.5	34/16.8

Notes: Each cell reports the contributions to the forecast error variance of the corresponding variable for the 1-year and 10-year forecast horizon, respectively. Short (Long)-term premium contributions show the result of adding the contributions of 1- and 3-year (5-year, 7-year and 10-year) term premium shocks.

The importance of disciplining short rate expectations and considering the Great Recession

The estimated term premium from the baseline AL model considers both data from the Great Recession period and SPF forecasts of the 3-month TB rate to discipline short-term interest rate expectations. We assess the importance of these two features in the estimation of the term premium by carrying out two sensitivity analysis. First, I remove SPF forecasts of the 3-month TB rate from the measurement equation. Second, I estimate the AL model using only data for the Great Moderation period.

More precisely, this subsection compares the estimated AL and the ACM 10-year term premium measures with that estimated in Dew-Becker (2014) using RE-DSGE models, the estimated AL obtained using only data from the Great Moderation period and the 10-year term spread. As pointed out by Rudebusch and Swanson (2012), the term spread is a directly observable, although imperfect, measure of term premium and as such it is a useful benchmark for comparing alternative measures of term premium. Figure 4 shows the three estimated term premia together with the 10-year term spread (i.e. the difference between the 10-year yield and the federal funds rate). In addition, I plot the estimated term premium when the SPF forecasts of the 3-month TB rate are not considered in the measurement equation (dotted line) in order to illustrate the importance of disciplining the short-term interest rate expectations in the characterization of the AL term premium.[17]

First of all, notice that the estimates of the Dew-Becker term premium in the first part of the sample (say until 1990) display an upward trend, which is at odds with the AL and ACM term premia and with the term spread. Interestingly, the estimated AL term premium obtained when the SPF forecasts of the 3-month interest rate are ignored also displays an upward trend. In order to avoid spurious correlations, I focus on the subsample period 1990:3-2004:4 in Table 5—for the sake of completeness this table also shows the correlation between alternative measures of term premium for the whole Great Moderation period (1983:3-2007:4) in italics. The first observation of the subsample period 1990:3-2004:4 is determined by the availability of data regarding the term premium estimated by Federal Reserve Board staff based on a simple three-factor arbitrage-free term structure model (Kim and Wright, 2005), which is also included in this comparison exercise. Thus, this table shows the cross-correlations between the four alternative measures of the term premium and the term spread associated with the 10-year yield considered in Figure 4 together with the Kim-Wright (KW) term premium.

[17] I thank Ian Dew-Becker for kindly sharing his estimated term premium time series. Dew-Becker (2014) uses the 3-month TB rate to define the term spread in his Figure 4 instead of the federal funds rate used here. Since these two short-term rates are highly correlated the associated term spreads are also very similar. Moreover, the term premium of Dew-Becker (2014) is estimated for the period 1983:1-2004:4, then, I consider the overlapping sample period (i.e. 1983:3-2004:4) of his sample and the sample considered in this paper when plotting the five time series together.

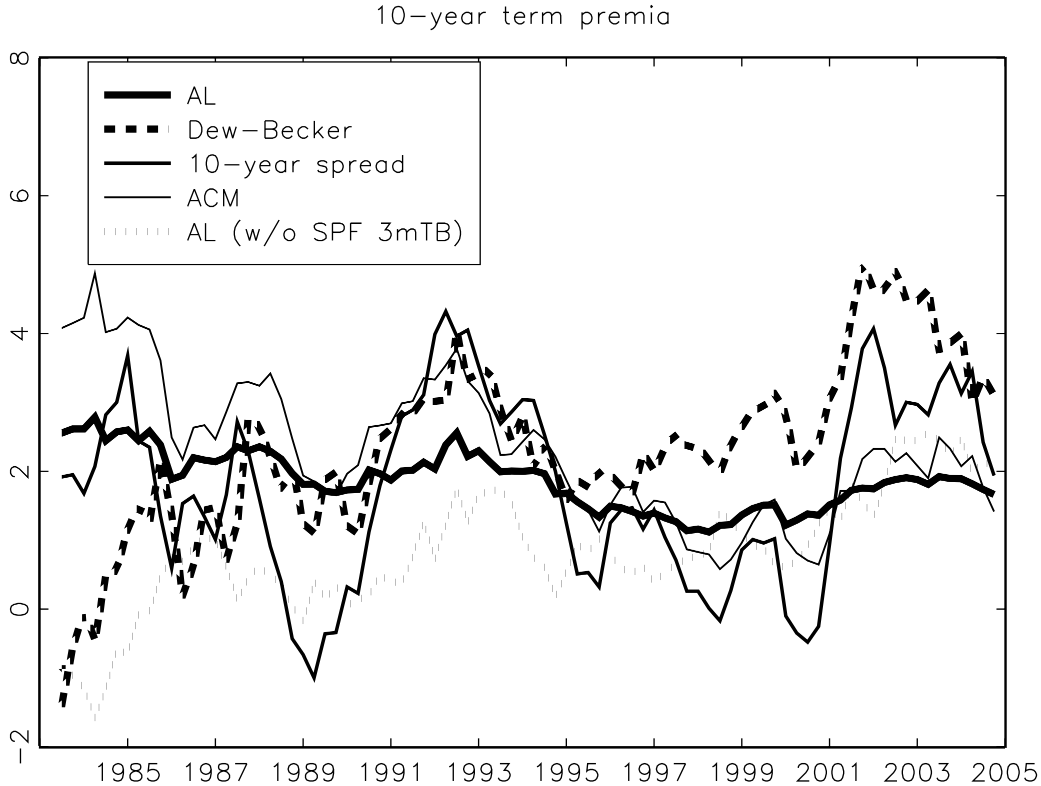

Figure 4. Term premium comparison during the Great Moderation

The strong correlation between ACM and AL term premia during this subsample (0.94) confirms the finding obtained above for the whole sample. Moreover, these two term premia are also closely correlated with the term spread (roughly 0.86), whereas the Dew-Becker term premium shows much lower correlation with both ACM and AL term premia and the term spread (0.36, 0.45 and 0.66, respectively). However, the Dew-Decker term premium exhibits close correlation with the AL term premium obtained without considering the SPF forecasts of the 3-month Treasury bill (3mTB) at 0.77. This finding may suggest that the Dew-Becker term premium is not capturing the risk neutral yield well since, according to our results, SPF forecasts of the short-term rate seem to be crucial for disciplining the yield implied by the expectations hypothesis of the term structure (i.e. the risk neutral yield) and the term premium. Moreover, Table 5 shows that there is a strong correlation between the

KW term premium and the AL term premium estimated using only data from the Great Moderation period (0.92), which is stronger than the correlations with the ACM and AL term premia obtained using the whole sample period and the term spread (0.79, 0.63 and 0.48, respectively).

Figure 5 shows that the estimated 10-year AL term premium obtained using only data from the Great Moderation (GM) subsample (1983:3-2007:4) displays a similar pattern to those followed by the AL and the ACM term premium both estimated using the whole sample. As shown in Table 5, the cross-correlation between the estimated AL term premium obtained with the Great Moderation sample period and the AL (ACM) term premium estimated with the whole sample period is also strong at 0.92 (0.93).

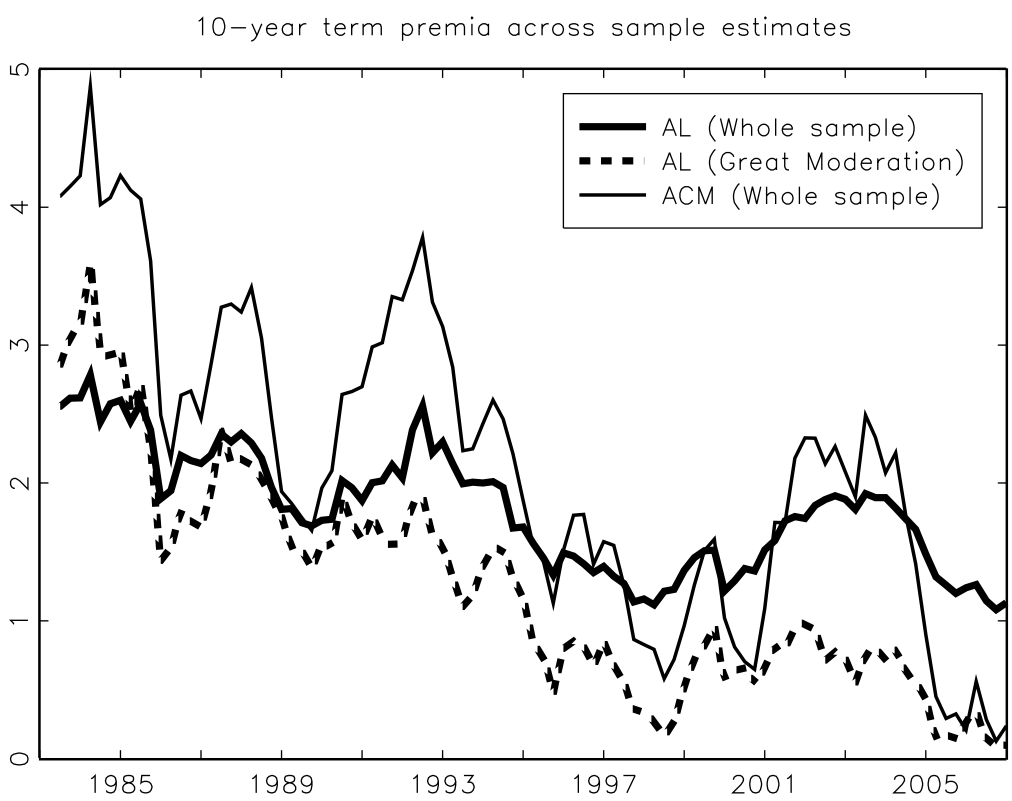

Figure 5. 10-year term premia across sample estimates

In sum, there main conclusions are drawn from these comparison exercises. First, the estimated term premium dynamics are not too sensitive to including data from the Great Recession period or just focusing on the Great Moderation period. Second, the disciplining of the short-term interest rate expectations with SPF forecasts proves to be crucial for estimating an AL term premium that resembles the ACM term premium. Finally, AL takes over other features needed to generate a sizable term premium under RE in the related literature (Rudebusch and Swanson, 2008, 2012; Dew-Becker, 2014; Kliem and Meyer-Gohde, 2017). Thus, the estimated AL term premium is based on a first-order approximation of a standard DSGE model characterized by a logarithmic utility function, which features low risk aversion.

Table 5. 10-year term premium cross-correlations (Great Moderation period)

	ACM	AL	Dew-Becker	AL (w/o SPF 3mTB)	AL GM	KW
AL	0.94 *0.96*					
Dew-Becker	0.36 –	0.45 –				
AL (w/o SPF 3mTB)	0.15 *-0.49*	0.35 *-0.40*	0.77 –			
AL (GM)	0.89 *0.93*	0.83 *0.92*	0.04 –	-0.23 *-0.72*		
KW	0.79 –	0.63 –	-0.14 –	-0.46 –	0.92 –	
Spread	0.87 *0.64*	0.86 *0.62*	0.66 –	0.48 *0.18*	0.61 *0.37*	0.48 –

Notes: Each cell contains two entries. The top entry shows the correlation associated with the subsample 1990:3-2004:4, whereas the bottom entry shows the same correlation coefficient for the whole Great Moderation period 1983:3-2007:4 in italics.

5 Term premium and the business cycle

This section studies the comovement of the estimated AL term premium with the business cycle. We compute the auto-correlations as well as the contemporaneous and lead- and lag-correlations of the 10-year term premium and three prominent indicators of the business cycle (GDP growth rate, inflation and the federal funds rate) from the estimation of a four-variable vector autoregression following methods described in Hamilton (1994, pp. 264-266) and popularized by Fuhrer and Moore (1995), among others. I consider the whole sample period in this comovement analysis. Figure 6 shows these correlations for two specifications of the VAR. One of them (solid lines) uses the estimated AL term premium whereas the other (dashed line) considers the ACM term premium. The main-diagonal graphs show the autocorrelation functions of the four variables considered in the VAR. At the same time, the off-diagonal graphs show lead and lag correlations between alternative pairs of variables. Although there are a few noteworthy differences, discussed below, between the lead- and lag correlations obtained with the two alternative term premium measures, in general they are fairly similar. The main-diagonal shows that the four variables exhibit a great deal of persistence. In particular, the two term premia are highly persistent, but the AL term premium is more persistent than the ACM, probably due to the strong persistence resulting from the AL process.

Focusing on the off-diagonal graphs involving lead/lag correlations of the term premium measures with the macroeconomic variables, three interesting features can be observed. First, both term premium measures are clearly procyclical. Thus the contemporaneous correlation between the AL (ACM) term premium and the GDP growth rate is 0.4 (0.55). Second, both a high (low) rate for GDP growth and federal funds tend to anticipate high (low) term premium. This finding suggests that at business cycle peaks, when the GDP growth rate and the policy interest rate are high, term premium is expected to increase 6-10 quarters ahead, which is anticipating a slowdown of economic activity. By contrast, at business cycle troughs low GDP growth rates and low policy interest rates anticipate a low term premium, which is consistent with a scenario where the economy is about to start a recovery. Notice that these lead patterns of both GDP growth rate and federal funds rate over the term premium are slightly larger in size (given by the maximum correlation value) and duration (given by the number of lags needed to reach the maximum correlation value) for the AL term premium than for the ACM term premium. These features are in line with a sluggish learning process. Second, the correlation between contemporaneous inflation and the 4-5 quarter lagged AL term premium is small, but negative (-0.25) suggesting that a low AL term premium somehow anticipates high inflation one year ahead.

The growth rate of GDP emphasizes the high-frequency components of economic activity. Therefore, I re-estimate the stationary 4-variable VAR by including the cyclical component of GDP computed with the HP filter (Hodrick and Prescott, 1997), which captures better than the GDP growth rate the business cycle frequencies. A comparison of Figures 6 and 7 shows that the estimated dynamic comovements are robust to the cyclical component of GDP used in the analysis, but those associated with the cyclical component of GDP chosen. In particular, the contemporaneous correlation of the term premium and GDP turns negative (around -0.35 for the estimated AL term premium) when HP output is considered. This countercyclicality of both the AL and the ACM term premium is consistent with much of the theoretical and empirical literature (among others, Campbell and Cochrane, 1999; Cochrane and Piazzesi, 2005) as emphasized by Bauer, Rudebusch and Wu (2014).

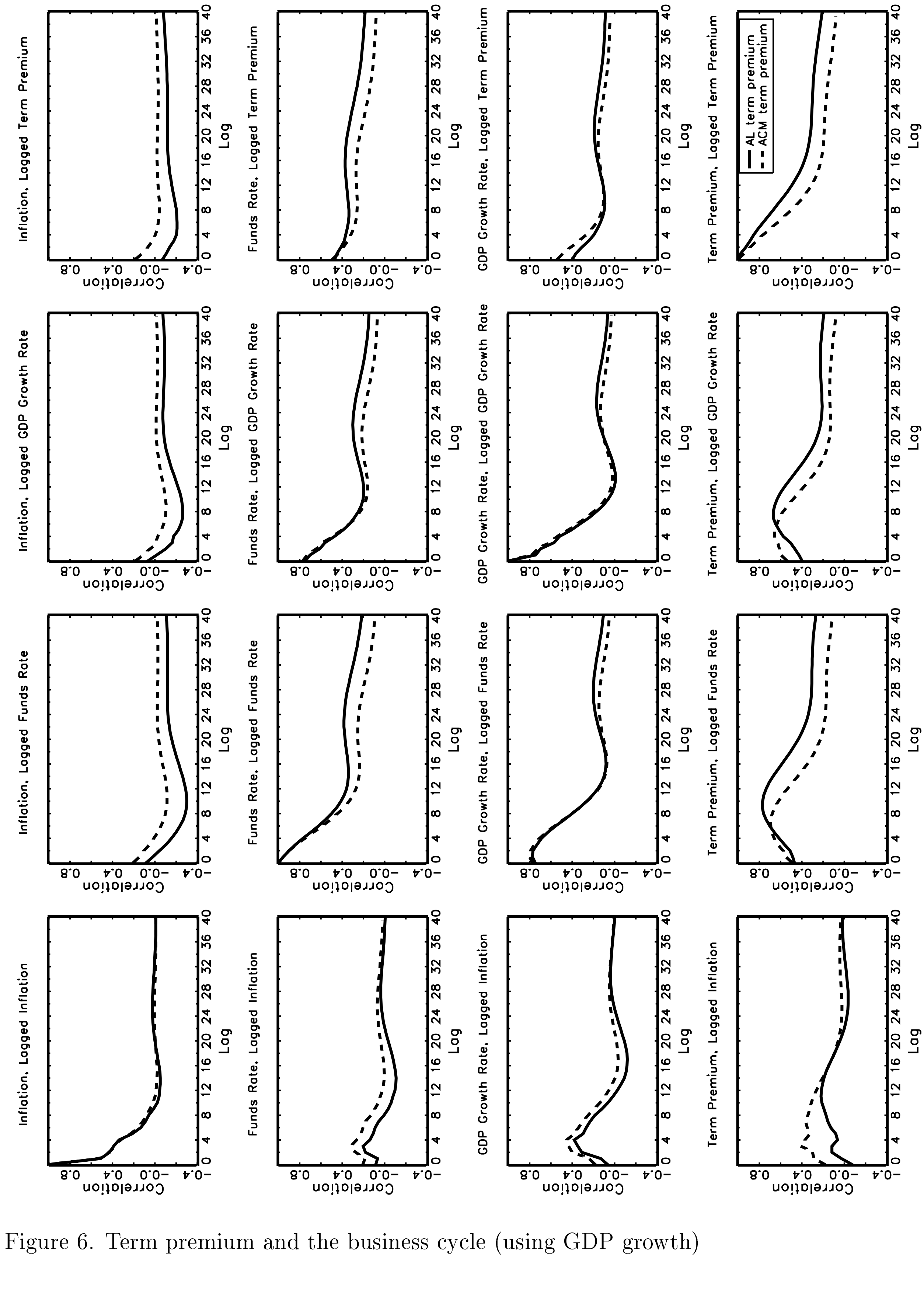

Figure 6. Term premium and the business cycle (using GDP growth)

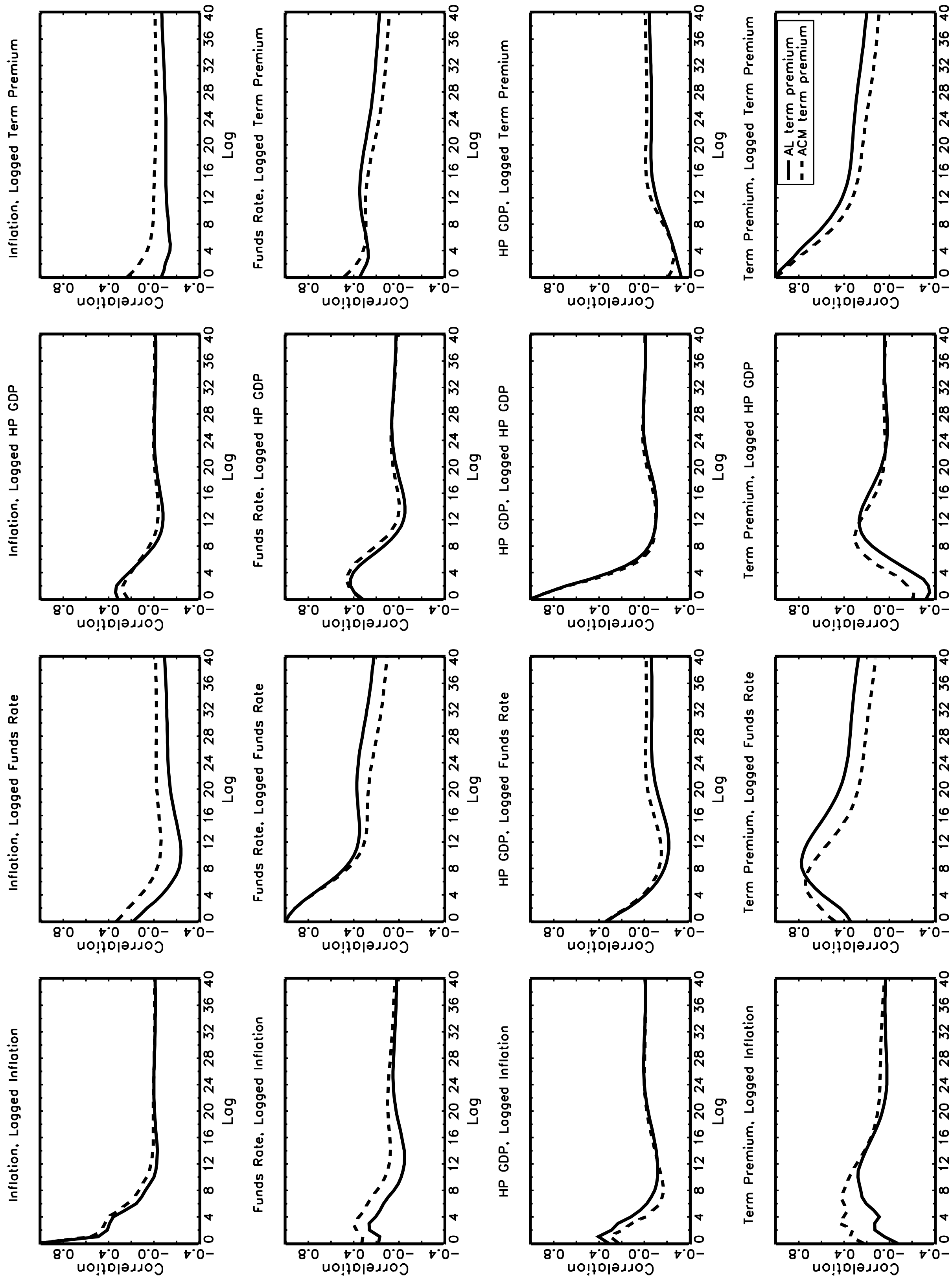

Figure 7. Term premium and the business cycle (using Hodrick-Prescott filtered GDP)

6 Conclusions

This paper provides an alternative measure of the bond term premium based on a DSGE model featuring adaptive learning (AL). In an AL framework, agents do not know the structure of the economy, so they face what I call a first-order uncertainty: they do not know what the true model is or what the alternative sources of fundamental uncertainty are, and they have to learn about how the economy behaves from the times series that they observe. Following Aguilar and Vázquez (2017, 2018), I extend the AL model of Slobodyan and Wouters (2012a) by introducing the term structure of interest rates. In addition to real-time inflation data, this extension enables the term structure of interest rates to fully characterize the expectations of all forward-looking variables of the model using only information available when expectations are formed in real time. I view the use of real-time information as a crucial step forward in characterizing the bond term premium because consumption and inflation risks associated with the concept of term premium are very much determined by the fact that investors' decisions are taken in a much more limited information scenario than the one assumed in rational expectations (RE) models.

The estimated AL term premia share important features with those estimated from no-arbitrage affine term structure models (e.g. Adrian, Crump and Moench, 2013) and those estimated from DSGE models under RE (Kliem and Meyer-Gohde, 2017). These findings reveal that the non-linear features introduced by AL take over other non-linear features needed to generate a sizable term premium under RE such as a large coefficient featuring risk aversion.

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Appendix 1

Set of the remaining log-linearized dynamic equations:

- Aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \varepsilon_t^g, \tag{15}$$

where $c_y = \frac{C}{Y} = 1 - g_y - i_y$, $i_y = \frac{I}{Y} = (\gamma - 1 + \delta)\frac{K}{Y}$, and $z_y = r^k \frac{K}{Y}$ are steady-state ratios. As in Smets and Wouters (2007), the depreciation rate and the exogenous spending-GDP ratio are fixed in the estimation procedure at $\delta = 0.025$ and $g_y = 0.18$.

- Investment equation:

$$i_t = i_1 i_{t-1} + (1 - i_1) E_t i_{t+1} + i_2 q_t + \varepsilon_t^i, \tag{16}$$

where $i_1 = \frac{1}{1+\overline{\beta}}$, and $i_2 = \frac{1}{(1+\overline{\beta})\gamma^2 \varphi}$ with $\overline{\beta} = \beta \gamma^{(1-\sigma_c)}$.

- Arbitrage condition (value of capital, q_t):

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r_{t+1}^k - (R_t - E_t \pi_{t+1}) + c_3^{-1} \varepsilon_t^b, \tag{17}$$

where $q_1 = \overline{\beta}\gamma^{-1}(1-\delta) = \frac{(1-\delta)}{(r^k + 1 - \delta)}$.

- Log-linearized aggregate production function:

$$y_t = \Phi \left(\alpha k_t^s + (1-\alpha) l_t + \varepsilon_t^a \right), \tag{18}$$

where $\Phi = 1 + \frac{\phi}{Y} = 1 + \frac{\text{Steady-state fixed cost}}{Y}$ and α is the capital-share in the production function.[18]

- Effective capital (with one period time-to-build):

$$k_t^s = k_{t-1} + z_t. \tag{19}$$

[18] From the zero profit condition in steady-state, it should be noticed that ϕ_p also represents the value of the steady-state price mark-up.

- Capital utilization:

$$z_t = z_1 r_t^k, \tag{20}$$

where $z_1 = \frac{1-\psi}{\psi}$.

- Capital accumulation equation:

$$k_t = k_1 k_{t-1} + (1-k_1) i_t + k_2 \varepsilon_t^i, \tag{21}$$

where $k_1 = \frac{1-\delta}{\gamma}$ and $k_2 = \left(1 - \frac{1-\delta}{\gamma}\right)\left(1+\overline{\beta}\right)\gamma^2 \varphi$.

- Marginal cost:

$$mc_t = (1-\alpha) w_t + \alpha r_t^k - \varepsilon_t^a. \tag{22}$$

- New-Keynesian Phillips curve (price inflation dynamics):

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 mc_t + \pi_4 \varepsilon_t^p, \tag{23}$$

where $\pi_1 = \frac{\iota_p}{1+\overline{\beta}\iota_p}$, $\pi_2 = \frac{\overline{\beta}}{1+\overline{\beta}\iota_p}$, $\pi_3 = \frac{A}{1+\overline{\beta}\iota_p}\left[\frac{\left(1-\overline{\beta}\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\right]$, and $\pi_4 = \frac{1+\overline{\beta}\iota_p}{1+\overline{\beta}\iota_p}$. The coefficient of the curvature of the Kimball goods market aggregator, included in the definition of A, is fixed in the estimation procedure at $\varepsilon_p = 10$ as in Smets and Wouters (2007).

- Optimal demand for capital by firms:

$$-\left(k_t^s - l_t\right) + w_t = r_t^k. \tag{24}$$

- Wage markup equation:

$$\mu_t^w = w_t - mrs_t = w_t - \left(\sigma_l l_t + \frac{1}{1-h/\gamma}\left(c_t - (h/\gamma)\, c_{t-1}\right)\right). \tag{25}$$

- Real wage dynamic equation:

$$w_t = w_1 w_{t-1} + (1-w_1)\left(E_t w_{t+1} + E_t \pi_{t+1}\right) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \varepsilon_t^w. \tag{26}$$

where $w_1 = \frac{1}{1+\overline{\beta}}$, $w_2 = \frac{1+\overline{\beta}\iota_w}{1+\overline{\beta}}$, $w_3 = \frac{\iota_w}{1+\overline{\beta}}$, $w_4 = \frac{1}{1+\overline{\beta}}\left[\frac{(1-\overline{\beta}\xi_w)(1-\xi_w)}{\xi_w((\phi_w-1)\varepsilon_w+1)}\right]$ with the curvature of the Kimball labor aggregator fixed at $\varepsilon_w = 10.0$ and a steady-state wage mark-up fixed at $\phi_w = 1.5$ as in Smets and Wouters (2007)

Appendix 2

This appendix provides a brief explanation of how AL expectation formation works.[19] A DSGE model can be represented in matrix form as follows:

$$A_0 \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + A_1 \begin{bmatrix} y_t \\ w_t \end{bmatrix} + A_2 E_t y_{t+j} + B_0 \epsilon_t = 0,$$

where y_t is the vector of endogenous variables at time t, $E_t y_{t+j}$ contains multi-period-ahead expectations, and w_t is the exogenous driving force following a VAR(1):

$$w_t = \Gamma w_{t-1} + \Pi \epsilon_t,$$

where ϵ_t is the vector of innovations.

Agents are assumed to have a rather limited view of the economy under AL. More precisely, their PLM process is generally defined as follows:

$$y_{t+j} = X_{t-1}\beta_{t-1}^{\{j\}} + u_{t+j},\ for\ j =\ 1,\ 2,\ ...,\ n,$$

where y is the vector containing the forward-looking variables of the model, X is the matrix of regressors, $\beta^{\{j\}}$ is the vector of updating parameters, which includes an intercept, and u is a vector of errors. These errors are linear combinations of the true model innovations. So, the variance-covariance matrices, $\Sigma = E[u_{t+j}u_{t+j}^T]$, are non-diagonal.

Agents are further assumed to behave as econometricians under AL. In particular, it is assumed that they use a linear projection scheme in which the parameters are updated to form their expectations for each forward-looking variable:

[19] For a detailed explanation see Slobodyan and Wouters (2012a,b).

$$E_t y_{t+j} = X_{t-1}\beta_{t-1}^{\{j\}}.$$

In line with Jordà (2005), we assume that agents make multi-period-ahead forecasts using local projections conditional on the information set available at the end of period $t-1$. Among the numerous advantages of using local projections for characterizing multi-period-ahead forecasts pointed out by Jordà (2005), we highlight two of them. First, they are easy to implement, which is a sensible approach when deviating from the RE hypothesis. Second, local projections are robust to model misspecifications. As discussed above, this is also a sensible feature to characterize agents' forecasts in a context where they face uncertainty about the true (highly non-linear) model economy.

The updating parameter vector, β, which results from stacking all the vectors $\beta^{\{j\}}$, is further assumed to follow an autoregressive process where agents' beliefs are updated through a Kalman filter. This updating expectation process can be represented as in Slobodyan and Wouters (2012a) by the following equation:

$$\beta_t - \bar{\beta} = F(\beta_{t-1} - \bar{\beta}) + v_t,$$

where F is a diagonal matrix with the learning parameter $\mid \rho \mid \leq 1$ on the main diagonal and v_t are i.i.d. errors with variance-covariance matrix V.

Once the expectations of the forward-looking variables,$E_t y_{t+j}$, are computed they are plugged into the matrix representation of the DSGE model to obtain a backward-looking representation of the model as follows

$$\begin{bmatrix} y_t \\ w_t \end{bmatrix} = \mu_t + T_t \begin{bmatrix} y_{t-1} \\ w_{t-1} \end{bmatrix} + R_t \epsilon_t,$$

where the time-varying matrices μ_t, T_t and R_t are nonlinear functions of structural parameters (entering in matrices A_0, A_1, A_2 and B_0) together with learning coefficients discussed below.

Updating expectations

The Kalman-filter updating and transition equations for the belief coefficients and the corresponding covariance matrix are given by

$$\beta_{t|t} = \beta_{t|t-1} + R_{t|t-1} X_{t-1} \left[\Sigma + X_{t-1}^T R_{t|t-1}^{-1} X_{t-1} \right]^{-1} \left(y_t - X_{t-1} \beta_{t|t-1} \right),$$

where $(\beta_{t+1|t} - \bar{\beta}) = F(\beta_{t|t} - \bar{\beta})$. $\beta_{t|t-1}$ is the estimate of β using the information up to time $t-1$ (but further considering the autoregressive process followed by β), $R_{t|t-1}$ is the mean squared error associated with $\beta_{t|t-1}$. Therefore, the updated learning vector $\beta_{t|t}$ is equal to the previous one, $\beta_{t|t-1}$, plus a correction term that depends on the forecast error, $\left(y_t - X_{t-1} \beta_{t|t-1} \right)$. Moreover, the mean squared error, $R_{t|t}$, associated with this updated estimate is given by

$$R_{t|t} = R_{t|t-1} - R_{t|t-1} X_{t-1} \left[\Sigma + X_{t-1}^T R_{t|t-1}^{-1} X_{t-1} \right]^{-1} X_{t-1}^T R_{t|t-1}^{-1},$$

with $R_{t+1|t} = F R_{t|t} F^T + V$.

Appendix 3

Table A.1.A: Priors and estimated posteriors of the structural parameters

	Priors			Posteriors					
				AL			RE		
Log-likelihood				815.09			569.95		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
φ: cost of adjusting capital	Normal	4.00	1.50	1.47	1.41	1.54	7.26	6.07	8.54
h: habit formation	Beta	0.70	0.10	0.49	0.46	0.52	0.32	0.30	0.36
σ_l : Frisch elasticity	Normal	2.00	0.75	3.07	3.02	3.16	1.15	0.69	1.45
ξ_p: price Calvo probability	Beta	0.50	0.10	0.69	0.66	0.72	0.911	0.906	0.917
ξ_w: wage Calvo probability	Beta	0.50	0.10	0.38	0.33	0.42	0.34	0.24	0.42
ι_w: wage indexation	Beta	0.50	0.15	0.64	0.58	0.70	0.35	0.17	0.53
ι_p: price indexation	Beta	0.50	0.15	0.19	0.13	0.25	0.12	0.08	0.17
ψ: capital utilization adjusting cost	Beta	0.50	0.15	0.52	0.47	0.56	0.78	0.74	0.83
Φ : steady state price mark-up	Normal	1.25	0.12	1.37	1.29	1.45	1.28	1.25	1.30
r_π: policy rule inflation	Normal	1.50	0.25	2.14	2.10	2.22	2.15	2.02	2.28
ρ_r: policy rule smoothing	Beta	0.75	0.10	0.57	0.50	0.63	0.84	0.82	0.85
r_{sp}: policy rule term spread	Normal	0.12	0.05	0.08	0.04	0.11	0.05	0.04	0.06
π: steady-state inflation	Gamma	0.62	0.10	0.67	0.64	0.71	0.74	0.72	0.76
$100(\beta^{-1}-1)$: steady-state rate of disc.	Gamma	0.25	0.10	0.11	0.08	0.16	0.10	0.07	0.13
l: steady-state labor	Normal	0.00	2.00	4.34	4.04	4.70	5.19	4.51	6.10
γ: one plus steady-state growth rate	Normal	0.40	0.10	0.51	0.49	0.53	0.40	0.39	0.42
α: capital share	Normal	0.30	0.05	0.12	0.10	0.15	0.16	0.15	0.17
ρ: learning parameter	Beta	0.50	0.29	0.66	0.61	0.72	—	—	—

Table A.1.B: Priors and estimated posteriors of the term structure parameters

	Priors			Posteriors					
				AL			RE		
$\bar{ts}^{\{4\}}$: steady-state 1-year premium	Normal	0.25	2.00	-0.16	-0.20	-0.12	0.06	0.04	0.08
$\bar{ts}^{\{12\}}$: steady-state 3-year premium	Normal	0.25	2.00	0.18	0.12	0.25	0.20	0.15	0.26
$\bar{ts}^{\{20\}}$: steady-state 5-year premium	Normal	0.25	2.00	0.35	0.32	0.39	0.31	0.17	0.44
$\bar{ts}^{\{28\}}$: steady-state 7-year premium	Normal	0.25	2.00	0.43	0.39	0.46	0.56	0.49	0.63
$\bar{ts}^{\{40\}}$: steady-state 10-year premium	Normal	0.25	2.00	0.50	0.47	0.54	0.66	0.56	0.74
λ_1: 1-year reciprocal of EH weight	Beta	0.50	0.29	0.63	0.57	0.69	0.09	0.08	0.11
λ_3: 3-year reciprocal of EH weight	Beta	0.50	0.29	0.72	0.64	0.81	0.30	0.24	0.36
λ_5: 5-year reciprocal of EH weight	Beta	0.50	0.29	0.88	0.83	0.95	0.73	0.68	0.83
λ_7: 7-year reciprocal of EH weight	Beta	0.50	0.29	0.82	0.78	0.88	0.996	0.993	1.0
λ_{10}: 10-year reciprocal of EH weight	Beta	0.50	0.29	0.77	0.70	0.83	0.91	0.88	0.94

Table A.1.C: Priors and estimated posteriors of the structural shock process parameters

	Priors			Posterior					
				AL			RE		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
σ_a: Std. dev. productivity innovation	Invgamma	0.10	2.00	0.44	0.40	0.48	0.42	0.41	0.43
σ_b: Std. dev. risk premium innovation	Invgamma	0.10	2.00	1.99	1.90	2.05	0.07	0.06	0.08
σ_g: Std. dev. exogenous spending innovation	Invgamma	0.10	2.00	0.38	0.34	0.41	0.35	0.34	0.36
σ_i: Std. dev. investment innovation	Invgamma	0.10	2.00	1.30	1.24	1.35	0.27	0.24	0.29
σ_R: Std. dev. monetary policy innovation	Invgamma	0.10	2.00	0.09	0.08	0.10	0.12	0.11	0.13
σ_p: Std. dev. price mark-up innovation	Invgamma	0.10	2.00	0.18	0.17	0.20	0.18	0.17	0.19
σ_w: Std. dev. wage mark-up innovation	Invgamma	0.10	2.00	0.99	0.92	1.05	0.74	0.69	0.80
$\sigma_{\eta\{2\}}$: Std. dev. 2-quarter yield innovation	Invgamma	0.10	2.00	0.65	0.60	0.71	0.05	0.04	0.06
$\sigma_{\eta\{3\}}$: Std. dev. 3-quarter yield innovation	Invgamma	0.10	2.00	0.10	0.03	0.17	0.08	0.02	0.15
$\sigma_{\eta\{4\}}$: Std. dev. 1-year yield innovation	Invgamma	0.10	2.00	0.57	0.51	0.65	2.43	2.30	2.60
$\sigma_{\eta\{12\}}$: Std. dev. 3-year yield innovation	Invgamma	0.10	2.00	0.94	0.86	1.02	3.53	3.17	3.88
$\sigma_{\eta\{20\}}$: Std. dev. 5-year yield innovation	Invgamma	0.10	2.00	0.52	0.47	0.57	3.00	2.84	3.10
$\sigma_{\eta\{28\}}$: Std. dev. 7-year yield innovation	Invgamma	0.10	2.00	0.05	0.03	0.07	2.98	2.88	3.07
$\sigma_{\eta\{40\}}$: Std. dev. 10-year yield innovation	Invgamma	0.10	2.00	1.35	1.26	1.41	3.36	3.27	3.47

Table A.1.D: Priors and estimated posteriors of the structural shock process parameters (*continued*)

	Priors			Posterior					
				AL			RE		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
ρ_a: Autoregressive coef. productivity shock	Beta	0.50	0.20	0.93	0.89	0.95	0.941	0.937	0.945
ρ_b: Autoregressive coef. risk-premium shock	Beta	0.50	0.20	0.91	0.89	0.92	0.969	0.958	0.980
ρ_g: Autoregressive coef. exog. spending shock	Beta	0.50	0.20	0.98	0.97	0.99	0.974	0.966	0.981
ρ_i: Autoregressive coef. investment shock	Beta	0.50	0.20	0.96	0.94	0.99	0.73	0.69	0.77
ρ_R: Autoregressive coef. monetary policy shock	Beta	0.50	0.20	0.87	0.81	0.94	0.025	0.004	0.044
ρ_p: Autoregressive coef. price markup shock	Beta	0.50	0.20	0.94	0.91	0.98	0.87	0.82	0.93
ρ_w: Autoregressive coef. wage markup shock	Beta	0.50	0.20	0.96	0.93	0.99	0.968	0.965	0.971
$\rho^{\{2\}}$: Autoregressive coef. 2-quarter yield shock	Beta	0.50	0.20	0.85	0.83	0.88	0.33	0.05	0.65
$\rho^{\{3\}}$: Autoregressive coef. 3-quarter yield shock	Beta	0.50	0.20	0.53	0.45	0.61	0.36	0.15	0.59
$\rho^{\{4\}}$: Autoregressive coef. 1-year yield shock	Beta	0.50	0.20	0.90	0.89	0.92	0.71	0.70	0.73
$\rho^{\{12\}}$: Autoregressive coef. 3-year yield shock	Beta	0.50	0.20	0.88	0.86	0.90	0.92	0.91	0.94
$\rho^{\{20\}}$: Autoregressive coef. 5-year yield shock	Beta	0.50	0.20	0.84	0.81	0.86	0.89	0.86	0.93
$\rho^{\{28\}}$: Autoregressive coef. 7-year yield shock	Beta	0.50	0.20	0.79	0.75	0.81	0.97	0.94	0.98
$\rho^{\{40\}}$: Autoregressive coef. 10-year yield shock	Beta	0.50	0.20	0.67	0.63	0.71	0.977	0.969	0.985
μ_p: MA coef. price markup shock	Beta	0.50	0.20	0.63	0.58	0.67	0.86	0.81	0.94
μ_w: MA coef. wage markup shock	Beta	0.50	0.20	0.51	0.47	0.56	0.81	0.76	0.87
ρ_{ga}: Interact. betw. product. and spending shocks	Beta	0.50	0.25	0.49	0.43	0.54	0.41	0.34	0.45
$\rho^{\{2\}}{}_{\xi}$: Interact. betw. 1- and 2-qtr yield shocks	Normal	0.50	0.25	0.73	0.69	0.76	0.20	-0.06	0.42
$\rho^{\{3\}}{}_{\xi}$: Interact. betw. 1- and 3-qtr yield shocks	Normal	0.50	0.25	0.85	0.76	0.91	0.27	0.16	0.40
$\rho^{\{4\}}{}_{\xi}$: Interact. betw. 1-qtr and 1-yr yield shocks	Normal	0.50	0.25	0.61	0.58	0.65	1.03	0.94	1.11
$\rho^{\{12\}}{}_{\xi}$: Interact. betw. 1-qtr and 3-yr yield shocks	Normal	0.50	0.25	0.71	0.66	0.77	0.58	0.42	0.69
$\rho^{\{20\}}{}_{\xi}$: Interact. betw. 1-qtr and 5-yr yield shocks	Normal	0.50	0.25	0.70	0.65	0.74	0.77	0.67	0.89
$\rho^{\{28\}}{}_{\xi}$: Interact. betw. 1-qtr and 7-yr yield shocks	Normal	0.50	0.25	0.70	0.66	0.74	0.83	0.75	0.90
$\rho^{\{40\}}{}_{\xi}$: Interact. betw. 1-qtr and 1-yr yield shocks	Normal	0.50	0.25	0.67	0.60	0.76	0.94	0.70	1.09

Table A.1.E: Estimated parameters of forecast deviations, inflation revisions and expectation rules

	Priors			Posterior					
				AL			RE		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
$\sigma_{\pi}^{\{1\}}$: Std. dev. 1-q-a inflation expt. innov.	Invgamma	0.10	2.00	0.09	0.08	0.10	0.087	0.082	0.094
$\sigma_{\pi}^{\{2\}}$: Std. dev. 2-q-a inflation expt. innov.	Invgamma	0.10	2.00	0.08	0.07	0.09	0.074	0.072	0.076
$\sigma_{\pi}^{\{3\}}$: Std. dev. 3-q-a inflation expt. innov.	Invgamma	0.10	2.00	0.067	0.060	0.074	0.069	0.065	0.074
$\sigma_{\pi}^{\{4\}}$: Std. dev. 4-q-a inflation expt. innov.	Invgamma	0.10	2.00	0.20	0.17	0.22	0.072	0.070	0.075
$\sigma_{\Delta c}^{\{1\}}$: Std. dev. 1-q-a cons. growth expt. innov.	Invgamma	0.10	2.00	0.70	0.61	0.77	0.24	0.22	0.25
$\sigma_{\Delta c}^{\{2\}}$: Std. dev. 2-q-a cons. growth expt. innov.	Invgamma	0.10	2.00	0.12	0.11	0.13	0.14	0.13	0.15
$\sigma_{\Delta c}^{\{3\}}$: Std. dev. 3-q-a cons. growth expt. innov.	Invgamma	0.10	2.00	0.105	0.094	0.114	0.119	0.115	0.123
$\sigma_{\Delta c}^{\{4\}}$: Std. dev. 4-q-a cons. growth expt. innov.	Invgamma	0.10	2.00	0.08	0.07	0.09	0.112	0.107	0.118
$\sigma_{r}^{\{1\}}$: Std. dev. 1-q-a interst. rate expt. innov.	Invgamma	0.10	2.00	0.14	0.13	0.15	0.10	0.09	0.11
$\sigma_{r}^{\{2\}}$: Std. dev. 2-q-a interst. rate expt. innov.	Invgamma	0.10	2.00	0.12	0.10	0.13	0.090	0.086	0.094
$\sigma_{r}^{\{3\}}$: Std. dev. 3-q-a interst. rate expt. innov.	Invgamma	0.10	2.00	0.13	0.12	0.15	0.092	0.087	0.098
σ_{π}^{r}: Std. dev. inflation revision innovation	Invgamma	0.10	2.00	0.21	0.18	0.24	0.33	0.31	0.35

Table A.1.E (*Continued*)

	Priors			Posterior					
				AL			RE		
	Distr	Mean	Std D.	Mean	5%	95%	Mean	5%	95%
$\rho_{\pi}^{\{1\}}$: persist. 1-q-a inflation expect. shock	Beta	0.50	0.20	0.75	0.71	0.79	0.74	0.71	0.77
$\rho_{\pi}^{\{2\}}$: persist. 2-q-a inflation expect. shock	Beta	0.50	0.20	0.83	0.75	0.88	0.81	0.75	0.84
$\rho_{\pi}^{\{3\}}$: persist. 3-q-a inflation expect. shock	Beta	0.50	0.20	0.91	0.87	0.94	0.89	0.84	0.95
$\rho_{\pi}^{\{4\}}$: : persist. 4-q-a inflation expt. shock	Beta	0.50	0.20	0.94	0.93	0.98	0.86	0.82	0.89
$\rho_{\Delta c}^{\{1\}}$: persist. 1-q-a cons. growth expt. shock	Beta	0.50	0.20	0.98	0.96	0.99	0.54	0.41	0.63
$\rho_{\Delta c}^{\{2\}}$: persist. 2-q-a cons. growth expt. shock	Beta	0.50	0.20	0.83	0.79	0.87	0.84	0.77	0.89
$\rho_{\Delta c}^{\{3\}}$: persist. 3-q-a cons. growth expt. shock	Beta	0.50	0.20	0.73	0.66	0.81	0.84	0.81	0.87
$\rho_{\Delta c}^{\{4\}}$: persist. 4-q-a cons. growth expt. shock	Beta	0.50	0.20	0.85	0.76	0.93	0.93	0.91	0.94
$\rho_{r}^{\{1\}}$: persist. 1-q-a interest rate expt. shock	Beta	0.50	0.20	0.90	0.86	0.94	0.67	0.63	0.72
$\rho_{r}^{\{2\}}$: persist. 4-q-a interest rate expt. shock	Beta	0.50	0.20	0.96	0.94	0.99	0.67	0.63	0.69
$\rho_{r}^{\{3\}}$: persist. 4-q-a interest rate expt. shock	Beta	0.50	0.20	0.79	0.71	0.85	0.84	0.82	0.86
$\rho_{\pi}^{\{r\}}$: persist. inflation revision shock	Beta	0.50	0.20	0.27	0.20	0.32	0.26	0.22	0.30
$b_{\pi}^{\{r\}}$: revision coef. of real-time inflation	Normal	0.0	0.50	-0.34	-0.39	-0.30	0.005	0.001	0.012
μ_{π}: persistence of inflation expt. rule	Beta	0.80	0.15	0.994	0.988	1.0	—	—	—
μ_{c}: persistence of consumption expt. rule	Beta	0.90	0.15	0.98	0.96	1.0	—	—	—
μ_{r}: persistence of interest rate expt. rule	Beta	0.90	0.15	0.98	0.95	1.0	—	—	—