


Metodología de la enseñanza de las matemáticas en Singapur



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+ ideas
- cuentas

Singapur es conocido en educación

Resultados en pruebas internacionales de referencia

TIMSS 2011 - 4º

1 Singapore	606 (3.2)
2 Korea, Rep. of	605 (1.9)
2 Hong Kong SAR	602 (3.4)
Chinese Taipei	591 (2.0)
Japan	585 (1.7)
† Northern Ireland	562 (2.9)
Belgium (Flemish)	549 (1.9)
Finland	545 (2.3)
England	542 (3.5)
Russian Federation	542 (3.7)
2 United States	541 (1.8)

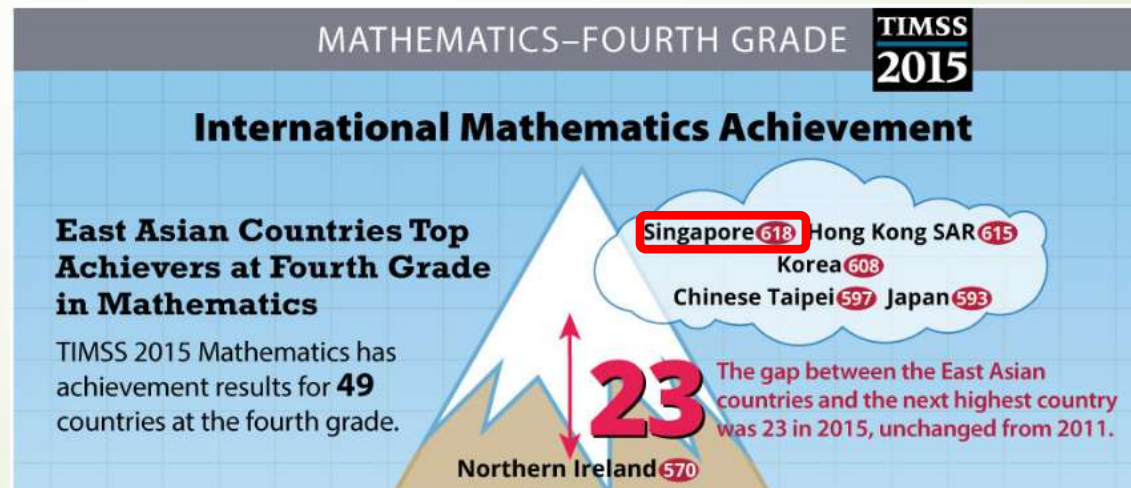
TIMSS 2011 - 8º

Korea, Rep. of	613 (2.9)
2 Singapore	611 (3.8)
Chinese Taipei	609 (3.2)
Hong Kong SAR	586 (3.8)
Japan	570 (2.6)
2 Russian Federation	539 (3.6)
3 Israel	516 (4.1)
Finland	514 (2.5)
2 United States	509 (2.6)
† England	507 (5.5)
Hungary	505 (3.5)

PISA 2012 Matemáticas

Shanghai-China	613
Singapore	573
Hong Kong-China	561
Chinese Taipei	560
Korea	554
Macao-China	538
Japan	536
Liechtenstein	535
Switzerland	531
Netherlands	523
Estonia	521

TIMSS 2015



No es “otro país asiático más”

Singapur en 1965



Apuestan por **la educación**





¿Y en matemáticas?





Los tres errores que cometían (hace 40 años)

- ❑ Exceso de cálculos tediosos.
- ❑ Aprendizaje rutinario de procedimientos, sin entenderlos.
- ❑ Aprendizaje memorístico.

El desarrollo de lo que se conoce como “método Singapur” fue la respuesta

Disclaimer: ¡No inventaron nada!

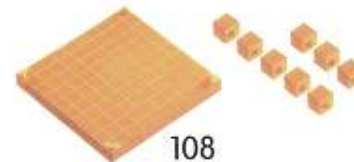
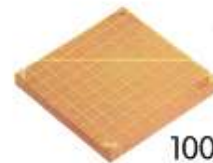
Fundamentos metodológicos

- ❖ El aprendizaje en tres etapas (Jerome Bruner)

(1)

Contando

- 1 Gugo usa 10  para formar una  .
Luego pone 10  juntas.
10 decenas = 100 unidades



10, 20, 30, 40, 50,
60, 70, 80, 90, 100.
¡Cien!



ciento ocho

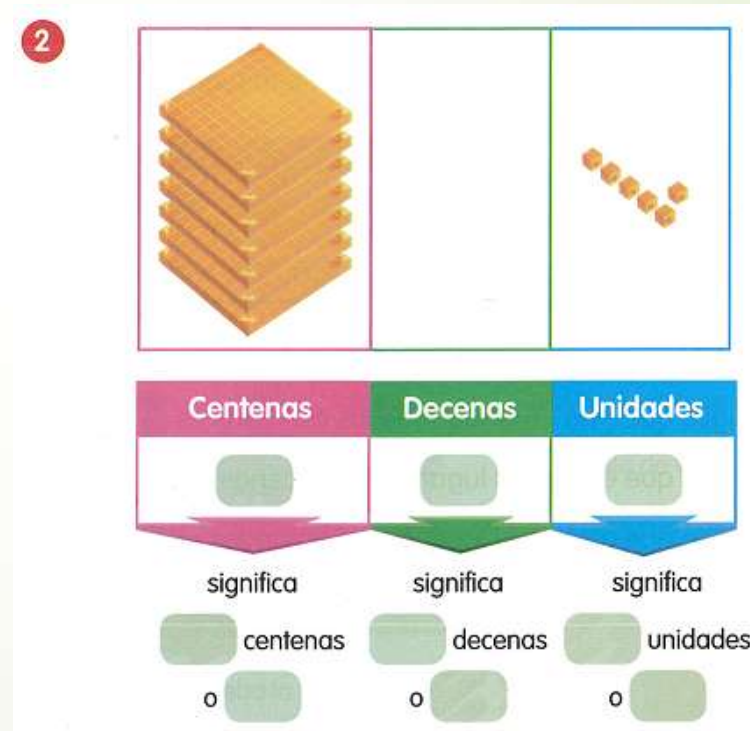


Concreto

Fundamentos metodológicos

- ❖ El aprendizaje en tres etapas (Jerome Bruner)

(2)



Pictórico (gráfico)

Fundamentos metodológicos

- ❖ El aprendizaje en tres etapas (Jerome Bruner)

(3)

b

Ordena los números.



Comienza por el mayor.

CPA

Abstracto

Fundamentos metodológicos

❖ Hay que superar la dicotomía

Comprensión de conceptos ↔ Aprendizaje de procedimientos

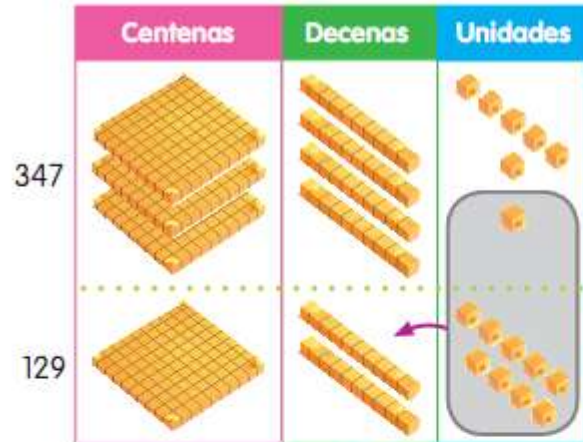
Ambos deben desarrollarse en paralelo

Richard Skemp:

Relational understanding and instrumental understanding (1976)

Sumar reagrupando las unidades

1 $347 + 129 = ?$



Primero, suma las unidades.

$$\begin{array}{r} 3 \ 4 \ 7 \\ + 1 \ 2 \ 9 \\ \hline \ 6 \end{array}$$

7 unidades + 9 unidades = 16 unidades

Reagrupa las unidades
16 unidades = 1 decena
6 unidades

Luego, suma las decenas.

$$\begin{array}{r} 3 \ 4 \ 7 \\ + 1 \ 2 \ 9 \\ \hline \ 7 \ 6 \end{array}$$

4 decenas + 2 decenas
+ 1 decena = 7 decenas

Finalmente, suma las centenas.

$$\begin{array}{r} 3 \ 4 \ 7 \\ + 1 \ 2 \ 9 \\ \hline 4 \ 7 \ 6 \end{array}$$

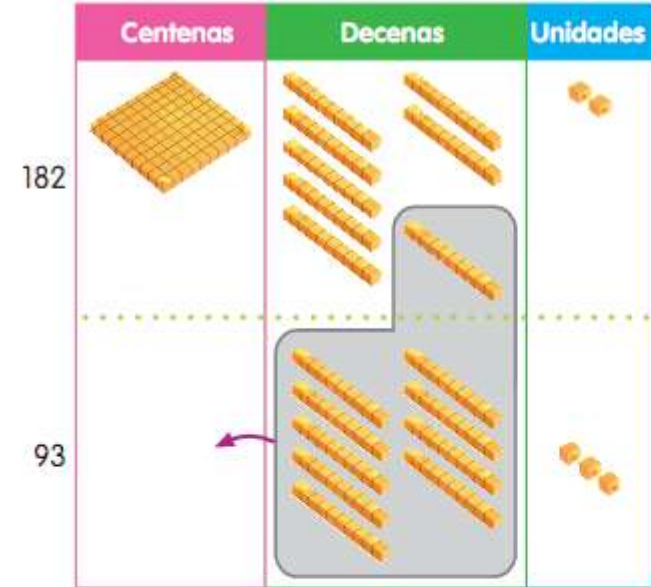
Entonces, $347 + 129 = 476$.

3 centenas + 1 decena
= 4 centenas

35

Sumar reagrupando las decenas

1 $182 + 93 = ?$



Primero, suma las unidades.

$$\begin{array}{r} 1 \ 8 \ 2 \\ + \ 9 \ 3 \\ \hline \ 5 \end{array}$$

2 unidades + 3 unidades = 5 unidades

Luego, suma las decenas.

$$\begin{array}{r} 1 \ 8 \ 2 \\ + \ 9 \ 3 \\ \hline \ 7 \ 5 \end{array}$$

8 decenas + 9 decenas = 17 decenas.

Reagrupa las decenas.
17 decenas = 1 decena + 7 decenas

Finalmente, suma las centenas.

$$\begin{array}{r} 1 \ 8 \ 2 \\ + \ 9 \ 3 \\ \hline 2 \ 7 \ 5 \end{array}$$

275

Entonces, $182 + 93 = 275$.

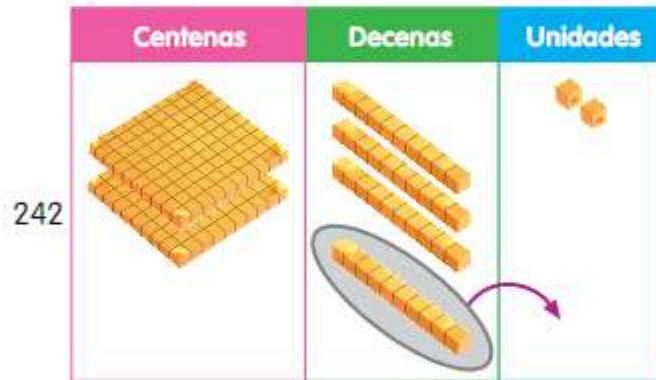
1 decena + 1 decena = 2 centenas

38

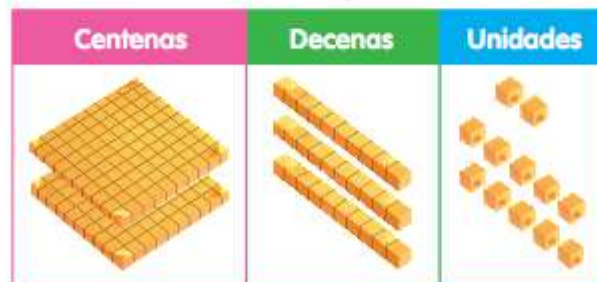
Restar reagrupando las decenas y las unidades

1 $242 - 128 = ?$

No podemos restar 8 unidades de 2 unidades. Entonces, reagrupamos las decenas y unidades.

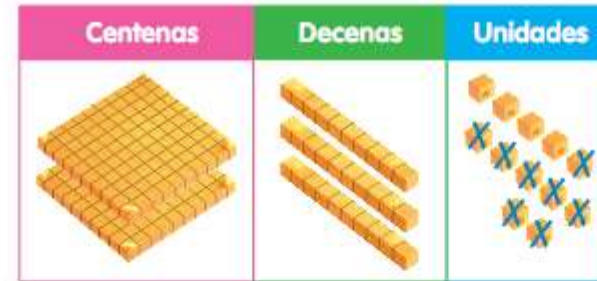


Reagrupa las decenas y unidades.



$$\begin{array}{r} 2 \ 34 \ 12 \\ - 1 \ 2 \ 8 \\ \hline 4 \end{array}$$

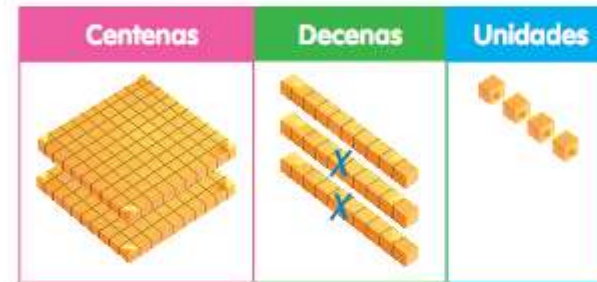
4 decenas 2 unidades
= 3 decenas
12 unidades



Primero, resta las unidades.

$$\begin{array}{r} 2 \ 34 \ 12 \\ - 1 \ 2 \ 8 \\ \hline 4 \end{array}$$

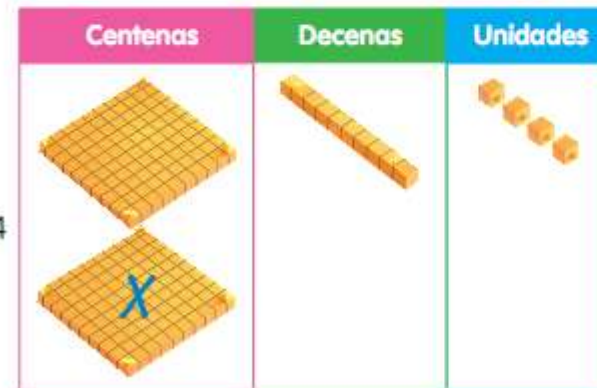
12 unidades - 8 unidades
= 4 unidades



Luego, resta las decenas.

$$\begin{array}{r} 2 \ 34 \ 12 \\ - 1 \ 2 \ 8 \\ \hline 1 \ 4 \end{array}$$

3 decenas - 2 decenas
= 1 decena



Finalmente, resta las centenas.

$$\begin{array}{r} 2 \ 34 \ 12 \\ - 1 \ 2 \ 8 \\ \hline 1 \ 1 \ 4 \end{array}$$

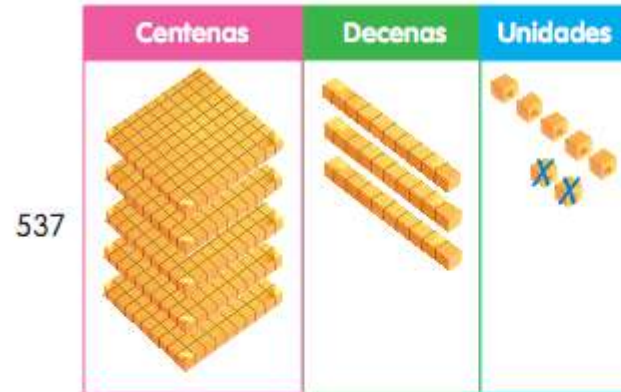
2 centenas - 1 centena
= 1 centena

114

Entonces, $242 - 128 = 114$.

Restar reagrupando las centenas y las decenas

1 $537 - 272 = ?$



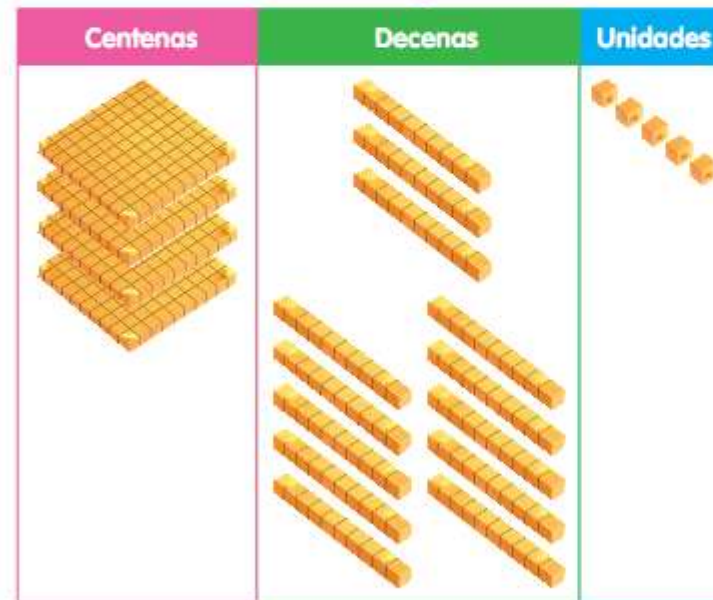
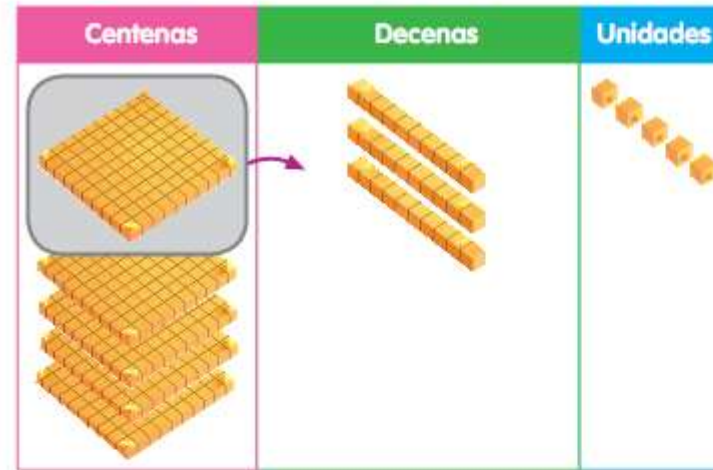
Primero, resta las unidades.

$$\begin{array}{r} 537 \\ - 272 \\ \hline 5 \end{array}$$

7 unidades - 2 unidades = 5 unidades

$$\begin{array}{r} 537 \\ - 272 \\ \hline 5 \end{array}$$

No podemos restar 7 decenas de 3 decenas. Entonces, reagrupamos las centenas y decenas.



Reagrupa las centenas y decenas.

$$\begin{array}{r} 4537 \\ - 272 \\ \hline 5 \end{array}$$

5 centenas
3 decenas
= 4 centenas
13 decenas

Recuerda reagrupar cuando no tengas lo suficiente para restar.



Centenas	Decenas	Unidades

Luego, resta las decenas.

$$\begin{array}{r} 537 \\ - 272 \\ \hline 65 \end{array}$$

13 decenas
- 7 decenas
= 6 decenas

Centenas	Decenas	Unidades

Finalmente, resta las centenas.

$$\begin{array}{r} 537 \\ - 272 \\ \hline 265 \end{array}$$

4 centenas
- 2 centenas
= 2 centenas

265

Entonces, $537 - 272 = 265$.

2 $719 - 383 = ?$

Primero, resta las unidades.

9 unidades - 3 unidades = unidades

Reagrupa las centenas y decenas.

7 centenas, 1 decena = 6 centenas, decenas

Luego, resta las decenas.

decenas - decenas = decenas

Luego, resta las centenas.

6 centenas - 3 centenas = centenas

Entonces, $719 - 383 =$.

Escribe $719 - 383$ de esta manera. Luego resta.

$$\begin{array}{r} 719 \\ - 383 \\ \hline \end{array}$$





¡Aprendamos!

Resta con números que tienen ceros

1 $200 - 18 = ?$

Centenas	Decenas	Unidades

Reagrupa las centenas.



$$\begin{array}{r} 200 \\ - 18 \\ \hline \end{array}$$

Centenas	Decenas	Unidades

Reagrupa las decenas.



$$\begin{array}{r} 200 \\ - 18 \\ \hline \end{array}$$

2 centenas = 1 centena y 1 decena
 = 1 centena 9 decenas 10 unidades

Continuación de la página anterior.

Centenas	Decenas	Unidades

Primero, resta las unidades.

$$\begin{array}{r} 200 \\ - 18 \\ \hline 2 \end{array}$$

10 unidades
 - 8 unidades
 = 2 unidades

Centenas	Decenas	Unidades

Luego, resta las decenas.

$$\begin{array}{r} 200 \\ - 18 \\ \hline 82 \end{array}$$

9 decenas
 - 1 decena
 = 8 decenas

Centenas	Decenas	Unidades

Finalmente, resta las centenas.

$$\begin{array}{r} 200 \\ - 18 \\ \hline 182 \end{array}$$

1 centena
 - 0 centenas
 = 1 centena

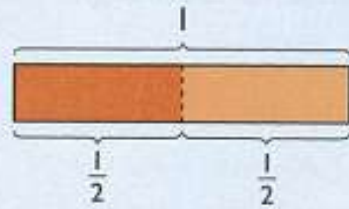
182

Entonces, $200 - 18 = 182$.

Un ejemplo de 6º: Dividir por una fracción

1 Lili cut a rectangular paper strip into a number of pieces. Each piece was $\frac{1}{2}$ of the paper strip. How many pieces did Lili cut the paper strip into?

$$\text{Number of pieces} = 1 \div \frac{1}{2}$$



$1 \div \frac{1}{2}$ means this: "How many halves are there in 1 whole?"



From the model, we see that there are 2 halves in 1 whole.

$$\text{So } 1 \div \frac{1}{2} = 2$$

Lili cut the rectangular paper strip into 2 pieces.

Un ejemplo de 6º: Dividir por una fracción

2



Carry out this activity.

Work in pairs.

Your teacher will provide each pair with 4 rectangular strips of paper. Each strip represents 1 whole.

a Use each strip to find:

i $1 \div \frac{1}{3}$

ii $1 \div \frac{1}{4}$

iii $1 \div \frac{1}{5}$

iv $1 \div \frac{1}{6}$

How many one-thirds, quarters, one-fifths and one-sixths are there in 1 whole?

Un ejemplo de 6º: Dividir por una fracción

5 Find by multiplication.

a $3 \div \frac{1}{5} = \square \times \square = \square$

b $7 \div \frac{1}{4} = \square \times \square = \square$

c $4 \div \frac{1}{2} = \square$

d $5 \div \frac{1}{3} = \square$

e $6 \div \frac{1}{5} = \square$

f $8 \div \frac{1}{8} = \square$

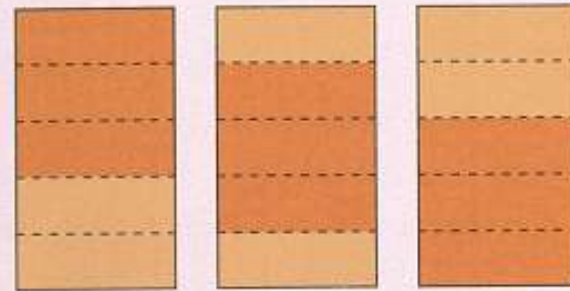
Un ejemplo de 6º: Dividir por una fracción

I Use the models to find the answers.

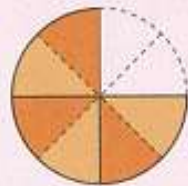
a $1 \div \frac{1}{4}$



b $3 \div \frac{3}{5}$




c $\frac{3}{4} \div \frac{1}{8}$



d $\frac{2}{3} \div \frac{2}{9}$





La idea de que la comprensión de los conceptos y los procedimientos deben trabajarse en paralelo me parece fundamental para contestar una pregunta básica:

¿En qué debería consistir la educación matemática básica en el siglo XXI?

- El aprendizaje de procedimientos quizá ya no tiene interés en sí mismo.
- Aprender **ciertos procedimientos** ayuda a la comprensión de los conceptos.

- Los algoritmos tradicionales fueron desarrollados con un objetivo: **eficiencia.**
- Un nuevo objetivo podría generar otros algoritmos. **¿Algoritmos significativos?**



Fundamentos metodológicos

- ❖ Variedad en las presentaciones (Zoltan Dienes)

La comprensión de un concepto es mejor si se presenta desde distintos puntos de vista.

Fundamentos metodológicos

- 10 Sally had 18 stamps. She sold $\frac{1}{3}$ of them.
How many stamps had she left?

Method 1

$$\frac{1}{3} \times 18 = 1 \times 6 = 6$$

She sold 6 stamps.

$$18 - 6 = 12$$

She had 12 stamps left.

Method 2

$$1 - \frac{1}{3} = \frac{3}{3} - \frac{1}{3} = \frac{2}{3}$$

She had $\frac{2}{3}$ of her stamps left.

$$\frac{2}{3} \times 18 = 2 \times 6 = 12$$

She had 12 stamps left.

First, find the number of stamps Sally sold.



First, find what fraction of the stamps she had left.



Un ejemplo de 4º

Method 3

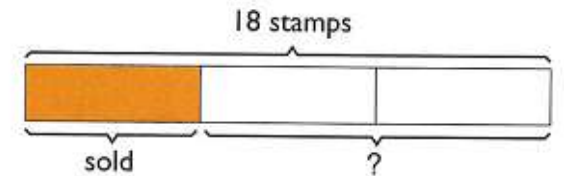
$$3 \text{ units} = 18$$

$$1 \text{ unit} = 18 \div 3 = 6$$

She sold 6 stamps.

$$2 \text{ units} = 6 \times 2 = 12$$

Sally had 12 stamps left.



1 unit → the number of stamps Sally sold

2 units → the number of stamps Sally had left



Fundamentos metodológicos

- ❖ El andamiaje y la zona de desarrollo próximo
(Lev Vygotsky)

En lugar de ir diciendo al alumno “esto se hace así”,
se le proponen actividades que estén en su

zona de desarrollo próximo.

Veamos un ejemplo de secuencia didáctica

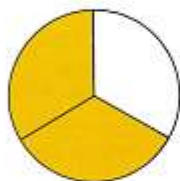
14 Fractions



Let's Learn!

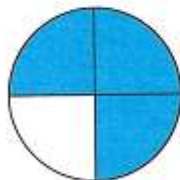
Numerator And Denominator

1



$\frac{2}{3}$ ← numerator
← denominator

In the fraction $\frac{2}{3}$, 2 is the **numerator**, and 3 is the **denominator**.



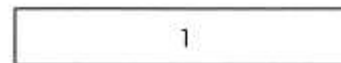
of the circle is shaded.
The numerator of the fraction is .
The denominator of the fraction is .

2

The numerator of a fraction is twice as large as 4.
The denominator of the fraction is 7 more than the numerator.
What is the fraction?

Understanding Equivalent Fractions

1 Googol has some fraction strips.



One whole



1 out of 2 equal parts = $\frac{1}{2}$



2 out of 4 equal parts = $\frac{2}{4}$



4 out of 8 equal parts = $\frac{4}{8}$

The fractions $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{4}{8}$ have different numerators and denominators.

But $\frac{1}{2}$ is equal to $\frac{2}{4}$.

$\frac{1}{2}$ is also equal to $\frac{4}{8}$.

$\frac{1}{2}$, $\frac{2}{4}$ and $\frac{4}{8}$ are **equivalent fractions**.



2 Name some equivalent fractions of $\frac{2}{3}$.




$\frac{2}{3}$ of the bar is shaded.



$\frac{2}{3} = \frac{\text{input}}{6}$

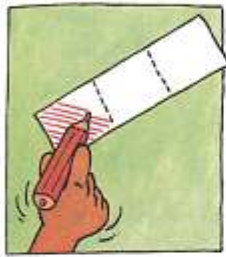


$\frac{2}{3} = \frac{\text{input}}{9}$

- 3  Carry out this activity.

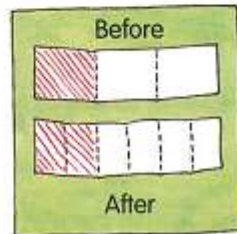
You will be given three paper strips of the same size.

- 1 Fold the first strip into three equal parts. Then unfold the strip and draw lines along the folds to divide it into three equal parts.



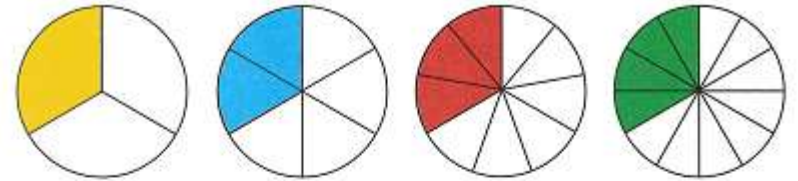
- 2 Shade one part of the first strip. You get the shaded fraction $\frac{1}{3}$.

- 3 Refold the first strip. Then fold it into half. You will find that an equivalent fraction of the shaded fraction $\frac{1}{3}$ is $\frac{2}{6}$.




- 4 Make the following shaded fractions with the remaining paper strips: $\frac{1}{4}$ and $\frac{3}{4}$. Then fold these strips further to find their equivalent fractions.

- 4 What are the missing numerators and denominators of these equivalent fractions?



$$\frac{1}{3} = \frac{2}{6} = \frac{3}{6} = \frac{4}{6}$$

- 5  Carry out this activity.



Use a suitable computer program.

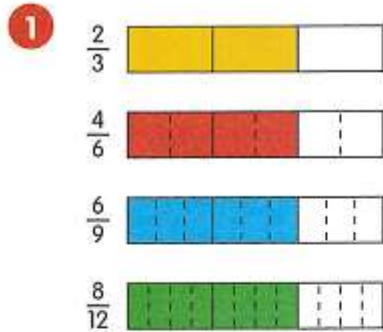
- 1 Draw a table with 1 row and 4 columns. Shade the first column.
- 2 Then draw a table with 1 row and 8 columns. Shade the first 2 columns.
- 3 Finally, draw a table with 1 row and 12 columns. Shade the first 3 columns.

What do you notice about the shaded parts?
What fraction of each table is shaded?

All three tables should be of the same width and height.



More Equivalent Fractions: Short Cut



$$\frac{2}{3} = \frac{4}{6} = \frac{6}{9} = \frac{8}{12}$$



I have a short cut!
To find an equivalent fraction,
multiply the numerator and the
denominator by the same number.

$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{2}{3} = \frac{6}{9}$$

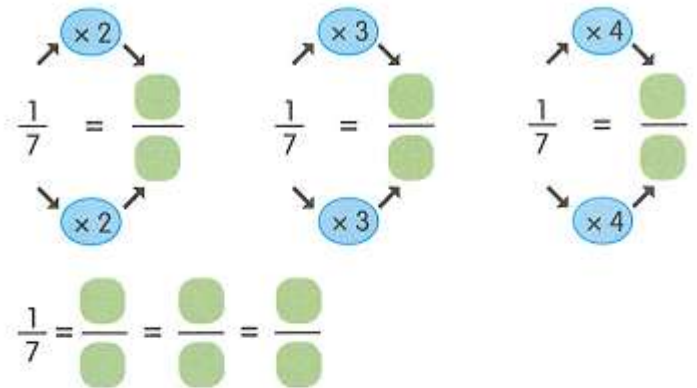


To get $\frac{8}{12}$, we **multiply**
the numerator and
denominator of $\frac{2}{3}$ by

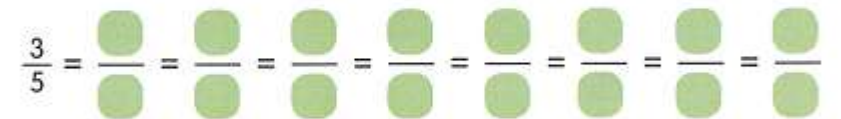


2 Use multiplication to find

a the first three equivalent fractions of $\frac{1}{7}$.



b the first eight equivalent fractions of $\frac{3}{5}$.



3 Complete the equivalent fractions of the following.

a $\frac{3}{4} = \frac{\quad}{8} = \frac{9}{\quad}$

b $\frac{2}{5} = \frac{4}{\quad} = \frac{\quad}{15}$

c $\frac{1}{3} = \frac{2}{\quad} = \frac{\quad}{9}$

4

Here is another way of finding equivalent fractions. **Divide** the numerator and the denominator by the same number.

$$\frac{6}{12} = \frac{3}{6}$$

Diagram showing the simplification of $\frac{6}{12}$ to $\frac{3}{6}$ by dividing both numerator and denominator by 2. Arrows point from the top and bottom of the original fraction to the top and bottom of the simplified fraction, with a circle containing $\div 2$ above and below the arrows.

$$\frac{6}{12} = \frac{2}{4}$$

Diagram showing the simplification of $\frac{6}{12}$ to $\frac{2}{4}$ by dividing both numerator and denominator by 3. Arrows point from the top and bottom of the original fraction to the top and bottom of the simplified fraction, with a circle containing $\div 3$ above and below the arrows.



5

Is $\frac{2}{4}$ the simplest equivalent fraction of $\frac{6}{12}$?

$$\frac{2}{4} = \frac{1}{2}$$

Diagram showing the simplification of $\frac{2}{4}$ to $\frac{1}{2}$ by dividing both numerator and denominator by 2. Arrows point from the top and bottom of the original fraction to the top and bottom of the simplified fraction, with a circle containing $\div 2$ above and below the arrows.

No, you can divide the numerator and denominator of $\frac{2}{4}$ further by the same number.

$\frac{1}{2}$ is the **simplest form** of $\frac{2}{4}$.

The simplest equivalent fraction of $\frac{6}{12}$ is $\frac{1}{2}$.

So, you use division when you want to find a fraction in its simplest form.



6

Complete the following equivalent fractions of $\frac{4}{12}$.

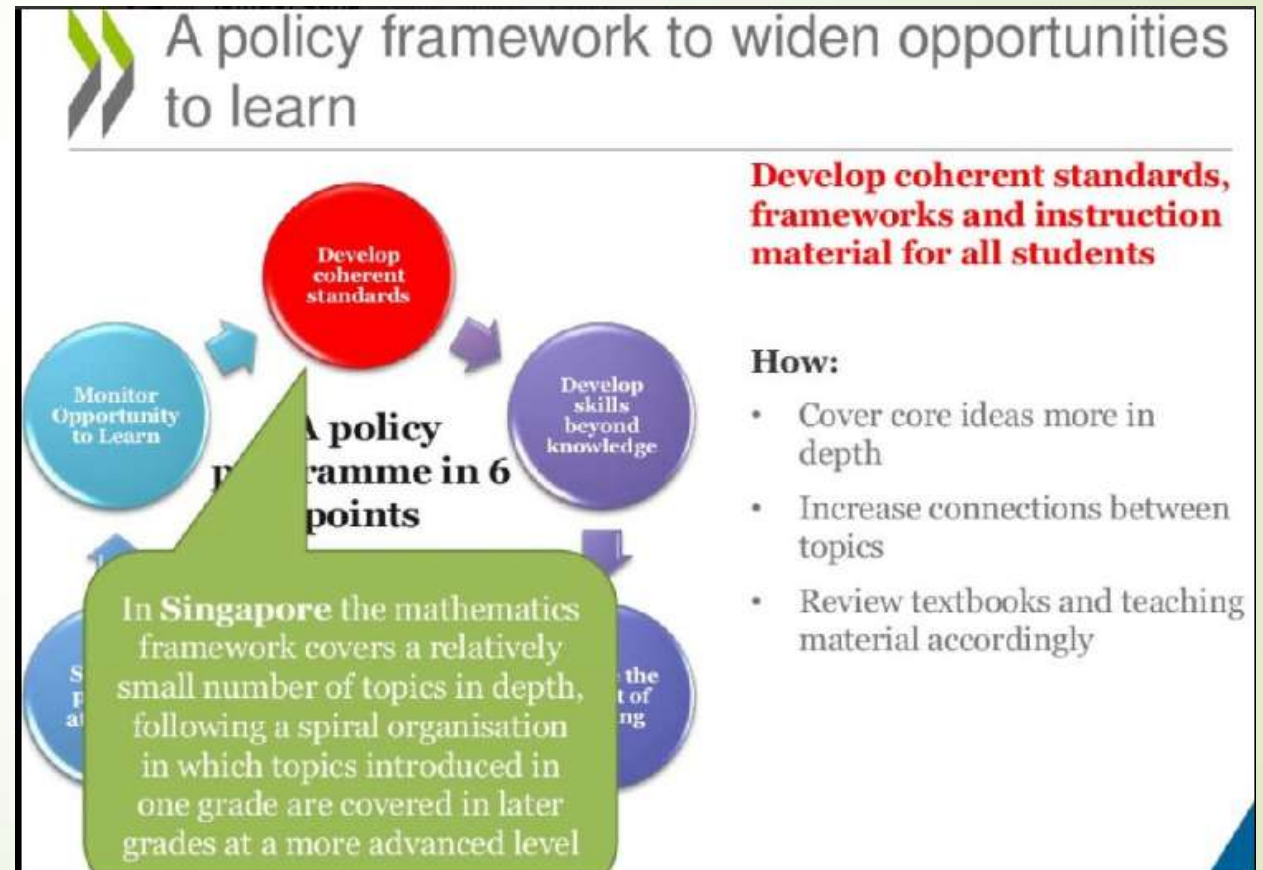
$$\frac{4}{12} = \frac{\text{green circle}}{6}$$

$$\frac{4}{12} = \frac{1}{\text{green circle}}$$

The simplest equivalent fractions of $\frac{4}{12}$ is $\frac{\text{green circle}}{\text{green circle}}$.

- Un inciso: trabajar la comprensión requiere **tiempo**.
- Uno de los factores del éxito de Singapur es una **profunda reforma del currículo**.

En [este informe](#) de la OCDE



Algunos ejemplos

(c) $q \overline{) 26480} D$

(d) $7 \overline{) 2184}$

Estas son las divisiones más complicadas que se pueden encontrar en Singapur en primaria

En nuestras aulas

Algoritmo "extendido"

$$\begin{array}{r} 6 \quad 4 \quad 0 \quad | \quad 2 \quad 3 \\ -4 \quad 6 \quad \quad | \quad 2 \quad 7 \\ \hline 1 \quad 8 \quad 0 \\ -1 \quad 6 \quad 1 \\ \hline 1 \quad 9 \end{array}$$

Algoritmo "usual"
("comprimido")

$$\begin{array}{r} 6 \quad 4 \quad 0 \quad | \quad 2 \quad 3 \\ 1 \quad 8 \quad 0 \quad | \quad 2 \quad 7 \\ \quad \quad 1 \quad 9 \end{array}$$

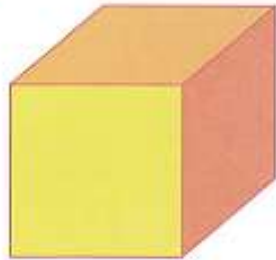
¿Cuál usamos? ¿Por qué?

Las potencias

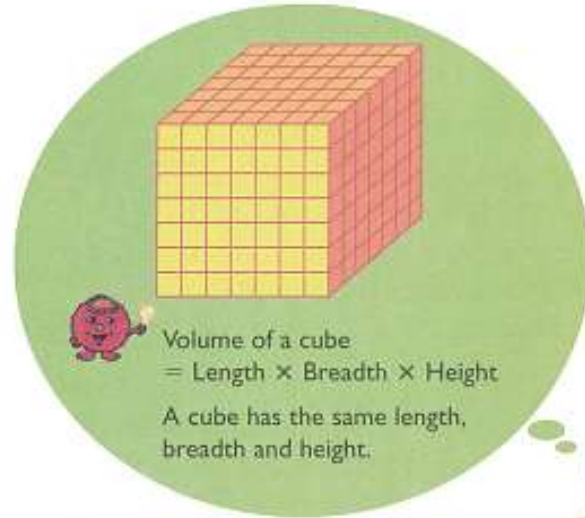
Volume Of Solids

Let us revise volume.

1



7 cm



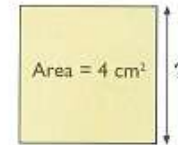
Volume of a cube
= Length \times Breadth \times Height
A cube has the same length,
breadth and height.



$$\begin{aligned} \text{Volume of the cube} &= \square \times \square \times \square \\ &= \square \text{ cm}^3 \end{aligned}$$

En 6°

- 9 The area of a square is 4 cm^2 . Find the length of one side of the square.



We write the **square root** of a number like this:
 $\sqrt{4}, \sqrt{9}, \sqrt{16}, \dots$



$$\text{Side} \times \text{Side} = 4$$

$4 = 2 \times 2$, so the length of one side of the square is 2 cm.

We say the **square root** of 4 is 2 and we write $\sqrt{4} = 2$.

10 a $9 = 3 \times \square$
 $\sqrt{9} = \square$


b $25 = \square \times \square$
 $\sqrt{25} = \square$

- 11 Find:

a $\sqrt{16}$

b $\sqrt{49}$

c $\sqrt{64}$

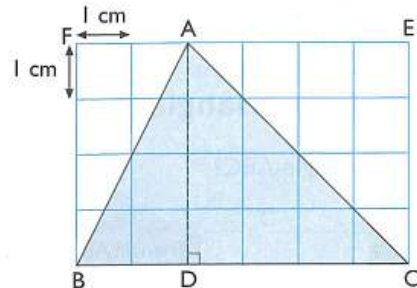
- 12  Find the square root of 225.

$$\sqrt{225} = \square$$

Press	Display
	0
   	225
	15

Un ejemplo de Geometría: Área del triángulo (5º)

2 What is the area of triangle ABC?



In triangle ABC, the base BC = 6 cm and the height AD = 4 cm.

Area of triangle ABC = area of triangle ABD + area of triangle ADC

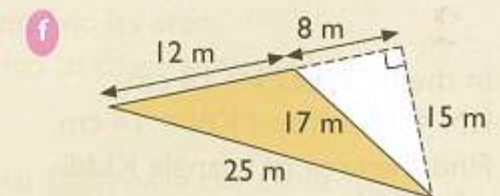
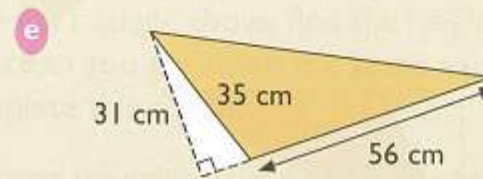
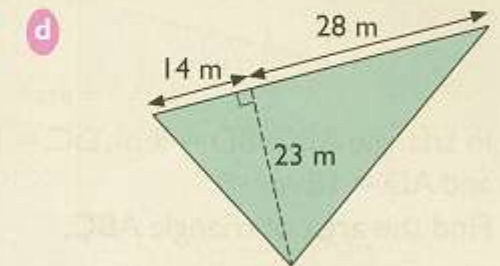
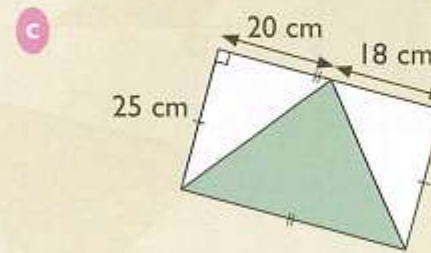
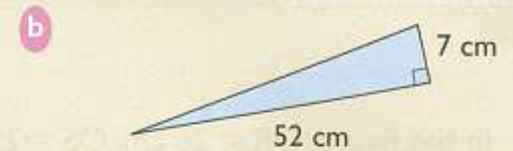
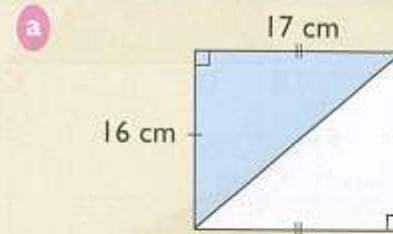
$$\begin{aligned} \text{Area of triangle ABD} &= \frac{1}{2} \times \text{area of rectangle FBDA} \\ &= \frac{1}{2} \times 2 \times 4 \\ &= 4 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of triangle ADC} &= \frac{1}{2} \times \text{area of rectangle ADCE} \\ &= \frac{1}{2} \times 4 \times 4 \\ &= 8 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{So, area of triangle ABC} &= 4 + 8 \\ &= 12 \text{ cm}^2 \end{aligned}$$

The lengths 6 cm and 4 cm of rectangle FBDA

8 Find the area of each shaded triangle.



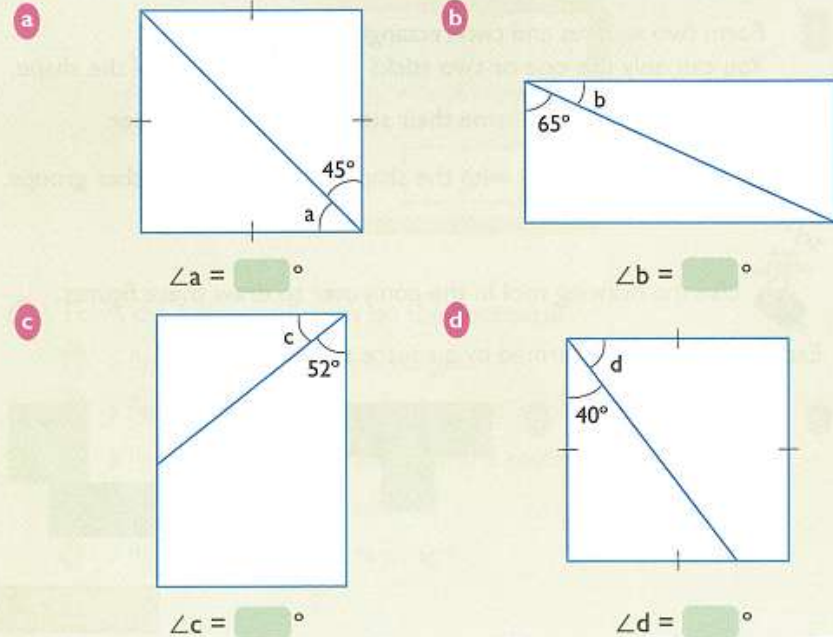
Geometría deductiva

- Poca presencia en nuestro currículo.
- Casi desaparecida de nuestras aulas de primaria.
- En Singapur: una herramienta fundamental para “enseñar a pensar”.

Algunos ejemplos:

4° de Primaria

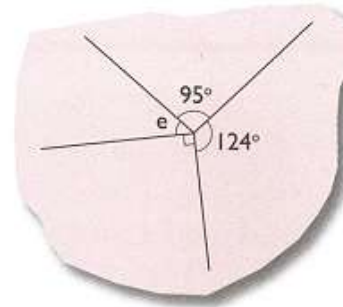
2 Find the unknown marked angles in the squares and rectangles below. The following figures are not drawn to scale.



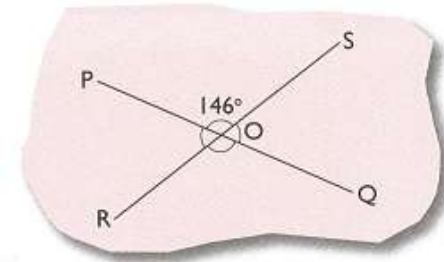
Geometría deductiva

5° de Primaria

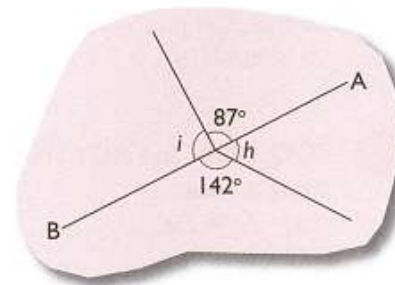
9 Find $\angle e$.



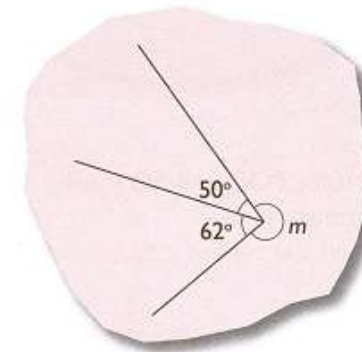
10 POQ and ROS are straight lines. Find $\angle POR$ and $\angle ROQ$.



11 AB is a straight line. Find $\angle h$ and $\angle i$.



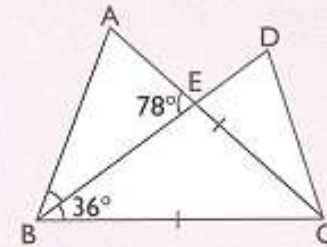
12 Find $\angle m$.



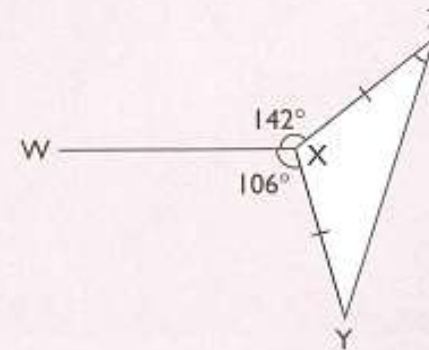
Geometría deductiva



6° de Primaria

- 2 ABC is an isosceles triangle where $AC = BC$. $\angle BEA = 78^\circ$ and $\angle CBE = 36^\circ$. Find $\angle ABE$.



- 3 XYZ is an isosceles triangle where $XY = XZ$. $\angle WXZ = 142^\circ$ and $\angle YXW = 106^\circ$. Find $\angle YZX$.

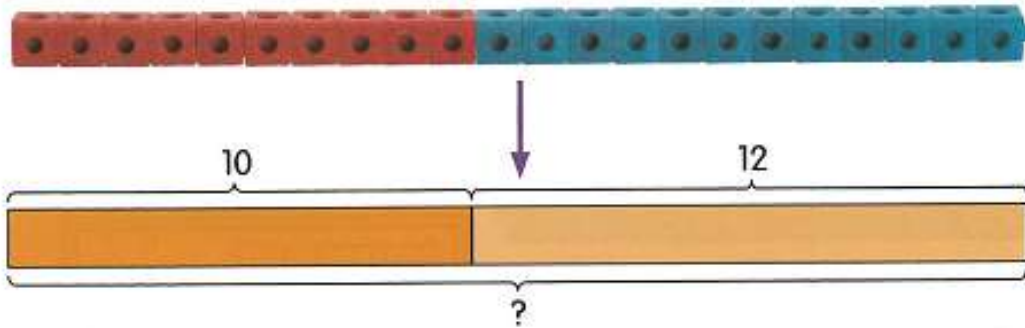


- 
- Hasta ahora: visión general de la metodología Singapur.
 - Una herramienta: el modelo de barras.
- 

El modelo de barras

Una herramienta para resolver problemas

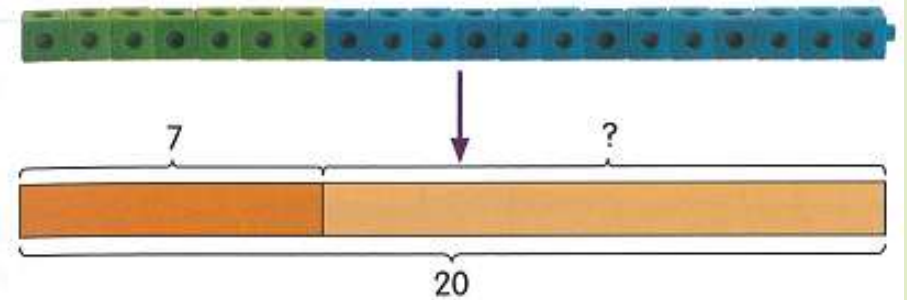
- 1** Gugo horneó 10 galletas de animales.
Aída horneó 12 galletas de animales.
¿Cuántas galletas de animales hornearon en total?



$$10 + 12 = 22$$

Ellos hornearon 22 galletas de animales en total.

- 3** Javier compró 20 huevos de gallina y codorniz.
Había 7 huevos de codorniz.
¿Cuántos huevos de gallina había?



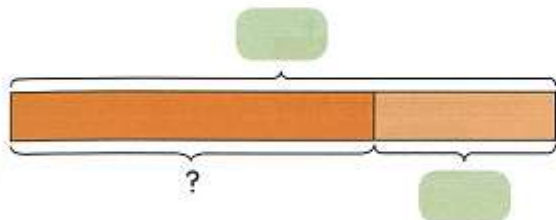
$$20 - 7 = 13$$

Había 13 huevos de gallina.

4 Gugo tiene en total 264 fotos de Francia e Italia.
93 fotos son de Italia.

- a ¿Cuántas fotos de Francia tiene Gugo?
- b ¿Cuántas fotos más de Francia que de Italia tiene Gugo?

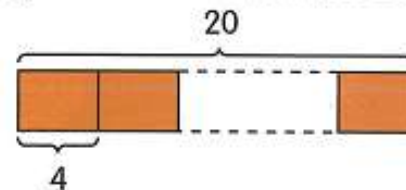
a



$$\text{[Green Box]} \cdot \text{[Green Box]} = \text{[Green Box]}$$

Gugo tiene [Green Box] fotos de Francia.

5 Bernardo tiene 20 trozos de madera para hacer patas de mesa.
Él necesita 4 patas para cada mesa.
¿Cuántas mesas hace Bernardo en total?



$$\text{[Green Box]} : \text{[Green Box]} = \text{[Green Box]}$$

Bernardo hace [Green Box] mesas en total.

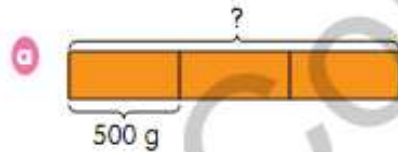
$$4 \times ? = 20$$



3°-B

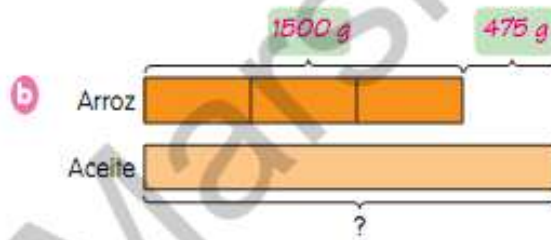
- 3 Gugo compró 3 bolsas de arroz y una botella de aceite para cocinar. El peso de cada bolsa de arroz era de 500 g. Las 3 bolsas de arroz eran 475 g más livianas que la botella de aceite.

- a ¿Cuál era el peso de las 3 bolsas de arroz?
b ¿Cuál era el peso de la botella de aceite?



$$500 \times 3 = 1500$$

El peso de las tres bolsas de arroz era de 1500 g.



$$1500 + 475 = 1975$$

El peso de la botella de aceite era de 1975 g.



Muy útil con las fracciones. Ejemplo de 4º-A

5

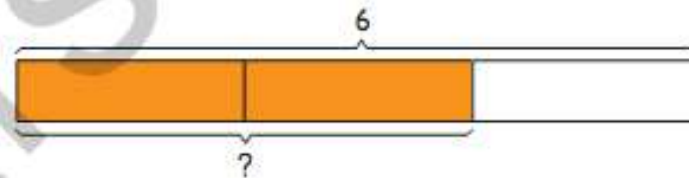


Realiza esta actividad.

Trabaja en parejas. Dibuja un modelo para encontrar el valor de lo siguiente:

Ejemplo:

$\frac{2}{3}$ de 6



3	partes	→ 6
1	parte	→ $6 : 3 = 2$
2	partes	→ $2 \times 2 = 4$

Por lo tanto, $\frac{2}{3}$ de 6 es 4.

Este es un modelo que muestra $\frac{2}{3}$ de 6.



a $\frac{2}{3}$ de 9 **6**

b $\frac{3}{5}$ de 30 **18**

2



Kim planted 312 carrots, tomatoes, and pumpkins in her garden. $\frac{2}{3}$ of her plants were carrots and $\frac{1}{4}$ of them were tomatoes. The rest of the plants were pumpkins. How many pumpkins did she plant?

En 5°

Kim plantó zanahorias, tomates y calabazas. $\frac{2}{3}$ de las plantas eran zanahorias, $\frac{1}{4}$ de las plantas eran tomates, y el resto calabazas. Si el total eran 312 plantas, ¿cuántas calabazas plantó?

Y en 6º ...

- 1** The ratio of the volume of water in Jug A to the volume of water in Jug B is 2 : 5.
- a** If half of the water in Jug A is poured into Jug B, what is the new ratio of the volumes of water in Jug A to Jug B?
 - b** If half of the water in Jug B is poured into Jug A, what is the new ratio of the volumes of water in Jug A to Jug B?
 - c** If $\frac{1}{3}$ of the water in Jug A is poured into Jug B, what is the new ratio of the volumes of water in Jug A to Jug B?

La **razón** entre el volumen de agua en el vaso A y el volumen de agua en el vaso B es 2:5.

- a) Si echamos la mitad del agua del vaso A en el vaso B, ¿cuál es ahora la razón entre los volúmenes de agua?
- b) Si echamos la mitad del agua del vaso B en el vaso A, ¿cuál es ahora la razón entre los volúmenes de agua?
- c) Si echamos $\frac{1}{3}$ del agua del vaso A en el vaso B, ¿cuál será la razón entre los volúmenes de agua en los vasos A y B?



Pongamos a prueba el modelo

Lisa y Pablo hicieron tarjetas durante dos días. El sábado Lisa hizo 19 tarjetas más que Pablo. El domingo, Lisa hizo 20 tarjetas, y Pablo hizo 15. Al acabar los dos días, Lisa hizo $\frac{3}{5}$ del total de las tarjetas. ¿Cuántas tarjetas hizo Pablo?

Un ejemplo de secundaria

5 Reduce a una sola potencia.

a) $x^5 \cdot \left(\frac{1}{x}\right)^3$

b) $\left(\frac{1}{z}\right)^6 \cdot z^4$

c) $\left(\frac{x}{y}\right)^2 \cdot \left(\frac{x}{y}\right)^3$

d) $\left(\frac{z}{m}\right)^4 \cdot \frac{z}{m}$

e) $\left(\frac{x}{y}\right)^4 \cdot \frac{y}{x}$

f) $\left(\frac{z}{m}\right)^6 \cdot \left(\frac{m}{z}\right)^4$

6 Reduce a una sola potencia.

a) $x^3 : \left(\frac{1}{x}\right)^2$

b) $\left(\frac{1}{z}\right)^3 : z$

c) $\left(\frac{x}{y}\right)^6 : \left(\frac{x}{y}\right)^5$

d) $\left(\frac{z}{m}\right)^8 : \left(\frac{z}{m}\right)^5$

e) $\left(\frac{x}{y}\right)^2 : \frac{y}{x}$

f) $\frac{z}{m} : \left(\frac{z}{m}\right)^3$

8. (a) Given that $x^{-2} = 4$, find the value of x^2 .

(b) If $y^{-3} = 27$, find the value of y .

9. (a) Find the value of r for which $3^9 \div 3^r = 3^2$.

(b) Find the value of s for which $3^s = 81$.

[N/89/P2]

10. Find the value of a if

(a) $a^{\frac{1}{2}} = 4$,

(b) $a^{\frac{1}{3}} = 3$,

(c) $a^{\frac{1}{5}} = 2$.

2° ESO

3° secundaria
(Singapur – vía académica)

Las fracciones en Primaria

La organización en Singapur:

2°

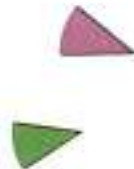


(12) Fractions



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More Fractions	38
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Adding And Subtracting Like Fractions	50
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3°



(14) Fractions

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4°

5 Fractions

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5°

3 Fractions (1)

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6°

4

Fractions

Four Operations With Fractions

70

Dividing By A Proper Fraction

72

Word Problems

87

12 Fractions



Let's Learn!

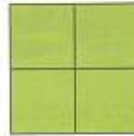
Understanding Fractions

- 1 Googol divides a square piece of paper into 4 parts.

Each part has the same size.
We say that each part is **equal**.



I can also divide the piece of paper this way.



Can you think of another way to divide the square piece of paper into 4 equal parts?

- 2 This square piece of paper has not been divided into 4 equal parts.



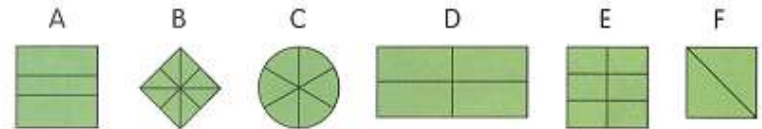
Each part does not have the same size.
We say that each part is **unequal**.

Can you think of other ways of dividing the square piece of paper into 4 unequal parts?

Are the parts equal?



- 3 Which of these shapes have been divided into equal parts?



- 4

This is a butter cookie.
It is one whole.



This is a piece of cake.
It is one whole too.



The pie is divided into 2 equal parts.



If Googol eats the 2 parts, we say that he eats 2 out of 2 equal parts.

We write it as $\frac{2}{2}$.

$\frac{2}{2}$ is a whole.

If Googol eats only 1 part, we say that he eats 1 out of 2 equal parts.

We write it as $\frac{1}{2}$.

$\frac{2}{2}$ and $\frac{1}{2}$ are some examples of **fractions**.



Let's Learn!

More Fractions

1

Let us use models to show fractions.



The model shows a whole with 5 equal parts.



2 parts are red and 3 parts are yellow.

What fraction of the whole is red?

Number of red parts = 2

Number of parts altogether = 5

Fraction of the whole in red is $\frac{2}{5}$.

Fraction of the whole in yellow is $\frac{3}{5}$.

$\frac{2}{5}$ and $\frac{3}{5}$ make 1 whole.



2 parts + 3 parts = 5 parts or 1 whole

2



The rectangle above is divided into equal parts.

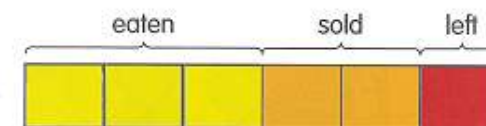
parts are shaded.

What fraction of the rectangle is shaded?

What fraction of the rectangle is **not** shaded?

and make 1 whole.

3



Sally's pie is cut into equal parts.

Fraction of Sally's pie that is eaten is .

Fraction of Sally's pie that is sold is .

Fraction of Sally's pie that is left is .

, , and make 1 whole.

- 4 Arrange the fractions in order.
Begin with the greatest.



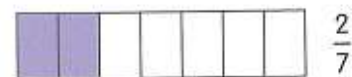
$\frac{3}{6}$ $\frac{2}{6}$ $\frac{1}{6}$
greatest

$\frac{3}{6}$ is the greatest.

$\frac{1}{6}$ is the smallest.



- 5 Arrange the fractions in order.
Begin with the smallest.



smallest

is the greatest.

is the smallest.

- 6 David eats $\frac{1}{2}$ of a cake.
Siva eats $\frac{1}{4}$ of the **same** cake.
Who eats more?



$\frac{1}{2}$ is greater than $\frac{1}{4}$.
David eats more.

- 7 Ahmad eats $\frac{1}{4}$ of a chocolate bar.
Liza eats $\frac{1}{3}$ of the **same** chocolate bar.
Who eats less?



$\frac{1}{4}$ is smaller than $\frac{1}{3}$.
Ahmad eats less.

- 8 Below are two rectangles of the same size.
Which fraction is greater?
Which fraction is smaller?



is greater than .

is smaller than .



Let's Learn!

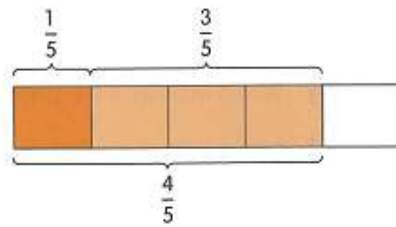
Adding And Subtracting Like Fractions

1 Jieming ate $\frac{1}{5}$ of a pizza.

Rani ate $\frac{3}{5}$ of it.

What fraction of the pizza did they eat altogether?

$\frac{1}{5}$ and $\frac{3}{5}$ are like fractions.
This number is the same.



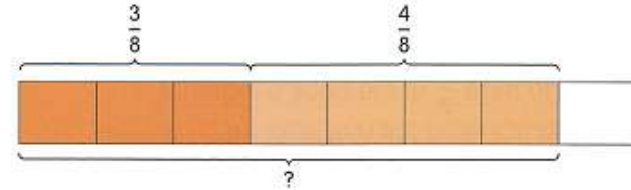
$\frac{1}{5} + \frac{3}{5}$
 $= 1 \text{ fifth} + 3 \text{ fifths}$
 $= 4 \text{ fifths}$



$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5}$$

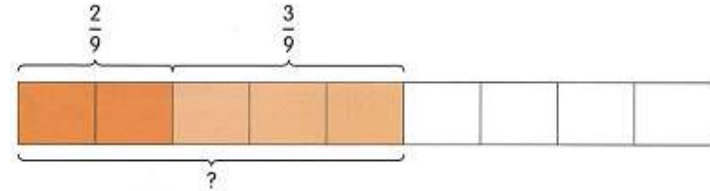
They ate $\frac{4}{5}$ of the pizza altogether.

2 Add $\frac{3}{8}$ and $\frac{4}{8}$.



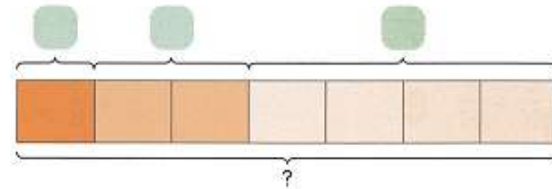
$$\frac{3}{8} + \frac{4}{8} = \text{[]}$$

3 What is $\frac{2}{9} + \frac{3}{9}$?



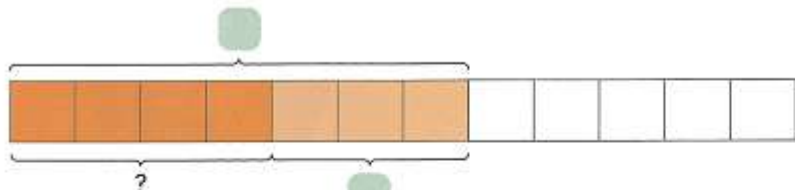
$$\frac{2}{9} + \frac{3}{9} = \text{[]}$$

4 What is $\frac{1}{7} + \frac{2}{7} + \frac{4}{7}$?



$$\frac{1}{7} + \frac{2}{7} + \frac{4}{7} = \text{[]}$$

- 5 Mrs Hooi gave $\frac{7}{12}$ of a pizza to Liza.
Liza gave some of her pizza to Milah and had $\frac{3}{12}$ of the pizza left.
What fraction of the pizza did Milah get?



\odot =
Milah got of the pizza.

- 6 $\frac{3}{8}$ of a class keep fish as pets.
Another $\frac{1}{8}$ of the class keep hamsters as pets.
The rest of the class do not keep any pets.
What fraction of the class keep fish and hamsters as pets?

\odot =
 of the class keep fish and hamsters as pets.

- 7 Karen ate $\frac{2}{9}$ of a pack of candy in the morning.
She continued eating the pack of candy in the evening.
She finished eating $\frac{8}{9}$ of the pack of candy altogether.
What fraction of the pack of candy did Karen eat in the evening?

\odot =
Karen ate of the pack of candy in the evening.

- 8 Raju drew a picture.
He coloured $\frac{4}{12}$ of the picture in red and $\frac{5}{12}$ in blue.
He did not colour the rest of the picture.
What fraction of the picture did Raju colour?

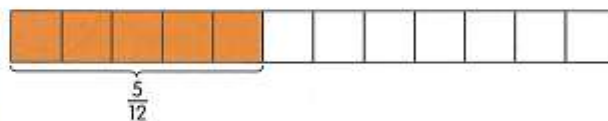
\odot =
Raju coloured of the picture.

WB 2B Part 1, p 57
Practice 5



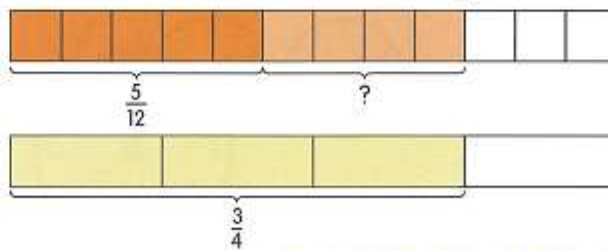
Put On Your Thinking Caps!

Look at the model below.
What fraction must be added to it to make 1 whole?



Now look at the model below.

What fraction must be added to it to make $\frac{3}{4}$?



WB 2B Part 1, p 59
Challenging Practice

WB 2B Part 1, p 60
Problem Solving

14 Fractions



Let's Learn!

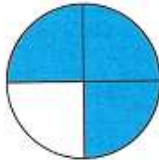
Numerator And Denominator

1



$\frac{2}{3}$ ← numerator
 $\frac{2}{3}$ ← denominator

In the fraction $\frac{2}{3}$, 2 is the **numerator**, and 3 is the **denominator**.



of the circle is shaded.
 The numerator of the fraction is .
 The denominator of the fraction is .

2

The numerator of a fraction is twice as large as 4.
 The denominator of the fraction is 7 more than the numerator.
 What is the fraction?

WB 3B Part 1, p 77
 Practice 1



Let's Learn!

Understanding Equivalent Fractions

1 Googol has some fraction strips.



One whole



1 out of 2 equal parts = $\frac{1}{2}$



2 out of 4 equal parts = $\frac{2}{4}$



4 out of 8 equal parts = $\frac{4}{8}$

The fractions $\frac{1}{2}$, $\frac{2}{4}$ and $\frac{4}{8}$ have different numerators and denominators.

But $\frac{1}{2}$ is equal to $\frac{2}{4}$.

$\frac{1}{2}$ is also equal to $\frac{4}{8}$.

$\frac{1}{2}$, $\frac{2}{4}$ and $\frac{4}{8}$ are **equivalent fractions**.



2

Name some equivalent fractions of $\frac{2}{3}$.




$\frac{2}{3}$ of the bar is shaded.



$\frac{2}{3} = \frac{\text{green circle}}{6}$



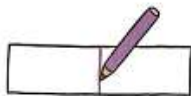
$\frac{2}{3} = \frac{\text{green circle}}{9}$

- 2  Carry out this activity.

You will be given two paper strips of the same size.

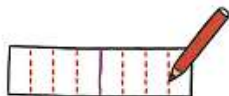
- 1 Fold the first strip into half.

- 2 Unfold the strip.
Using a colour pencil, draw a line along the fold.



- 3 Refold the strip.
Then fold it into half twice.

- 4 Unfold the strip.
Using a different colour pencil, draw lines along the new folds.



- 5 Shade to show a fraction greater than $\frac{1}{2}$.

The shaded fraction is $\frac{3}{4}$.



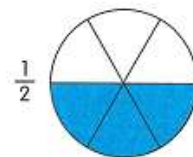
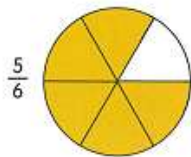
- 6 Now fold the second strip into half and repeat steps 2 to 4.

- 7 Shade a fraction which is smaller than $\frac{1}{2}$.

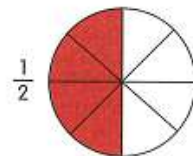
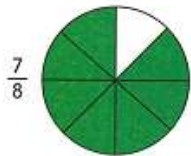
The shaded fraction is $\frac{1}{4}$.



- 3 Which is greater, $\frac{5}{6}$ or $\frac{1}{2}$?



- Which is smaller, $\frac{7}{8}$ or $\frac{1}{2}$?

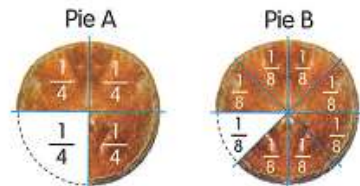



- 4 Pie A and Pie B are of the same size.

Mrs Lim cut $\frac{3}{4}$ of Pie A for Sue.

She cut $\frac{7}{8}$ of Pie B for Tim.

Who got a bigger portion?
Who got a smaller portion?



- 10  Carry out this activity.

Which fraction is greater?
Use fraction discs to help you!



- a $\frac{4}{9}$ or $\frac{2}{3}$ b $\frac{2}{4}$ or $\frac{2}{12}$
- c $\frac{3}{8}$ or $\frac{2}{4}$ d $\frac{2}{3}$ or $\frac{3}{5}$

- 11 Do these.

- a Which is smaller, $\frac{2}{3}$ or $\frac{7}{9}$?

$$\frac{2}{3} = \frac{6}{9}$$

- b Which is greater, $\frac{5}{6}$ or $\frac{3}{4}$?

$$\frac{5}{6} = \frac{10}{12} \qquad \frac{3}{4} = \frac{9}{12}$$

- c Which is smaller, $\frac{5}{6}$ or $\frac{3}{8}$?

- d Which is greater, $\frac{6}{7}$ or $\frac{1}{3}$?

- e Write any three fractions, two of which are smaller than $\frac{3}{4}$.

- f Write any three fractions, two of which are greater than $\frac{1}{2}$.



Compare the fractions to see which is greater or smaller than $\frac{1}{2}$.

- 12 Arrange the fractions in order, beginning with the smallest.

$$\frac{1}{2}, \frac{5}{6}, \frac{1}{12}$$

Method 1

Let's compare $\frac{5}{6}$ and $\frac{1}{12}$ with $\frac{1}{2}$.



$\frac{5}{6}$ is greater than $\frac{1}{2}$.

$\frac{1}{12}$ is smaller than $\frac{1}{2}$.

$$\frac{1}{12}, \frac{1}{2}, \frac{5}{6}$$

↑
smallest

Method 2

Express all the fractions with the same denominator 12.

$$\frac{1}{2} = \frac{6}{12}$$

$$\frac{5}{6} = \frac{10}{12}$$

$\frac{1}{12}$ is smaller than $\frac{1}{2}$.

$\frac{5}{6}$ is greater than $\frac{1}{2}$.

$$\frac{1}{12}, \frac{1}{2}, \frac{5}{6}$$

↑
smallest

- 13 Arrange the fractions in order, beginning with the

a greatest: $\frac{7}{8}, \frac{1}{4}, \frac{1}{2}$

b smallest: $\frac{1}{2}, \frac{9}{10}, \frac{2}{5}$



Let's Learn!

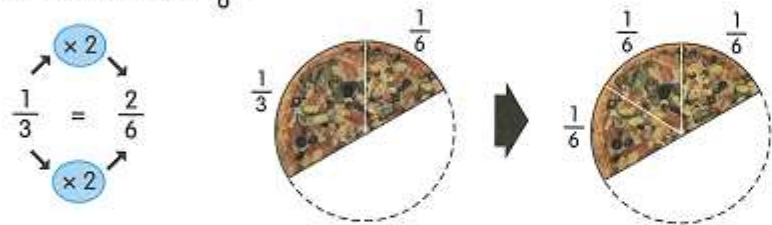
Adding Fractions

1 Lisa ate $\frac{1}{3}$ of a pizza.

Kaixing ate $\frac{1}{6}$ of the same pizza.

What fraction of the pizza did they eat altogether?

First, find an equivalent fraction of $\frac{1}{3}$ that has the same denominator as $\frac{1}{6}$.



Then add.

$$\begin{aligned} \frac{1}{3} + \frac{1}{6} &= \frac{2}{6} + \frac{1}{6} \\ &= \frac{3}{6} \\ &= \frac{1}{2} \end{aligned}$$

Always remember to write your answer in its simplest form.



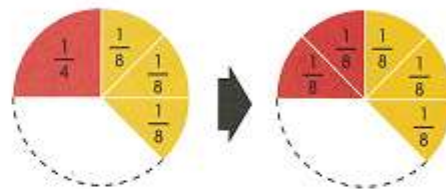
They ate $\frac{1}{2}$ of the pizza altogether.

To add fractions, first change them to fractions with the same denominator.

2 Add $\frac{1}{4}$ and $\frac{3}{8}$.

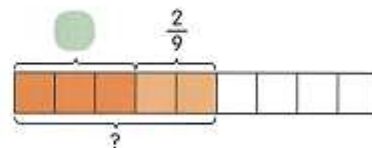
$$\begin{aligned} \frac{1}{4} + \frac{3}{8} &= \frac{2}{8} + \frac{3}{8} \\ &= \frac{5}{8} \end{aligned}$$

What fraction is equal to $\frac{1}{4}$ and has the same denominator as $\frac{3}{8}$?



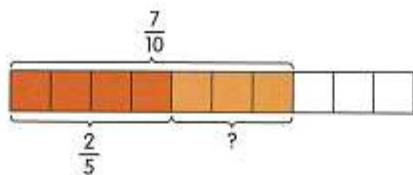
3 Find the equivalent fraction. Complete the model. Then add the fractions.

$$\begin{aligned} \frac{1}{3} + \frac{2}{9} &= \frac{3}{9} + \frac{2}{9} \\ &= \frac{5}{9} \end{aligned}$$



- 2 Subtract $\frac{2}{5}$ from $\frac{7}{10}$.

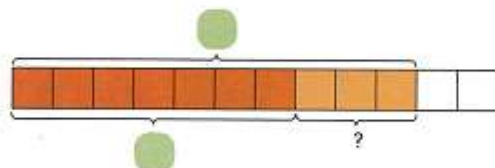
$$\frac{2}{5} = \frac{\overset{\times 2}{2}}{\underset{\times 2}{5}}$$



$$\frac{7}{10} - \frac{2}{5} = \frac{7}{10} - \frac{4}{10} = \frac{3}{10}$$

- 3 Find the equivalent fraction.
Complete the model. Then subtract the fractions.

$$\frac{5}{6} = \frac{\overset{\times 2}{5}}{\underset{\times 2}{6}}$$



$$\frac{5}{6} - \frac{7}{12} = \frac{5}{6} - \frac{7}{12} = \frac{10}{12} - \frac{7}{12} = \frac{3}{12} = \frac{1}{4}$$

- 4 Find the difference.

a $1 - \frac{3}{4} = \frac{4}{4} - \frac{3}{4} = \frac{1}{4}$

b $\frac{1}{2} - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} = \frac{1}{4}$

c $\frac{2}{3} - \frac{5}{9} = \frac{4}{9} - \frac{5}{9} = -\frac{1}{9}$

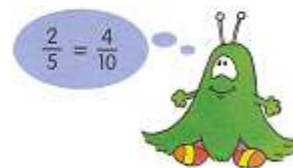
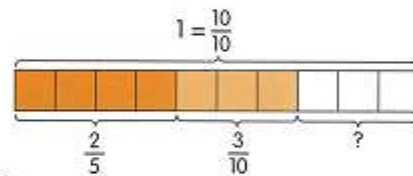
- 5 Cheryl ate $\frac{2}{5}$ of a pie.

Dennis ate $\frac{3}{10}$ of the same pie.

What fraction of the pie was left?

$$1 - \frac{2}{5} - \frac{3}{10} = \frac{10}{10} - \frac{4}{10} - \frac{3}{10} = \frac{3}{10}$$

$\frac{3}{10}$ of the pie was left.



- 6 Subtract.

a $1 - \frac{2}{7} - \frac{3}{7}$

b $1 - \frac{2}{11} - \frac{5}{11}$

c $1 - \frac{3}{4} - \frac{1}{2}$

d $1 - \frac{5}{12} - \frac{1}{3}$

4°



Put On Your Thinking Caps!

1 The tables below show the number of eggs sold by Mr Ali and Mr Cheng from Monday to Thursday last week.

Eggs sold by Mr Ali

Day	Monday	Tuesday	Wednesday	Thursday
Number of eggs sold	125	150	180	240

Eggs sold by Mr Cheng

Day	Monday	Tuesday	Wednesday	Thursday
Number of eggs sold	160	235	110	185

Study the tables and answer the following questions.

- a How many eggs did Mr Ali and Mr Cheng sell altogether on Tuesday?
- b How many eggs did Mr Ali and Mr Cheng sell altogether from Monday to Thursday?
- c On which days did Mr Ali sell more eggs than Mr Cheng?
- d On which days did Mr Ali sell more than 150 eggs?
- e On which days did Mr Cheng sell more than 180 eggs?
- f How many more eggs would Mr Ali have to sell on Tuesday in order to match the number of eggs sold by Mr Cheng on the same day?



WB 4A, p 69
Challenging Practice

5 Fractions

Let's Learn!



Mixed Numbers

1



1 whole



1 whole



1 half

$$2 + \frac{1}{2} = 2\frac{1}{2}$$

There are $2\frac{1}{2}$ watermelons.

$2\frac{1}{2}$ is a mixed number.

I have two and a half watermelons.



When you add a whole number and a fraction, you get a mixed number.

2

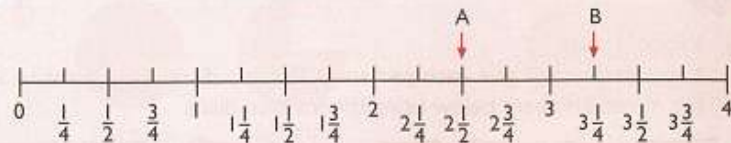
Googol drank 3 bottles of milk. Gary drank $\frac{1}{4}$ bottle of milk. How much milk did they drink altogether?

$$3 + \frac{1}{4} = \square$$

They drank \square bottles of milk altogether.



- 6 What number does each letter represent?



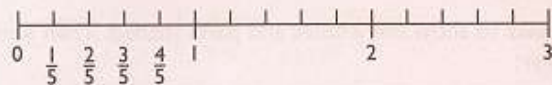
A represents $2\frac{1}{2}$ on the number line.

B represents on the number line.

You can show mixed numbers on a number line.

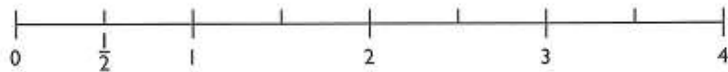


- 7 Where are $1\frac{4}{5}$ and $2\frac{1}{5}$ on the number line?

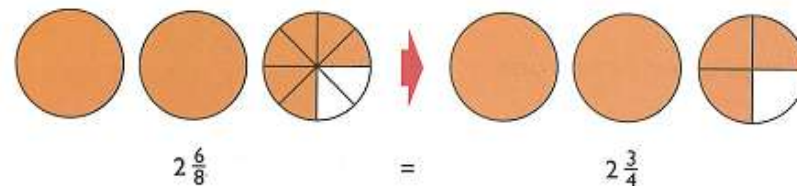


- 8 Mark the following mixed numbers on the number line.

$1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$

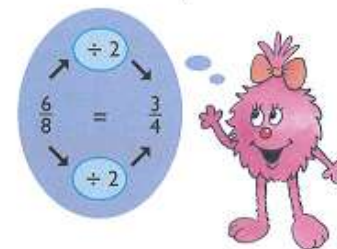


- 9 Simplify the fraction shown by the shaded parts.



$$2\frac{6}{8} = 2\frac{3}{4}$$

Cancellation is another way of dividing both the numerator and denominator by the same number.



- 10 Express the mixed number in its simplest form.

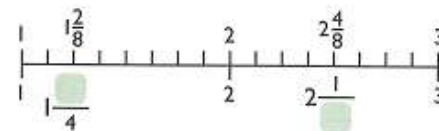
a $3\frac{8}{10} = 3\frac{\quad}{\quad}$

b $1\frac{9}{12} = 1\frac{\quad}{\quad}$

c $1\frac{4}{6} = 1\frac{\quad}{\quad}$

d $4\frac{6}{9} = 4\frac{\quad}{\quad}$

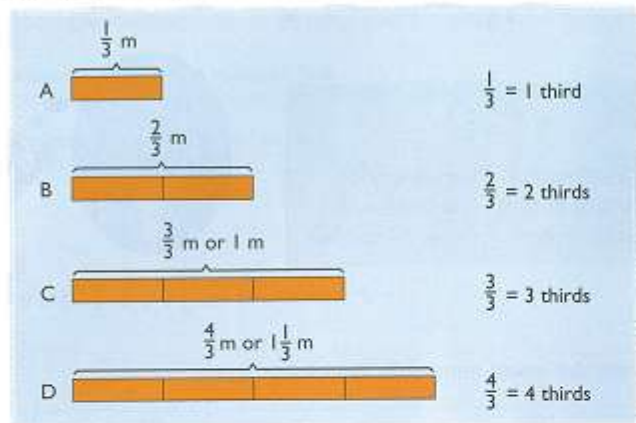
- 11 What are the missing numerator and denominator?





Improper Fractions

1 Mr Lim has some strips of wire.

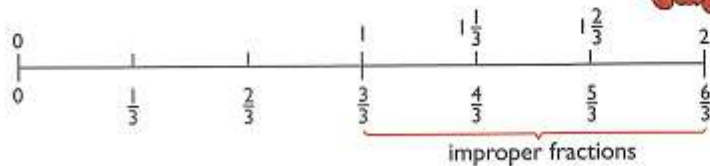


Look at Strip D. It is $1\frac{1}{3}$ m long.

There are 4 thirds in $1\frac{1}{3}$.

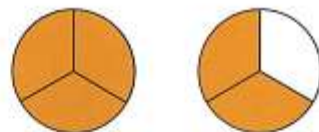
$$1\frac{1}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{4}{3}$$

$$1 = \frac{3}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$



$\frac{3}{3}$, $\frac{4}{3}$, $\frac{5}{3}$ and $\frac{6}{3}$ are equal to or greater than 1. They are called **improper fractions**.

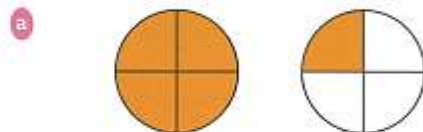
2 Write an improper fraction for the shaded parts.



There are 5 thirds in $1\frac{2}{3}$.

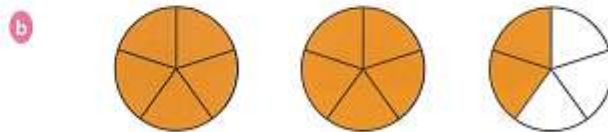
$$1\frac{2}{3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{5}{3}$$

3 Write an improper fraction for the shaded parts.



There are quarters in $1\frac{1}{4}$.

$$1\frac{1}{4} = \text{} + \text{} + \text{} + \text{} + \text{} = \text{}$$



There are fifths in $2\frac{2}{5}$.

$$2\frac{2}{5} = \text{}$$

3 Add and express the answer in its simplest form.

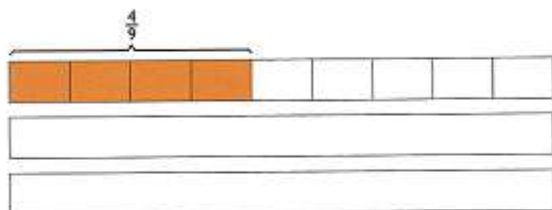
a $\frac{7}{9} + \frac{2}{3}$

b $\frac{5}{6} + \frac{1}{12} + \frac{2}{12}$

c $\frac{1}{3} + \frac{5}{12}$

d $\frac{3}{4} + \frac{3}{8}$

4 Frederick had 3 similar swiss rolls. He ate $\frac{4}{9}$ of a swiss roll. What fraction of the swiss rolls were left?



Method 1

$$3 - \frac{4}{9} = 2\frac{9}{9} - \frac{4}{9} = 2\frac{5}{9}$$

$$3 = 2 + 1 = 2 + \frac{9}{9} = 2\frac{9}{9}$$



Method 2

$$3 - \frac{4}{9} = \frac{27}{9} - \frac{4}{9} = \frac{23}{9} = 2\frac{5}{9}$$

$$\begin{array}{r} 2 \\ 9 \overline{)23} \\ \underline{18} \\ 5 \end{array}$$

$$3 = \frac{9}{9} + \frac{9}{9} + \frac{9}{9} = \frac{27}{9}$$

or

$$3 = \frac{3}{1} = \frac{3 \times 9}{1 \times 9} = \frac{27}{9}$$

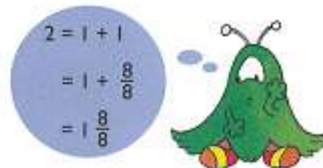


$2\frac{5}{9}$ of the swiss rolls were left.

5 Find the difference between 2 and $\frac{3}{8}$.

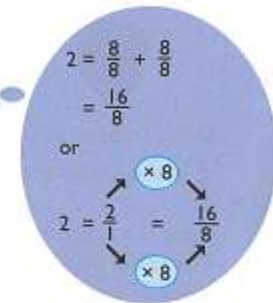
Method 1

$$2 - \frac{3}{8} = 1\frac{8}{8} - \frac{3}{8} = 1\frac{5}{8}$$

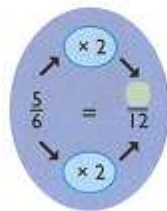


Method 2

$$2 - \frac{3}{8} = \frac{16}{8} - \frac{3}{8} = \frac{13}{8} = 1\frac{5}{8}$$



6 Find the difference between $\frac{5}{6}$ and $\frac{7}{12}$.



$$\frac{5}{6} - \frac{7}{12} = \frac{10}{12} - \frac{7}{12} = \frac{3}{12} = \frac{1}{4}$$

7 Subtract. Express the answer in its simplest form.

a $2 - \frac{5}{12}$

b $5 - \frac{2}{9}$

c $\frac{3}{4} - \frac{5}{12}$

d $\frac{8}{9} - \frac{1}{3}$



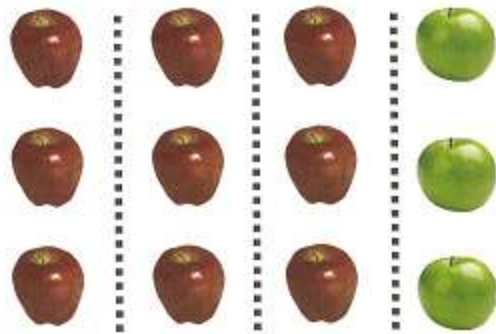
Fraction Of A Set

- 1 There are 4 apples.
3 out of the 4 apples are red.



What fraction of the apples are red?
 $\frac{3}{4}$ of the apples are red.

Here is a set of 12 apples.
The set of apples is divided into 4 equal groups.
3 out of 4 groups of apples are red.



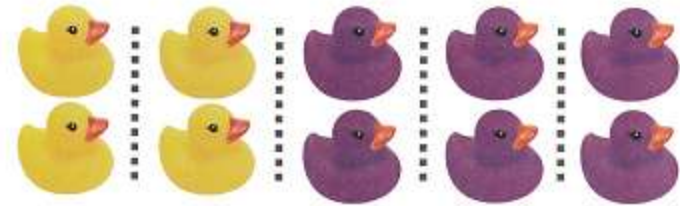
What fraction of the apples are red?
 $\frac{3}{4}$ of the apples are red.

$\frac{3}{4}$ is 3 out of 4 equal groups.



Encourage your child to talk about fraction of a set. For example, if you have bought 3 oranges and 5 apples, ask "What fraction of the fruits are oranges?"

2

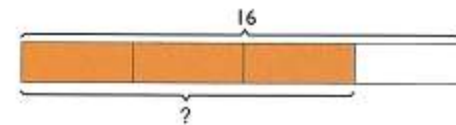


- a What fraction of the ducks are yellow?
 of the ducks are yellow.
- b What fraction of the ducks are purple?
 of the ducks are purple.

3



There are 16 cups in the set. 12 out of the 16 cups in the set are blue. $\frac{3}{4}$ of the cups are blue. So, $\frac{3}{4}$ of 16 is 12.



The shaded parts make up $\frac{3}{4}$ of the set.

What is $\frac{3}{4}$ of 16?

$$4 \text{ units} = 16$$

$$1 \text{ unit} = 16 \div 4 = 4$$

$$3 \text{ units} = 4 \times 3 = 12$$

So, $\frac{3}{4}$ of 16 is 12.

You can show fraction of a set using a model.



Put a tick (✓) to show which part(s) of this chapter you like most.
Put a cross (X) to show which part(s) of this chapter you find difficult.

- Mixed Numbers
- Improper Fractions
- Conversion Of Fractions
- Adding And Subtracting Fractions
- Fraction Of A Set
- Word Problems

Write three or four sentences on how fractions can help you in your daily life.

**Put On Your Thinking Caps!**

Jessie had a whole chocolate bar.
Minah had only part of a similar chocolate bar.
Jessie gave $\frac{1}{4}$ of her chocolate bar to Minah.



In the end, both girls had the same amount of chocolate.
What fraction of a chocolate bar had Minah at first?



Here are 2 equal bars to show that both girls had an equal amount of chocolate in the end.



Work backwards to find the fraction of the chocolate bar Minah had at first.

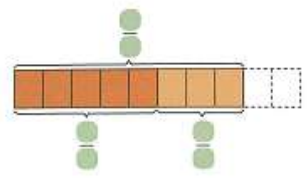


5°

Let's Practise! 3b



1 Complete the model with the fractions $\frac{1}{2}$, $\frac{3}{10}$ and $\frac{4}{5}$. Then, write two subtraction sentences.



2 Subtract. Draw models to help you.

a $\frac{5}{8} - \frac{1}{2}$

b $\frac{4}{5} - \frac{1}{4}$

3 Subtract. Express your answer in its simplest form.

a $\frac{5}{6} - \frac{1}{12}$

b $\frac{9}{10} - \frac{3}{5}$

c $\frac{8}{9} - \frac{5}{6}$

d $\frac{11}{12} - \frac{7}{8}$

e $\frac{4}{5} - \frac{2}{7}$

f $\frac{7}{9} - \frac{3}{4}$

g $\frac{4}{7} - \frac{1}{6}$

h $\frac{2}{3} - \frac{3}{8}$

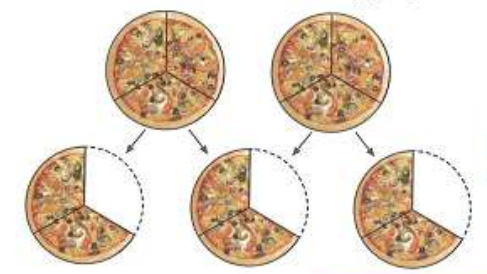
WB 5A, p 69
Practice 2

Let's Learn!



Fractions And Division

1 2 similar pizzas are shared equally among 3 pupils. What fraction of a pizza will each pupil get?



Each pizza is divided into 3 parts equally.
Each part is $\frac{1}{3}$ of a pizza.



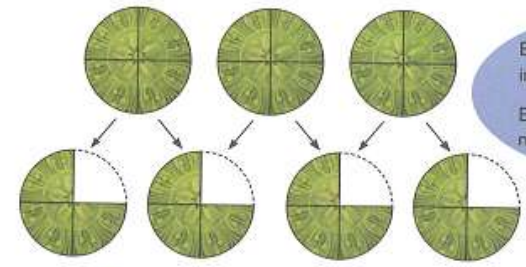
$2 \div 3 = \frac{2}{3}$

2 divided by 3 is the same as $\frac{2}{3}$.

Each pupil will get $\frac{2}{3}$ of a pizza.



2 3 similar mooncakes were cut and shared among 4 children during a family reunion dinner. What fraction of a mooncake did each child get?



Each mooncake is divided into 4 equal parts.
Each part is $\frac{1}{4}$ of a mooncake.



$3 \div 4 = \frac{3}{4}$

Each child got $\frac{3}{4}$ of a mooncake.

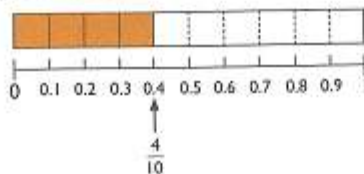


Converting Fractions To Decimals

Converting Tenths, Hundredths and Thousandths

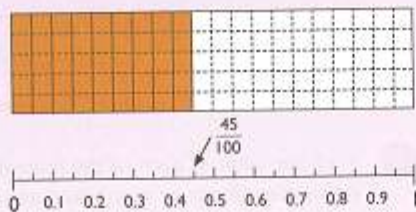
- 1 Express $\frac{2}{5}$ as a decimal.

$$\begin{aligned}\frac{2}{5} &= \frac{2 \times 2}{5 \times 2} \\ &= \frac{4}{10} \\ &= 0.4\end{aligned}$$



- 2 Express $\frac{9}{20}$ as a decimal.

$$\begin{aligned}\frac{9}{20} &= \frac{9 \times 5}{20 \times 5} \\ &= \frac{45}{100} \\ &= 0.45\end{aligned}$$



- 3 Express $\frac{1}{8}$ as a decimal.

$$\begin{aligned}\frac{1}{8} &= \frac{1 \times 125}{8 \times 125} \\ &= \frac{125}{1000} \\ &= 0.125\end{aligned}$$

8 is a factor of 1000.
 $8 \times 125 = 1000$
 By converting $\frac{1}{8}$ to $\frac{125}{1000}$,
 we can express the fraction
 in decimal easily.



- 4 Convert each fraction to a decimal.

a $\frac{4}{5} = \frac{8}{10} = \square$

b $\frac{7}{20} = \frac{35}{100} = \square$

c $\frac{2}{8} = \square$

d $\frac{6}{8} = \square$

Converting Using Long Division

- 5 Express $\frac{3}{7}$ as a decimal. Round off your answer to 2 decimal places.

$$\begin{aligned}\frac{3}{7} &= 3 \div 7 \\ &\approx 0.43\end{aligned}$$

$$\begin{array}{r} 0.428 \\ 7 \overline{) 3} \\ \underline{-28} \\ 20 \\ \underline{-14} \\ 60 \\ \underline{-56} \\ 4 \end{array}$$

- 6 Express $\frac{2}{9}$ as a decimal. Round off your answer to 2 decimal places.

$$\begin{aligned}\frac{2}{9} &= 2 \div 9 \\ &\approx \square\end{aligned}$$

$$\begin{array}{r} 0.22 \\ 9 \overline{) 2} \\ \underline{-18} \\ 20 \\ \underline{-18} \\ 2 \\ \underline{-} \end{array}$$

- 7 Convert each fraction to a decimal. Round off your answers to 2 decimal places.

a $\frac{5}{7} \approx \square$

b $\frac{1}{6} \approx \square$

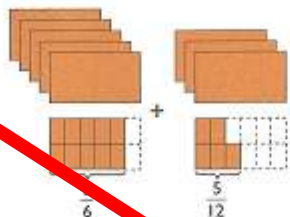
c $\frac{2}{3} \approx \square$

d $\frac{8}{9} \approx \square$

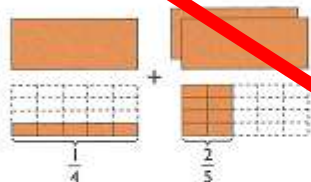


1 Add. Express your answer in its simplest form. Then, check your answer with a calculator.

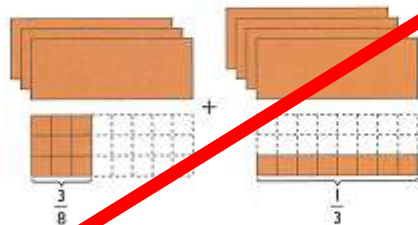
a $5\frac{5}{6} + 3\frac{5}{12}$



b $1\frac{1}{4} + 2\frac{2}{5}$



c $3\frac{3}{8} + 4\frac{1}{3}$



2 Find the sum of the mixed numbers. Express your answer as:

i a mixed number

ii a decimal correct to 2 decimal places

a $1\frac{3}{5} + 2\frac{3}{5}$

b $3\frac{3}{4} + 5\frac{2}{7}$

c $5\frac{1}{6} + 2\frac{2}{9}$

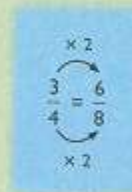


Subtracting Mixed Numbers

1 Kim bought $2\frac{3}{4}$ m of cloth. She cut $1\frac{1}{8}$ m to make a dress. How much cloth did she have left?

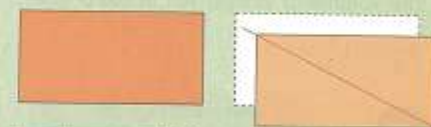


To subtract, change $\frac{1}{8}$ and $\frac{3}{4}$ to like fractions first.

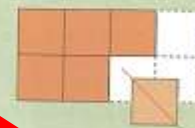


$$2\frac{3}{4} - 1\frac{1}{8} = 2\frac{6}{8} - 1\frac{1}{8}$$

$$= 1\frac{5}{8} \text{ m}$$



Kim had $1\frac{5}{8}$ m of cloth left.



4 Fractions (2)

Let's Learn!

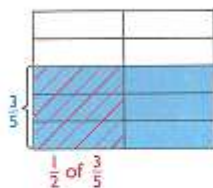


Product Of Proper Fractions

- 1 Margie draws a rectangle and colours $\frac{3}{5}$ of it blue.

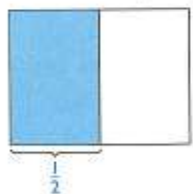


She then draws red stripes over $\frac{1}{2}$ of the coloured parts.

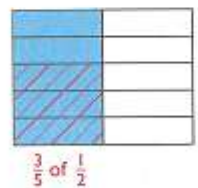


$$\begin{aligned} \frac{1}{2} \text{ of } \frac{3}{5} &= \frac{1}{2} \times \frac{3}{5} \\ &= \frac{1 \times 3}{2 \times 5} \\ &= \frac{3}{10} \end{aligned}$$

- Paul draws an identical rectangle and colours $\frac{1}{2}$ of it blue.



He then draws red stripes over $\frac{3}{5}$ of the coloured part.



$$\begin{aligned} \frac{3}{5} \text{ of } \frac{1}{2} &= \frac{3}{5} \times \frac{1}{2} \\ &= \frac{3 \times 1}{5 \times 2} \\ &= \frac{3}{10} \end{aligned}$$

Do Margie and Paul get the same answer?

We say $\frac{1}{2}$ of $\frac{3}{5}$ $\frac{3}{5}$ of $\frac{1}{2}$.

There are parts in each of Margie's and Paul's rectangles.
 coloured parts in each rectangle have red stripes.
 of each rectangle has red stripes.



Chapter 4: Fractions (2)

- 2 Find the product.

Method 1

$$\begin{aligned} \frac{3}{4} \times \frac{8}{9} &= \frac{3 \times 8}{4 \times 9} \\ &= \frac{24}{36} \\ &= \frac{2}{3} \end{aligned}$$

Method 2

$$\begin{aligned} \frac{3}{4} \times \frac{8}{9} &= \frac{\overset{3}{\cancel{3}}}{4} \times \frac{8}{\underset{3}{\cancel{9}}} \\ &= \frac{\overset{3}{\cancel{3}}}{4_1} \times \frac{8^2}{\underset{3}{\cancel{9}_3}} \\ &= \frac{1 \times 2}{1 \times 3} \\ &= \frac{2}{3} \end{aligned}$$

Divide both the numerator and denominator by the common factor, 3.

Divide both the numerator and denominator by the common factor, 4.

- 3 Find the product.

a $\frac{3}{10}$ of $\frac{5}{9} = \text{img alt="green circle"}$

b $\frac{4}{10} \times \frac{5}{12} = \text{img alt="green circle"}$

- 4 Carry out this activity.

Work in pairs.

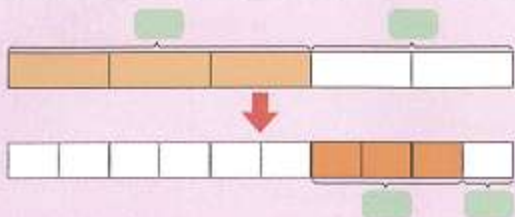
Your teacher will give each group a sheet of grid paper:

- 1 Draw a rectangle on the grid paper.
- 2 Divide the rectangle into 4 equal parts. Colour $\frac{3}{4}$ of it.
- 3 Draw crosses on $\frac{1}{4}$ of the coloured parts.
 How many coloured parts have crosses on them?
 How many parts are there altogether?
 What fraction of the whole rectangle has crosses on it?
 $\frac{1}{4}$ of $\frac{3}{4} = \text{img alt="green circle"}$
- 4 Now draw a rectangle identical to the first one.
- 5 Colour $\frac{1}{4}$ of it.

- 4 Yee Lian cut $\frac{3}{5}$ of a pie for her children. She gave $\frac{3}{4}$ of the remainder to her neighbour.

- a What fraction of the pie did she give to her neighbour?
b What fraction of the pie did she have left?

Method 1



From the model, we see that:

Number of units given to the neighbour =

Total number of units in 1 whole =

- a She gave $\frac{\text{ }}{\text{ }}$ of the pie to her neighbour.
b She had $\frac{\text{ }}{\text{ }}$ of the pie left.

Method 2

a $1 - \frac{\text{ }}{\text{ }} = \frac{\text{ }}{\text{ }}$ (remainder)

$\frac{3}{4} \times \frac{\text{ }}{\text{ }} = \frac{\text{ }}{\text{ }}$

She gave $\frac{\text{ }}{\text{ }}$ of the pie to her neighbour.

b $1 - \frac{\text{ }}{\text{ }} - \frac{\text{ }}{\text{ }} = \frac{\text{ }}{\text{ }}$

She had $\frac{\text{ }}{\text{ }}$ of the pie left.



Let's Practise! 4b



Solve these word problems. Draw models to help you where necessary.

- Mrs Smith had a plot of land. She planted flowering plants on $\frac{3}{4}$ of the land. $\frac{2}{3}$ of the flowering plants were sunflowers. What fraction of the land was planted with sunflowers?
- Jun Jie spent $\frac{7}{9}$ of his time in the morning studying Chinese and English. He spent $\frac{4}{7}$ of this time studying Chinese. What fraction of the total time did he spend studying English?
- Priya has a piece of string of length $\frac{5}{6}$ m. She cuts out $\frac{3}{5}$ of it to sew a button on her dress. What is the length of string left?
- Ben sold $\frac{7}{12}$ of his poultry. Of his remaining poultry, $\frac{3}{5}$ were chickens and the rest were ducks. What fraction of all the poultry was the unsold ducks?
- Jeff ate $\frac{1}{6}$ of a cake. He gave $\frac{1}{5}$ of the remainder to his children. He kept the rest of the cake. What fraction of the cake did he keep?
- Mrs Kong used $\frac{1}{3}$ of a stick of butter to make some biscuits. Then she used $\frac{5}{8}$ of the remaining butter to make some tarts. What fraction of the butter was left?
- Lisa spent $\frac{2}{5}$ of her money on a blouse. She then spent $\frac{4}{9}$ of her remaining money on a pair of shoes. What fraction of her money was left?

- 5 Anne used 3 bottles of syrup to make some desserts. Each bottle contained $1\frac{1}{2}$ ℓ of syrup. The cost of 1 ℓ of syrup was \$5. Find the total cost of the syrup she used.

$$\begin{aligned} 1 \text{ bottle} &\rightarrow \square \text{ ℓ} \\ 3 \text{ bottles} &\rightarrow 3 \times \square \text{ ℓ} \\ &= \square \text{ ℓ} \end{aligned}$$

3 bottles contained \square ℓ of syrup.

$$\begin{aligned} 1 \text{ ℓ of syrup} &\rightarrow \$5 \\ \square \text{ ℓ of syrup} &\rightarrow \square \times \$5 \\ &= \$\square \end{aligned}$$

The total cost of the syrup she used was \$ \square .

Let's Practise! 4e



Solve these word problems. Show your working clearly.

- 1 Ben has 6 children. He gives each child $2\frac{1}{3}$ pies. How many pies does he need?
- 2 Amin cuts a ball of string into 15 equal pieces. The length of each piece of string is $15\frac{1}{4}$ cm. What is the total length of the string?
- 3 Ken bought 9 packets of meat. Each packet of meat was $7\frac{1}{2}$ kg. The cost of 1 kg of meat is \$3. How much did he pay for all the meat he bought?
- 4 Belinda bought a plot of land 12 m long and $5\frac{2}{5}$ m wide. The cost of 1 m² of land is \$2200. How much did Belinda pay for the plot of land?

WB 5A, p 107
Practise 5

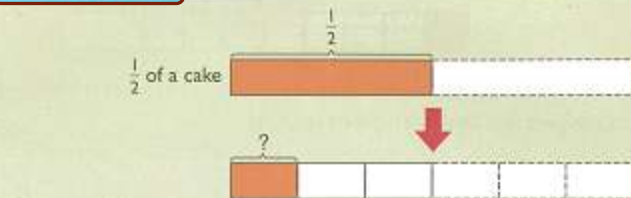
Let's Learn!



Dividing A Fraction By A Whole Number

- 1 Half of a rectangular cake is shared among 3 children. What fraction of the cake will each child get?

Method 1



$$\frac{1}{2} \div 3 = \frac{1}{6}$$

From the model, we see that each child will get $\frac{1}{6}$ of the cake.

Method 2

$$\begin{aligned} \frac{1}{2} \div 3 &= \frac{1}{3} \text{ of } \frac{1}{2} \\ &= \frac{1}{3} \times \frac{1}{2} \\ &= \frac{1}{6} \end{aligned}$$

Each child will get $\frac{1}{6}$ of the cake.

Each child will get $\frac{1}{3}$ of $\frac{1}{2}$ of the cake.



Method 3

$$\begin{aligned} \frac{1}{2} \div 3 &= \frac{1}{2} \times \frac{1}{3} \\ &= \frac{1}{6} \end{aligned}$$

Each child will get $\frac{1}{6}$ of the cake.

Multiply $\frac{1}{2}$ by $\frac{1}{3}$.

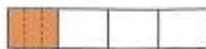


- 5 Carry out this activity.

Work in pairs.

Use the models to help you find the division of a fraction by a whole number. Then check your answer using the multiplication method.

- a Divide $\frac{1}{4}$ by 3.



- b Divide $\frac{1}{3}$ by 5.



Check: $\frac{1}{4} \div 3 = \frac{1}{12}$ of $\frac{1}{4}$

$$= \frac{1}{12} \times \frac{1}{4}$$

$$= \frac{1}{48}$$

Let's Practise! 4f



- 1 Divide.

a $\frac{2}{7} \div 4 = \frac{2}{7} \times \frac{1}{4}$

$$= \frac{2}{28}$$

b $\frac{2}{3} \div 8 = \frac{2}{3} \times \frac{1}{8}$

$$= \frac{2}{24}$$

c $\frac{3}{4} \div 12$

d $\frac{6}{7} \div 9$

- 2 Draw a model to solve each division sentence.

a $\frac{6}{11} \div 3$

b $\frac{8}{9} \div 4$

c $\frac{2}{5} \div 4$

d $\frac{3}{7} \div 2$

- 3 Solve these word problems. Draw models to help you where necessary.

- a A box contained red and green apples. $\frac{4}{5}$ of the apples were red. All the red apples were shared equally among 8 pupils. What fraction of all the apples from the box did each pupil get?

- b Melvin cut out $\frac{9}{10}$ of a cake. This portion of the cake was shared equally among Li Chen, Mi Lan and Yan Fen. What fraction of the cake did each of them get?

- c The area of a rectangular piece of cloth is $\frac{4}{9}$ m². Julie cuts the cloth into 3 smaller pieces of the same size. What is the area of each small piece of cloth?

- d A plank of wood $\frac{3}{5}$ m long is cut into 4 pieces of the same length. Find the length of each piece of wood.



- e Mrs Feng gave $\frac{1}{3}$ of her money to Lisa and $\frac{5}{12}$ of the money to James. Then she deposited the rest of the money equally in 3 accounts. What fraction of her money did she put in each account?

- f Ting Ting bought $\frac{3}{8}$ ℓ of rose syrup. She poured the rose syrup equally into 6 similar cups. Find the amount of rose syrup, in litres,

- i in each cup.
ii in 5 cups.

- g Christine bought $\frac{5}{9}$ kg of flour. She repacked them equally into 15 packets.

- i Find the mass of 1 packet of flour in kg.
ii She sold 7 packets of flour. How many kilograms of flour did she sell?

4 Fractions

Let's Learn!



Four Operations With Fractions

1 Recall that:

a **i** $\frac{1}{4} + \frac{1}{8} = \frac{\quad}{8} + \frac{\quad}{8}$
 $= \frac{\quad}{8}$

ii $\frac{2}{3} + \frac{1}{2} = \frac{\quad}{\quad} + \frac{\quad}{\quad}$
 $= \frac{\quad}{\quad}$
 $= \frac{\quad}{\quad}$

b **i** $\frac{5}{6} - \frac{1}{12} = \frac{\quad}{12} - \frac{\quad}{12}$
 $= \frac{\quad}{12}$
 $= \frac{\quad}{\quad}$

ii $\frac{4}{7} - \frac{1}{3} = \frac{\quad}{\quad} - \frac{\quad}{\quad}$
 $= \frac{\quad}{\quad}$
 $= \frac{\quad}{\quad}$

c $\frac{2}{5} \times \frac{3}{4} = \frac{\quad}{\quad} \times \frac{\quad}{\quad}$
 $= \frac{\quad}{\quad}$
 $= \frac{\quad}{\quad}$

I can also do this:
 $\frac{2}{5} \times \frac{3}{4} = \frac{2^1}{5} \times \frac{3}{4^2}$
 $= \frac{\quad}{\quad}$
 $= \frac{\quad}{\quad}$



d $\frac{13}{4} \times \frac{4}{5} = \frac{\quad}{\quad}$

e $\frac{6}{7} \div 3 = \frac{6^2}{7} \times \frac{1}{3^1}$
 $= \frac{\quad}{\quad}$

f Mrs Lim bought $2\frac{3}{4}$ kg of minced meat on Monday and another $4\frac{1}{6}$ kg on Tuesday. She mixed the meat together and repacked some of it into 5 packets of $1\frac{1}{4}$ kg each. She used the rest of the meat for cooking. How much meat did she use for cooking?

$2\frac{3}{4} + 4\frac{1}{6} = \frac{\quad}{\quad}$

She bought $\frac{\quad}{\quad}$ kg of meat.

$1\frac{1}{4} \times 5 = \frac{\quad}{\quad}$

She repacked $\frac{\quad}{\quad}$ kg of meat.

$\frac{\quad}{\quad} - \frac{\quad}{\quad} = \frac{\quad}{\quad}$

She used $\frac{\quad}{\quad}$ kg of meat.

WB 6A, p 75
Practice 1

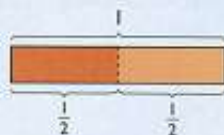


Dividing By A Proper Fraction

Dividing A Whole Number By A Proper Fraction

- 1 Lili cut a rectangular paper strip into a number of pieces. Each piece was $\frac{1}{2}$ of the paper strip. How many pieces did Lili cut the paper strip into?

$$\text{Number of pieces} = 1 \div \frac{1}{2}$$



$1 \div \frac{1}{2}$ means this: "How many halves are there in 1 whole?"



From the model, we see that there are 2 halves in 1 whole.

$$\text{So } 1 \div \frac{1}{2} = 2$$

Lili cut the rectangular paper strip into 2 pieces.

- 2 Carry out this activity.

Work in pairs.

Your teacher will provide each pair with 4 rectangular strips of paper. Each strip represents 1 whole.

- a Use each strip to find:

i $1 \div \frac{1}{3}$

ii $1 \div \frac{1}{4}$

iii $1 \div \frac{1}{5}$

iv $1 \div \frac{1}{6}$

How many one-thirds, quarters, one-fifths and one-sixths are there in 1 whole?

- b How many one-tenths are there in 1 whole?

$$1 \div \frac{1}{10} = \square$$

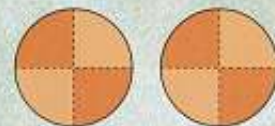
- c How many one-twelfths are there in 1 whole?

$$1 \div \frac{1}{12} = \square$$

- 3 Lee cut 2 pies into a number of pieces. Each piece was $\frac{1}{4}$ of a pie. How many pieces did Lee cut the 2 pies into?

$$\text{Number of pieces} = 2 \div \frac{1}{4}$$

$2 \div \frac{1}{4}$ means this: "How many quarters are there in 2 wholes?"



From the model, we see that:

Number of quarters in 1 pie = 4

Number of quarters in 2 pies = 2×4

$$\text{So } 2 \div \frac{1}{4} = 2 \times 4 = 8$$

Dividing by $\frac{1}{4}$ is the same as multiplying by 4.

Lee cut the 2 pies into 8 pieces.



Let's Explore!



Work out the following:

a $4 \div \frac{2}{5}$ and $\frac{2}{5} \div 4$

b $\frac{1}{4} \div \frac{2}{3}$ and $\frac{2}{3} \div \frac{1}{4}$

c $\frac{4}{5} \div \frac{3}{10}$ and $\frac{3}{10} \div \frac{4}{5}$

d $\frac{5}{8} \div \frac{3}{4}$ and $\frac{3}{4} \div \frac{5}{8}$

What do you observe about the answers to each pair of division sums?

Given that $\frac{6}{7} \div 9 = \frac{2}{21}$ and $\frac{10}{11} \div \frac{5}{6} = \frac{12}{11}$, find without further working:

i $9 \div \frac{6}{7}$

ii $\frac{5}{6} \div \frac{10}{11}$



Maths Journal

1 Explain in words, the meaning of:

a $5 \div \frac{2}{5}$

b $\frac{4}{7} \div \frac{1}{2}$

2 Julie and Sunny worked out $\frac{3}{4} \div \frac{1}{8}$ like this:

Julie	Sunny
$\frac{3}{4} \div \frac{1}{8} = \frac{4}{3} \times \frac{1}{8}$	$\frac{3}{4} \div \frac{1}{8} = \frac{4}{3} \times 8$
$= \frac{4}{24}$	$= \frac{32}{3}$
$= \frac{1}{6}$	$= 10\frac{2}{3}$

Explain what they did wrongly.

Let's Practise! 4a

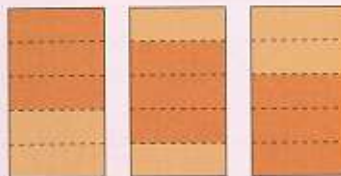


1 Use the models to find the answers.

a $1 \div \frac{1}{4}$



b $3 \div \frac{3}{5}$



c $\frac{3}{4} \div \frac{1}{8}$



d $\frac{2}{3} \div \frac{2}{9}$



2 Find by multiplication.

a $4 \div \frac{1}{7}$

b $12 \div \frac{1}{3}$

c $9 \div \frac{3}{4}$

d $10 \div \frac{4}{5}$

e $\frac{1}{2} \div \frac{1}{8}$

f $\frac{1}{4} \div \frac{1}{2}$

g $\frac{3}{5} \div \frac{11}{15}$

h $\frac{2}{3} \div \frac{10}{13}$

Un vistazo a la geometría

1. Área y perímetros

3°-B

18 Area And Perimeter



Let's Learn!

Area

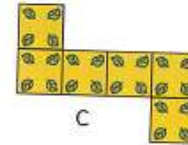
1 Googol makes these figures with some square tiles.



A



B



C

Count the number of square tiles in each figure.

Figure A is made up of 4 square tiles.

Figure B is made up of 5 square tiles.

Figure C is made up of 6 square tiles.

The amount of surface covered by the tiles is the **area** of each figure.



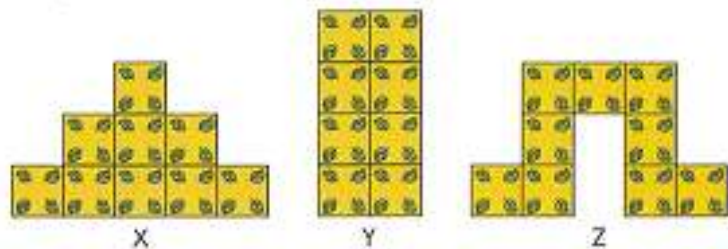
The area of Figure B is 5 tiles. What is the area of Figure A and Figure C?



Area is the amount of surface covered.

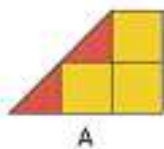



- 2 Googol makes some more figures using square tiles.






- a How many square tiles does he use to make each figure?
 b What is the area of each figure?
 Which figures have the greatest area?
 c Which two figures have the same area?

3



 is 1 square unit.

 is $\frac{1}{2}$ square unit.

 is equal to .


 make 1 square unit.



Figure A is made up of squares  and half-squares .

Figure A is made up of 4 square units. Its area is 4 square units.



4

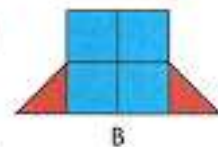
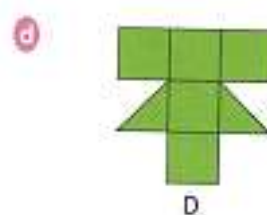
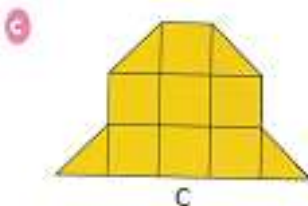
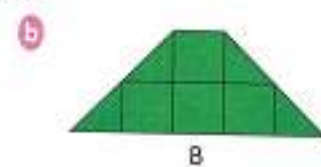
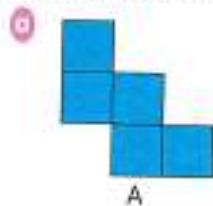


Figure B is made up of square units.

The area of Figure B is square units.

5

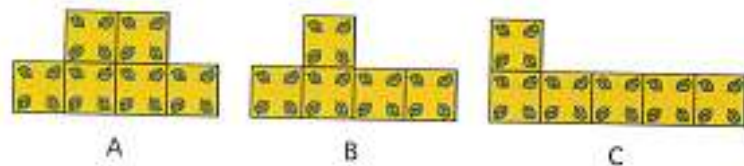
Find the area of each figure. Give your answer in square units.



Which figure has the smallest area?
 Which figure has the greatest area?
 Which figures have the same area?

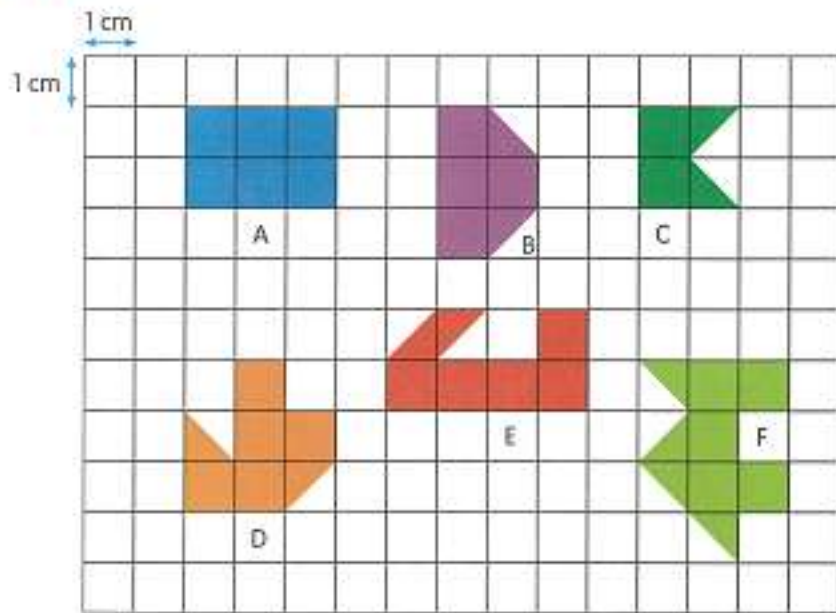
6

a These figures are made of tiles. Which two figures have the same area?



b You want all of the figures to have the same area. Give two ways of doing this.

6 a Find the area of each figure.

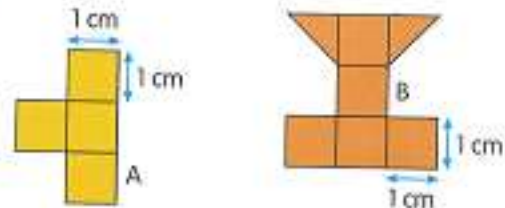


- b Which figure has the smallest area?
- c Which figure has the greatest area?
- d Which figures have the same area?



Show your child what an actual 1-cm square looks like by drawing the 1-cm square using a ruler.

7 a What is the area of each figure?



- b Which figure has a greater area?
- c You want both figures to have the same area. Give two ways of doing this.

WB 30 Part 2, p 35
Practice 7



Let's Explore!

How many 1-cm squares are there in each figure?

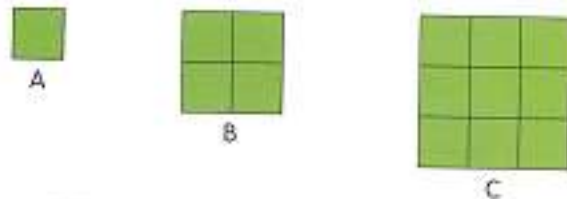


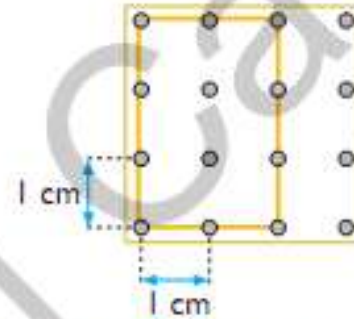
Figure	Number of squares
A	<input type="text"/> = 1
B	<input type="text"/> = 1 + <input type="text"/>
C	<input type="text"/> = 1 + <input type="text"/> + <input type="text"/>

Do you see a pattern?

4°-B

Perímetro y área

1 Gugo hace un rectángulo con un elástico en un geoplano.

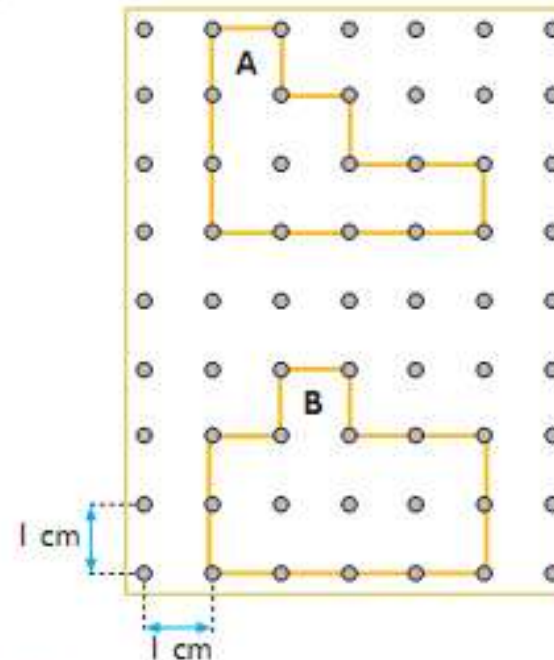


El perímetro del rectángulo es la medida de su contorno.



El perímetro del rectángulo es 10 cm.
El área del rectángulo es 6 cm^2 .

2



Gugo hace dos figuras en el geoplano.

El perímetro de la figura A es **14** cm.

El perímetro de la figura B es **14** cm.

El área de la figura A es **7** cm^2 .

El área de la figura B es **8** cm^2 .

Las figuras A y B tienen el mismo **perímetro**, pero distinta **área**.

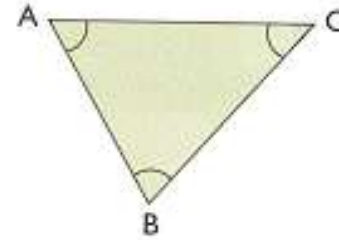
2. Los triángulos

5°-A

P 133

Base And Height Of A Triangle

1 ABC is a triangle.

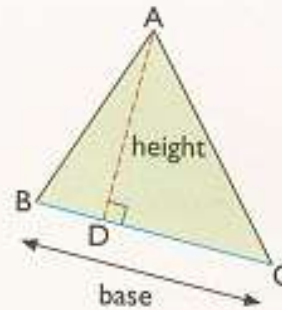


Let's recall the parts of a triangle. It has three sides and three angles.

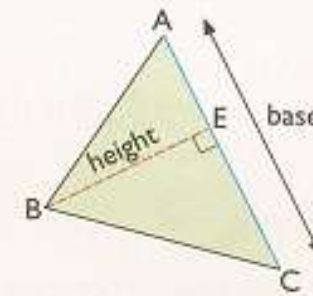


The three sides are AB, BC and CA.

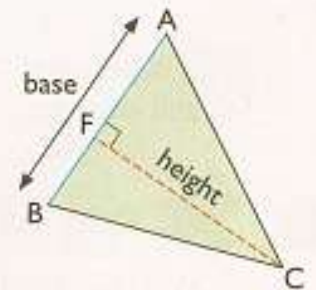
2 In triangle ABC,



AD is perpendicular to BC. BC is called the **base** and AD is called the **height**.

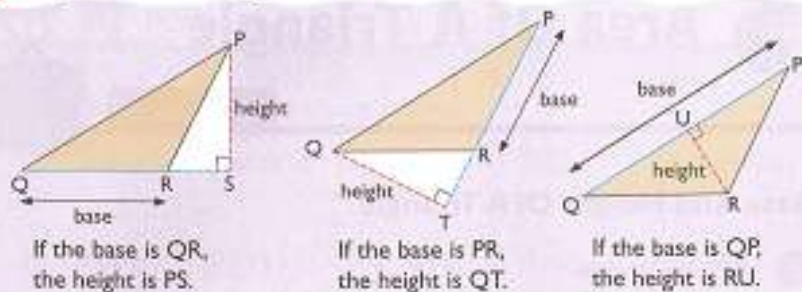


BE is perpendicular to AC. In this case, AC is the **base** and BE is the **height**.



CF is perpendicular to AB. In this case, AB is the **base** and CF is the **height**.

3 PQR is another triangle.



Any side of a triangle can be the base.



The height is always perpendicular to the base.

4 Each of these triangles is named XYZ.



is perpendicular to XY.
In each case, is the height and is the base.

5 Carry out this activity.

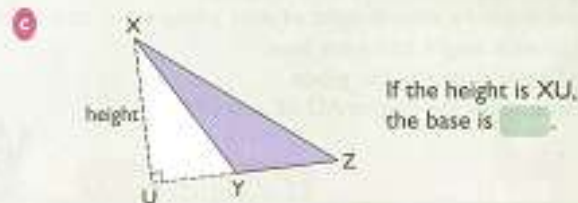
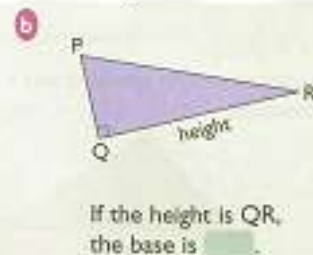


Use the drawing tool in your computer to draw a pair of perpendicular lines that meet at:

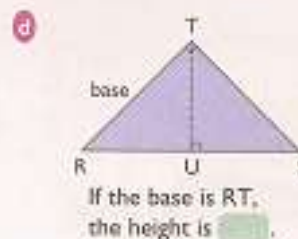
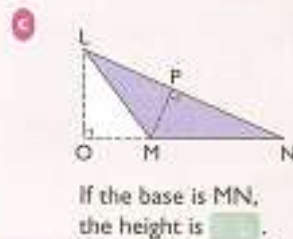
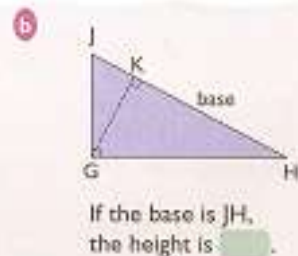
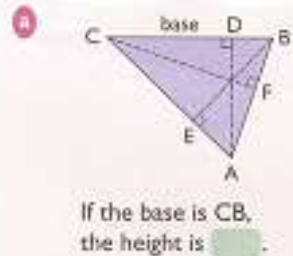
- a one right angle b two right angles


In a, label your two lines AB and BC. In b, label your two lines AB and CD. In each case, add a line or lines to each pair of the perpendicular lines that you have drawn above to form a triangle. For each triangle, name a base and its related height. What do you notice about the base and height?

6 Name the base for the given height in each triangle.



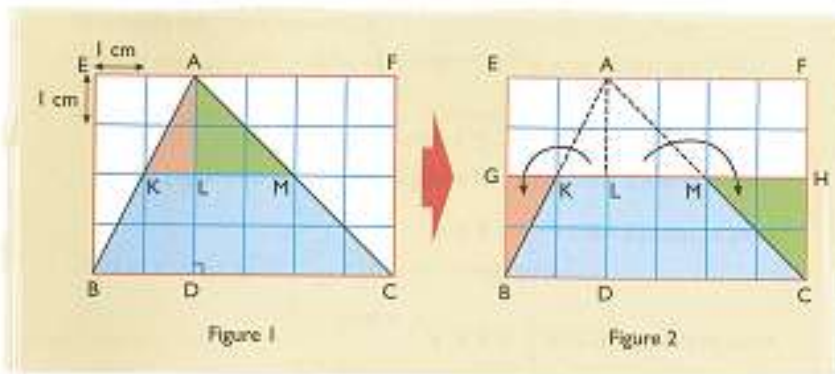
7 Name the height for the given base in each triangle.



- 4  Carry out this activity.

In triangle ABC, BC is the base and AD is the height.

- 1 Copy Figure 1 on a piece of square grid paper.
- 2 Then, cut out triangles AKL and ALM.
- 3 Rearrange the two triangles as shown in Figure 2.

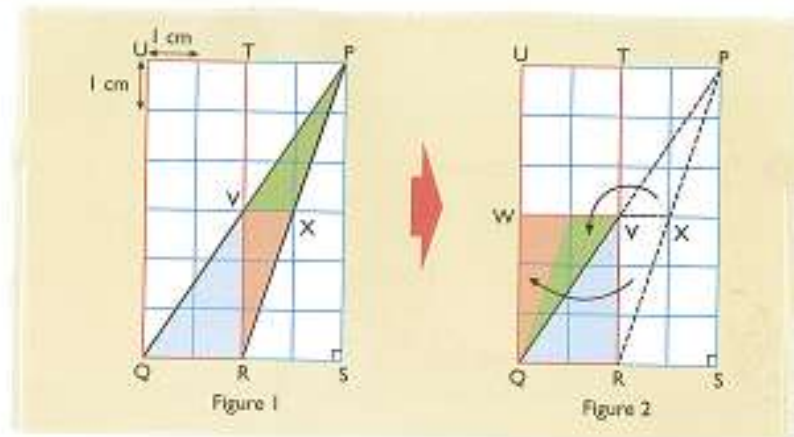


$$\begin{aligned}
 \text{Area of triangle ABC} &= \text{area of rectangle } \square \\
 &= \frac{1}{2} \times \text{area of rectangle } \square \\
 &= \frac{1}{2} \times BC \times EB \\
 &= \frac{1}{2} \times BC \times \square \\
 &= \frac{1}{2} \times \text{base} \times \square
 \end{aligned}$$

- 5  Carry out this activity.

In triangle PQR, QR is the base and PS is the height.

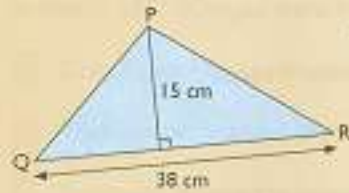
- 1 Copy Figure 1 on a piece of square grid paper.
- 2 Then, cut out triangles PVX and VRX.
- 3 Rearrange the two triangles as shown in Figure 2.



$$\begin{aligned}
 \text{Area of triangle PQR} &= \text{area of rectangle } \square \\
 &= \frac{1}{2} \times \text{area of rectangle } \square \\
 &= \frac{1}{2} \times QR \times TR \\
 &= \frac{1}{2} \times QR \times \square \\
 &= \frac{1}{2} \times \text{base} \times \square
 \end{aligned}$$

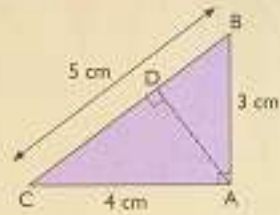
$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

- 6 Find the area of triangle PQR.



$$\begin{aligned} \text{Area of triangle PQR} &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times 38 \times 15 \\ &= \end{aligned}$$

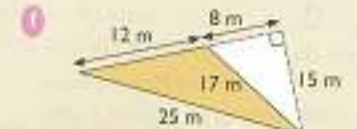
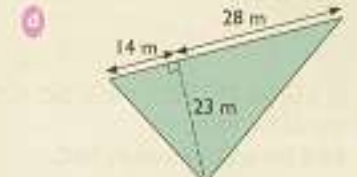
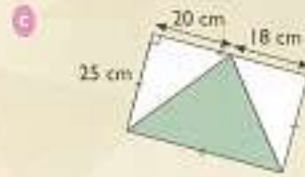
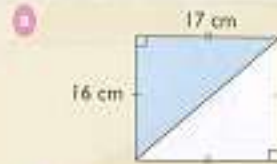
- 7 Carry out this activity.



Work in pairs.
ABC is a triangle. $\angle BAC$ is a right angle and AD is perpendicular to BC.

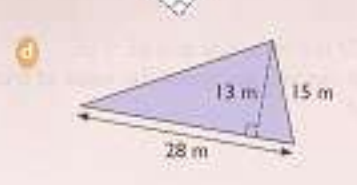
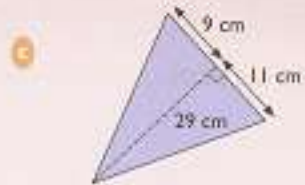
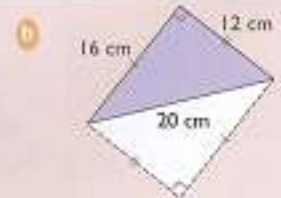
- 1 Measure the height AD in centimetres to 1 decimal place.
- 2 In turn, take each side of the triangle, AB, AC, BC as the base.
- 3 Work out the area of triangle ABC. Do you get the same area?

- B Find the area of each shaded triangle.



Let's Practise! 5b

- I Find the area of each shaded triangle.



Let's revise!

ABCD is a rectangle of perimeter 48 cm, AB = 6 cm and CD = DE.



- a Find the length of the rectangle.

$$\text{Perimeter} = 48 \text{ cm}$$

$$AB + BC = 48 \div 2$$

$$= 24 \text{ cm}$$

$$\text{Length } BC = 24 - 6$$

$$= 18 \text{ cm}$$

- b Find the area of the shaded triangle ACE.

Method 1

$$DE = CD = 6 \text{ cm}$$

$$AD = BC = 18 \text{ cm}$$

$$\begin{aligned} \text{Area of } \triangle CDE &= \frac{1}{2} \times 6 \times 6 \\ &= 18 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of } \triangle ACD &= \frac{1}{2} \times 18 \times 6 \\ &= 54 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of shaded triangle ACE} &= \text{Area of } \triangle ACD - \text{Area of } \triangle CDE \\ &= 54 - 18 \\ &= 36 \text{ cm}^2 \end{aligned}$$

Method 2

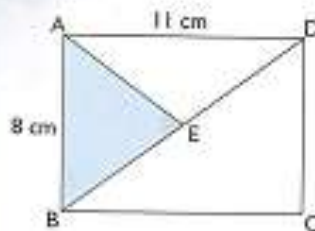
$$\begin{aligned} \text{Area of shaded triangle ACE} &= \frac{1}{2} \times AE \times CD \\ &= \frac{1}{2} \times (18 - 6) \times 6 \\ &= \frac{1}{2} \times 12 \times 6 \\ &= 36 \text{ cm}^2 \end{aligned}$$



Put On Your Thinking Caps!

ABCD is a rectangle. BE = ED.

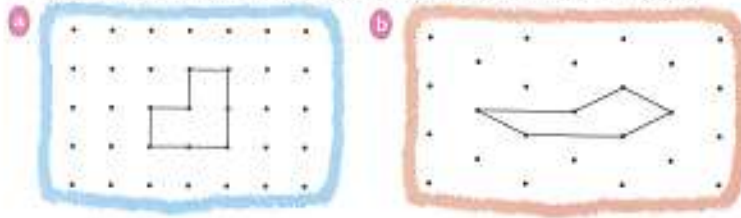
Find the area of the shaded triangle ABE.



Otra geometría - Teselaciones

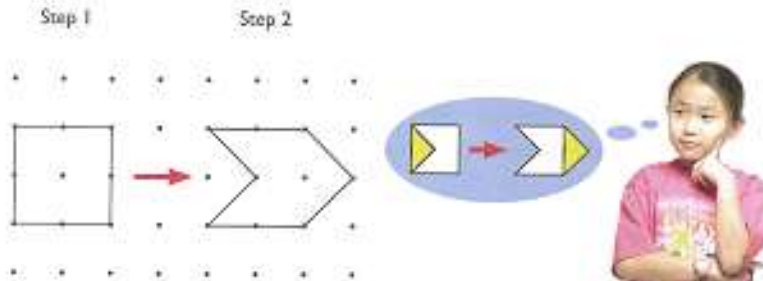
4°-B

3 Draw and cut out the shapes below. Make ten copies of each shape and use them to make as many different tessellations as you can for each shape.



4 Cassandra designs a wallpaper pattern for an art competition. She shows how she designs the unit shape for her wallpaper from a square.

Cassandra's design:

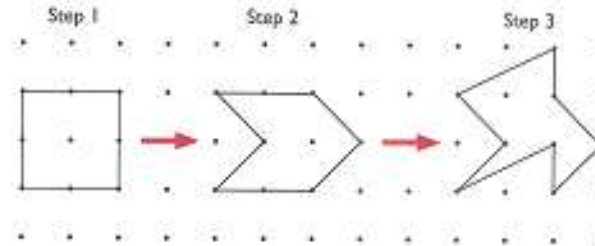


She cuts out the unit shape and makes copies of it. She colours half of the shapes blue and the other half of the shapes yellow. Then she tessellates the unit shape to make a wallpaper pattern.



Cassandra decides to improve on her unit shape. She shows how she designs another unit shape for her wallpaper.

Cassandra's second design:



She cuts out this unit shape and makes copies of it. She colours the shapes. Then she tessellates the unit shape to make another wallpaper pattern.



Prueba final de primaria

2

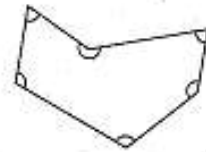
Questions 1 to 10 carry 1 mark each. Questions 11 to 15 carry 2 marks each.
For each question, four options are given. One of them is the correct answer.
Make your choice (1, 2, 3 or 4) and shade your answer on the Optical Answer Sheet.
(20 marks)

1 Redondea 31 804 al millar más cercano

- (1) 30 000
- (2) 31 000
- (3) 31 800
- (4) 32 000

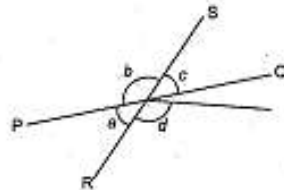
2 La figura tiene 6 ángulos. ¿Cuántos son mayores que un ángulo recto?

- (1) 5
- (2) 2
- (3) 3
- (4) 4



3 En la figura PQ y RS son rectas.
¿Cuál de esas afirmaciones es cierta?

- (1) $\angle a = \angle c$
- (2) $\angle b = \angle d$
- (3) $\angle a + \angle c = 180^\circ$
- (4) $\angle b + \angle d = 180^\circ$



4

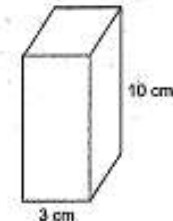
Calcula el valor de $2g-4+2g$
si $g=6$

- (1) 18
- (2) 38
- (3) 46
- (4) 62

5

Un ortoedro de altura 10 cm tiene una base cuadrada de lado 3 cm.
¿Cuál es su volumen?

- (1) 30 cm³
- (2) 90 cm³
- (3) 180 cm³
- (4) 300 cm³



6

¿Cuál dirías que es el peso total de 8 monedas de 1 euro?

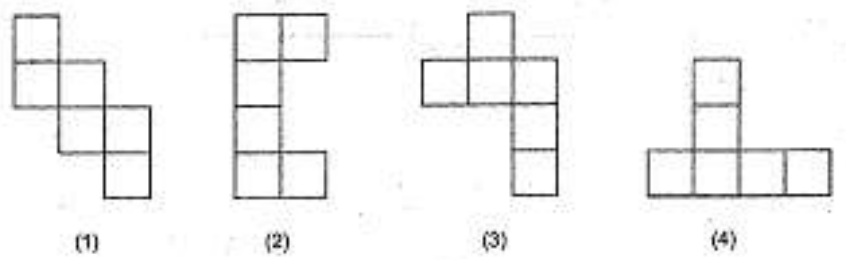
- (1) 6 g
- (2) 60 g
- (3) 600 g
- (4) 6000 g



7



¿Cuál de los siguientes es el desarrollo de un cubo?

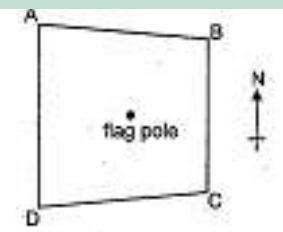


8 Tai estuvo en el colegio desde las 7 am hasta las 4 pm. ¿Cuántas horas estuvo en el colegio?

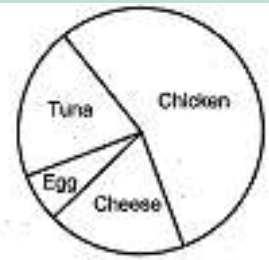
- (1) 7
- (2) 9
- (3) 10
- (4) 11

La figura muestra la posición de una bandera en el campo ABCD. ¿Qué vértice del campo está al sureste de la bandera?

- (1) A
- (2) B
- (3) C
- (4) D



Usa esta información para las preguntas 10 y 11. El diagrama muestra los diferentes tipos de bocadillos en un mostrador. 1/5 de los bocadillos son de atún y 1/4 de los bocadillos son de queso o de huevo. Había 3 veces más bocadillos de queso que de huevo.



¿Qué fracción de los bocadillos son de pollo?

- (1) $\frac{1}{2}$
- (2) $\frac{3}{4}$
- (3) $\frac{9}{20}$
- (4) $\frac{11}{20}$

¿Qué fracción de los bocadillos son de huevo?

- (1) $\frac{1}{12}$
- (2) $\frac{1}{16}$
- (3) $\frac{1}{3}$
- (4) $\frac{1}{4}$

12

Ordena estas distancias de menor a mayor

3.15 km, $3\frac{1}{5}$ km, 3 km 105 m

- | | <u>Shortest</u> | | <u>Longest</u> |
|-----|--------------------|--------------------|-------------------|
| (1) | 3 km 105 m, | 3.15 km, | $3\frac{1}{5}$ km |
| (2) | 3 km 105 m, | $3\frac{1}{5}$ km, | 3.15 km |
| (3) | $3\frac{1}{5}$ km, | 3.15 km, | 3 km 105 m |
| (4) | 3.15 km, | $3\frac{1}{5}$ km, | 3 km 105 m |

13

La figura 1 es un trapecio de perímetro 36 cm.
La figura 2 está formada por 4 de esos trapecios.
El perímetro de la figura 2 es 96 cm.

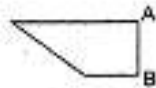


Figure 1

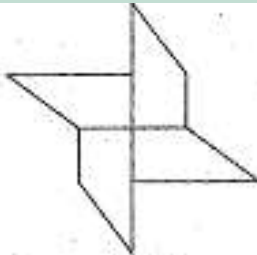


Figure 2

¿Cuánto mide el lado AB del trapecio?

- (1) 15 cm
- (2) 12 cm
- (3) 3 cm
- (4) 6 cm

14

Al dividir un número entre 30 el resto es 8.
¿Cuánto hay que sumarle al número para que sea múltiplo de 6?

- (1) 6
- (2) 2
- (3) 5
- (4) 4

15

Ling y Juni hicieron tarjetas durante dos días.
El sábado Ling hizo 19 tarjetas más que Juni.
El domingo, Ling hizo 20 tarjetas, y Juni hizo 15.
Al acabar los dos días, Ling hizo $\frac{3}{5}$ del total de las tarjetas. ¿Cuántas tarjetas hizo Juni?

- (1) 24
- (2) 26
- (3) 48
- (4) 78

(Fin primera parte – 50 minutos)

Questions 16 to 25 carry 1 mark each. Write your answers in the spaces provided. For questions which require units, give your answers in the units stated. (10 marks)

Do not write in this space

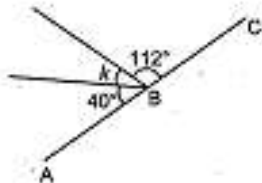
16 Calcula $8020 : 5$

Ans: _____

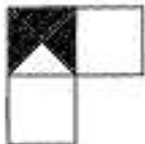
17 Calcula la media de 9 y 14

Ans: _____

18 En la figura ABC es una línea recta. Calcula x y k

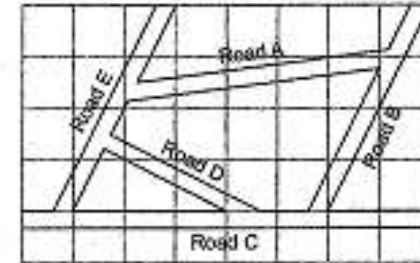


19 La figura está formada por 3 cuadrados. Uno de los cuadrados está dividido en 4 triángulos iguales. ¿Qué fracción de la figura está sombreada?



Ans: _____

Usa la figura para las preguntas 20 y 21. La figura muestra un mapa con 5 calles.



20 Nombra dos calles que sean paralelas.

Ans: _____ and _____

21 Nombra dos calles que sean perpendiculares

Ans: _____ and _____

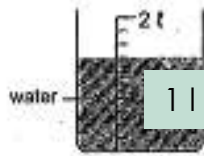
22 ¿Cuál será el precio del reloj después de añadirle el 7% de IVA?



Ans: \$ _____

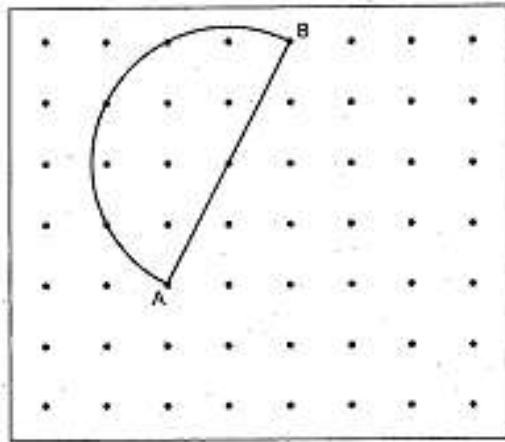
23

¿Cuánta agua (en ml) hay en el vaso?



Ans: _____ ml

Usa esta figura para las preguntas 24 y 25. Hemos dibujado un semicírculo.



24 Mide y escribe la longitud del radio.

Ans: _____ cm

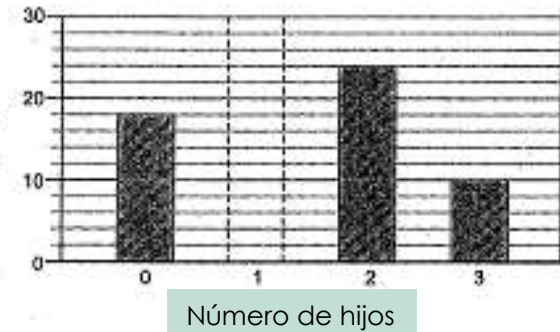
25

Elige un punto C dentro del recuadro y dibuja dos segmentos AC y BC para formar un triángulo ABC tal que $AB = AC$

Do not write in this space

Questions 26 to 30 carry 2 marks each. Show your working clearly and write your answers in the spaces provided. For questions which require units, give your answers in the units stated. (10 marks)

El diagrama de barras muestra el número de hijos en las familias de un bloque de apartamentos. $\frac{1}{3}$ de las familias tienen 1 hijo. Dibuja la barra que muestra esas familias en el diagrama.



Número de familias

La tabla muestra el precio de unos trabajos de limpieza

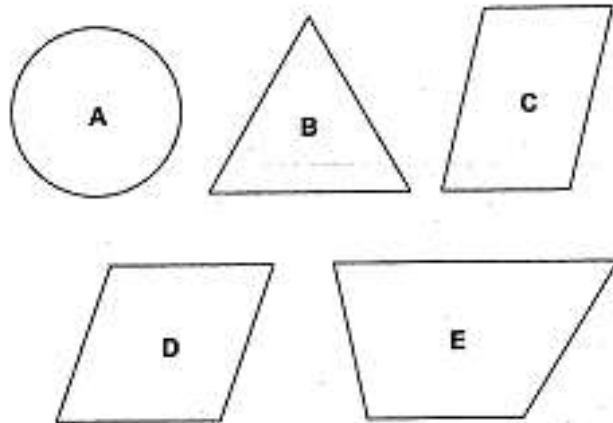
Tres primeras horas	\$80
Cada hora adicional	\$20

La Sra Menon pagó a la empresa \$200. ¿Cuántas horas duró la limpieza?

Ans: _____ h

28

Sam dibujó estas figuras. A es una circunferencia, B un triángulo equilátero, C un paralelogramo, D un rombo y E un trapecio.




Nombra las figuras que tienen al menos una recta de simetría

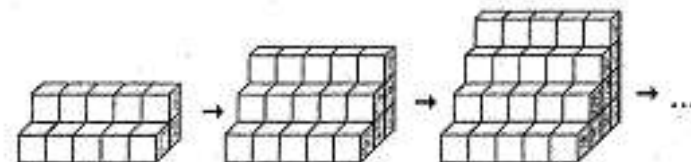
Ans: _____

2 Una bolsa contiene pajitas de tres colores distintos. $\frac{1}{4}$ de las pajitas son azules. La razón del número de pajitas rojas y el número de verdes es 2:3. ¿Cuál es la razón del número de pajitas azules y el número de pajitas verdes?

Ans: _____

Meng quiere construir una escalera con cubos de 1 cm

 The figures are 3 cm to 4 cm.



Las figuras muestran la construcción de 2 cm, luego 3 cm y luego 4 cm.

Si continúa de esta forma, ¿cuál será la altura de la escalera formada por 140 cubos?

Ans: _____ cm

Un comentario final

Muchos factores:

- ✓ Diseño curricular.
- ✓ **Metodología.**
- ✓ Formación profesorado.
- ✓ Aspectos sociológicos.

El experimento (nada científico) está en marcha ...

