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State-uncertainty preferences and the risk premium in the exchange rate market

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1. Introduction

According to the standard uncovered interest rate parity condition, the expected variation in an exchange rate should be equal to the interest rate differential between foreign and domestic risk-free bonds. Instead, empirical work usually shows that the slope coefficient from the linear projection of the change in the foreign exchange rate on the interest rate differential is often significantly negative, which implies that the domestic currency is expected to appreciate when domestic nominal interest rates exceed foreign interest rates. This is puzzling because economic intuition suggests that international investors would demand higher interest rates on currencies that are expected to depreciate.

Among the explanations of this anomaly is that there exists a timevarying risk premium in currency markets. Attempts to account for the forward premium anomaly by time varying risk premium have mostly focused on exploring dynamic, stochastic general equilibrium models with identical consumers endowed with isoelastic expected utility preferences. Engle (1982) provides an excellent survey of this literature and shows that most of these models are unable to explain the risk premiums observed in actual financial markets. The problem resides in the smoothness of implied consumption growth, relative to the volatility of the risk premium embedded in asset prices.

Inside the representative agent framework, several authors have attempted to rationalize asset pricing through state-dependent pre-

ABSTRACT

This paper shows that state-uncertainty preferences help to explain the observed exchange rate risk premium. In the framework of Lucas (1982) economy, state-uncertainty preferences amount to assuming that a given level of consumption will yield a higher level of utility the lower is the level of uncertainty perceived by consumers. Under these preferences we can distinguish between two factors driving the exchange rate risk premium: "macroeconomic risk" and "the risk associated with variation in the private agents' perception on the level of uncertainty". Empirical evidence from three main European economies in the transition period to the euro provides empirical support for the model. The model is more successful in accounting for the observed currency risk premium than models with more standard preferences, and the general perception of risk by private agents is shown to be a more important determinant of risk premium than macroeconomic uncertainty.

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ferences. Examples include papers where the utility produced by a given level of consumption depends on the previous level of consumption (habit formation), (as in Constantinides, 1990 and Campbell and Cochrane, 1999), relative social standing (as in Bakshi and Chen, 1996), or stochastic subsistence consumption levels (Campbell and Viceira, 2002). We take an alternative avenue that considers the perception by consumers on the current level of uncertainty as the state variable in preferences. A given level of consumption would then yield a higher level of utility when the consumer feels relatively certain about his future income stream than in periods when the range of possible income streams is wider. Such preferences are bound to induce real effects from changes in the perception on the level uncertainty through shifts in aggregate demand.

That this effect can improve the explanation of the observed behaviour in currency premium relative to previous specifications is shown here in a model taken from Lucas (1982). First order conditions for the time aggregate, expected utility maximization problem under standard distributional assumptions lead to an analytical expression that allows us to examine the effect on risk premium of both, private agents' perception on the level of uncertainty or state-uncertainty, and the uncertainty produced by the time evolution of macroeconomic aggregates. That way, we can discuss the relative importance of each type of uncertainty to explain excess returns in the exchange rate market.

We take advantage of the unique experiment provided by the convergence process to a monetary union in Europe to test our model. Becoming a member of the currency union would suggest higher credibility, with low inflation and increased stability, the opposite being the case if the country does not enter the union. We assume that the level of uncertainty in the economy is adequately represented by

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private agents' perceptions about the probability of their country entering the Eurozone. Changes in the perceived probability of that event will alter the level of uncertainty on future economic policy and hence, changes in the marginal utility of consumption and in the allocation of resources throughout the economy. For robustness, we also consider an alternative representation for state-uncertainty, in which filtering techniques are used to construct a proxy for the perception of uncertainty in the economy with no explicit link to the possibility of becoming a member of the Eurozone.

We proceed as follows: we present the theoretical model in section two, describing the relationships implied by optimality conditions among risk premium, the volatility of fundamental variables and the level of state-uncertainty. We also derive an analytical expression for the risk premium that allows for statistical tests to be performed. In the third section, we use our model to account for the risk premium during the transition period to the European currency. Section four presents the main conclusions.

2. Optimal decisions, the level of uncertainty and the foreign exchange risk premium

Fama (1984) defines the foreign exchange risk premium, RP_{t+1}^e , as the difference between the market expectation of currency depreciation and the current one-period forward premium, fp_t^{t+1} :

$$RP_{t+1}^{e} = [E_{t}(s_{t+1}) - s_{t}] - fp_{t}^{t+1}$$

$$= [E_{t}(s_{t+1}) - s_{t}] - [f_{t}^{t+1} - s_{t}] = E_{t}(s_{t+1}) - f_{t}^{t+1}$$
(1)

where s_t and f_t denotes the logarithm of spot and forward rates. The exchange rate risk premium can be interpreted as the excess return of a domestic investor who borrows one unit of domestic currency, buys $1/S_t$ worth of foreign currency, lends it on the foreign market for one period, and reconverts his earnings to the domestic currency.

Traditional families of preferences are generally incapable of delivering enough volatility in consumption to explain the empirically observed risk premium, but state-dependent preferences may be able do so. In particular, preferences that depend on the general level of uncertainty can deliver a significant and time-varying currency risk even if the fundamental shocks have low variance. In fact, the goal of this paper is to search for evidence on the role of private agents' perceptions on the level of uncertainty to explain the currency market anomaly in a basic representative agent, consumption-based asset pricing model [Lucas (1982) and Hu (1997)]. The model considers two countries (domestic and foreign) and two perishable commodities. In each country, a different currency is used to pay for transactions in their respective commodities. Each period t, the domestic (foreign) country receives an exogenous stochastic endowment Y_t^D (Y_t^F), and zero units of the other commodity. The domestic (foreign) country also receives an exogenous stochastic endowment $M_t^D(M_t^F)$ of its own currency.

Consumers are identical in both countries. The model is written from the perspective of the domestic country. The representative consumer maximizes time aggregate, discounted expected utility:¹

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} U\left(c_{is}^D, c_{is}^F, Z_{is}\right) \quad 0 < \beta < 1$$
⁽²⁾

where E_t denotes the conditional expectation based on information known at the beginning of period *t*. c_{is}^{D} and c_{is}^{F} represent the consumption levels of the domestic and foreign goods by the

representative agent of country *i* at period *s*, and Z_{is} denotes the perceived level of uncertainty in country *i*, i = D, *F*, at time *s*. We assume the utility function U(., .) to be bounded, continuously differentiable, increasing in the consumption of domestic and foreign goods, decreasing in the level of uncertainty, and strictly concave. The cross derivative U_{CZ} can take any sign, and β is the constant time discount factor.

The equilibrium exchange rate, in units of domestic currency per unit of foreign currency, is:

$$S_{t} = \frac{P_{t}^{D}}{P_{t}^{F}} \cdot \frac{U_{c_{D}^{E}}(c_{Dt}^{D}, c_{Dt}^{F}, Z_{it})}{U_{c_{D}^{D}}(c_{Dt}^{D}, c_{Dt}^{F}, Z_{it})}$$
(3)

Therefore, if we denote by q_{t+1}^{j} the intertemporal marginal rate of substitution (IMRS):

$$q_{t+1}^{j} = \beta \frac{U_{c_{D}^{j}}(c_{Dt+1}^{D}, c_{Dt+1}^{F}, Z_{it+1})P_{t}^{j}}{U_{c_{D}^{j}}(c_{Dt}^{D}, c_{Dt}^{F}, Z_{it})P_{t+1}^{j}}, \text{ for } j = D, F.$$
(4)

Then the rate of change in the equilibrium foreign exchange rate is given by:

$$\frac{S_{t+1}}{S_t} = \frac{q_{t+1}^F}{q_{t+1}^D}.$$
(5)

Additionally, a *forward* contract specifies at date *t* the number of units of domestic currency F_t^{t+1} to be exchanged at time t + 1 for one unit of foreign currency. *Forward* contracts allow consumers to insure themselves against the uncertainty on the future purchasing power of their own currencies. This contract specifies a net flow of $F_t^{t+1} - S_{t+1}$ units of domestic currency at date t + 1. Since it involves no payments at date *t*, the fair (absence of arbitrage) pricing relationship implies [see Backus et al. (2001)]:

$$E_t \Big[q_{t+1}^D \Big(F_t^{t+1} - S_{t+1} \Big) \Big] = 0.$$
(6)

Dividing Eq. (6) by S_t and using Eq. (5), we obtain:

$$\left(F_{t}^{t+1}/S_{t}\right)E_{t}\left(q_{t+1}^{D}\right) = E_{t}\left(q_{t+1}^{D}(S_{t+1}/S_{t})\right) = E_{t}\left(q_{t+1}^{F}\right),$$

so that, we get for the forward premium fp_t^{t+1} :

$$\frac{F_t^{t+1}}{S_t} = \frac{E_t(q_t^F+1)}{E_t(q_{t+1}^D)}.$$
(7)

Thus, given Eqs. (1), (5) and (7) the risk premium RP_{t+1}^{e} becomes equal to the difference between "the expectation of the log" and the "log of the expectation" of the IMRS for the foreign and domestic goods:

$$RP_{t+1}^{e} = E_t \Big(\log q_{t+1}^{F} \Big) - E_t \Big(\log q_{t+1}^{D} \Big) - \Big[\log \Big(E_t \left(q_{t+1}^{F} \right) \Big) - \log \Big(E_t \left(q_{t+1}^{D} \right) \Big) \Big].$$
(8)

As it is standard in the literature,² we assume that, conditional on information available at time t, Ω_t , stochastic discount factors follow a log-normal distribution: log $q_{t+1}^i/\Omega_t N$ ($\mu_{t+1}^i, \sigma_{q_{t+1}}^2$), i = D,F. Then,

$$RP_{t+1}^{e} = \mu_{t+1}^{F} - \mu_{t+1}^{D} - \left(\mu_{t+1}^{F} + \frac{1}{2}\sigma_{q_{t+1}}^{2} - \left(\mu_{t+1}^{D} + \frac{1}{2}\sigma_{q_{t+1}}^{2}\right)\right)$$
$$= \frac{1}{2}\sigma_{q_{t+1}}^{2} - \frac{1}{2}\sigma_{q_{t+1}}^{2},$$
(9)

¹ This specification is in the spirit of formulations proposed for state-dependent preferences with different rationalizations for the state variable. In Bakshi and Chen (1996) the state depends on social standing, while Campbell and Cochrane (1999) use state-dependent preferences with habits.

² See Backus et al. (2001) and Alvarez et al. (2007), among many others.

showing that a currency risk premium will arise only under a significantly different volatility on the inflation-adjusted intertemporal rate of substitution across countries

To analytically illustrate the link between the level of uncertainty and the risk premium, we need to impose some additional assumptions on the joint stochastic properties of real and nominal endowments, the probability distribution of the state variable, and the utility function.

2.1. An analytical expression for the risk premium

We assume the utility function to be time separable as well as separable in the consumption of domestic and foreign goods:

$$U(c_{it}^{D}, c_{it}^{F}) = \frac{\left(c_{it}^{D}\right)^{1-\alpha}}{1-\alpha} Z_{t}^{\lambda^{D}} + \frac{\left(c_{it}^{F}\right)^{1-\gamma}}{1-\gamma} Z_{t}^{\lambda^{F}} \quad \alpha, \gamma \ge 0, \ \lambda^{D}, \ \lambda^{F} \le 0 \quad \text{and} \quad \alpha \neq 1, \gamma \neq 1,$$
(10)

where α and γ are intertemporal elasticity of substitution parameters, Z_t is the state variable measuring the perceived level of uncertainty, and λ^j , for j = D, F, indicates the extent to which uncertainty affects the utility of the consumption of domestic and foreign goods. Additionally, it is necessary to make some assumptions on the joint stochastic behaviour of real and nominal endowments, as well as on the probability distribution of the level of uncertainty before obtaining a tractable expression for the risk premium.

If we impose standard cash-in-advance constraints and exploit the conditions for the perfectly pooled equilibrium in Lucas (1982) that consumption is equal in each country to half of the domestic and foreign production, the domestic and foreign IMRS from Eq. (4) under our assumed preferences, become:

$$q_{t+1}^{D} = \left(y_{t+1}^{D}\right)^{1-\alpha} \left(m_{t+1}^{D}\right)^{-1} z_{t+1}^{\lambda^{D}},$$

$$q_{t+1}^{F} = \left(y_{t+1}^{F}\right)^{1-\gamma} \left(m_{t+1}^{F}\right)^{-1} z_{t+1}^{\lambda^{F}},$$
(11)

where $y_{t+1}^i \equiv Y_{t+1}^i / Y_t^i$, $m_{t+1}^i \equiv M_{t+1}^i / M_t^i$, for i = D, *F* and $z_{t+1} \equiv Z_{t+1} / Z_t$.

Together with Eq. (9), these relationships allow us to relate the risk premium to the main sources of uncertainty in the economy, coming from the time evolution of macroeconomic variables, like money supply and output, $M_{t+1}^i, Y_{t+1}^i, i=D, F$, or from alternative sources, not reflected in observed variables, that we summarize in Z_{t+1} . We assume the rates of growth of output, the money supply and the level of uncertainty to be conditionally jointly log–normal. From Eq. (11), this is a sufficient condition for the log–Normality of IMRS. Taking logs in Eq. (11), IMRS volatility can be seen to depend on the average change in the perceived level uncertainty, given by $\sigma_{z_{t+1}}$, the size of that effect being determined by $\lambda^i, j=D, F$:

$$\begin{split} \sigma_{q_{t+1}^{p}}^{2} &= (1-\alpha)^{2} \sigma_{y_{t+1}^{p}} + \sigma_{m_{t+1}^{p}}^{2} + \left(\lambda^{D}\right)^{2} \sigma_{z_{t+1}}^{2} - 2(1-\alpha) \sigma_{y_{t+1}^{p} m_{t+1}^{p}} \\ &+ 2\lambda^{D}(1-\alpha) \sigma_{z_{t+1}y_{t+1}^{p}} - 2\lambda^{D} \sigma_{z_{t+1} m_{t+1}^{p}}, \\ \sigma_{q_{t+1}^{p}}^{2} &= (1-\gamma)^{2} \sigma_{y_{t+1}^{p}} + \sigma_{m_{t+1}^{p}}^{2} + (\lambda^{F})^{2} \sigma_{z_{t+1}}^{2} - 2(1-\gamma) \sigma_{y_{t+1}^{F} m_{t+1}^{F}} \\ &+ 2\lambda^{F}(1-\gamma) \sigma_{z_{t+1}y_{t+1}^{p}} - 2\lambda^{F} \sigma_{z_{t+1} m_{t+1}^{F}}. \end{split}$$

From Eqs. (9) and (11), the risk premium can be written in terms of conditional variances and covariances of output growth, monetary aggregates, and the level of uncertainty:

$$RP_{t+1}^{e} = \frac{1}{2}(1-\alpha)^{2}\sigma_{y_{t+1}^{o}}^{2} - \frac{1}{2}(1-\gamma)^{2}\sigma_{y_{t+1}^{e}}^{2} + \frac{1}{2}\sigma_{m_{t+1}^{o}}^{2} - \frac{1}{2}\sigma_{m_{t+1}^{e}}^{2} + \frac{1}{2}\left((\lambda^{p})^{2} - \left(\lambda^{F}\right)^{2}\right)\sigma_{z_{t+1}}^{2} - (1-\alpha)\sigma_{y_{t+1}^{o}m_{t+1}^{o}} + (1-\gamma)\sigma_{y_{t+1}^{e}m_{t+1}^{e}} + (1-\alpha)\lambda^{p}\sigma_{z_{t+1}y_{t+1}^{e}} - (1-\gamma)\lambda^{F}\sigma_{z_{t+1}y_{t+1}^{e}} - \lambda^{p}\sigma_{z_{t+1}m_{t+1}^{e}} + \lambda^{F}\sigma_{z_{t+1}m_{t+1}^{e}},$$

$$(12)$$

where $o_{x_{t+1}}^i \equiv \operatorname{var}_t (\log(x_{t+1}^i))$ and $o_{x_{t+1}p_{t+1}} \equiv \operatorname{cov}_t (\log(x_{t+1}^i), \log(p_{t+1}^i))$. The expected risk premium is determined by macroeconomic uncertainty through (i) the conditional variance of domestic and foreign output, (ii) the conditional variance of domestic and foreign money supply, (iii) the conditional covariance between output and money supply, (iv) the conditional variance of the uncertainty indicator, and (v) the conditional covariance between money supply and output with the uncertainty indicator.

Conditions i), ii), and iii) capture the effect of macroeconomic uncertainty on the forward risk premium. As shown in Eq. (11), an increase in the volatility of money supply or real income in the domestic country or a decrease in the positive covariance between these two variables will increase IMRS volatility and hence the forward the risk premium, from Eq. (9).³ An increase in the volatility of money supply or real income in the foreign country or a decrease in their covariance would lead to the opposite effect on the risk premium. Conditions iv) and v) have to do with the uncertainty indicator. Under the maintained assumption that $|\lambda^D| > |\lambda^F|$, an increase in the volatility of state-uncertainty changes will increase the difference between the volatility of the domestic and the foreign IMRS, and this effect will raise the forward premium.

Thus our model generalizes Hu (1997) with the exchange rate risk premium having a second source of risk associated to the private agents' perception on the level of uncertainty. This additional argument might provide the additional volatility needed to reproduce the empirically observed high currency risk premium without requiring unreasonable coefficients of relative risk aversion, which is the main goal of this paper. We are particularly interested in the evolution of the observed risk premium during the transition period to the European currency using bilateral exchange rates between the French franc, British pound, and Spanish peseta, all relative to the German mark, and we want to estimate the relevance of fundamental uncertainty, relative to macroeconomic uncertainty, to explain the observed risk premium.

3. Testing the model

We start the empirical analysis of Eq. (12) by⁴ estimating the conditional variances and covariances for the exogenous variables as well as by constructing proxies for the perception of uncertainty by private agents. To estimate the level of state-uncertainty we follow two different approaches: First, a structural approach that considers a specific type of uncertainty, emerging from the possibility of joining the Eurozone at the outset. The second approach is mostly empirical, and uses a filtering technique to infer the evolution over time of the perception of the general level of uncertainty in the economy. Since our main interest is to evaluate

³ This effect arises because, under the cash-in-advance constraint, the referred changes in second order moments of money supply or income will increase the conditional variance of the price level. It is this increase in future price volatility that produces the increase in IMRS volatility.

⁴ We consider the bilateral relationships between Spanish peseta (SPA), Deutsche mark (DEM), Sterling pound (GBP) and French franc (FRF). The sample starts on January 1, 1986 after Spain became a member of the European Economic Community and ends in April 1998. In May 1998 the European Council announced the countries that would form the euro area on January 1, 1999. We use monthly data for Spain (SP), Germany (GER), France (FR) and United Kingdom (UK). The industrial production index (IP) is used as an indicator of economic activity and M2 as the monetary aggregate. We also use interest rates on 3- and 10-year maturity *swaps* for all countries, from 1992:1 to 1998:04. Finally, *spot* and *forward* exchange rates are taken for the last day of the month. Preliminary data analysis using unit root tests and intervention analysis [Box and Tiao (1975)] shows that all variables, except the risk premium, are I(1). Therefore, all variables are differenced when estimating the model for the conditional variance. These preliminary results are not reported here but are available upon request.

the importance of state-uncertainty, relative to macroeconomic uncertainty, to explain the currency risk premium, it is especially relevant that we check for the robustness of our results under widely different approaches to estimating the unobserved level of state-uncertainty.

3.1. A structural approach to estimating the perception of uncertainty

The main difficulty in estimating expression (12) for the exchange risk premium is that the level of uncertainty is unobserved, and a popular approach to dealing with this problem is to postulate a specific law of motion for changes in the level of uncertainty. In our model, Z_t represents the type of uncertainty which is not already captured by macroeconomic aggregates as industrial production or the money supply. In consistency with that view, we identify Z_t in this section with the perceived level of uncertainty on the success of the convergence process to the euro. It seems natural to assume that the effect of unexpected news depends on the level of uncertainty in the economy: in an economy where agents are almost sure that they will enter the Eurozone, a piece of negative news will not induce expectations of future policy changes, and hence, it will not alter consumers' decisions. The same could be said about a piece of positive news arriving to an economy where private agents are almost sure that they will not join the euro area. In both situations, the level of uncertainty is low. If, on the other hand, private agents believe that there is a 50-50 chance that their country may join the euro, then any piece of negative or positive news may have a large contribution to the general level of uncertainty with a significant influence on consumers' decisions.

We formalize this view by considering a regime indicator x to be realized at time T, when the decision of joining the Eurozone is to be made. At that time, x would take a value of 1 if the economy enters the euro system, being equal to 0 otherwise. At each point in time, the representative agent in the economy associates a probability p_t of joining the euro area, i.e., to the event x = 1, and a probability of $(1 - p_t)$ of being left out, i.e., to the event x = 0. The probability p_t should be expected to change over time as a function of the information that private agents receive on some economic indicators that agents consider relevant when predicting future policy decisions. At time t, the expected value of x is: $E_t(x) = p_t$, and its variance: $(\sigma_t^x)^2 = \operatorname{var}_t(x) = p_t(1 - p_t)$. The variance of x indicates the level of uncertainty on the event of entering the Eurozone.

To capture the possibility that the impact of a given piece of news will be larger the higher the level of uncertainty prevailing in the economy, we assume that changes in the level of uncertainty, z_t , are driven by:

$$z_{t+1} = \sigma_t^{x} \xi_{t+1}, \quad \xi_{t+1} / p_t, p_{t-1}, p_{t-2}, \dots : N(0, \kappa)$$
(13)

where ξ_t represents the arrival of new information regarding the fulfillment of Maastricht criteria. The variance of z_{t+1} , an indicator of the expected size of changes in uncertainty, is:

$$\sigma_{z_{t+1}}^2 = var_t z_{t+1} = \kappa (\sigma_t^x)^2 = \kappa p_t (1-p_t).$$
(14)

Therefore, the variance of z_{t+1} is zero when p_t is either 0 or 1, reflecting absolute certainty about being OUT or IN, a situation of zero euro-uncertainty. The variance of changes in uncertainty reaches its maximum value for intermediate values of p_t . Whenever $p_t < 1/2$, an increase in the probability of entering the Eurozone will increase the level of uncertainty, whereas, for $p_t > 1/2$, an increase in the probability of joining the euro would reduce the variance of z_{t+1} , and the opposite would arise for reductions in p_t .

Under this specification, adding the assumption that money supply and production are conditionally independent of the level of uncertainty, *Z*, Eq. (12) can be written:

$$\begin{aligned} RP_{t+1}^{e} &= \lambda p_{t}(1-p_{t}) + \frac{1}{2}\sigma_{m_{t+1}}^{2} - \frac{1}{2}\sigma_{m_{t+1}}^{2} + \frac{1}{2}(1-\alpha)^{2}\sigma_{y_{t+1}}^{2} \\ &- \frac{1}{2}(1-\gamma)^{2}\sigma_{y_{t+1}}^{2} - (1-\alpha)\sigma_{y_{t+1}}m_{t+1}^{p} + (1-\gamma)\sigma_{y_{t+1}}m_{t+1}^{F} \end{aligned} \tag{15}$$
with: $\lambda = \frac{1}{2}\left((\lambda^{D})^{2} - (\lambda^{F})^{2}\right)\kappa.$

3.2. An empirical approach to filtering for the uncertainty in risk premia

In our second approach, we start by extracting from the risk premium the effect of macroeconomic uncertainty, represented by the conditional variances and covariances of money supply and income that we used in the previous approach. The residual from such a projection should be the sum of two elements: the level of unobservable uncertainty $\frac{1}{2}((\lambda^p)^2 - (\lambda^F)^2)\sigma_7^2$ and the residual in Eq. (12), and we would like to identify both components. Luckily enough, the structure of our theoretical model provides us with identification restrictions, since the $\frac{1}{2}((\lambda^p)^2 - (\lambda^F)^2)\sigma_z^2$ term is a conditional variance and hence, a function of state variables that are observable at time t. On the other hand, the residual in Eq. (12) should be an innovation with zero autocorrelation. This allows for the following identification strategy: we first compute the residual from a least squares projection of the risk premium on the conditional second order moments of macroeconomic indicators. That residual is then projected onto state variables known at t: lagged conditional variances and covariances of industrial production and money supply, lagged interest rates at 3 and 10-year maturities, all of them for the two countries, and one lag of the risk premium itself.⁵ The fitted values are a function of information available at time *t*, so they can be safely interpreted as proportional to the conditional variance $\sigma_{z_{t+12}}$. The remainder is serially uncorrelated and it can be safely interpreted as the innovation term in Eq. (12). This procedure can only lead to underestimation of the level of uncertainty, since it could also incorporate an unpredictable component which our approach will include into the estimate of the innovation component.⁶

4. Macroeconomic versus state-uncertainty in explaining the observed risk premium

4.1. The structural approach

The exchange rate risk premium in Eq. (15) depends on the perceived probability of convergence. To substitute for the unobserved probability assigned by the financial markets to the event that the country may belong to the euro area by January 1999, we use a procedure similar to the *JP Morgan EMU calculator* (J.P. Morgan, 1997).⁷ The basic feature of such a *calculator* is that the observed interest rate spread at time *t*, *IR_SPR*, is supposed to be a weighted average of the *IN* spread, *IR_SPR^{IN}*, which would apply if the country adopts the

⁵ We are using information provided by the rest of the variables considered in our analysis. One lag seems to be enough to capture the dependence of the conditional variance on past information.

⁶ With a longer sample, we could try to implement a full filtering approach by recursively estimating each time period the conditional second order moments for the macroeconomic variables. The residual obtained every period from a linear regression of the risk premium on those second order moments could then be projected on past state variables to split it into the conditional variance component of z_t and the serially uncorrelated innovation. However, the shortness of our sample, with 52 observations, does not allow us to estimate recursively with enough precision.

⁷ Extracting market expectations on a given event from asset prices is a question that has attracted a great deal of interest [see Dillén and Edlund (1997), Favero et al. (2000), as well as the review essays by Soderlind and Svensson (1997), and Bates (1998)].

single currency and the *OUT* spread, *IR_SPR^{OUT}*, corresponding to the case when the country is out of the Eurozone. The weights are the corresponding probabilities for each event,

$$IR_SPR_t = p_t IR_SPR_t^{IN} + (1-p_t)IR_SPR_t^{OUT}.$$
(16)

In a monetary union, financial instruments from different countries sharing the same maturity, liquidity and credit risk must have the same yield. Hence, if a country fulfills the convergence criteria⁸ in January 1999 and enters into the euro area, its riskless interest rate should be equal to those in the other countries of the monetary union. On the other hand, if the country does not enter into the union, its interest rate will be determined by a variety of factors, including its own monetary policy, and it will generally maintain a positive spread relative to countries in the union. Hence, assuming $IR_SPR_t^{IN}=0$ and $IR_SPR_t^{OUT}=\theta>0$ in Eq. (16), we can estimate the probability assigned by the financial markets at time *t* to the event that the country belongs to the euro area by January 1999:

$$p_t = 1 - IR_SPR_t / \theta. \tag{17}$$

We estimate Eq. (15) under a more flexible functional form for p_t , using Eq. (17) just to suggest a negative relationship between the convergence probability and the interest rate spread:

$$p_t = \alpha_0 - \alpha_1 I R_SPR_t, \tag{18}$$

Then, $p_t(1 - p_t) = (\alpha_0 - \alpha_1 IR_SPR_t) - (\alpha_0 - \alpha_1 IR_SPR_t)^2$, and Eq. (15) becomes:

$$\begin{aligned} RP_{t+1}^{e} &= (\alpha_{0} - \alpha_{1} IR_SPR_{t}) - (\alpha_{0} - \alpha_{1} IR_SPR_{t})^{2} + \frac{1}{2} \sigma_{m_{t+1}}^{D} - \frac{1}{2} \sigma_{m_{t+1}}^{2} + \\ &+ \frac{1}{2} (1 - \alpha)^{2} \sigma_{y_{t+1}}^{2} - \frac{1}{2} (1 - \gamma)^{2} \sigma_{y_{t+1}}^{2} - (1 - \alpha) \sigma_{y_{t+1}}^{D} m_{t+1}^{D} + (1 - \gamma) \sigma_{y_{t+1}}^{e} m_{t+1}^{e} + \end{aligned}$$

Since fundamental variables are measured differently in each country, their volatilities are not directly comparable and it is not possible to estimate the model under the constraints imposed by international symmetry. Therefore, in the next section we estimate a regression version of this equation, using the "*ex-post realized risk premium*", RP_{t+1} as dependent variable:

$$\begin{aligned} RP_{t+1} \equiv & s_{t+1} - f_t^{t+1} = \beta_0 + \beta_1 IR_SPR_t + \beta_2 IR_SPR_t^2 + \beta_3 \sigma_{m_{t+1}}^2 + \beta_4 \sigma_{m_{t+1}}^2 + \\ & + \beta_5 \sigma_{y_{t+1}}^2 + \beta_6 \sigma_{y_{t+1}}^2 + \beta_7 \sigma_{y_{t+1}}^2 m_{t+1}^2 + \beta_8 \sigma_{y_{t+1}}^r m_{t+1}^r + u_t, \end{aligned}$$

$$(19)$$

where the residual captures the forecast error in future exchange rates. Probability estimates of entering the Eurozone can be recovered by solving the system:

$$\beta_0 = \lambda \alpha_0 (1 - \alpha_0); \quad \beta_1 = -\lambda \alpha_1 (1 - 2\alpha_0); \quad \beta_2 = -\lambda \alpha_1^2,$$
 (20)

The quadratic polynomial in (*IR_SPR*) captures the effect on the risk premium of the probability of joining the euro, which we will be able to recover after estimation. To estimate the conditional variances ($\sigma_{mf_{+},1}^2$, $\sigma_{yf_{+},1}^2$ and $\sigma_{yf_{+},1}^2$ and covariances ($\sigma_{yf_{+},1}^D$, mt_{+}) and $\sigma_{yf_{+},1}^2$, mt_{+}) for each country we additionally assume that the dynamics of real and monetary variables, money supply and industrial production can be summarized by a VARMA model in logged differences with GARCH innovations, which allows for some possible nonlinear

dependence among them. Appendix A shows the specification and estimation of the conditional second order moments.⁹

The unquestionable participation of Germany in the euro area makes it reasonable to focus the analysis on interest rate differentials with respect to Germany. Under this convention, our model predicts that the probability of a given country adopting the single currency at the outset of the Eurozone will be inversely related to the spread of interest rates with Germany. To accommodate the criticism in Favero et al. (2000), we work with two different sets of interest rates. We initially consider the spread in 3-year swap rates as a proxy to capture expectations of convergence to the euro area for a given country. Their behaviour is similar to those of the 5- and 10-year rates, while the 1-year rate is influenced by monetary policy decisions. We prefer them to interest rates for government bonds, that trade in often narrow and not very liquid markets, and is subject to a different tax treatment of returns across countries. The swap market is very liquid, contracts are standardized across currencies, including the tax treatment of returns, and it is not affected by default risk.

Favero et al. (2000) remark the potential sensitivity of the J.P. Morgan Calculator to the set of interest rates used. Following De Grauwe (1996) and Weidman (1996), these authors suggest using the differential of instantaneous forward rates for December 31, 1998 relative to Germany, as an indicator of beliefs on the probability that a given country joins the euro. We follow their recommendation and follow their same approach by estimating a Nelson-Siegel specification for the zero coupon curve for each country at each point in time, to infer from it the instantaneous forward rate.¹⁰ For Germany, we used data for 1-week, and 1 to 12-month LIBOR rates from the interbank market, together with interest rate swap rates for 2 to 10-year plus the 30-year rate. For France we use the same rates, except for the 30-year rate. For the UK we used 1- and 2-week, and 1-, 3-, 6- and 12-month LIBOR rates from the interbank market, plus 2-, 3-, 4-, 5-, 7- and 10-year swap rates. For Spain we used the same rates as for the UK except for the two shortest maturities.

Interest rate spreads with Germany using both, 3-year swap rates and instantaneous forward rates, are shown for France, Spain, and United Kingdom in Fig. 1 for 1994:01–1998:04 together with currency risk premia. In the three countries, the risk premium is clearly more volatile than the two interest rate spreads. Over 1994 and the first part of 1995, interest rate spreads were increasing for Spain, approaching 6 percentage points, and reflecting the increased belief on the fact that the country could not possibly meet Maastricht convergence criteria. The situation drastically changed after the summer of 1995, when it experienced a continuous and rapid decrease in interest rate spreads, reflecting a growing probability that this country could adopt the single currency at the outset. The spread for France widened in the spring of 1995 from zero to about 1 percentage point, remaining at that level until the end of 1996, when it fell back to zero. This is consistent with a high probability of this country adopting the single currency from the beginning. On the other hand, the spread showed a positive trend since the beginning of 1994 for the United Kingdom, with the swap spread stabilizing after 1996 but without the sharp decrease observed for Spain and France.

We start by examining in Appendix B Table 1 the explanatory power of swap interest rate spreads for the currency risk premium over the 1994:01–1998:04 period, and for the three bilateral relationships with Germany, ignoring the potential effect from macroeconomic uncertainty. We use risk premium data corrected from extreme values, which sharply decreases the evidence of residual autocorrelation. Estimated coefficients are significant and take the expected sign for France and

⁸ To enter into the Eurozone, the Maastricht Treaty indicated that candidates should lower their inflation rate to within 1.5% of the lowest three in the European Community, push budget deficits below 3% of GDP, and decrease debt-to-GDP ratios to 60%, while maintaining a stable currency.

⁹ To gain precision, VARMA-GARCH models are estimated with the longer 1986:1–1998:4 sample.

¹⁰ In Nelson-Siegel model, zero coupon forward rates behave according to: $f_{kt} = \beta_0 + \beta_1 exp(-\frac{k}{\tau}) + \beta_2 \frac{k}{\tau} exp(-\frac{k}{\tau})$, where *k* denotes maturity, so that instantaneous forward rates are given by $\beta_0 + \beta_1$.



Fig. 1. Interest rate spreads and observed risk premium Sample: 1994:01-1998:04.

Spain, suggesting that the linear probability term would not adequately capture all the information regarding the risk premium in that currency. Estimated coefficients turn out not to be significant for the UK. The negative sign for the Sterling pound is the consequence of an upward trending interest rate spread with Germany over 1994–1998, together with a slightly decreasing forward risk premium. Interest rate coefficients are not statistically significant in this case due to colinearity. In fact, the joint null hypothesis that the two coefficients of the quadratic polynomial are equal to zero is rejected at 5% and 10% significance levels.

Appendix B Table 2 presents the estimation results for the full model, including macroeconomic uncertainty and the uncertainty on convergence, over the 1994:01–1998:04 period, using data on interest

rate swaps. Conditional second order moments of fundamental macroeconomic variables are not significant. On the contrary, uncertainty on convergence to the Eurozone, as captured by the quadratic function on the interest rate spread, is significant for France and Spain. The comparison of adjusted R^2 coefficients in Appendix B Tables 1 and 2 suggests that the uncertainty on whether the country fulfils the Maastricht convergence criteria may be more important than the uncertainty on macroeconomic indicators to explain the exchange rate risk premium. However, this evidence arises only after 1994,¹¹ suggesting that it was the formal approval of the Maastricht criteria, more than the Maastricht agreement itself, the starting point for exchange rate markets to incorporate the uncertainty on the convergence process in the determination of the risk premium. Appendix B Table 3 presents the estimates obtained using instantaneous forward rates. Results are very similar to those in Appendix B Table 2, in terms of the structure of signs in the quadratic polynomial on interest rate spreads as well as in terms of statistical significance of individual coefficients.

We now have all the information needed to use Eq. (20) to estimate the probability attached by the market to the event that each country joins the European monetary union at a given date, as we will explain in Section 4.3.

4.2. A filtering approach to measuring uncertainty

As described above, our second approach to measuring uncertainty consists on filtering the residual from a linear least squares regression that explains the risk premium using the level of uncertainty in macroeconomic indicators. Our goal is to decompose that residual into a proxy for the conditional variance of the general level of uncertainty in the economy, not captured by fluctuations in macroeconomic indicators, and a pure innovation. The conditional variance that we obtain can be considered a proxy for the size of potential variations in the perceived level of uncertainty and hence, an indicator of the type of risk with non-macroeconomic origin. After that, we can project the risk premium onto this risk proxy, the indicators of macroeconomic uncertainty and the innovation. Since the three components are essentially uncorrelated, that projection will allow us to compute a decomposition of the variance of the risk premium that can be used to evaluate the relative importance of each component.

Estimates for that projection in Appendix B Table 4 are fairly robust across currencies. The proxy for non-macroeconomic risk enters the risk premium equations with a coefficient close to one, and it is always statistically significant. Some conditional variance terms also turn out to be significant. The left panel in Fig. 2 presents the fitted and the observed risk premium, thereby providing a detailed view of the fit of the model in each country. The central and right columns show that the influence from macroeconomic uncertainty to the fitted risk premium (middle column) is less important than the effect of the proxy for non-macroeconomic risk (right column). That is also reflected in the linear correlations with the fitted risk premium, which are noticeably higher for the general perception of uncertainty in the economy (between 0.75 and 0.94) than for macroeconomic uncertainty (between 0.48 and 0.61).

To compare the relative quantitative importance, we use the decomposition of variance of the currency risk premium, which is shown in Appendix B Table 4. The proxy for the perceived level of uncertainty accounts for about 25% of the variance in risk premium in

¹¹ Using the longer available sample for interest rate swaps, 1992:02–1998:04, we obtained a poor fit (not shown in the paper), probably because of including the period prior to formal approval of the Union Treaty. Convergence criteria were part of the European Union Treaty, which was approved at the European Council celebrated at Maastricht in February 1992. However, their final approval at the level of the Congress of each country took place in November 1993. Hence, even though governments could consider in 1992 the possibility of implementing policy with a goal of achieving convergence, it is just since 1994 that convergence criteria had a formal validity.



(d) Correlation between variables in scatter diagrams

Fig. 2. Scatter diagrams: fitted risk premium (horizontal axis) versus observed risk premium, macroeconomic and state-uncertainty contribution^{(a)(b)(c)}. Sample: 1994:01–1998:04.

the three currencies. Macroeconomic uncertainty plays a minor role, with a weight between 6% and 17% in the variance decomposition. Fig. 3 illustrates the relevance of the two types of uncertainty to explain the currency risk premium. From left to right, each panel compares the fitted risk premium and the components accounted for by macroeconomic uncertainty, $\hat{\beta}_3 \ \sigma_{M_{f+1}}^2 + \hat{\beta}_4 \ \sigma_{M_{f+1}}^2 + \hat{\beta}_5 \ \sigma_{y_{f+1}}^2 + \hat{\beta}_8 \ \sigma_{y_{f+1}}^F + n m d$ by state- uncertainty, $\hat{\beta}_8 \ filtering uncertainty_t$, with the observed risk premium.¹²

4.3. Numerical estimates of convergence probabilities

From estimates in Appendix B Table 4 we can recover estimates for the probabilities that France, Spain, and UK joined the euro area in April 1999. We take estimates for $\beta_0, \beta_1, \beta_2$ to Eq. (20) obtained when using instantaneous forward rates¹³ and compute the implied values

 $^{^{12}}$ Note that when we use filtering uncertainty as a proxy for the state-uncertainty we estimate the next regression:

 $[\]begin{aligned} RP_{t+1} &= \beta_1 + \beta_2 \text{ filtering Uncertainty}_t + \beta_3 \sigma_{y_{t+1}^F m_{t+1}^F}^F + \beta_4 \sigma_{y_{t+1}^D m_{t+1}^D}^P + \beta_5 \sigma_{m_{t+1}^P}^2 \\ &+ \beta_6 \sigma_{y_{t+1}^P}^2 + u_t. \end{aligned}$

¹³ Estimates obtained using 3-year swap rate spreads lead to an increasing probability of joining the eurozone for the UK, which does not correspond to reality. But coefficient estimates are non-significant in this case, so a strict structural interpretation is not justified. Probability estimates for France are similar under instantaneous forward and 3-year swap rates, even though in the former case, coefficients are non-significant.

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Left column: Observed exchange risk premium versus the fitted value from the model in Table 4 (a)

(b) Middle column: Observed exchange risk premium versus macroeconomic uncertainty contribution

(c) Right column: Observed exchange risk premium versus EURO-uncertainty contribution

Fig. 3. Observed risk premium versus fitted risk premium, macroeconomic and State-uncertainty^{(a)(b)(c)} Sample: 1994:01–1998:04.

for α_0, α_1 in Eq. (18). Estimated probabilities of joining the EMU, normalized so that p_t [0,1], are shown in Fig. 4, and they look fairly reasonable. For France, our estimated probability rapidly increased since April 1995, while for Spain the probability increased very fast from January 1996. The upward trend suggests the perception of an increased probability that Spain and France would adopt the single currency from the beginning. According to our estimates, the United Kingdom was viewed at the beginning of 1994 to have a high probability of entering the Eurozone, but that probability collapsed during the general wave of pessimism on the future of the currency union in the second part of 1994. After that, our estimate of the probability of joining the euro area decreases until the end of 1997 suggesting, as it finally was the case, that the likelihood of this country in joining the euro area was not considered to be particularly high.

5. Conclusions

We have proposed a general equilibrium model to characterize risk premium in the exchange rate market. The model has as a main feature the state dependency of preferences on the perceived level of uncertainty. As a consequence, the excess return in exchange rates is a function of two factors: i) the volatility of fundamental variables (money and output), and ii) the perception by private agents on the general level of uncertainty in the economy, or state-uncertainty. The stochastic discount factor is then connected to the properties of money, output and a broad uncertainty index, and the presence of the latter increases the sensitivity of the stochastic discount factor to even small variations in consumption. Therefore, our model does not rely as much as more standard models on consumption risk when explaining the currency risk premium.

We have used two different proxies for the private sector perception of risk: a quadratic function of interest rate spreads meant to capture euro uncertainty and a proxy obtained by filtering techniques. The first approach is performed twice, using either swap interest rates or instantaneous forward rates, to address some criticism that has been raised in the literature. Both proxies suggest an acceptable explanatory power for interest rate spreads in the 1994-1998 period, once national Parliaments approved the Maastricht



Fig. 4. Probability indicator for Spain, France and the United Kingdom Sample: 1994:01–1998:04.

criteria. Regarding macroeconomic uncertainty, the relevance of the conditional variances of money supply and industrial production as well as the conditional covariance between these two variables is rather limited, accounting for 7% to 15% of the volatility in the currency risk premium. This is in consistency with results reached by other authors. Hence, it seems that forces other than those from money and goods markets were important sources of uncertainty during this period, a fact that was reflected in the exchange rate risk premium. In fact, we have shown state-uncertainty to account for up to 25% of the volatility in the currency risk premium. Hence, the perception by private agents on the level of uncertainty is quantitatively important for reproducing the observed risk premium along the convergence process to the European currency union. Not only is state-uncertainty more relevant than macroeconomic uncertainty, but it also achieves a significant gain in explanatory power compared to that obtained in previous research under a more standard approach. However, there is still some room for searching for additional explanatory factors of the risk premium in currency markets.

Three western European countries in the EMS, United Kingdom, Denmark and Sweden, belong to the European Union but have not yet adopted the euro. Eight central and eastern European countries have joined the European Union in recent years, and our model could be used to explain the behavior of exchange rate risk premium in these countries, as they moved towards joining the eurozone. A similar analysis could eventually be applied if common currency areas in Latin America or South East Asia are eventually approved.

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Appendix A. Specification and estimation of conditional second order moments

With regard to fundamental uncertainty we follow Hu (1997) to assume that, conditional on available information, growth rates in fundamental variables (m_{t+1}^i and y_{t+1}^i ; i=D, F) follow a joint lognormal distribution. We assume that the dynamics of real and monetary variables, represented by the money supply and industrial production, can be summarized by a VARMA model in logged differences with GARCH innovations, which allows for possible nonlinear dependence among them.¹⁴

Standard specification tools¹⁵ suggest a VARMA(1, 1) model for $(\ln(m_t), \ln(y_t))$ for Spain, a VAR(3) for Germany, a VAR (2) with a seasonal VAR(1) component for the UK, and a VAR (3) with a seasonal VAR (2) for France.¹⁶ All these models are special cases of:

$$\begin{split} I &+ \begin{bmatrix} \varphi_{11}^{1} & \varphi_{12}^{1} \\ \varphi_{21}^{1} & \varphi_{22}^{1} \end{bmatrix} B &+ \begin{bmatrix} \varphi_{11}^{2} & \varphi_{12}^{2} \\ \varphi_{21}^{2} & \varphi_{22}^{2} \end{bmatrix} B^{2} &+ \begin{bmatrix} \varphi_{11}^{3} & \varphi_{12}^{3} \\ \varphi_{21}^{3} & \varphi_{22}^{3} \end{bmatrix} B^{3} \\ &\times \left(I + \begin{bmatrix} \Phi_{1}^{1} & 0 \\ 0 & \Phi_{2}^{1} \end{bmatrix} B^{12} &+ \begin{bmatrix} \Phi_{1}^{2} & 0 \\ 0 & \Phi_{2}^{2} \end{bmatrix} B^{24} \right) \begin{pmatrix} ln(m_{t}) \\ ln(y_{t}) \end{pmatrix} = \\ &= \begin{bmatrix} \lambda_{1} \\ \lambda_{2} \end{bmatrix} &+ \begin{pmatrix} I + \begin{bmatrix} \theta_{11}^{1} & \theta_{12}^{1} \\ \theta_{21}^{1} & \theta_{22}^{1} \end{bmatrix} B \end{pmatrix} \begin{pmatrix} \varepsilon_{m_{t}} \\ \varepsilon_{y_{t}} \end{pmatrix}, \end{split}$$
(21)

$$\begin{pmatrix} \varepsilon_{m_t} \\ \varepsilon_{y_t} \end{pmatrix} / I_{t-1} \sim N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{m_t}^2 & \sigma_{m_t y_t} \\ \sigma_{m_t y_t} & \sigma_{y_t}^2 \end{pmatrix} \right)$$
(22)

with *B* being the backshift operator, and ε_t the innovation vector.

As initial conditions, we used estimates obtained under the assumption of lack of heteroskedasticity. Lagrange multiplier and Ljung-Box statistics on the residuals point out to possible conditional heteroskedasticity in the money supply for France and the UK, as well as for an autoregressive structure for the covariance between the money supply and the industrial production in France. These tests led us to specifying a GARCH(1, 1) model for the conditional variances and covariance in Eq. (22):

$$\begin{vmatrix} \sigma_{m_{t}y_{t}}^{\sigma} \\ \sigma_{m_{t}y_{t}}^{\sigma} \end{vmatrix} = \begin{bmatrix} c_{01} \\ c_{02} \\ c_{03} \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_{m_{t-1}}^{\sigma} \\ \varepsilon_{m_{t-1}} \\ \varepsilon_{y_{t-1}}^{\sigma} \end{bmatrix} + \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix} \begin{bmatrix} \sigma_{m_{t-1}}^{\sigma} \\ \sigma_{m_{t-1}y_{t-1}} \\ \sigma_{y_{t}}^{\sigma} \end{bmatrix}.$$

$$(23)$$

We imposed diagonality constraints, so that $\sigma_{m,r}^2$, $\sigma_{y,r}^2$, and $\sigma_{m,yt}^2$ depend only on their own lags and lags of $\varepsilon_{m,r}^2$, $\varepsilon_{y,r}^2$, and $\varepsilon_{m,yt}$ respectively. These restrictions are made to avoid the numerical difficulties that would arise when estimating an over-parametrized model. For estimation, we used an alternative VARMA(1, 1) representation of the GARCH (1, 1) model: Let us considerer the 3×1 stochastic vector:

$$\xi_t = \operatorname{vech}(\varepsilon_t \varepsilon'_t) - \operatorname{vech}\Sigma_t \tag{24}$$

where vech $(\varepsilon_t \varepsilon'_t) = (\varepsilon_{m_t}^2 \varepsilon_{m_t} \varepsilon_{y_t} \varepsilon_{y_t}^2)'$, vech $\sum_t = (\sigma_{m_t}^2 \sigma_{m_t y_t} \sigma_{y_t}^2)'$ and ξ_t is a white noise process.

 $^{^{\}rm 14}$ As proposed by Bollerslev (1986) and Baba et al. (1991), among many others.

¹⁵ Partial and simple autocorrelation functions as well as the criteria proposed by Akaike, Hannan and Quinn, and Schwarz.

 $^{^{16}}$ Evidence of seasonal components shows up in spite of using seasonally adjusted time series data.

Substituting Eq. (24) in Eq. (23) and rearranging:

$$\begin{pmatrix} I - \begin{bmatrix} a_{11} + g_{11} & 0 & 0 \\ 0 & a_{22} + g_{22} & 0 \\ 0 & 0 & a_{33} + g_{33} \end{bmatrix} B \end{pmatrix} \begin{bmatrix} \varepsilon_{m_t}^2 \\ \varepsilon_{m_t} \varepsilon_{y_t} \\ \varepsilon_{y_t}^2 \end{bmatrix}$$

$$= \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{bmatrix} + \begin{pmatrix} I - \begin{bmatrix} g_{11} & 0 & 0 \\ 0 & g_{22} & 0 \\ 0 & 0 & g_{33} \end{bmatrix} B \end{pmatrix} \begin{pmatrix} \xi_{m_t} \\ \xi_{y_t} \end{pmatrix}$$

$$(25)$$

in which the presence of the sum $a_{ii} + g_{ii}$ allows us to test for stationarity in variance, whenever $|a_{ii} + g_{ii}| < 1$ [Bollerslev (1986)].

In the estimated models (not shown in the paper), conditional variances for the money supply and the industrial production actually depend on their own innovations, while their conditional covariance depends on innovations in both variables. Conditional heteroskedasticity seems to be present in all countries. As suggested by the previous tests, we estimated heteroskedastic effects for the money supply in France and the UK and the covariance between the money supply and the industrial production in France. We also obtained a statistically significant autoregressive structure for the conditional covariance between both variables in Spain and Germany. No conditional heteroskedasticity in the variances of the money supply or the industrial production was found for these two countries.

Appendix **B**

Table 1

^(a)Least squares estimation of the risk premium associated to euro-uncertainty. Sample: 1994:01–1998:04 $RP_{t+1} = \gamma_0 + \gamma_1 IR_SPR_t + \gamma_2 IR_SPR_t^2 + u_t$.

С	ESP/DEM ^(e)	FRF/DEM	GBP/DEM
	-0.0061* (-1.83)	-0.0016 (-1.46)	0.0242 (1.25)
IR_SPR	0.465* (1.81)	0.876*(1.87)	-1.531 (-0.80)
IR_SPR ²	-10.017* (-2.33)	-101.110* (-2.84)	6.358 (0.13)
Euro-Uncertainty ^(b)	0.022	0.002	0.022
R^2	0.144	0.223	0.144
Adj. R ²	0.109	0.192	0.109
$COR(1)^{(c)}$	0.87	0.24	0.53
COR(12) ^(e)	0.90	0.59	0.30
ARCH(6) ^(d)	0.16	0.64	0.57

Notes: ^(a)*t*-statistics in parentheses. ^(b)*p*-value for *F*-statistics for the null hypothesis: H_0 : $\gamma_1 = \gamma_2 = 0$. ^(c)*p*-value for Breusch-Godfrey test statistics for residual serial correlation up to lag order *p*. ^(d)*p*-value of LM test statistics for an ARCH structure of order 6. ^(e)An asterisk denotes a coefficient significant at the 10% level.

Table 2

Least squares estimation of the risk premium equation^(a) Interest rate swap rates Sample: 1994:01–1998:04 $RP_{t+1} = \beta_0 + \beta_1 IR_SPR_t + \beta_2 IR_SPR_t^2 + \beta_3 \sigma_{R_{t+1}}^{a_p} + \beta_4 \sigma_{R_{t+1}}^{a_t} + \beta_5 \sigma_{y_{t+1}}^{a_p} + \beta_6 \sigma_{y_{t+1}}^{a_t} + \beta_6 \sigma_{y_{t+1}}^{a_t} + \beta_8 \sigma_{y_{t+1}}^{a_t} + \mu_t$

	ESP/DEM ^(g)	FRF/DEM	GBP/DEM
С	-0.0080 (-1.62)	-0.0003 (-0.05)	0.0074 (0.14)
IR_SPR	0.497* (1.88)	1.044* (2.00)	-1.400(-0.68)
IR_SPR ²	-9.866* (-2.22)	-98.427* (-2.56)	-1.662 (-0.03)
$\hat{O}_{y^{GER}_m GER}$	0.0121 (0.86)	0.0096 (1.11)	0.0263* (1.31)
$\hat{O}_{m'y'}^{2(b)}$	0.0152 (0.39)	0.0409 (0.69)	
$\hat{O}_{m^i}^2$		-0.0061 (-0.32)	0.1609* (1.31)
$\hat{\sigma}_{y^i}^2$			0.0176 (0.23)
Euro-Uncertainty ^(c)	0.076	0.040	0.156
Fundamental	0.636	0.666	0.420
Uncertainty ^(d)			
R^2	0.160	0.249	0.195
Adj. R ²	0.089	0.167	0.103
COR(1) ^(e)	0.76	0.25	0.36
COR(12)	0.91	0.31	0.33
ARCH(6) ^(f)	0.21	0.76	0.40

Notes: ^(a)*t*-statistics in parentheses. ^(b) $\hat{\sigma}_{x^{i}}^{2} \equiv \operatorname{var}_{t}(\log(x_{t+1}^{i})) \, \sigma_{x_{t+1}^{i}p_{t+1}^{i}} \equiv \operatorname{cov}_{t}(\log(x_{t+1}^{i}) \times \log(p_{t+1}^{i})), \text{ for } i = \operatorname{FR}, \operatorname{SP}, \operatorname{UK}. ^(c)$ *p*-value for*F* $-statistics for the null hypothesis: <math>H_{0}$: $\beta_{1} = \beta_{2} = 0$. ^(d)*p*-value for *F*-statistics for the null hypothesis: H_{0} : $\beta_{3} = \beta_{4} = \beta_{5} = \beta_{6} = \beta_{7} = \beta_{8} = 0$. ^(e)*p*-value of Breusch-Godfrey test statistics for residual serial correlation up to lag order *p*, in brackets. ^(f)Autoregressive conditional heteroskedasticity test. ARCH(6) is the *p*-value of LM test statistics for an ARCH structure of order 6. ^(g)An asterisk denotes a coefficient significant at the 10% level.

Table 3

Least squares estimation of the risk premium equation^(a) Instantaneous forward rates Sample: 1994:01–1998:04 $RP_{t+1} = \beta_0 + \beta_1 IR_SPR_t + \beta_2 IR_SPR_t^2 + \beta_3 \sigma_{RP_{+1}}^2 + \beta_4 \sigma_{RF_{+1}}^2 + \beta_5 \sigma_{yP_{+1}}^2 + \beta_6 \sigma_{yP_{+1}}^2 + \beta_6$

	ESP/DEM ^(g)	FRF/DEM	GBP/DEM
С	-0.0079 (-1.18)	0.0067 (0.65)	0.0166 (-0.33)
IR_SPR	0.6497* (1.62)	0.2944 (1.21)	-2.783* (-1.93)
IR_SPR ²	-14.693* (-2.21)	-11.308 (-1.03)	60.479* (1.70)
$\hat{O}_{y^{GER}m^{GER}}$	0.0039 (0.26)	0.0164 (1.64)	-0.0049 (-0.14)
$\hat{O}_{m^i \gamma^i}^{2(b)}$	-0.0024(-0.06)	0.1261* (1.78)	
$\hat{O}_{m^i}^2$		-0.0268 (-1.25)	0.1677* (1.30)
$\hat{O}_{V^i}^2$			-0.0286 (-0.31)
Euro- Uncertainty ^(c)	0.031	0.478	0.201
Fundamental	0.963	0.203	0.114
Uncertainty ^(d)			
R^2	0.191	0.164	0.146
Adj. R ²	0.122	0.073	0.054
$COR(1)^{(e)}$	0.513	0.407	0.196
COR(12)	0.944	0.845	0.340
ARCH(6) ^(f)	0.005	0.420	0.864

Notes: ^(a)*t*-statistics in parentheses. ^(b) $\hat{\sigma}_{x^i}^2 \equiv_t (\log(x_{t+1}^i)) \quad \sigma_{x_{t+1}^i} p_{t+1}^i \equiv_t (\log(x_{t+1}^i) \times \log(p_{t+1}^i))$, for i = FR, SP, UK. ^(c)*p*-value for *F*-statistics for the null hypothesis: $H_0: \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$. ^(a)*p*-value for *F*-statistics for the null hypothesis: $H_0: \beta_3 = \beta_4 = \beta_5 = \beta_6 = 0$. ^(e)*p*-value of Breusch-Godfrey test statistics for residual serial correlation up to lag order *p*, in brackets. ^(f)*p*-value of LM test statistics for an ARCH structure of order 6. An asterisk denotes a coefficient significant at the 10% level.

Table 4

Least squares estimation of the risk premium equation^(a) Filtering proxy for stateuncertainty Sample: 1994:01–1998:04 $RP_{t+1} = \beta_1 + \beta_2$ filtering Uncertainty $+\beta_3 \sigma_{m_{t+1}}^2 + \beta_4 \sigma_{m_{t+1}}^2 + \beta_5 \sigma_{y_{t+1}}^2 + \beta_7 \sigma_{y_{t+1}}^$

	ESP/DEM ^(g)	FRF/DEM	GBP/DEM
C Uncertainty	-0.0026 (-0.87) 1.026* (4.38)	0.0054 (1.07) 1.016* (4.37)	-0.0713* (-3.24) 1.0423* (3.96)
$\hat{O}_{y^{GER}m^{GER}}$	0.0167* (1.60)	0.0138* (2.02)	-0.0050 (-0.19)
$\hat{O}_{m^i y^i}^{2(b)}$	-0.0106 (-0.31)	0.0957* (2.36)	
$\hat{O}_{m^i}^2$		-0.0228 (-1.49)	0.1331* (2.93)
$\hat{G}_{y^i}^2$			0.1091 (0.95)
Euro-Uncertainty ^(c)	0.000	0.000	0.000
Fundamental	0.263	0.009	0.036
Uncertainty ^(d)			
Variance decomposit	ion		
Euro-Uncertainty	26.5%	24.5%	20.9%
Fundamental	6.1%	13.7%	12.7%
Uncertainty			
Remainder	65.5%	61.6%	66.3%
\mathbb{R}^2	0.348	0.386	0.346
Adj. R ²	0.307	0.334	0.290
$COR(1)^{(e)}$	0.62	0.76	0.48
COR(12)	0.51	0.81	0.09
ARCH(6) ^(f)	0.17	0.18	0.91

Notes: ^(a)*t*-statistics in parentheses. ^(b) $\Theta_{\mathbf{x}^{i}}^{2} \equiv \operatorname{var}_{t}(\log(\mathbf{x}^{i}_{t-1})) \quad \mathcal{O}_{\mathbf{x}^{i}_{t}-p^{i}_{t-1}}^{i} \equiv \operatorname{cov}_{t}(\log(\mathbf{x}^{i}_{t-1}) \times \log(p^{i}_{t-1}))$ for $i = \operatorname{FR}$, SP, UK. ^(c)p-value for *F*-statistics for the null hypothesis: H_{0} : $\beta_{1} = \beta_{2} = 0$. ^(d)p-value for *F*-statistics for the null hypothesis: H_{0} : $\beta_{3} = \beta_{4} = \beta_{5} = \beta_{6} = \beta_{7} = \beta_{8} = 0$. ^(e)p-value of Breusch-Godfrey test statistics for residual serial correlation up to lag order p, in brackets. ^(f)Autoregressive conditional heteroskedasticity test. ARCH(6) is the p-value of LM test statistics for an ARCH structure of order 6. An asterisk denotes a coefficient significant at the 10% level.

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1052

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