Abstract
In several optical techniques, two (or more) simultaneous events can be analyzed by coding their information in a crossed fringe pattern. In this work, two phase extraction methods for the processing of crossed fringe patterns are presented, and compared with previously established methods.

Introduction: What is a crossed fringe pattern?
Depending on the optical technique used, the resulting crossed fringe patterns are formed by either the multiplicative or the additive superposition of the separate fringe patterns.

Additive
- Fringe projection
- Ronchi test with square grid
- Experimental-mechanic system

Multiplicative
- Photelasticity
- Deflectometry moiré with square grid

Problem: Demodulation of Crossed Fringe Pattern
The phase extraction is complex because:
- two, instead of one, phase informations must be estimated and,
- these phase informations are spatially superposed.

Previous solutions
- Multichannel Fourier Transform (FT)
  - FT in crossed fringe patterns has been applied in an experimental mechanics systems and in a fringe projection technique.
  - It is necessary to introduce a high-frequency carrier.
  - The two side-lobes must be processes separately in the frequency space.
  - The properties of the bandpass filter must be carefully selected.
- Spatial Carrier Phase Shift (SCPS)
  - It is a phase-shifting technique applied to sequential pixels.
  - Requires the introduction of a carrier with a specific selected frequency.
  - Although the phase obtained is noisy, the application of a low-pass phase filter permits the obtention of "acceptable" phase maps.

Proposed solution 1: Modified phase-shifting method (PS)
- The system must be able to apply phase shifts in two different directions.
- For Additive superposition fringe patterns a standard phase-shift method is able to extract correctly the phase data, however, for Multiplicative superposition fringe patterns, the spatial overlapping of the low intensity zones results in low-modulation areas where the phase information is lost!
- We solve this problem by acquiring two times each phase map, but shifting by \( \pi \) the low-modulation zones between acquisitions. The "bad" zones are suppressed by calculating the contrast of this two phase maps, using either a weighted average method or a maximum contrast method.

Proposed solution 2: Digital Multiplicative Moiré (DMM)
- It is necessary to introduce a high-frequency-carrier in the fringe pattern.
- The method is based in making a moiré effect between the fringe pattern and a computer generated grating. The moiré fringes are selected with a low-pass filter.
- Two computer gratings must be generated with a direction and frequency similar to the carriers introduced in the fringe patterns.
- By numerically displacing the gratings and using a phase-shifting algorithm, the phase maps are obtained.

Comparative Analysis of The Methods

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Legend: + = High, − = Medium, = Low.
* = Depends on the phase shifting algorithm used.

Example: Progressive ophthalmic lens characterization by crossed gratings

Note the relation between the deflectograms obtained with Ronchi gratings and with crossed gratings.

The carrier frequency must be high-enough to separate the two phase informations.

Comparison of the informations obtained with the different methods

Demonstration of the low-modulation problem with simulated and real data