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Income taxes, public investment and welfare in a growing economy

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Abstract

In a growth model with public capital and a spillover externality from private capital, we find that income taxes as part of an optimal fiscal policy is a more common result than usually thought. The commitment to finance an exogenous component of public expenditures in the form of an exogenous fraction of output may lead to the optimality of positive income taxes. This result is robust to alternative assumptions on depreciation rates and preferences. We show that welfare losses from deviations from the optimal policy are always smaller when compensated with changes in income taxes than when adjusting lump-sum taxes.

JEL classification: E13; E62; H21; O41

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1. Introduction

Clarifying whether factor rents should be taxed or subsidized continues to be a central issue when characterizing optimal fiscal policy. In a Ramsey-type setting with

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no externalities, taxing production factor incomes will generally undermine growth and welfare. In such a setup, Judd (1985), Chamley (1986) and Lucas (1990) propose zero taxes on capital in the long-run. Jones et al. (1997) and Milesi-Ferretti and Roubini (1998) extend the zero tax rate result to labor and consumption taxes in models with human capital. A government may even want to subsidize capital income when the presence of some externalities leads the competitive mechanism to an underaccumulation of physical capital (Turnovsky, 2000; Cassou and Lansing, 1997).

However, taxes on factor rents represent more than 50% of total tax receipts in developed countries. An answer to this apparent *puzzle* can be found in the literature on economies with heterogenous agents.[1] In a representative agent setting, the existence of externalities inducing over-accumulation of physical capital can also lead to the optimality of capital income taxes. Often, externalities of this kind have been introduced in the production function (Turnovsky, 1996; Fisher and Turnovsky, 1998; Corsetti and Roubini, 1996) or as a credit market imperfection in an stochastic environment (Chamley, 2001).[2] We attempt to contribute along this line of research. In a simple growing economy with infinitely lived agents, we find that positive income taxes as part of an optimal fiscal policy may be a more common result than usually thought. This comes about in spite of the fact that lump-sum taxes are allowed and that a spillover externality from aggregate private capital (Romer, 1986) and the presence of public capital in the production function (Glomm and Ravikumar, 1994, 1999; Turnovsky, 2000, 2004) both tend to support subsidies on the production factors.

Most papers leading to a zero distortionary tax rate in the long-run allow for debt issuing. Optimal policy then usually begins with a large capital tax levy that raises enough resources to finance public expenditures and allows for income taxes to go to zero in the long-run. We consider an income tax rate (i.e., labor and capital are uniformly taxed) in addition to a Ricardian-equivalent lump-sum tax, which makes our long-run tax results comparable to those obtained in the referred papers. What is new in our framework is the requirement that the government must finance a constant and exogenous fraction of public expenses to output every period. This commitment produces an important budget distortion in the form of a lower bound for government revenues. Since neither households nor firms internalize the fact that higher income will lead to extra public consumption, financing public expenditures through lump-sum taxes may lead to an excessive crowding-out of private consumption, so that collecting additional resources from income taxation may turn out to be preferable.[3] This simple but important effect has not been seriously considered when characterizing optimal tax policy in dynamic settings. Obviously,

[1] Aiyagari (1995) and Domeij and Heathcote (2004), among others, show that the optimal income tax rate is positive and far from zero in an heterogenous agents framework with asset market incompleteness. Jones and Manuelli (1992) found a similar result in an overlapping generations framework.

[2] Caballé (1998) also found the optimality of capital income taxes in a framework with altruistic preferences and low elasticity of intertemporal substitution, without intergenerational transfers.

[3] This result is in line with Jones et al. (1997), who pointed out that certain *public policy constraints* could imply that taxing productive factors positively in the long-run might be optimal.

the optimal tax/subsidy result depends on the interaction and relative strength of the spillover and the public capital externalities, on the one hand, and the requirement on public expenditures on the other, but we show that taxing factor incomes may be part of an optimal fiscal policy for standard parameterizations of the model economy. Finally, we show that welfare losses from permanent deviations from the optimal expenditure policy are always smaller when compensated with changes in income taxes than when adjusting lump-sum taxes.

A broad classification of public expenditures distinguishes between productive and non-productive concepts. Roughly, we consider as public investment the productive type of public expenditure components, like infrastructures and education. Among the non-productive concepts, it seems safe to assume that some of them (defense, health, police services, etc.) increase consumers' welfare and can be endogenously determined each period. However, some other concepts, such as public wages, the payment of interest on public debt or bureaucratic costs, can be seen as a type of public expenditures that were previously committed and that any government should take as exogenous. The number of public servants ultimately in charge of implementing government policy is mostly given, as it is the need to provide current funding to pay for maintenance of infrastructures for transportation and other public services, like education and health. In addition, the political cost of cutting down on most public consumption items is very high, even if their current levels are clearly above their optimum values. In fact, the public expenditure components that might be seen as pre-committed in actual economies are far from zero and have remained fairly stable, as a percentage of output, over the last decades.[4] This empirical observation suggests the existence of some restrictions that lead actual governments into a second-best choice of public investment and consumption, as well as of their appropriate financing scheme. Hence, there is a variety of reasons motivating the interest and convenience of assuming that part of total public expenditures could be seen as being exogenous, having been previously committed by policy decisions made over a period of time. The main contribution of the paper is to analyze the effects of this realistic public expenditure restriction in a simple growth economy, and emphasize how its size may be determinant for the design of optimal fiscal policy.

The paper is organized as follows. The basic framework is described in Section 2. In Section 3 we analyze the competitive equilibrium allocation, obtain the welfare-maximizing ratios of public consumption and investment to output, and characterize the optimal tax-mix financing government expenditures. Section 4 discusses policy implications of previous results and illustrates the main conclusions of the paper with numerical simulations. Section 5 shows the robustness of our main results. Finally, Section 6 concludes.

[4]Over the last decades, public wages and interest payments on outstanding debt represent about 10% of total GDP (OECD statistics), while total public consumption and public investment amount to about 20% and 3% of GDP, respectively (Easterly and Rebelo, 1993). In addition, the standard deviation of public investment is similar to its mean, the ratio being half as much for public consumption and salaries. This observation supports the idea that the time path of some public expenditure concepts can be much more easily altered than some others.

2. The economy

The model draws on work by Glomm and Ravikumar (1994), Cassou and Lansing (1998), Turnovsky (2004) and Marrero and Novales (2005). The economy consists of a government, a continuum of identical firms and a representative household. Several assumption enable us to obtain closed form solutions of the competitive equilibrium[5]: log utility, Cobb–Douglas technology, uniform tax rates on capital and labor income, and full depreciation of private and public capital. Under these assumptions, the economy reduces to a version of the AK-model, not displaying transitional dynamics. In order to make the policy analysis more transparent, we maintain these assumptions in the following sections. Nevertheless, in a section below we generalize the model regarding the utility function and the depreciation rates.

We assume zero population growth and population size is normalized to one. The representative consumer is endowed with a unit of time every period. She is the owner of physical capital, and allocates her resources between consumption, C, and investment in physical capital, I. Private capital, K, fully depreciates in one period, thus

$$K_{t+1} = I_t, \tag{1}$$

where K_{t+1} denotes the stock of physical private capital at the end of time t. The consumer obtains utility from private consumption and *valued* public consumption, C^g, according to

$$U(C_t, C_t^g) = \ln\, C_t + \lambda \ln\, C_t^g, \tag{2}$$

which is discounted over time at a constant rate $\beta \in (0,1)$; $\lambda > 0$ determines the relative appreciation for public and private consumption. The consumer will offer her unit endowment of time inelastically every period, since she does not receive any utility from leisure. She faces the budget constraint:

$$C_t + K_{t+1} + X_t = (1 - \tau_t)(w_t L_t + r_t K_t), \tag{3}$$

each period, where w_t, r_t are real wages and interest rates, X_t denotes lump-sum taxes and τ_t is a uniform tax rate applied to both, labor and capital rents. Since labor is perfectly inelastically supplied, the income tax really only has an impact on capital, and not on labor.

Each firm rents every period physical capital, k, and labor, l, from households, and produces y units of the consumption commodity. The capital stock used on the aggregate by all firms, K, is taken as a proxy for the index of knowledge available to each single firm (Romer, 1986). Public capital, G, enters as an additional input in the technology available to firms (Glomm and Ravikumar, 1994, 1999; Turnovsky, 1996, 2000). Output is produced according to a *Cobb–Douglas* function

$$y_t = A l_t^{1-\alpha} k_t^{\alpha} K_t^{\phi} G_t^{\theta}, \quad \alpha, \theta \in (0,1), \quad \phi \in [0,1], \tag{4}$$

[5]See footnote 4 in Glomm and Ravikumar (1994).

where α is the share of private capital in output, θ and ϕ are the constant elasticities of output with respect to public capital and the knowledge index, and A is a technological scale factor. Since all firms are identical, we can aggregate on (4) to obtain total output, Y,

$$Y_t = AL_t^{1-\alpha} K_t^{(\alpha+\phi+\theta)} \left(\frac{G_t}{K_t}\right)^{\theta}, \tag{5}$$

where L is the total amount of labor used by all firms. In what follows, we restrict our attention to economies with $\alpha + \theta + \phi = 1$, a condition needed for endogenous growth.

The public sector provides three types of goods which are financed through lump-sum and income taxes. We consider as *public investment*, I^g, the productive type of public expenditures. We assume fully depreciation of public capital, hence

$$G_{t+1} = I_t^g, \tag{6}$$

where G_{t+1} denotes the stock of public capital at the end of time t. Government expenditures also include purchases of non-productive goods and services, that we refer to as *public consumption*. We assume that some of these concepts contribute to consumers' welfare, C^g, and their amount is a policy choice, while some *rigid* public consumption items, denoted by C^w, are pre-committed and exogenous. These three expenditure concepts are assumed to be a constant fraction of total output,[6]

$$G_{t+1} = I_t^g = \varkappa_i Y_t, \tag{7}$$

$$C_t^g = \varkappa_c Y_t, \tag{8}$$

$$C_t^w = \varkappa_w Y_t. \tag{9}$$

Thus, for simplicity, we assume that C^w does not affect consumers' welfare. The relevant assumption here is that C^w is a constant and exogenous fraction of output when deciding on the other policy variables. For analytical convenience, we denote by υ the ratio of X to output, although it should be clear that it is X, rather than υ, that is chosen each period. Without loss of generality, debt issuing is not allowed in the economy, so that the government's budget balances every period,

$$I_t^g + C_t^g + C_t^w = \tau_t Y_t + X_t \iff \varkappa_i + \varkappa_c + \varkappa_w = \tau_t + \upsilon_t. \tag{10}$$

Assuming complete financial markets, the level of the lump-sum tax can be seen throughout the paper as the present value of outstanding public debt. Decisions by the government and consumers would be the same in the two economies.[7] In our setting, a feasible fiscal policy is defined by a vector $\Pi = \{\varkappa_i, \varkappa_c, \varkappa_w, \upsilon_t, \tau_t\}_{t=0}^{\infty}$ satisfying (10), with $\varkappa_i, \varkappa_c, \varkappa_w \geqslant 0$ and $\varkappa_i + \varkappa_c + \varkappa_w < 1$.

[6]Under this assumption, the exogenous component of public consumption will grow at the same rate than aggregate output, which allows us to analyze the long-run limitations that public consumption impose on the choice of fiscal policy. It has no sense to assume a constant path for C^w in a growing economy.

[7]Changes in the level of the tax would correspond to the government issuing or retiring some debt.

3. The optimal policy

Under the competitive equilibrium mechanism, consumers and firms take prices and fiscal policy as given, maximizing time-aggregate utility and profits, respectively. The representative consumer chooses $\{C_t, K_{t+1}\}_{t=0}^{\infty}$ to maximize the discounted time-aggregate utility (2), subject to the budget constraint (3), the transversality condition

$$\lim_{t\to\infty} \beta^t K_{t+1} \frac{\partial U(\cdot)}{\partial C_t} = 0, \tag{11}$$

and $K_{t+1} \geqslant 0$, $C_t \geqslant 0$, for any period t, with K_0 and G_0 given. The solution to this problem leads to the standard intertemporal optimality condition

$$\frac{C_{t+1}}{C_t} = \beta(1-\tau_{t+1})r_{t+1} \quad \text{for } t = 0, 1, 2, \ldots\ . \tag{12}$$

In period t, each firm would pay the competitively determined real wage w_t on the labor it hires and the real interest rate r_t on the capital it rents. The profit-maximizing problem of the firm turns out to be static, leading to the usual marginal product conditions

$$r_t = \alpha A l_t^{1-\alpha} k_t^{\alpha-1} K_t^{\phi} G_t^{\theta} = \alpha \frac{Y_t}{K_t} \quad \text{for } t = 0, 1, 2, \ldots, \tag{13}$$

$$w_t = (1-\alpha) A l_t^{-\alpha} k_t^{\alpha} K_t^{\phi} G_t^{\theta} = (1-\alpha) \frac{Y_t}{L_t} \quad \text{for } t = 0, 1, 2, \ldots, \tag{14}$$

where we have used the fact that each firm treats its own contribution to the aggregate capital stock as given.

Definition 1. Given a feasible policy Π and K_0, $G_0 > 0$, a Π-competitive equilibrium is a vector of allocations $\{C_t, C_t^g, C_t^w, K_{t+1}, G_{t+1}, I_t, I_t^g, L_t, Y_t\}_{t=0}^{\infty}$ and prices $\{r_t, w_t\}_{t=0}^{\infty}$ such that, given the vector of prices: (i) $\{L_t, K_{t+1}\}_{t=0}^{\infty}$ solve the profit-maximizing problem of firms (i.e., (13)–(14) hold), (ii) $\{C_t, L_t, K_{t+1}\}_{t=0}^{\infty}$ maximize the utility of households, i.e., (3), (11) and (12) hold, together with $C_t, K_{t+1} \geqslant 0$, (iii) the technology constraints (5), (1), (6) hold and (iv) markets clear every period:

$$L_t = 1, \tag{15}$$

$$Y_t = I_t + I_t^g + C_t + C_t^g + C_t^w. \tag{16}$$

The simplicity of the model allows for the Π-competitive equilibrium to be analytically characterized.[8] Plugging into the equilibrium conditions (12), (3), (15), (10), (13), (14), (5) a linear guess for the dependence of C_t and K_{t+1} on output, $C_t = aY_t$ and $K_{t+1} = bY_t$, we get

$$C_t = [(1 - \varkappa_i - \varkappa_c - \varkappa_w) - \beta\alpha(1-\tau)] A G_t^{\theta} K_t^{1-\theta}, \tag{17}$$

[8]The way these conditions are obtained is available upon request. In a similar framework, an explicit argument on existence and uniqueness of a linear competitive equilibrium allocation is made in Glomm and Ravikumar (1994) and Marrero and Novales (2005).

$$K_{t+1} = \alpha\beta(1-\tau)AG_t^{\theta}K_t^{1-\theta}. \tag{18}$$

From (7), G_{t+1} is also a proportion $\varkappa_i$ of output every period.[9] The economy is of the AK-type, jumping from initial conditions to equilibrium values C_0, K_1 and G_1, the three variables growing from that time on at the common rate γ, characterized below.

A *balanced growth path* (*BGP*) is a trajectory along which aggregate variables grow at a zero or positive constant rate. With $\alpha+\theta+\phi=1$, a standard argument (e.g., Barro, 1990; Rebelo, 1991; Jones and Manuelli, 1997) shows that, under a stationary fiscal policy, the paths for Y_t, C_t, K_t, G_t, C_t^g, C_t^w solving the Π-competitive equilibrium all grow at the same constant rate. This growth rate is obtained by combining (5), (18), (7) and (10),

$$1+\gamma = Y_{t+1}/Y_t = A\varkappa_i^{\theta}[\alpha\beta(1-\tau)]^{1-\theta}. \tag{19}$$

From (17), it is clear that increasing public expenditures crowds out private consumption, this negative effect on consumption being more intense when the increased expenditures are financed through lump-sum taxation. When that is the case, the crowding-out effect on consumption increases with $\varkappa_w$. On the other hand, (18) and (19) show that capital accumulation and economic growth are affected only if the increase in expenditures is financed through income taxation. The trade-off between lump-sum and income taxes is clear: income taxes reduce growth, with a lower negative impact on initial consumption, the opposite being the case under lump-sum taxes. An excessive crowding-out on private consumption due to large lump-sum taxes is the key argument leading to the possibility that income taxes may be part of an optimal fiscal policy, as we will see in the next section.

3.1. The Ramsey problem

Given $K_0, G_0>0$ and $\varkappa_w \geqslant 0$, the Ramsey problem reduces to searching among the set of Π-competitive equilibrium allocations, for the vector $(\varkappa_i, \varkappa_c, \upsilon, \tau)$, maximizing

$$V(C_0, C_0^g, \gamma) = \left[\frac{1}{1-\beta}(\ln\, C_0 + \lambda\, \ln\, C_0^g) + \frac{\beta(1+\lambda)}{(1-\beta)^2}\ln(1+\gamma)\right]. \tag{20}$$

[9]As shown by (17)–(18), the optimal time allocation of private capital responds only to the income tax rate, while consumption responds to both the income and lump-sum taxes. Moreover, private consumption and capital do not depend on public consumption. Among other simplifications, these facts come about because of the inelastic supply of labor and the separability of the utility function. In addition to the convenience of starting with a simple model to better grasp the intuitions behind our results, these assumptions are needed for the competitive equilibrium to be analytically tractable. Modelling leisure in preferences and non-trivial interactions between private consumption and public consumption would alter the marginal rate of substitution between present and future consumption, as well as that between consumption and leisure. Moreover, private consumption and capital would then depend, in a complex and non-linear way, on both income and lump-sum taxes, and numerical simulations would be required to obtain any conclusion on optimal fiscal policy under these circumstances. In this more general framework, conclusions would depend on the elasticities and cross-elasticities of the arguments in the utility function. This analysis goes beyond the scope of this paper, and it is left for future research.

Proposition 2 shows the solution to this problem, $(\varkappa_i^+, \varkappa_c^+, \upsilon^+, \tau^+)$. Moreover, the allocation of resources is the same under the Ramsey solution than under the planner's problem (Proposition 3), hence the tax system is complete.

Proposition 2. *The welfare-maximizing fiscal policy in the competitive equilibrium resource allocation is given by* $\varkappa_c^+ = \lambda/(1+\lambda)(1-\beta)(1-\varkappa_w)$, $\varkappa_i^+ = \beta\theta(1-\varkappa_w)$, $\upsilon^+ = ((1-\theta)(1-\alpha\beta)/\alpha - (1-\beta)/(1+\lambda))(1-\varkappa_w)$, $\tau^+ = 1-(1-\varkappa_w)(1-\theta)/\alpha$ *and* $\upsilon^+ + \tau^+ = 1 - [(1-\theta)\beta + (1-\beta)/(1+\lambda)](1-\varkappa_w)$.

Proof. See Appendix A.

Proposition 3. *The resource allocation emerging from the fiscal policy that solves the Ramsey problem is the optimal allocation of resources.*

Proof. First, the benevolent planner's problem is solved to find the efficient allocation of resources for any given public expenditures policy. Welfare maximization then leads to the optimal public expenditures policy and the implied allocation of resources. Taking the Ramsey policy of public expenditures and revenues to the competitive equilibrium conditions, we obtain the same optimal allocation of resources. □

4. Policy implications

The government determines the percentage of output devoted to public investment and public consumption, given a committed and exogenous stream of public services. It collects funds from two alternative sources and it could decide to use a fraction of lump-sum revenues to subsidize factor rents. In this section, we first discuss whether factor rents should be taxed or subsidized. Second, we characterize how the optimal mixture of income and lump-sum taxation, as well as the public expenditure composition, depend on the economic environment. Finally, we assess the welfare consequences of deviating from the optimal policy and the best way to adjust that deviation.

4.1. Taxing or subsidizing factor rents?

Not perceiving the spillover externality or the fact that the stock of public capital next period depends on current investment, the consumer takes into account a private marginal productivity of capital below its social product, leading to underinvestment in the competitive equilibrium allocation. Because of these externalities, a government will generally be interested in subsidizing capital rents, financing these subsidies through lump-sum taxation.

A third relevant externality arising in our economy comes from the need to finance an exogenous fraction of public consumption. As mentioned above, since the crowding-out impact on private consumption of raising tax revenues is larger under lump-sum than under income taxes, and this difference is more important for a larger $\varkappa_w$, this externality will tend to reduce subsidies on productive factors and may even lead to income taxation being optimal.

The optimal fiscal policy attempts to reduce the effects of these externalities. Choosing the ratio of public investment to output appropriately, the government mitigates the underinvestment produced by the public capital externality (Barro, 1990). However, the government lacks a direct instrument to correct the other two externalities, so it has to introduce either positive or negative income taxes. Since labor supply is perfectly inelastic, the income tax only has an effect on capital, but not on labor, and it can be seen as a corrective tax to get the right relative prices between consumption and investment. The spillover externality suggests subsidizing production factors, while the constraint imposed by the exogenous component of public expenditures points to taxing factor incomes. Which option prevails will depend on their relative strength.

Under an inelastic labor supply, the following proposition shows that, whenever the exogenous stream of public consumption goes beyond a given proportion of output, taxing factor incomes becomes optimal, against the alternative of subsidizing them. This critical share only depends on the two technological externalities. That labor is inelastically supplied is a potentially important assumption. Indeed, taxing labor income in an elastic labor supply framework would incentive leisure and discourage consumption, which could reverse the result that income taxes may increase consumption. However, the global effect on initial utility is unclear, since although consumption falls, leisure raises, so the relative elasticities of supply and demand schedules should be an important element in determining results. A promising extension of the paper is to study this issue in more detail.[10]

Proposition 4. *There is a threshold level $\bar{\varkappa}_w$ for the ratio of exogenous public consumption to output, above which taxing factor incomes is part of an optimal fiscal policy,*

$$\bar{\varkappa}_w = 1 - \frac{\alpha}{1-\theta} = \frac{\phi}{1-\theta}. \tag{21}$$

Proof. It comes directly from the expression for τ^+ in Proposition 2. □

The proposition can also be read as follows: taxing factor incomes is optimal whenever the aggregate strength of the two technological externalities is not too large, for a given level of $\varkappa_w$. From (21), several remarks are in order. First, the spillover externality is necessary for subsidies on factor incomes to be part of an optimal fiscal policy: when $\phi = 0$, taxing factor incomes is always optimal for any positive level of $\varkappa_w$. Second, the presence of an externality from public capital is not

[10] Turnovsky (2000) studies the relationship between fiscal policy and elastic labor supply in the AK growth model. If government expenditures are set optimally, capital should not be taxed and consumption and leisure should be taxed uniformly. However, if government expenditures are not at their optimal level, all three tax rates are positive, to correct the distortions induced. This latter result is in accordance with ours, since we assume an exogenous (and sub-optimal) positive level of $\varkappa_w$. In a two sector endogenous growth model with physical capital as an input in the education sector and endogenous labor decision, de Hek (2006) finds that taxing capital income alone may have a positive impact on growth if the indirect (positive) effect through more education and labor compensates the direct (and negative) impact on private capital.

Table 1
Sign effects on main variables

	$\varkappa_i^+$	$\varkappa_c^+$	$\varkappa^+$	υ^+	τ^+	$\upsilon^+ + \tau^+$	a^+	b^+
$\varkappa_w \uparrow$	$\downarrow$	$\downarrow$	$\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$
$\theta \uparrow (\Delta\alpha = -\Delta\theta)$	$\uparrow$	$=$	$\uparrow$	$\uparrow$	$\downarrow$	$\uparrow$	$=$	$\downarrow$
$\phi \uparrow (\Delta\alpha = -\Delta\phi)$	$=$	$=$	$=$	$\uparrow$	$\downarrow$	$=$	$=$	$=$

enough, by itself, to imply the optimality of subsidizing production factor incomes: when $\theta = 0$, taxes on factor incomes would be optimal for any ratio of the exogenous public spending to output above $1 - \alpha$. Third, a positive and exogenous fraction of public consumption is necessary for factor income taxes to be part of an optimal fiscal policy.[11]

4.2. The optimal tax mix and the composition of public expenditures

Our analysis reveals important aspects regarding the relationship between the tax mix and the optimal composition of public expenditures. Proposition 4 shows that taxing factor incomes is optimal whenever $\varkappa_w > \bar{\varkappa}_w$. However, revenues from income taxes are never greater than the exogenous public expenditure items and consequently, optimal lump-sum taxes are always positive. These results are summarized in the following propositions:

Proposition 5. *Along the optimal tax policy, income tax revenues are never above the exogenous public consumption stream.*

Proof. It comes directly from the expression for τ^+ in Proposition 2. □

Proposition 6. *It is never optimal to implement negative lump-sum taxes.*

Proof. It comes directly from the expression for υ^+ in Proposition 2. □

Table 1 shows the effects on optimal fiscal policy of changes in $\varkappa_w$, as well as in either θ or ϕ. As a percentage of output, income tax revenues move in the same direction as $\varkappa_w$, while lump-sum taxes and optimal public investment and consumption move in the opposite direction. The ratio of total tax revenues to output increases. To maintain the constant returns to scale assumption, changes in θ and ϕ are compensated by a change of equal size and opposite sign in α. A higher θ leads to a higher public investment ratio, without affecting public consumption. The optimal tax-mix changes, with a higher ratio for lump-sum taxes and a lower one for income taxes, and higher tax revenues overall, as a percentage of output. The optimal tax mix would change in the same direction with a higher spillover of aggregate private capital in production, in this case without any effect on optimal

[11] The standard AK economy emerges as a special case when $\theta = \phi = 0$, and income taxes are then optimal for any $\varkappa_w > 0$.

public investment and consumption, or total tax revenues, as a percentage of output.[12]

A final issue concerns the relationship between different types of revenues and expenditures. Proposition 5 shows that the optimal fiscal structure is such that lump-sum taxation allows for financing endogenous public consumption and public investment, as well as for a fraction of the exogenous public consumption item. On the revenue side, Proposition 2 shows that the optimal income tax rate depends only on technology parameters, while lump-sum taxes depend on preference parameters as well. On the expenditure side, the optimal choice of the public investment ratio, given $\varkappa_w$, depends only on technology parameters, while public consumption expenditures depend only on parameters in preferences. This double duality is produced mainly by the separability of the utility function, and has some incidence on the optimal structure of government financing: for a given size of the exogenous component of public consumption, any desired increase in other public consumption concepts will generally be financed through lump-sum taxes, while an increase in public investment will be financed through a combination of lump-sum and income taxation.

4.3. Deviations from optimal policy

The relevance of income taxes goes beyond the optimal structure of fiscal policy. In the standard time evolution of actual economies, perturbations of different kinds will produce deviations between observed tax revenue and expenditure ratios and their optimal values. We show in this section that such deviations should better be accommodated using income taxes than lump-sum taxes.

Starting at the optimal policy, $\Pi^+ = \{\varkappa_i^+, \varkappa_c^+, \varkappa_w, \tau^+, \upsilon^+\}$, we change $\varkappa_i$ by an amount ε, $\varkappa_i = \varkappa_i^+ + \varepsilon$, which we alternatively assume to be financed either by a change in τ, to $\tau = \tau^+ + \varepsilon$, or by a change in υ, to $\upsilon = \upsilon^+ + \varepsilon$. Values of ε in the interval $-\varkappa_i^+ < \varepsilon < (1-\varkappa_w)(1-\beta)/(1+\lambda)$ guarantee non-negativity of public investment and private consumption. Even though we focus on deviations in the public investment ratio, a similar argument can be used to establish that the optimality of income tax financing also applies to the case when it is $\varkappa_c$ that deviates from its optimal level. The result would apply again to the case of a deviation from the optimal tax mix, i.e., when compensating changes are simultaneously introduced in income and lump-sum taxes, while keeping the total public expenditures ratio to output unchanged.

Expression (20) decomposes welfare into a term emerging from initial private and public consumption, and a second term which depends on the rate of growth. The analysis in the previous sections showed that a change in public investment would affect C_0 and γ, but not C_0^g. Hence, if we denote by $V^{(\tau)}(\varepsilon) = V(\varkappa_i^+ + \varepsilon, \varkappa_c^+, \varkappa_w, \tau^+ + \varepsilon, \upsilon^+)$ and $V^{(\upsilon)}(\varepsilon) = V(\varkappa_i^+ + \varepsilon, \varkappa_c^+, \varkappa_w, \tau^+, \upsilon^+ + \varepsilon)$, the levels of welfare under each financing alternative, expression (39) in Appendix B can be used to write the

[12]To see this, substitute $(1-\phi-\alpha)$ for θ and $(\phi+\alpha)$ for $(1-\theta)$ in Proposition 4, and take derivatives with respect to ϕ.

difference $D(\varepsilon)$ between them as

$$(1-\beta)D(\varepsilon) = V^{(\tau)}(\varepsilon) - V^{(v)}(\varepsilon) = D^c(\varepsilon) + D^\gamma(\varepsilon)$$

with

$$D^c(\varepsilon) = \ln\left(1 + \frac{\beta\alpha(1+\lambda)}{(1-\varkappa_w)(1-\beta) - \varepsilon(1+\lambda)}\varepsilon\right), \tag{22}$$

$$D^\gamma(\varepsilon) = \frac{\beta(1+\lambda)}{(1-\beta)}(1-\theta)\ln\left(1 - \frac{\alpha}{(1-\varkappa_w)(1-\theta)}\varepsilon\right),$$

where $D^c(\varepsilon)$ and $D^\gamma(\varepsilon)$ capture the relative welfare effects of a deviation from $\varkappa_i^+$ on C_0 and γ, respectively. Both effects are zero for $\varepsilon = 0$. $D^c(\varepsilon)$ shares the same sign than ε: an increase in $\varkappa_i$ above $\varkappa_i^+$, financed by higher lump-sum taxes, will lower C_0 by more than if income taxes are raised, so $D^c(\varepsilon)$ will be positive, and the opposite effect would apply if $\varkappa_i$ falls below $\varkappa_i^+$. On the other hand, $D^\gamma(\varepsilon)$ has the opposite sign to ε: an increase in τ to finance a higher $\varkappa_i$ will lower γ more than if lump-sum taxes are used. As a consequence, $D^\gamma(\varepsilon)$ will be negative for $\varepsilon > 0$, and the opposite would happen when $\varepsilon < 0$, the observed $\varkappa_i$ falling below $\varkappa_i^+$. That the effects on C_0 and γ have opposite sign illustrates the trade-off embedded in each tax policy. Hence, in principle, the sign of the net effect on $D(\varepsilon)$ of a change on ε is unclear. The following proposition shows that the net effect is always favorable to compensating a deviation from $\varkappa_i^+$ using income taxes. This is an important result, that complements the characterization of optimal policy we made above. It states that if the government is forced to run into a sub-optimal public expenditure policy, the deviation in expenditures should be compensated with movements in the income tax rate, whileleaving non-distortionary taxation unchanged.

Proposition 7. *From the point of view of welfare maximization, it is always better to compensate deviations from the optimal expenditure policy using factor income taxes than using lump-sum taxes.*

Proof. See Appendix B.

4.4. A numerical illustration

Further insights into the optimal policy can be obtained by illustrating our fiscal policy results with numerical examples. We keep $\lambda = 0.40$ and $\beta = 0.90$ constant in all calibrations, and change other more relevant parameters to find stationary equilibrium values and optimal policy variables. Aggregate variables are shown as a percentage of output.

4.4.1. Optimal policy

Table 2 shows numerical results for various optimal policies under alternative calibrations.[13] The first four columns show the parameterization used. Columns five

[13]The full depreciation assumption restrains us from any strict comparison between the optimal ratios implied by the calibrated economy and the ones observed in actual economies. To make a sound

Table 2
Optimal policy and equilibrium values under alternative parameterizations

Parameterizations $\beta = 0.90; \lambda = 0.40$				As % of total output					Taxes in %			Ratio to $\varkappa_w$ in %	
$\varkappa_w$	α	ϕ	θ	C	I	I^g	C^g	C^w	υ	τ	$\upsilon + \tau$	υ	τ
0.15	**0.75**	**0.00**	**0.25**	**6.1**	**57.4**	**19.1**	**2.4**	**15.0**	**21.6**	**15.0**	**36.6**	**143.7**	**100**
0.15	**0.75**	**0.10**	**0.15**	**6.1**	**65.0**	**11.5**	**2.4**	**15.0**	**25.2**	**3.7**	**28.9**	**168.2**	**24.4**
0.15	0.75	0.25	0.00	6.1	76.5	0.0	2.4	15.0	30.8	−13.3	17.4	205.1	−88.9
0.30	**0.75**	**0.00**	**0.25**	**5.0**	**47.3**	**15.8**	**2.0**	**30.0**	**17.8**	**30.0**	**47.8**	**59.2**	**100.0**
0.30	**0.75**	**0.10**	**0.15**	**5.0**	**53.6**	**9.5**	**2.0**	**30.0**	**20.8**	**20.7**	**41.5**	**69.3**	**68.9**
0.30	**0.75**	**0.25**	**0.00**	**5.0**	**63.0**	**0.0**	**2.0**	**30.0**	**25.3**	**6.7**	**32.0**	**84.4**	**22.2**
0.15	0.50	0.20	0.30	6.1	53.6	23.0	2.4	15.0	59.4	−19.0	40.4	395.9	−126.7
0.15	0.40	0.20	0.40	6.1	45.9	30.6	2.4	15.0	75.5	−27.5	48.0	503.5	−183.3
0.15	0.30	0.20	0.50	6.1	38.3	38.3	2.4	15.0	97.3	−41.7	55.7	649.0	−277.8
0.30	**0.50**	**0.20**	**0.30**	**5.0**	**44.1**	**18.9**	**2.0**	**30.0**	**48.9**	**2.0**	**50.9**	**163.0**	**6.7**
0.30	0.40	0.20	0.40	5.0	37.8	25.2	2.0	30.0	62.2	−5.0	57.2	207.3	−16.7
0.30	0.30	0.20	0.50	5.0	31.5	31.5	2.0	30.0	80.2	−16.7	63.5	267.2	−55.6

Note: Bold case figures show those economies where taxing factor rents is optimal.

and six show private consumption and investment decisions, followed by optimal decisions on public consumption and public investment. In general, investment is high in our economy, and consumption levels are relatively small, because of the full depreciation assumption on both types of capital. The next columns show lump-sum taxes and income taxes, both as a proportion of output, followed by total revenues and by the relevance of each tax concept relative to C^w.

The first panel considers $\alpha = 0.75$, which suggests a broad interpretation of capital as incorporating physical as well as human capital. Income taxes amount to 15.0% of output in the absence of spillover, reducing their significance to 3.7% of output when $\phi = 0.10$. For $\phi = 0.25$, it is optimal to subsidize factor incomes. A comparison between the first and the second panel shows that a higher level of $\varkappa_w$ leads to higher income taxes, while lump-sum revenues decrease, as a percentage of output. Total taxes increase, although less than proportionally, while the public investment and consumption ratios both decrease. All this goes as mentioned in Sections 4.1 and 4.2,[14] and it reflects the crowding-out effect of a higher $\varkappa_w$ on public consumption and investment characterized in Table 1. This is a rather broad crowding-out effect, with private and public decisions on consumption and investment all being affected. The third and fourth panels show that, for given ϕ,

(*footnote continued*)
comparison, other economic fundamentals not included in the model, such as the presence of heterogenous agents, capital markets imperfections, and the presence of congestion effects in public infrastructures, should be taken into account when characterizing optimal fiscal policy.

[14]The same result arises in the comparisons between panels 3 and 4.

an increase in θ, together with a corresponding decrease in α to maintain $\alpha+\theta+\phi=1$, induces higher public investment and lump-sum revenues, while income taxes falls. Thus, the higher elasticity of public capital leads to increased factor subsidies, and the comparison between rows 10 and 11 in the table shows how a shift may arise in optimal policy from taxing to subsidizing factor rents.

Finally, it is never optimal to give positive net lump-sum transfers to the private sector, as shown in Proposition 6, and tax revenues are never above exogenous public expenditures, as shown in Proposition 5.

4.4.2. *Deviations from optimal policy: uncompensated and compensated changes*

Table 3 summarizes the effects of deviations from optimal policy. We examine the welfare effects of *uncompensated changes* of 0.01 in either $\varkappa_c$ or $\varkappa_i$ from their optimal levels. Results depend on the way deviations from optimal policy are financed, and we alternatively consider a raise in lump-sum taxes, income taxes, or a simultaneous increase of equal size in both types of revenues. Effects are shown in the table as the percentage increase in private consumption that would be needed each period to reach the same level of welfare as before the deviation from the optimal policy.

We take again $\lambda=0.4$ and $\beta=0.9$ in all cases, and the technological constant A is chosen so that growth is 4.0% under the optimal policy in all simulated economies. We consider $\varkappa_w=0.15$ in economies 1, 2, and 5, and $\varkappa_w=0.30$ in economies 3 and 4. In economy 1 there is no spillover from aggregate private capital (i.e., $\phi=0$), while ϕ is large in economies 2 and 3, and low in economies 4 and 5. θ is higher in economies 4 and 5. In economies 1 and 2, with $\varkappa_w=0.15$, $\varkappa_i^+$ and $\varkappa_c^+$ turn out to be 19.1% and 2.4%, respectively. The difference is that optimal policy leads to financing total expenditures by a combination of lump-sum and income taxes in economy 1, whereas, in the economy 2, optimal financing is in the form of strong subsidies to factor incomes, together with high lump-sum taxes. From this optimal situation, an increase of 0.01 in $\varkappa_c$ would lead to a welfare loss similar to a decrease of 4.12% in consumption every period if the change is financed with lump-sum taxes, or 2.85% of consumption if it would be financed through income taxes. Any combination of taxes would produce intermediate results. The nature of effects of a 0.01 increase in $\varkappa_i$ is similar: the welfare loss amounts to 1.92% or 0.63% of consumption depending on whether the deviation is financed with lump-sum or income taxes, respectively. Finally, notice that effects are more important in economies 3 and 4, who share a higher level of $\varkappa_w$.

The first two rows in the *Compensated changes* panel refer to simultaneous changes in both public expenditure ratios maintaining total expenditures constant, as a proportion of output. Alternatively, in the last two rows, the label refers to simultaneous changes in lump-sum and income taxes, maintaining total revenues constant as a percentage of output. An increase of 0.01 in $\varkappa_i$, compensated with an equal decrease in $\varkappa_c$, produces in economies 1 and 2 a welfare loss comparable to a decrease in consumption of 5.04% every period. An increase of 0.01 in $\varkappa_c$, compensated with an equal decrease in $\varkappa_i$, has always a smaller welfare effect than the reverse deviation. The welfare effects of deviations from optimal policy are asymmetric. Effects are again higher in the two economies with the higher level of

Table 3
Welfare effects of deviations from the optimal policy

Economies	1	2	3	4	5
Parameterizations					
α	0.75	0.40	0.40	0.40	0.40
ϕ	0.00	0.35	0.35	0.15	0.15
θ	0.25	0.25	0.25	0.45	0.45
$\varkappa_w$	0.15	0.15	0.30	0.30	0.15
Optimal policies					
$\varkappa_i$	0.191	0.191	0.158	0.284	0.344
$\varkappa_c$	0.024	0.024	0.020	0.020	0.024
τ	0.150	−0.594	−0.313	0.038	−0.169
υ	0.216	0.959	0.790	0.566	0.687
$1+\gamma$	1.040	1.040	1.040	1.040	1.040
Uncompensated changes[a]					
Higher $\varkappa_c$, higher υ	−4.116	−4.116	−5.914	−5.914	−4.116
Higher $\varkappa_c$, higher τ	−2.850	−3.240	−4.597	−4.606	−3.247
Higher $\varkappa_c$, higher τ and υ	−3.281	−3.618	−5.163	−5.165	−3.620
Higher $\varkappa_i$, higher υ	−1.924	−1.924	−2.881	−2.624	−1.746
Higher $\varkappa_i$, higher τ	−0.629	−1.028	−1.522	−1.271	−0.855
Higher $\varkappa_i$, higher τ and υ	−1.069	−1.414	−2.106	−1.849	−1.237
Compensated changes[a]					
Higher $\varkappa_i$, lower $\varkappa_c$	−5.039	−5.039	−7.997	−7.753	−4.867
Higher $\varkappa_c$, lower $\varkappa_i$	−3.075	−3.075	−4.347	−4.058	−2.879
Higher τ, lower υ	−0.640	−0.188	−0.275	−0.285	−0.194
Higher υ, lower τ	−0.730	−0.201	−0.299	−0.309	−0.208

[a] % increase in C_t needed to match the level of welfare attained under the optimal policy.

$\varkappa_w$. An increase of 0.01 in τ, compensated with an equal decrease in υ leads in the first economy to a welfare loss comparable to a fall of 0.64% in consumption every period, whereas the reverse deviation has the same effect as a 0.73% reduction in consumption every period. Welfare effects are again asymmetric. Effects are smaller in the other parametric cases considered, most likely because tax rates are highest in the first economy.

5. Robustness of results

In Section 3.1, several assumptions were needed to solve the Ramsey problem analytically: log utility, Cobb–Douglas technology, uniform income tax rates and full depreciation of private and public capital. The main result of the paper shows that a positive income tax rate may be optimal, even when allowing for lump-sum taxation as well as for a positive spillover of aggregate capital and for public capital externalities. In this section we analyze the robustness of this result when utility moves away from the logarithmic specification and when public and private capital are long-lived.[15] We show that these generalizations do not affect the optimal income tax rate, although they may have important implications on optimal public investment and lump-sum tax policies. Unfortunately, departing from the mentioned assumptions precludes the possibility of a full analytical discussion. For simplicity and since C^g does not affect the optimal tax structure, we do not consider C^g as an argument in utility and we set $\varkappa_c = 0$ throughout this section. Thus, public expenditure is either productive or wasteful.

Conditions (2), (1), (6) and (10) are now

$$U(C_t) = \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-1/\sigma} - 1}{(1-1/\sigma)}, \quad \sigma > 0, \tag{23}$$

$$K_{t+1} = (1-\delta)K_t + I_t, \tag{24}$$

$$G_{t+1} = (1-\delta^g)G_t + I_t^g, \tag{25}$$

$$I_t^g + C_t^w = \tau_t Y_t + X_t \;\Leftrightarrow\; \varkappa_i + \varkappa_w = \tau_t + \upsilon_t, \tag{26}$$

where σ is the constant elasticity of intertemporal substitution and δ and δ^g are the linear depreciation rates of private and public capital, respectively, both between zero and one and not necessarily equal to each other. The consumption-saving condition (12) for the representative household becomes

$$C_{t+1}/C_t = \beta^\sigma[1-\delta+r_{t+1}(1-\tau_{t+1})]^\sigma. \tag{27}$$

The Ramsey problem turns out to be

$$\max_{\{C_t,K_{t+1},G_{t+1},\tau_t,\upsilon_t\}_0^\infty} \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-1/\sigma} - 1}{(1-1/\sigma)}, \quad \text{s.t.} \tag{28}$$

$$C_t^{-1/\sigma} = \beta C_{t+1}^{-1/\sigma}[1-\delta+\alpha A(G_{t+1}/K_{t+1})^\theta(1-\tau_{t+1})], \tag{29}$$

$$G_{t+1} = (\upsilon_t + \tau_t - \varkappa_w)A(G_t/K_t)^\theta K_t - (1-\delta^g)G_t, \tag{30}$$

$$C_t + K_{t+1} - (1-\delta)K_t = (1-\tau_t-\upsilon_t)A(G_t/K_t)^\theta K_t, \tag{31}$$

where the first constraint corresponds to the household's consumption-saving condition, the second one represents the accumulation of public capital while the third is the global constraint of resources in the economy.[16]

Fiscal policy must be stationary along the BGP with all aggregate variables growing at the same constant rate γ. We define the stationary ratios: $\tilde{G} = G_t/K_t$ and $\tilde{u}_c = (C_{t+1}/C_t)^{1/\sigma} = (1+\gamma)^{1/\sigma}$, the marginal rate of substitution between next period and current consumption. For the BGP, the solution of the above Ramsey problem

[15] As in Marrero (2005, 2006a,b).

[16] As under $\sigma, \delta, \delta^g = 1$, it is not hard to show that the resource allocation emerging in competitive equilibrium under the fiscal policy that solves the Ramsey problem reproduces that of the optimal planner's problem.

leads to the following conditions on the public-to-private capital ratio[17]:

$$\beta[1+(1-\varkappa_w)\theta A\tilde{G}^{\theta-1}-\delta^g]=(1+\gamma)^{1/\sigma}, \tag{32}$$

$$\varkappa_i(\alpha+\phi)A\tilde{G}^{\theta}+[1-\delta+(1-\tau-\upsilon)(\alpha+\phi)A\tilde{G}^{\theta}]=(1+\gamma)^{1/\sigma}/\beta. \tag{33}$$

In addition, from the public capital accumulation rule (30) and the Euler condition (29),

$$\theta A\tilde{G}^{\theta-1}=\theta\frac{\delta^g+\gamma}{\varkappa_i}, \tag{34}$$

$$1-\delta+\alpha A\tilde{G}^{\theta}(1-\tau)=(1+\gamma)^{1/\sigma}/\beta. \tag{35}$$

Plugging (34) into (32), we obtain an equation for the optimal public investment ratio

$$\varkappa_i^+=\beta\theta(1-\varkappa_w)\frac{\delta^g+\gamma}{(1+\gamma)^{1/\sigma}-\beta(1-\delta^g)}. \tag{36}$$

In general, since γ depends on policy variables, an explicit expression for $\varkappa_i^+$ cannot be obtained. Only under full depreciation ($\delta^g=1$) and a logarithmic utility function ($\sigma=1$) do we get an explicit expression for the optimal public investment ratio, $\varkappa_i^+=\beta\theta(1-\varkappa_w)$, the same one in Proposition 2.

Combining (34) with (32) yields the optimal income tax rate

$$\tau^+=1-(1-\varkappa_w)\frac{1-\theta}{\alpha}, \tag{37}$$

the same as that shown in Proposition 2, so Proposition 4 holds with generality. Finally, using (36), we obtain the optimal stationary level of lump-sum taxes as a fraction of output

$$\upsilon^+=(1-\varkappa_w)\left(\theta\frac{\gamma+\delta^g}{(1+\gamma)^{1/\sigma}/\beta-1+\delta^g}+\frac{\phi}{\alpha}\right). \tag{38}$$

For $\sigma\neq 1$, $\varkappa_i^+$ is positive and strongly related with δ^g and σ, while the relationship with δ is also positive, but less intense.[18] The same dependence on these two parameters is held by υ^+, since τ^+ is not affected by them. It is easy to show that $\varkappa_i^+<\beta\theta(1-\varkappa_w)$ whenever $\sigma<1$, or when $\sigma=1$ but $\delta^g<1$. Moreover, $\varkappa_i^+$ can never be higher than the ratio maximizing the long-run growth rate, $\theta(1-\varkappa_c)$ (Marrero, 2006a). Needless to say, under full depreciation ($\delta^g=1$) and a logarithmic utility function ($\sigma=1$) we get explicit expressions for the optimal fiscal variables, $\varkappa_i^+=\beta\theta(1-\varkappa_w)$, $\tau^+=1-(1-\varkappa_w)(1-\theta)/\alpha$, $\upsilon^+=(1-\varkappa_w)(\theta\beta+\phi/\alpha)$, the same ones in Proposition 2 when setting $\lambda=0$ in υ^+.

[17] See Marrero (2006b) for a detailed description of the solution of the Ramsey problem in this setup.
[18] See Marrero (2006a) for more details about this point.

6. Final remarks

We have characterized the optimal tax and expenditure policies in an economy where public capital is productive and there is a spillover effect from aggregate private capital. We assume debt cannot be issued, so that the government has three decisions to make every period: (i) investment on public capital, (ii) public consumption and (iii) the tax mixture between income and lump-sum taxation to finance public expenditures, allowing for the possibility of negative taxes, i.e., subsidies on factors' rents. The presence of public capital as an input in the production function and the spillover externality, both tend to favor subsidizing factor rents. In addition, we incorporate into our model economy the fact that the government is pre-committed to finance an exogenous component of public expenditures, due to commitments made in the past. This is assumed to be a constant proportion of output. The crowding-out effect on private consumption of raising tax revenues is larger when it is lump-sum taxes that are increased, rather than income taxes. This difference turns out to be more important the larger the requirement of the exogenous component of public expenditures. Hence, this constraint on public expenditures will tend to reduce subsidies on productive factors and may even lead to income taxation being optimal.

The main result is that taxing factor incomes may be optimal for reasonable parameterizations in the presence of the public capital externality and the spillover effect from aggregate private capital, in spite of the fact that both externalities tend to favor subsidizing production factors. Moreover, we have also shown that welfare losses from deviations from the optimal policy are always smaller when compensated with changes in income taxes than when adjusting lump-sum taxes. Income taxes are always optimal in the absence of the spillover externality, but they can also be optimal in cases when that externality is sizeable, if combined with the constraint to pay for a relatively high level of the exogenous component of public expenditures. The optimality of factor income taxes is more likely in economies with a low elasticity of public capital, a weak spillover from private capital, and also when exogenous public expenditures claim a large proportion of output. This result is shown to be robust to changes in the intertemporal elasticity of substitution and in the rate at which public and private capital depreciate.

Two extensions should allow us to discuss further relevant policy issues, even though a full characterization of optimal policy would require of numerical solution methods in each case. The first one would look at the welfare effects of different expenditure components by moving away from the separability between private and public consumption in consumers' preferences. In our model economy, that kind of separability implies a strong dichotomy between factors affecting the structure of revenues and expenditures and, as a consequence, the presence of public consumption in utility function does not play a significant role in the optimality of income taxes. A second extension should consider leisure as an argument in the utility function. An elastic labor supply could lead to an effect of income taxes on initial consumption of opposite sign to that described in this paper, so the optimality of income taxes might depend on the relative elasticities of supply and demand schedules. Characterizing the optimal combination of lump-sum taxes, labor and capital income taxes to finance public investment and consumption is hence a natural extension of this research.

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Appendix A. Solution to the Ramsey problem. Proof of Proposition 2

Plugging (17) for C_0, (19) for γ and $C_0^g = \varkappa_c Y_t$ into (20) we get

$$V(\varsigma) = \frac{1}{1-\beta}\left\{\begin{array}{l} \ln[(1-\varkappa_i-\varkappa_c-\varkappa_w)-\beta\alpha(1-\varkappa_i-\varkappa_c-\varkappa_w+\upsilon)]+\lambda\ln\varkappa_c \\ \quad +(1+\lambda)\ln Y_0+\dfrac{\beta(1+\lambda)}{1-\beta}\ln\{A\varkappa_i^{\theta}[\alpha\beta(1-\varkappa_i-\varkappa_c-\varkappa_w+\upsilon)]^{1-\theta}\} \end{array}\right\}. \tag{39}$$

It is easy to show that $V(\cdot)$ is non-monotonic in either $\varkappa_i, \varkappa_c$ and υ. Moreover, for any given υ, we have $\lim_{\varkappa_i,\varkappa_c\to 0^+} V = \lim_{\varkappa_i,\varkappa_c\to 1^-} V = -\infty$, so the welfare-maximizing levels of $\varkappa_i$, $\varkappa_c$ fall in the interior of $[0,1]\times[0,1]$. This is not the case for υ, whose optimal value may fall above 1 or below 0. $V(\zeta)$ defined in (39) is strictly concave in ζ, so condition $\nabla V(\zeta)=0$ is necessary and sufficient to characterize a global maximum

$$\frac{\partial V}{\partial \varkappa_i} = \frac{-(1-\beta\alpha)}{1-\varkappa-\beta\alpha(1-\varkappa+\upsilon)}+\frac{\beta(1+\lambda)\theta}{(1-\beta)\varkappa_i}-\frac{\beta(1+\lambda)(1-\theta)}{(1-\beta)(1-\varkappa+\upsilon)}=0, \tag{40}$$

$$\frac{\partial V}{\partial \upsilon} = -\frac{\beta\alpha}{1-\varkappa-\beta\alpha(1-\varkappa+\upsilon)}+\frac{\beta(1+\lambda)(1-\theta)}{(1-\beta)(1-\varkappa+\upsilon)}=0, \tag{41}$$

$$\frac{\partial V}{\partial \varkappa_c} = -\frac{1-\beta\alpha}{1-\varkappa-\beta\alpha(1-\varkappa+\upsilon)}+\frac{\lambda}{\varkappa_c}-\frac{\beta(1+\lambda)(1-\theta)}{(1-\beta)(1-\varkappa+\upsilon)}=0, \tag{42}$$

where $\varkappa = \varkappa_i + \varkappa_c + \varkappa_w$. Adding up (41) and (42), we get

$$\upsilon = (1-\varkappa)\left(\frac{1}{\alpha\beta} - 1\right) - \frac{\varkappa_c}{\lambda}\frac{1}{\alpha\beta}, \tag{43}$$

and thus

$$1 - \varkappa + \upsilon = 1 - \tau = \left(1 - \varkappa - \frac{\varkappa_c}{\lambda}\right)\frac{1}{\alpha\beta}. \tag{44}$$

Plugging (44) into (41) we get

$$-\frac{\beta\alpha}{1 - \varkappa - \left(1 - \varkappa - \varkappa_c/\lambda\right)} + \frac{\beta(1+\lambda)(1-\theta)}{(1-\beta)\left(1 - \varkappa - \varkappa_c/\lambda\right)(1/\beta\alpha)} = 0,$$

from where we get the relationship between the welfare-maximizing levels of $\varkappa_i$ and $\varkappa_c$,

$$\varkappa_c^+ = \frac{\lambda}{1+\lambda}\frac{1-\beta}{1-\beta+\beta(1-\theta)}(1 - \varkappa_i^+ - \varkappa_w). \tag{45}$$

On the other hand, combining (40) and (42), we get an alternative relationship between $\varkappa_i^+$ and $\varkappa_c^+$,

$$\varkappa_i^+ = \beta\frac{1+\lambda}{1-\beta}\frac{\theta}{\lambda}\varkappa_c^+. \tag{46}$$

Combining (45) with (46), we get those expressions for $\varkappa_c^+$ and $\varkappa_i^+$ in Proposition 2. Using (43) we obtain that for υ^+ and τ^+ comes from $\varkappa_c^+ + \varkappa_i^+ + \varkappa_w - \upsilon^+$.

Appendix B. Proof of Proposition 7

Lemma 8. *$\frac{\partial|D^C(\varepsilon)|}{\partial\varepsilon} > \frac{\partial|D^\gamma(\varepsilon)|}{\partial\varepsilon}$ for $0 < \varepsilon < \frac{(1-\varkappa_w)(1-\beta)}{(1+\lambda)}$, while $\frac{\partial|D^C(\varepsilon)|}{\partial\varepsilon} < \frac{\partial|D^\gamma(\varepsilon)|}{\partial\varepsilon}$ for $-\varkappa_i^+ < \varepsilon < 0$.*

Proof. First order derivatives are: $\partial D^c(\varepsilon)/\partial\varepsilon = \big(\beta\alpha(1+\lambda)(1-\varkappa_w)(1-\beta)/[(1-\varkappa_w)(1-\beta) - \varepsilon(1+\lambda)]^2\big)[1 + \beta\alpha(1+\lambda)\varepsilon/((1-\varkappa_w)(1-\beta) - \varepsilon(1+\lambda))]^{-1}$, $\partial D^\gamma(\varepsilon)/\partial\varepsilon = -[\beta(1+\lambda)/(1-\beta)]\alpha(1-\theta)/([(1-\varkappa_w)(1-\theta) - \alpha\varepsilon])$. Since $1-\beta < 1+\lambda$ and $\alpha \leqslant 1-\theta$, we have that $\partial D^c(\varepsilon)/\partial\varepsilon$ is positive, and is negative for any ε in the range $-\varkappa_i^+ < \varepsilon < (1-\varkappa_w)(1-\beta)/(1+\lambda)$.

(i) For $0 < \varepsilon < (1-\varkappa_w)(1-\beta)/(1+\lambda)$, we have: $\partial|D^c(\varepsilon)|/\partial\varepsilon > \partial|D^\gamma(\varepsilon)|/\partial\varepsilon \Leftrightarrow \partial D^C(\varepsilon)/\partial\varepsilon > -\partial D^\gamma(\varepsilon)/\partial\varepsilon \Leftrightarrow (1-\beta)(1-\varkappa_w)/([(1-\beta)(1-\varkappa_w) - \varepsilon(1+\lambda)(1-\beta\alpha)][(1-\beta)(1-\varkappa_w) - \varepsilon(1+\lambda)]) > (1-\theta)/((1-\beta)[(1-\varkappa_w)(1-\theta) - \alpha\varepsilon])$. Since $\lambda \geqslant 0$ and $\alpha \leqslant 1$, we have $(1-\varkappa_w)(1-\beta)/(1+\lambda) < (1-\varkappa_w)(1-\beta)/\alpha$, so that $\varepsilon < (1-\varkappa_w)(1-\beta)/(1+\lambda)$ implies $\alpha\varepsilon < (1-\varkappa_w)(1-\theta)$, and the previous inequality will be equivalent to

$$(1-\beta)^2(1-\varkappa_w)\alpha < (1-\theta)(1-\beta)(1-\varkappa_w)(1+\lambda) + \Phi$$

with

$$\Phi = (1-\theta)(1+\lambda)(1-\beta\alpha)(1-\beta)(1-\varkappa_w) - (1-\theta)(1+\lambda)^2(1-\beta\alpha)\varepsilon.$$

Since Φ is positive for the range of ε considered, and $(1-\beta)\alpha < (1+\lambda)(1-\theta)$, the inequality above holds.

(ii) For $-\varkappa_i^+ < \varepsilon < 0$ we have $\partial|D^C(\varepsilon)|/\partial\varepsilon < |\partial D^\gamma(\varepsilon)|/\partial\varepsilon \Leftrightarrow -\partial D^C(\varepsilon)/\partial\varepsilon < \partial D^\gamma(\varepsilon)/\partial\varepsilon \Leftrightarrow \partial D^C(\varepsilon)/\partial\varepsilon > -\partial D^\gamma(\varepsilon)/\partial\varepsilon$ which, by the same previous argument, it is seen to hold for any negative ε.

Proposition 7 follows directly from the previous lemma.

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