# Testing the expectations hypothesis in Eurodeposits 

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#### Abstract

Analyzing data on Euro-rates for 1978-1998, we find some consistent evidence in favor of the Expectations Hypothesis (EH) of the term structure: a) interest rates offered on deposits in a given currency form a cointegrated system, b) the restrictions of the EH on the cointegrating relationships are not rejected, c) forward rates contain significant explanatory power on future interest rates, unbiasedness being an acceptable hypothesis as a cointegrating relationship between forward rates and the appropriate future value of the corresponding short term interest rate.

However, we also provide evidence that past rates contain information additional to that in forward rates to predict future short-term rates, against the rational expectations version of the EH and market efficiency. © 2000 Elsevier Science Ltd. All rights reserved.


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## 1. Introduction

Characterizing the properties of the term structure in markets where a given asset is offered at different maturities is a central issue in financial economics, for a variety of sound reasons. Apart from its relevance for monetary policy implementation, or from the possible ability of the term structure slope to predict future changes in

[^0]economic activity, it has been discussed for a number of years that some characteristics of the term structure contain significant information on future interest rate changes. Specifically, according to the Expectations Hypothesis (EH) of the term structure, long-term interest rates are an average of current and expected future shortterm rates over the life of the investment. An implication is that there is a close link between short- and long-term rates, to the point that their spread contains all relevant information on future changes in short-term rates. That would be of utmost interest for market participants, who could otherwise hope to design profitable investment strategies using information currently available.

Interest rates on Eurodeposits, known as Euro-rates, provide an interesting data set on which to test these issues. They share important characteristics, not being distorted by differences in the fiscal treatment of returns or in the timing of interest payments, and not being affected by possible capital controls or other government regulations. That makes them more comparable than domestic rates, so testing the EH with Euro-rates should lead to fairly robust conclusions on the relationship between short- and long-term returns.

The ability of the EH to explain the behavior of interest rates over the term structure has been controversial for a long time. Even though the initial evidence on US data (as presented in Shiller et al., 1983; Fama, 1984; Fama and Bliss, 1987; Shiller, 1990), consistently rejected the restrictions implied by the EH, some of these authors obtained evidence of explanatory power in the short/long-term interest rate spread on future short-term rates. Fama (1990) and Mishkin (1988) both found that the spread does contain information on short-term rates several periods into the future. Mankiw and Summers (1984) and Mankiw and Miron (1986) analyzed 3- and 6month US rates, concluding that the term structure had important explanatory power for future interest rates, although it seems to have faltered after the founding of the Federal Reserve System. Campbell and Shiller $(1987,1991)$ found again that the restrictions of the EH do not hold, but that the US spread explains the direction of changes in short-term rates. However, the predicted changes are small, suggesting a possible time varying risk premium or a term premium. Similar results were obtained by Jorion and Mishkin (1991).

Some evidence has recently been brought up in favor of the EH: Hardouvelis (1994) uses quarterly data from the G-7 countries, and rates of return on three month and 10 year bonds, to conclude that the cumulative movements in future short term rates roughly agree with the implications of the theory, and strongly rejecting the hypothesis that the spread lacks any explanatory content. Even more recently, Gerlach and Smets (1997) have obtained evidence in favor of both, the restrictions of the EH , and the explanatory power of the spread on future short-term rates. An additional result from Hardouvelis (1994) and Gerlach and Smets (1997) is that the EH tends not to do very well in the US so that the hypothesis should be tested with international data.

The goal of this paper is to test several implications of the EH in the market for Eurodeposits, using data on interest rates at $1-$, $3-$, 6 - and 12-months, for the US dollar, Japanese yen, German mark, British pound, Spanish peseta, French franc, Italian lira and Swiss franc, between January 1978 and December 1996. We first
test whether there is a tight connection between short- and long-term rates over a given term structure. Given their non-stationary nature at all the different maturities and for all currencies considered, we view the vector of returns offered in a given currency as a system of possibly cointegrated variables, and estimate and interpret the number of cointegrating relationships between them. After that, we proceed to test for the restrictions that the EH imposes on the possible cointegrating relationships among interest rates.

We then analyze a second implication of the EH regarding the information content in the long/short-term spread on future short-term interest rate fluctuations. In particular, we pay special attention to testing whether forward rates are unbiased predictors of short-term rates. To do so, we again take into account the non-stationary nature of forward rates, and look for a possible cointegrating relationship between forward rates and the corresponding future short-term spot rate.

Finally, we search for possible deviations from the rational expectations version of the EH that could point at some violation of market efficiency. We implement two tests, alternatively searching for whether recent interest rates, or interest rates very distant in the past, contain information additional to that in forward rates, to anticipate future interest rate fluctuations.

In Section 2 we review some concepts relating to the EH, and analyze the term structure as a system of possibly cointegrated rates of return. In Section 3 we characterize the information content in implicit forward rates on future short-term rates and test for unbiasedness. Possible deviations from the EH that could suggest market inefficiency are discussed in Section 4. The paper closes with some conclusions.

## 2. The term structure as a cointegrated system of rates of return

According to the EH , the return on an $n$-period investment, $r_{t}^{n}$, should be equal to the average expected return on a roll-over strategy over that period, plus possibly a time invariant term or risk premium $\pi^{n, 1}$,

$$
\begin{equation*}
r_{t}^{n}=\frac{1}{n} \sum_{j=0}^{n-1} E_{t} r_{t+j}^{1}+\pi^{n, 1} \tag{1}
\end{equation*}
$$

$E_{t} r_{t+j}^{1}$ being the current expectation, based on information available at time $t$, of the one-period interest rate prevailing in the market at time $t+j$. We work with annualized, continuously compounded rates of return, for which Eq. (1) is an exact expression. Under risk neutrality, the risk premium would be zero, although $\pi^{n, 1}$ might still represent some constant term premium. The stronger version of the EH implies that there is no premium of any kind, $\pi^{n, 1}=0$, long-term rates being just the average of current and expected future short-term rates, while the weaker version of the EH would allow for a significant constant in Eq. (1).

This expression can be generalized to consider rates of return on $n$ - and $m$-period investments, $n$ being a multiple of $m$,

$$
\begin{equation*}
r_{t}^{n}=\frac{m^{\frac{n}{m}-1}}{n} \sum_{j=0} E_{t} r_{t+j m}^{m}+\pi^{n, m} \tag{2}
\end{equation*}
$$

An interesting special case occurs when $n=2 m$, as in the comparison between returns on 3- and 6-month investments, or between returns on 6- and 12-month investments. Then,

$$
\begin{equation*}
r_{t}^{n}=\frac{1}{2}\left(r_{t}^{m}+E_{t} r_{t+m}^{m}\right)+\pi^{n, m}, \tag{3}
\end{equation*}
$$

so that in the case of a 3-month reference period, the rate of return on a 6-month investment should be equal to the average of the rate of return on a 3-month investment and the rate of return on a 3-month deposit expected to prevail 3 months hence, plus a possible term premium.

Under rational expectations we have,

$$
\begin{equation*}
r_{t+m}^{m}=E_{t} r_{t+m}^{m}+\varepsilon_{t+m}^{m}, \tag{4}
\end{equation*}
$$

where $\varepsilon_{t+m}^{m}$, the rational expectations error in forecasting $r_{t+m}^{m}$ at time $t$, has a MA ( $m-1$ ) structure. Finally, substituting Eq. (4) into Eq. (3) and subtracting $r_{t}^{m}$ from both sides, we get,

$$
\begin{equation*}
r_{t}^{n}-r_{t}^{m}=\frac{1}{2}\left(r_{t+m}^{m}-r_{t}^{m}\right)-\frac{1}{2} \varepsilon_{t+m}^{m}+\pi^{n, m}, \tag{5}
\end{equation*}
$$

so that the current spread between long- and short-term interest rates (left hand side) should be a good predictor of future changes in the short-term rate (right hand side).

So long as interest rates are $I(1)$ variables, their first order difference will be stationary. But so is the rational expectations error, which has a finite order moving average structure so, unless we believe that a risk/term premium may exist which is non-stationary, Eq. (5) shows that the spread $r_{t}^{n}-r_{t}^{m}$ will also be stationary. Hence, an implication of the rational expectations version of the EH is that long- and shortterm interest rates in any maturity comparison for a given currency should be cointegrated, with cointegrating vector $(1,-1)$. Cointegration between interest rates over the term structure of a currency is consistent with the idea that market forces continuously adjust to correct any temporary disequilibrium, so that risk adjusted rates of return on different maturities do not drift apart permanently, which would otherwise give rise to arbitrage opportunities.

The previous argument can be replicated for each pair of short- and long-term rates, so if the EH holds, there should be $k-1$ independent cointegrating vectors across the term structure of $k$ rates of return. Additionally, Engsted and Tangaard (1994) show that under the EH , the coefficients in each of the $k-1$ cointegrating relationships should add up to zero. Equivalently, by linear transformations, the $k-1$ cointegrating relationships could be written as differences between interest rates at any two maturities and, in particular, as differences between returns on successive maturities, as already shown in Eq. (5). In our sample of interest rates on 1-, 3-, 6-
and 12-month deposits, the EH would imply the existence of 3 cointegrating vectors. Along the paper, we will pay attention to two possible reasons why this and other implications of the EH could fail to hold: first, the comparison of 1- versus 3-month rates cannot be written in the form of Eq. (5) unless some assumptions are made. Secondly, the volume of deposits at 12 months is much lower than those at other maturities, which could distort somewhat the relationship between the 6- and 12month rates.

Using different specifications for short- and long-term interest rates, Engle and Granger (1987), Stock and Watson (1988), Campbell and Shiller (1987) and Bradley and Lumpkin (1992), among many others, have, in fact, found long- and short-term US interest rates to be cointegrated variables $\mathrm{CI}(1,1)$. The possible cointegration of Euro-rates has also been considered. Using daily Eurocurrency bid rates for 1-, 3-, 6 - and 12-month between 1980 and 1990 for the Canadian dollar, Japanese yen, Swiss franc, British pound, and US dollar, Mougoué (1992) found evidence of a single cointegration vector among the returns offered over the term structure of each currency. With a mixture of cointegration techniques and ARIMA specifications, Chiang and Chiang (1995) used monthly data for 1977-1992 on Euro-rates at the same mentioned maturities, finding evidence of a single cointegrating vector for interest rates on the British pound and German mark interest rates, and two cointegrating vectors for interest rates on the US and Canadian dollars, Swiss franc and yen. They found more evidence of cointegration when they tested for stationarity of the residuals of regressions of the 1 - on the 3 -month rate of return, the 3 - on the 6month, and the 6 - on the 12 -month rate, rejecting the null hypothesis of a unit root in all cases for the currencies they considered.

We hope to confirm these results and provide additional evidence on the EH in a sample enlarged over time and across currencies. To that end, we use monthly average bid rates on 1-, 3-, 6- and 12-month deposits from the London eurocurrency market for the US dollar, Japanese yen, German mark, British pound, Spanish peseta, French franc, Italian lira and Swiss franc, between January 1978 and December 1998, in their annualized, continuous equivalent form. Fig. 1 shows the time pattern of 1month Euro-rates, clearly much more volatile during the first years of the sample, due in part to some central parity changes that took place in the European Monetary System, following periods of turbulence in the financial markets. Unit root tests (not shown here) suggest that all the Euro-rates we consider are $I(1)$ variables, while there is no evidence of $I(2)$ structure in any currency or maturity. So, according to our previous discussion, $(1,-1)$ should be the cointegrating vector between any shortand long-term rate in a given term structure.

To test for cointegration over the term structure, we use the maximum likelihood procedure by Johansen $(1988,1991)$ to estimate the cointegration relationships linking a set of variables, and derive a likelihood ratio test for the null hypothesis that there is a given number of these relationships. For that, an unrestricted VAR for each currency is specified

$$
X_{t}=c+\sum_{i=1}^{k} \pi_{i} X_{t-i}+\varepsilon_{t}
$$







Fig. 1. 1-month interest rates: 1979-1998.
Fig
where $X_{t}$ is an $n \times 1$ vector of $I(1)$ variables, $\pi_{i}$ is an $n \times n$ matrix of coefficients, $c$ is an $n \times 1$ vector of constants, and $t$ is an $n \times 1$ vector white noise with a variancecovariance matrix $\Sigma$, not necessarily diagonal. In our application, $X_{t}$ is the vector of interest rates on 1-, 3-, 6- and 12-month deposits on a given currency. The model can be written in a vector error correction form:

$$
\Delta X_{t}=c+\Gamma_{1} \Delta X_{t-1}+\ldots+\Gamma_{k-1} \Delta X_{t-k+1}+\Pi X_{t-k}+\varepsilon_{t},
$$

in the first differences of the rates of return on deposits at the four maturities considered, where:

$$
\Gamma_{m}=-I+\sum_{j=1}^{m} \pi_{j}, m=1,2, \ldots, k-1, \quad \Pi=-I+\sum_{j=1}^{k} \pi_{j}
$$

In this representation, $\Gamma_{i}, i=1,2, \ldots, k-1$ contains the short-run dynamics and $I$ is an identity matrix. $\Pi$ is the matrix of long-run coefficients, whose rank $r$ determines the number of stationary linear combinations of $X_{t}$. For $r<n$, there exist $r$ cointegrating vectors, and $\Pi$ can be factorized as $\alpha \beta^{\prime}$, with $\alpha$ and $\beta$ both being $n \times r$ matrices. Each of the $n$ variables in $X_{t}$ can then be represented as a linear combination of $n-r$ common trends or factors and an $I(0)$ component.

The hypothesis of cointegration is formulated as a reduced rank of the $\Pi$-matrix:

$$
H_{1}(r): \Pi=\alpha \beta^{\prime}
$$

where $\alpha$ and $\beta$ are $n \times r$ full-rank matrices. Under $H_{1}(r)$, the process $\Delta X_{t}$ is stationary, $X_{t}$ is non-stationary, but $\beta^{\prime} X_{t}$ are stationary relations among nonstationary variables (Johansen, 1991).

The likelihood function is first concentrated with respect to $\Gamma_{1}, \ldots, \Gamma_{k-1}$ and $c$, by running regressions of $\Delta X_{t}$ and $X_{t-k}$ on $\Delta X_{t-1}, \ldots, \Delta X_{t-k+1}$ and a vector of ones. These define the residuals $R_{0 t}$ and $R_{k t}$, and the residual cross-moment matrices: $S_{i j}=$ $T^{-1} \sum_{t=1}^{T} R_{i t} R_{j t}^{\prime}, i, j=0, k$. The following eigenvalue problem is then solved:

$$
\left|\lambda S_{k k}-S_{k 0} S_{00}^{-1} S_{0 k}\right|=0,
$$

having $n$ solutions $1>\lambda_{1}>\ldots>\lambda_{n}>0$, with corresponding eigenvectors $V=\left(v_{1}, \ldots, v_{n}\right)$ normalized by: $V^{\prime} S_{k k} V=I_{n}$. For a given hypothetical rank $r, r \leq n$, the vector of cointegrating relationships $\beta$ is: $\beta=\left(v_{1}, \ldots, v_{r}\right)$, the first $r$ columns of matrix $\beta$, and the maximized value of the likelihood function is:

$$
L_{\max }^{-2 / T}=\left|S_{00}\right| \prod_{i=1}^{r}\left(1-\lambda_{i}\right) .
$$

The likelihood ratio test for the reduced rank hypothesis of at most $r$ cointegrating vectors, $0 \leq r<n$, and hence, $n-r$ unit roots, in the full VAR model, against the alternative of more than $r$ cointegrating vectors, is:

$$
L R_{\text {Trace }}=-T \sum_{i=r+1}^{n} \ln \left(1-\lambda_{i}\right)
$$

which is called the Trace statistic. An alternative statistic, the Maximum eigenvalue,

$$
L R_{\mathrm{Max}}=-T \ln \left(1-\lambda_{r+1}\right),
$$

tests the null hypothesis of $r$ cointegrating vectors, against the alternative of $r+1$ of such relationships. With some differences in the specification of the hypothesis, both tests examine whether $\lambda_{r+1}=\lambda_{r+2}=\ldots=\lambda_{n}=0$, which means that the system has $n-r$ unit roots. To determine the cointegration rank, a sequence of tests is used, starting with the hypothesis of $r=0$, i.e., $n$ unit roots $(r=0)$. If this hypothesis is rejected, it implies that $\lambda_{1}>0$, and one continues to test the hypothesis: $\lambda_{2}=\lambda_{3}=\ldots=\lambda_{n}=0$. Rejection of this hypothesis implies $\lambda_{2}>0$, and so on. When the hypothesis $r \leq r_{0}$ cannot be rejected, having rejected previously the hypothesis $r \leq r_{0}-1$, we have an estimate of $r_{0}$ cointegrating vectors, i.e., $n-r_{0}$ unit roots.

We are interested in the number of cointegrating relationships along the term structure, but also on testing for the $(1,-1)$ structure implied by the EH. Johansen (1991) have shown that the maximum likelihood estimates for $\beta$ follow asymptotically a mixed Gaussian distribution, which implies that the likelihood ratio test for a set of linear restrictions on $\beta$ is asymptotically distributed as a chi-square.

According to the Maximum eigenvalue and Trace statistics in Table 1 there seem to be in fact three cointegrating vectors among the four interest rates considered for the Spanish peseta, French franc and Italian lira, in full consistency with the EH. Three cointegrating relationships amount to a single trend common to all the returns in a given currency. That trend could be interpreted as being the rate of inflation, which would determine the general level of interest rates, the term structure then determining the relationships between interest rates at different maturities. In fact, prices seem to follow an integrated process of order two in most international empirical studies.

The statistics point out to four cointegrating relationships for the US dollar, which is not acceptable since, as we have already mentioned, interest rates are all $I(1)$ variables. The evidence of two cointegrating relationships for the yen, Deutsche mark and British pound, and of just one cointegrating relationship for the Swiss franc, goes against a strict interpretation of the EH. As pointed out above, this may be due to possible distortions produced by a low volume of transactions at the 12 -month horizon, as well as to some aspects specific of the relationship between 1- and 3month rates. Before we move to a more detailed discussion of these issues, it seems safe to conclude that Table 1 provides strong evidence against the EH just for the Swiss franc. The possibilities of either zero or just one cointegrating relationships are strongly rejected for all currencies. The number of cointegrating relationships turns out not to be very sensible to the number of lags chosen for the VAR. To obtain Table 1 we used fourth order VAR systems, which left no residual autocorrelation left in all cases. These results have also proven to be quite robust to considering the whole 1978-1998 sample. The only change concerns the US term structure, that then shows three cointegrating vectors, as implied by the EH.
Table 1
Cointegration tests among 1-, 3-, 6-12-month interest rates in each currency

| Hypothesized number of cointegrating relationships: $r$ | Maximum eigenvalue and trace statistics for existence of r-cointegrating relationships over a currency term structurea |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  | Critical | values ${ }^{\text {b }}$ |  |  |  |  |
|  | US dollar | Yen | Deutsche mark | British pound | Spanish <br> peseta | French franc | Italian lira | Swiss Franc | $\begin{aligned} & \mathrm{LR}_{\text {Max }} \\ & 90 \% \end{aligned}$ | 95\% | 99\% | 90\% | 95\% | $\begin{aligned} & \mathrm{LR}_{\text {Tace }} \\ & 99 \% \end{aligned}$ |
| None | 45.3 $3^{* * /} / 93.4 *$ | 50.2**/106.9** | 37.6**74.3********) | 38.2**m7.2*** | $31.5^{* \prime} / 70.5^{\text {** }}$ | 473.3*/108.7 ${ }^{\text {"** }}$ | 38.4**/84.2******* | 44.7**/80.8*** | 25.6 | 28.1 | 33.2 | 49.7 | 53.1 | 60.2 |
| At most 1 | 21.8*/48.1** | $42.3{ }^{* * / 56.6 * *}$ | $23.8{ }^{\text {"1/ }} 36.7{ }^{\text {" }}$ | $28.6{ }^{\prime \prime \prime} 138.9^{* *}$ | 21.5*/39.0** | 38.0**/61.3 ${ }^{\text {+**}}$ | $24.4{ }^{* *} 45.7^{\text {** }}$ | 16.1/25.9 | 19.8 | 22.0 | 26.8 | 31.9 | 34.9 | 41.1 |
| At most 2 | $16.9{ }^{* / 26.16 .4 * * * ~}$ | 12.3/14.0 | 11.0/13.0 | 8.3/10.1 | 14.2\%17.5 | 17.9 - $123.44^{* *}$ | 17.3 "/21.4* | 8.2/9.8 | 13.8 | 15.7 | 20.2 | 17.8 | 20.0 | 24.6 |
| At most 3 | 9.5.9/9.5* | 1.7/1.7 | 2.0/2.0 | 1.8/1.8 | 3.4/3.4 | 5.5/5.5 | 4.1/4.1 | 1.611 .6 | 7.5 | 9.2 | 13.0 | 7.5 | 9.2 | 13.0 |

${ }^{a}$ Maximum eigenvalue (left) and Trace statistics (right), as defined in Johansen (1988). An (two, three) asterisk denotes significance at the $90 \%$ ( $95 \%$, $99 \%$ ) confidence level. Four lags were included in the vector autoregression for each currency. A constant was always included in the cointegrating vectors, but not in the estimated VAR in first differences. No trend was included in either one.
${ }^{\mathrm{b}}$ Critical values for testing for the presence of $r$ cointegrating relationships, at the $90 \%, 95 \%$ and $99 \%$ confidence levels.

Our results are in line with Mougoué (1992) and Chiang and Chiang (1995), but differ from them in that we detect more evidence in favor of cointegration and hence, an even stronger evidence in favor of the EH. The fact that interest rates have followed stable, decreasing paths in all countries in the more recent years, which are included in our sample, may explain these differences.

Table 2 presents the three estimated cointegrating vectors for each term structure, appropriately normalized although, as it is known, we can identify just a basis of the cointegrating space. It is not hard to find a linear transformation of the cointegrating vectors in Table 2 to rewrite them for most currencies to approximately imply that the differences between the longest rate and any other rate is stationary or, equivalently, that the spread between interest rates at each two successive maturities is stationary. As shown by Engsted and Tangaard (1994), these would be the implied cointegrating relationships under the EH. The last column in Table 2 contains likelihood ratio statistics to jointly test for the set of restrictions implied by the EH, that the matrix of coefficients in the three cointegrating relationships is,

$$
\beta^{\prime}=\left(\begin{array}{rrrr}
1 & -1 & 0 & 0 \\
0 & 1 & -1 & 0 \\
0 & 0 & 1 & -1
\end{array}\right)
$$

for the 1984-98 sample. This joint hypothesis is rejected at the $95 \%$ confidence level only for the yen and the Swiss franc. To test whether a low volume of transactions on 12-month deposits may produce a behavior of its rate of return different from those on shorter maturities, we also test for the joint hypothesis that $(1,-1)$ is the cointegrating vector between 1 - and 3-month interest rates, and also between 3-and 6 -month rates. Relative to the previous test, we leave unrestricted the third cointegrating relationship. The right side of the column shows that in the case of the Swiss franc, the relationship between the 6 - and 12 -month rates seems to be the cause for rejection, since the $P$-value now increases to 0.86 . This joint hypothesis would not be rejected at the $99 \%$ confidence level for the yen either. The restrictions on the three cointegrating relationships are rejected at the $99 \%$ confidence level in half of the countries for the full 1978-1998 sample, although if we leave the 6-/12-month relationship unconstrained, the restrictions implied by the EH would be rejected just for the Italian lira. These tests provide support for the EH which is more evident in the more recent sample, although with some possible deviations at the longer maturities.

We also tested for non-stationarity of the differences between returns on successive maturities for a given currency, with results similar to those in Chiang and Chiang (1995). Table 3 presents Augmented Dickey-Fuller and Phillips-Perron statistics. Twelve lags, a constant and a trend were initially included in the models for both tests. The deterministic components and some of the lags were later excluded if they turned out not to be significant. At the $95 \%$ confidence level, we reject the null
Table 2
Normalized maximum-likelihood estimates of the cointegrating vectors within each term structure ${ }^{\mathrm{a}}$

|  | Estimated coefficients on interest rates ${ }^{\text {b }}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1-month | 3-month | 6-month | 12-month | Constant | 1979:1-1998:12 |  | 1984:1-1998:12 |  |
|  |  |  |  |  |  | $r=3$ | $r=2$ | $r=3$ | $r=2$ |
| US | 1.0 | -1.673 | 0.576 | 0.098 | -0.020 |  |  |  |  |
|  | -0.075 | 1.0 | -2.681 | 1.696 | 0.060 | 20.1 | 3.6 | 2.07 | 0.32 |
|  | -0.082 | -0.657 | 1.0 | -1.347 | 0.089 | (0.00) | (0.16) | (0.55) | (0.85) |
| Yen | 1.0 | -3.044 | 2.182 | -0.110 | -0.076 |  |  |  |  |
|  | -0.800 | 1.0 | -0.083 | -0.127 | -0.002 | 22.5 | 1.2 | 14.1 | 10.6 |
|  | 0.087 | -0.502 | 1.0 | -0.600 | 0.050 | (0.00) | (0.56) | (0.00) | (0.01) |
| DM | 1.0 | -2.046 | 1.217 | -0.160 | -0.032 |  |  |  |  |
|  | 0.345 | 1.0 | -1.826 | 0.492 | 0.037 | 5.1 | 5.0 | 7.27 | 7.22 |
|  | 0.008 | -0.506 | 1.0 | -0.516 | 0.083 | (0.17) | (0.08) | (0.06) | (0.03) |
| BP | 1.0 | -2.227 | 1.572 | -0.342 | 0.056 |  |  |  |  |
|  | 0.920 | 1.0 | -2.759 | 0.792 | 0.415 | 6.2 | 5.3 | 5.94 | 3.67 |
|  | -0.196 | -0.151 | 1.0 | -0.660 | 0.108 | (0.10) | (0.07) | (0.11) | (0.16) |
| SP | 1.0 | -1.634 | 0.800 | -0.137 | -0.179 |  |  |  |  |
|  | -0.196 | 1.0 | -0.895 | 0.062 | 0.234 | 5.4 | 5.4 | 1.93 | 0.78 |
|  | -0.103 | -0.341 | 1.0 | -0.586 | 0.330 | (0.14) | (0.07) | (0.59) | (0.68) |
| FF | 1.0 | -0.500 |  | -1.480 | -0.814 |  |  |  |  |
|  | -0.345 | 1.0 | -1.074 | 0.418 | -0.022 | 13.3 | 7.8 | 1.16 | 1.10 |
|  | -0.119 | -0.353 | 1.0 | -0.546 | 0.141 | (0.00) | (0.02) | (0.76) | (0.58) |
| LI | 0.029 | -0.069 | 1.0 | -0.411 | 0.103 |  |  |  |  |
|  | -0.636 | 1.0 | -0.203 | -0.136 | 0.040 | 25.5 | 18.6 | 7.88 | 7.86 |
|  | -0.286 | 1.0 | 1.574 | 0.843 | 0.104 | (0.00) | (0.00) | (0.05) | (0.03) |
| SF | 1.0 | 0.402 | -1.898 | 0.477 | 0.351 |  |  |  |  |
|  | -1.078 | 1.0 | 0.183 | -0.098 | -0.152 | 9.7 | 0.7 | 16.6 | 0.30 |
|  | -0.082 | 0.033 | 1.0 | -0.992 | 0.170 | (0.02) | $(0.69)$ |  | (0.86) |

a US, Yen, DM, BP, SP, FF, LI, and SF denote the US dollar, Japanese yen, Deutsche mark, British pound, Spanish peseta, French franc, Italian lira and Swiss franc, respectively.
${ }^{\text {b }}$ Each row gives the coefficients of one of the cointegrating vectors. Four lags were included in the vector autoregression for each currency, 1984:1-1998:12 ${ }^{\mathrm{c}}$ Likelihood Ratio statistic to test the restrictions that the EH imposes on the cointegrating space, with $p$-values in brackets. Left column: test for restrictions on 3 cointegrating relationships. Right column: test for restrictions on 2 cointegrating relationships.

Table 3
Augmented Dickey-Fuller/Phillips-Perron statistics for stationarity of spreads ${ }^{\text {a }}$

|  | $r_{t}^{3}-r_{t}{ }^{1}$ | $r_{t}^{6}-r_{t}^{3}$ | $r_{t}^{12}-r_{t}^{6}$ |
| :--- | :--- | :--- | :--- |
| US | $-4.86^{* *}(12) c, t /-9.00^{* * *} c, t$ | $-5.31^{* *}(9) c /-9.47^{* *} c$ | $-1.90(12) /-5.78^{* *}$ |
| Yen | $-4.59^{* * *}(12) /-6.33^{* *}$ | $-4.38^{* * *}(12) c, t /-4.45^{* *}$ | $-4.45^{* * *}(12) c, t /-3.91^{* *}$ |
| GM | $-2.65^{* *}(12) /-7.69^{* *}$ | $-2.86^{* *}(12) /-4.44^{* *}$ | $-3.79^{* *}(12) /-3.31^{* *}$ |
| BP | $-4.11^{* *}(9) /-11.37^{* *}$ | $-4.33^{* *}(2) /-3.75^{* *}$ | $-3.23^{* *}(0) /-3.23^{* *}$ |
| SP | $-6.23^{* *}(7) c, t /-8.32^{* *}$ | $-3.95^{* *}(9) /-9.33^{* *}$ | $-5.40^{* * *}(0) /-5.40^{* *}$ |
| FF | $-2.20^{*}(8) /-9.06^{* *}$ | $-6.57^{* *}(12) /-7.40^{* *}$ | $-6.32^{* *}(8) /-15.50^{* *}$ |
| LI | $-4.03^{* * *}(11) c, t /-13.51^{* *} c, t$ | $-3.28^{* *}(6) /-7.09^{* *}$ | $-2.54^{* *}(10) /-4.87^{* *}$ |
| SF | $-3.83^{* *}(9) /-6.74^{* *}$ | $-3.14^{*}(9) c, t /-1.74$ | $-2.67^{* * *}(8) /-3.18^{* *}$ |

[^1]hypothesis of non-stationarity in all cases. For the 12/6-month spread on the US dollar and the $6 / 3$-month spread on the Swiss franc the two tests provided disparate information, although the presence of a unit root would be rejected at the $90 \%$ confidence for the $12 / 6$-month US dollar spread. Imposing the $(1,-1)$ restriction on the cointegrating vector rather than estimating it, seems to lead to an increase in the power of the cointegration test, which allows for more evidence of stationarity in the spreads to show up.

Summarizing, we have found that interest rates are cointegrated over the term structure of each currency. In joint tests with the vector of returns at the four maturities considered, we have found evidence of either two or three cointegrating relationships in all currencies but the Swiss franc. However, none of the 24 spreads between interest rates on successive maturities seem to contain a unit root. A joint likelihood ratio test of the set of restrictions implied by the EH on the cointegrating relationships is rejected for just two currencies for the 1984:1-1998:12 sample, but a less restrictive set of constraints is not rejected for any currency. Overall, these results provide quite strong preliminary evidence in favor of the EH. Having shown that a close connection exists among the returns offered over the term structure in each of the eight currencies considered, we now analyze a further implication of the term structure under EH, summarized in forward rates.

## 3. Forward rates as predictors of future spot rates

With continuously compounded rates of return, implicit forward rates are defined by $(n-m) f_{t, t+m}^{n-m}=n r_{t}^{n}-m r_{t}^{m}$. Hence, with $n=2 m$, we have: $f_{t, t+m}^{m}=2 r_{t}^{2 m}-r_{t}^{m}$ so that using Eqs. (3) and (4),

$$
\begin{equation*}
r_{t+m}^{m}=f_{t, t+m}^{m}-2 \pi^{2 m, m}+\varepsilon_{t+m}^{m} \tag{6}
\end{equation*}
$$

The rational expectations version of the EH of the term structure of interest rates has often been discussed by analyzing whether its implication (Eq. (6)) holds in a particular market. To that end,

$$
\begin{equation*}
r_{t+m}^{m}=\alpha+\beta f_{t, t+m}^{m}+u_{t+m} \tag{7}
\end{equation*}
$$

is usually estimated, testing the hypothesis $H_{0}: \alpha=0, \beta=1$, which is referred to as the forward rate being an unbiased predictor of the future spot rate. Under the EH, the error term in Eq. (7) is a rational expectations error which has a MA $(m-1)$ structure as already indicated. Under the stronger version of the EH (incorporating neutrality) there is no risk or term premia, so $H_{0}$ should hold. In that case Eqs. (4) and (6) imply that forward rates, which are known at time $t$, are just expectations of future short term rates: $f_{t, t+m}^{m}=E_{t} r_{t+m}^{m}$. A weaker version of the EH allows for a constant risk/term premium and suggests testing: $H_{0}^{\prime}: \beta=1$ in Eq. (7). When significant, $\alpha$ will be a negative multiple of the possible risk/term premium $\pi^{2 m, m}$. This analysis is specially interesting in the comparisons of 3- versus 6-month rates, and 6- versus 12 -month rates, since the 3 - and 6 -month are some of the more actively traded maturities in most financial markets. The one-month interest rate is also of great interest, but it needs an assumption to relate expectations one and two periods ahead, of the form: $E_{t} r_{t+1}^{1}=E_{t} r_{t+2}^{1}$, since this comparison does not exactly fit our framework. With that, and the definition of the 2-month forward rate: $2 f_{t, t+1}^{2}=3 r_{t}^{3}-r_{t}^{1}$, a regression similar to Eq. (7) can be run to test unbiasedness of the 2-month forward rate, relative to the future one-month spot rate. In this case, the intercept will be equal to $-(3 / 2) \pi^{3,1}$.

Hence, if the EH holds true, implicit forward rates should summarize all information contained in the term structure, relevant to forecast future spot rates. In recent work, Gerlach and Smets (1997) have estimated regressions of cumulative changes in short-term rates on current spreads, finding general evidence in favor of a unit slope, in consistency with the EH, although results differ widely over countries. Those regressions include, in special cases, Eq. (7).

Fig. 2 shows three-month forward rates to be apparently nonstationary. In fact, Augmented Dickey-Fuller (ADF) and Phillips-Perron tests for the presence of a unit root in forward rates $f_{t, t+1}^{2} f_{t, t+3}^{3}$ and $f_{t, t+6}^{6}$ in the eight currencies we consider (not shown to save space) provided evidence in favor of that hypothesis, at the same time the null hypothesis of two unit roots was rejected in favor of the alternative of a single root.

Since spot and forward interest rates are $I(1)$ variables for all maturities and currencies, Eq. (7) must be interpreted as a possible cointegrating relationship between current forward and future spot rates, on which to test the restrictions implied by the EH. Estimation of a cointegrating vector between two variables can be easily done in the least-squares framework initially proposed by Engle and Granger (1987). However, the resulting $t$-ratios do not follow a standard $t$-distribution, so tests on the estimated coefficients cannot be performed easily. On the other hand, the more complex maximum-likelihood estimation framework suggested by Johansen (1988,





Fig. 2. 3-month forward rates: 1979-1988.


1991), allows for a rigorous implementation of those tests. From that point of view, the latter procedure would dominate, but not much is known about the finite sample properties of the resulting estimates and test statistics in either case. Since we want to get conclusions as sharp as possible on the validity of the EH , we use both methods to analyze the relationship between forward and future spot interest rates.

### 3.1. Least-squares estimates

We first present in Table 4 least-squares estimates of Eq. (7) for 1981:1-1998:12 together with ADF statistics to test for stationarity of the residuals. There is uniform evidence in favor of stationary residuals in all regressions except for the regression of the 6 -month rate for the US dollar, yen, British pound and the 3- and 6-month rates on the Swiss franc. This may not be independent of the weaker evidence of 3 cointegrating vectors found in Table 1 for some of these currencies. Another interpretation of the result may be the significant loss of power which may arise in unit root tests because of conditionally heteroskedastic residuals, as pointed out by Alexakis and Apergis (1996). We found evidence of $\operatorname{GARCH}(1,1)$ in the mean structures for the residuals from regressions in Table 4. When incorporated into the regression model, we found ADF statistics for the normalized residuals of the 6month rate regressions on the US dollar, yen, British pound and Swiss franc, of $-3.66(7),-3.76(6),-3.73(11)$ and $-3.59(6)$, where the number of lags is shown in brackets, and of $-2.50(3)$ for the residuals of the regression of the 3-month rate on the Swiss franc. With $95 \%$ confidence, these tests would produce evidence of nonstationarity just for the latter case.

Stationary residuals mean that if there is any risk or term premium, it is stationary, and we can think of regressions in Table 4 as cointegrating relationships between current forward and future interest rates, estimated as suggested in Engle and Granger (1987). Standard errors shown in the table have been computed to be robust to the possible presence of heteroskedasticity and autocorrelation, following Newey and West (1987), and they tend to increase with maturity. Slope estimates are always significant and quite close to one, except in the regression of the 3-month interest rate on the Swiss franc. Even though the $t$-ratios do not follow a standard $t$-distribution, they suggest that $(1,-1)$ may be the cointegrating vector between the two interest rates, in favor of the rational expectations version of the EH.

As explained above, according to the EH, the intercept in Eq. (7) will be a negative multiple of the risk/term premium, if it exists. Since spot and forward interest rates have similar sample means, the intercept tends to be positive when the estimated slope is below one, and negative when the slope is above one. But again, since the distribution of the $t$-ratios is non-standard, not much can be inferred from these leastsquares estimates, although they do not suggest much evidence of significant premia.

### 3.2. Maximum-likelihood estimates

Even though least-squares estimates have been quite favorable to the EH, they provide an informal and not well justified discussion of this hypothesis. Column 2
Table 4
Estimated regression $r_{t}^{m}=\alpha+\beta f_{t-m, t}^{m}+u_{t}, m=1,3,6$

|  | Maturity | Parameter estimates: 1981:1-1998:12 |  |  |  | Significance tests for lagged interest rates ${ }^{\text {c }}$ |  | Estimates of differencing parameter ${ }^{\text {d }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\alpha^{a}$ | $\beta^{\text {a }}$ | $R^{2}$ | $\mathrm{ADF}^{\text {b }}$ | A | B | C | D |
| US | 1 m. | 0.103 (0.220) | 0.963 (0.033) | 0.97 | -5.26 (9) | 3.0 (0.08) | 28.3 (0.00) | $-0.255^{\circ}$ | $0.295^{\circ}$ |
|  | 3 m . | 0.485 (0.466) | 0.898 (0.065) | 0.84 | -4.75 (6) | 1.2 (0.28) | 1.0 (0.36) | 0.154 | $0.323^{\circ}$ |
|  | 6 m . | 0.427 (0.542) | 0.925 (0.075) | 0.77 | -2.96 (12) | 7.4 (0.00) | 25.0 (0.00) | $0.459^{\text {o,a }}$ | $0.476^{\text {oa }}$ |
| Yen | 1 m . | -0.035 (0.026) | 0.991 (0.007) | 0.98 | -13.05 (0) | 0.0 (0.86) | 9.3 (0.00) | 0.017 | $0.398^{\circ}$ |
|  | 3 m . | -0.131 (0.071) | 1.013 (0.017) | 0.95 | -7.78 (1) | 1.3 (0.26) | 8.5 (0.00) | $0.574^{\text {o,a }}$ | $0.369^{\circ}$ |
|  | 6 m . | -0.214 (0.169) | 1.010 (0.038) | 0.90 | -2.85 (7) | 1.9 (0.17) | 9.2 (0.00) | $0.548^{\text {o,a }}$ | $0.819^{\text {o,a }}$ |
| DM | 1 m . | -0.010 (0.071) | 0.981 (0.013) | 0.98 | -11.35 (0) | 0.3 (0.59) | 14.9 (0.00) | 0.094 | 0.206 |
|  | 3 m . | 0.134 (0.211) | 0.960 (0.038) | 0.92 | -4.53 (6) | 6.4 (0.01) | 9.1 (0.00) | 0.055 | 0.125 |
|  | 6 m . | 0.613 (0.349) | 0.845 (0.061) | 0.75 | -5.20 (4) | 7.9 (0.01) | 7.0 (0.00) | -0.222 | $0.193^{\circ}$ |
| BP | 1 m . | -0.120 (0.098) | 1.003 (0.011) | 0.98 | -11.89 (0) | 3.3 (0.07) | 8.4 (0.00) | 0.076 | $0.577^{\text {o,a }}$ |
|  | 3 m . | 0.253 (0.273) | 0.965 (0.032) | 0.87 | -3.89 (6) | 1.0 (0.31) | 5.9 (0.00) | $0.894^{\text {o,a }}$ | $0.805^{\text {oad }}$ |
|  | 6 m . | 0.877 (0.435) | 0.881 (0.039) | 0.72 | -2.73 (12) | 0.1 (0.79) | 0.8 (0.45) | $0.443^{\text {a }}$ | $0.891^{\text {o,a }}$ |
| SP | 1 m . | 0.519 (0.508) | 0.940 (0.049) | 0.87 | -4.34 (9) | 0.7 (0.39) | 6.7 (0.00) | 0.055 | 0.059 |
|  | 3 m . | -0.142 (0.698) | 1.015 (0.071) | 0.76 | -4.12 (9) | 0.2 (0.66) | 2.8 (0.06) | 0.235 | 0.233 |
|  | 6 m . | 0.248 (0.889) | 0.970 (0.086) | 0.71 | -5.26 (10) | 0.0 (0.88) | 19.6 (0.00) | 0.280 | 0.272 |
| FF | 1 m . | 0.674 (0.464) | 0.881 (0.060) | 0.85 | -4.10 (12) | 3.8 (0.05) | 6.3 (0.00) | $0.428^{\text {o,a }}$ | $0.389^{\circ}$ |
|  | 3 m . | 0.584 (0.402) | 0.899 (0.057) | 0.87 | -3.73 (9) | 3.0 (0.09) | 0.4 (0.64) | $0.577^{\text {o,a }}$ | $0.744^{\text {o,a }}$ |
|  | 6 m . | 0.154 (0.449) | 0.931 (0.056) | 0.90 | -4.17 (9) | 0.6 (0.43) | 0.6 (0.57) | 0.276 | $0.432^{\text {o,a }}$ |
| LI | 1 m . | 0.552 (0.315) | 0.940 (0.022) | 0.90 | -11.67 (0) | 31.5 (0.00) | 16.9 (0.00) | $-0.498^{\text {o.a }}$ | -0.244 |
|  | 3 m . | 0.852 (0.472) | 0.891 (0.035) | 0.91 | -4.72 (9) | 3.6 (0.06) | 3.3 (0.03) | $0.324^{\circ}$ | $0.761^{\text {o,a }}$ |
|  | 6 m . | 0.914 (0.397) | 0.865 (0.035) | 0.78 | -4.27 (7) | 1.3 (0.25) | 2.1 (0.12) | $0.491^{\text {a }}$ | $0.731^{\text {o,a }}$ |
| SF | 1 m . | 0.036 (0.228) | 0.937 (0.050) | 0.91 | -4.04 (12) | 0.1 (0.70) | 163.7 (0.00) | $0.310^{\circ}$ | $0.321^{\circ}$ |
|  | 3 m . | 3.411 (0.205) | 0.316 (0.034) | 0.26 | -1.73 (9) | 1.2 (0.69) | 68.1 (0.00) | $0.842^{\text {o,a }}$ | $0.930^{\text {o,a }}$ |
|  | 6 m . | 0.730 (0.179) | 0.867 (0.078) | 0.76 | -2.04 (12) | 5.8 (0.02) | 11.2 (0.00) | 0.205 | 0.167 |

[^2]in Table 5 contains the Maximum eigenvalue and Trace statistics to test for the number of cointegrating relationships between spot and lagged forward rates, which seems to be one in all cases, except at the 3-month horizon for the US dollar, Deutsche mark, peseta and Swiss franc, and the 6-month returns on deposits in Deutsche marks. There is some ambiguity for the 1-month interest rate for the peseta and the 6 -month rates on the Swiss franc. There cannot be two cointegrating vectors, since both variables are $I(1)$. These statistics can again be biased due to conditionally heteroskedastic residuals, but incorporating such structures in Johansen-type analysis is not as simple as we shown in the Engle-Granger estimates. Maximum likelihood estimates of the single cointegrating vector are shown in the middle panel, even in the cases where the test failed, together with the number of lags used in the VAR specification. Slope estimates are again very close to one, being above that level in about half of the cases.

Looking at the estimated maximum-likelihood standard deviations, we would reject the null hypothesis that the slope is equal to one for the 3- and 6-month interest rate models for the yen, British pound and Italian lira, the 1 -month interest rate on the Deutsche mark and the 6 -month rate on the Swiss franc. A more formal, likelihood ratio test of the unit slope hypothesis (column 4 in Table 5) leads to rejection again for the 3- and 6-month interest rate models for the yen, the 3-month rate on the British pound, and the 6 -month rate on the Swiss franc, 4 of the 24 cases at the $1 \%$ significance level and in 6 cases at the $5 \%$ level, since we have to add the 6month rate regression for the Deutsche mark and British pound. Four of the six rejections of this implication of the EH arise in the 6-month horizon, again suggesting that the lower liquidity at the 12-month maturity may explain most of the deviations from the EH.

Rejection of the unit slope hypothesis at the 5\% significance level comes together with a significant negative constant in most cases, since it always arises for slope estimates above one. Even when the hypothesis is not rejected, negative estimates for the intercept are obtained in all but five cases, which would be consistent with the existence of term/risk premia. Besides, the supposed premia seem to increase with maturity in most countries, as it should be expected, however, our intercept estimates are not significant in most cases, although especially for the peseta and French franc, lack of significance arises from estimates not being very precise. By and large, we cannot claim to have found consistent evidence of constant risk premia.

If we impose the restrictions of the EH in the form of a unit slope on forward rates and test for stationarity of the differences $r_{t}^{m}-f_{t-m, t}^{m}, m=1,3,6$ (last column in Table 5), we reject the unit root hypothesis at the $95 \%$ confidence level for all currencies and maturities, although the evidence on the 6-month Swiss franc rate is not totally clear. With this qualification, these tests suggest that $(1,-1)$ may be considered to be the approximate cointegrating vector between each of the 1-, 3- and 6-month returns and the corresponding forward rate, appropriately lagged, in support of the EH. As in the case of interest rates in Section 2, it could be that imposing the restrictions on the cointegrating vector increases the power of the test, allowing for more evidence of cointegration to emerge. However, preference for the likeli-
Table 5
Estimated cointegrating relationship: $r_{t}^{m}=\alpha+\beta f_{t-m, t}^{{ }^{m}}+u_{t}, m=1,3,6$

| Maturity |  | $\mathrm{LR}_{\text {Max }} / \mathrm{LR}_{\text {Trace }}{ }^{\text {a }}$ | Parameter estimates: 1984-1998 |  |  |  | ADF and PhillipsPerron statistics ${ }^{\mathrm{e}}$ :$r_{t}^{m}-f_{t-m, t}^{n}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\alpha^{\text {b }}$ | $\beta^{\text {b }}$ | $n^{\text {c }}$ | $H_{0}: \beta=1^{\text {d }}$ |  |
| US | 1 m . | $15.0^{*} / 22.1^{* *}$ | -0.024 (0.251) | 0.979 (0.010) | 4 | 1.28 (0.26) | $-5.6(2) /-10.0$ |
|  | 3 m . | 10.9/14.1 | -0.138 (0.183) | 0.983 (0.019) | 12 | 0.04 (0.84) | -4.8(9)/-5.7 |
|  | 6 m . | 14.8*/19.0* | -0.415 (0.296) | 0.986 (0.037) | 12 | 0.36 (0.55) | -3.7(12)/-4.3 |
| Yen | 1 m . | 29.6 ***/31.8*** | -0.067 (0.039) | 0.998 (0.008) | 9 | 0.29 (0.59) | -7.2(3)/-13.0 |
|  | 3 m . | $23.8^{* * *} / 26.5^{* * *}$ | -0.137 (0.042) | 1.029 (0.008) | 6 | 13.56 (0.00) | $-5.9(3) /-6.9$ |
|  | 6 m . | $17.5^{* * *} / 19.4^{*}$ | -0.278 (0.135) | 1.081 (0.027) | 12 | 12.58 (0.00) | -4.8(2)/-4.1 |
| DM | 1 m . | $121.9^{* * *} / 25.0^{* *}$ | 0.324 (0.182) | 0.918 (0.030) | 12 | 0.02 (0.90) | $-3.8(12) /-9.9$ |
|  | 3 m . | 12.8/16.4 | -0.151 (0.182) | 0.996 (0.028) | 4 | 2.20 (0.14) | $-3.8(6) /-5.6$ |
|  | 6 m . | 11.0/13.8 | -0.568 (0.286) | 1.071 (0.044) | 12 | 4.46 (0.03) | -4.2(12)/-4.0 |
| BP | 1 m . | $18.7{ }^{* *} / 20.8^{* *}$ | -0.335 (0.176) | 1.025 (0.017) | 4 | 4.38 (0.04) | -8.8(1)/-9.4 |
|  | $3 \mathrm{~m}$ | $22.1^{* * *} / 25.6^{* * *}$ | -0.606 (0.293) | 1.064 (0.029) | 6 | $8.45 \text { (0.00) }$ | $-4.2(6) /-5.5$ |
|  | 6 m . | $16.2^{* *} / 18.6^{*}$ | $-1.500(0.434)$ | 1.138 (0.043) | 7 | 7.55 (0.01) | -5.0(1)/-4.1 |
| SP | 1 m . | $14.1 * / 17.7$ | 0.375 (0.454) | 0.954 (0.038) | 6 | 0.13 (0.72) | -4.8(4)/-10.2 |
|  | 3 m . | 11.5/13.9 | 0.381 (0.505) | 0.973 (0.042) | 6 | 0.48 (0.49) | $-4.4(4) /-6.1$ |
|  | 6 m . | $17.9^{* *} / 21.1^{* *}$ | -0.702 (0.476) | 1.043 (0.039) | 10 | 1.29 (0.26) | -5.9(4)/-4.0 |
| FF | 1 m . | $19.4 * / 23.1^{* *}$ | -0.259 (0.404) | 1.011 (0.049) | 9 | 0.10 (0.75) | -3.7(6)/-12.1 |
|  | 3 m . | $16.2^{* *} / 19.6^{*}$ | -1.103 (1.018) | 1.146 (0.133) | 12 | 1.20 (0.27) | $-9.4(9) /-5.3$ |
|  | 6 m . | 22.1***/26.2*** | -0.750 (1.145) | 1.081 (0.141) | 12 | 1.44 (0.23) | $-4.2(7) /-5.7$ |
| LI | 1 m . | $17.0^{* *} / 22.0^{*}$ | -0.417 (0.348) | 0.951 (0.030) | 12 | 2.02 (0.16) | -4.3(10,c)/12.0 |
|  | 3 m . | $17.2^{* *} / 21.0^{*}$ | 0.938 (0.440) | 0.882 (0.037) | 12 | 1.22 (0.27) | -3.1(12)/-5.8 |
|  | 6 m . | $25.3^{* * *} / 29.9^{* * *}$ | 0.876 (0.314) | 0.882 (0.025) |  | 2.84 (0.09) | -4.2(7,c)/-4.0 |
| SF | 1 m . | 19.1**/20.5** | -0.230 (0.137) | 1.013 (0.026) | 12 | 1.41 (0.24) | $-6.3(3, \mathrm{c}) /-8.8$ |
|  | 3 m . | 13.2/15.3 | -0.235 (0.121) | 1.019 (0.022) | 6 | 1.06 (0.30) | -4.7(7,c)/-4.2 |
|  | 6 m . | 14.0* / 15.6 | -0.455 (0.189) | 1.105 (0.037) | 7 | 8.29 (0.00) | $-2.4(12) /-3.2$ |

[^3]hood-ratio test or the $\mathrm{ADF} / \mathrm{PP}$ tests should rely on their finite sample properties, for which not much is known.

To summarize the results in this section: a) there is overwhelming evidence in favor of forward rates having explanatory power for future short term spot rates, b) unbiasedness of the forward rate is an acceptable hypothesis, having found just some ambiguous evidence for some of the 6- versus 12 -month comparisons, and c) we have not found consistent evidence of constant risk premia.

## 4. The expectations hypothesis and market efficiency

An implication of the rational expectations version of the Expectations Hypothesis $(\mathrm{EH})$ is that forward rates are unbiased predictors of future short-term rates. Forward rates are then expectations of future interest rates conditional on current information, so that there should not be any information in current or past interest rates which could be used to forecast future short-term rates, once forward rates are already being used. This implies, in particular, that markets are efficient, since current prices capture all the available information which is relevant to forecast future prices, and forward rates become a simple way to summarize that information.

We explore in this section two directions for possible deviations from efficiency. First, we examine again the regressions of short-term rates on lagged forward rates in Section 3, and test for whether additional lags of either the forward rate or the interest rate have any explanatory power, additional to the single lag of the forward rate which should be included as the only regressor, according to the EH. Secondly, we examine for possible evidence of fractional integration of the residuals of regression (Eq. (7)). From Table 4, we already know that those residuals do not contain a unit root except in the 6-month case in some currencies, but they could still be long-memory processes, reflecting the fact that there is information in the very far past that could be useful to predict future short-term rates. A fractional differenced white noise is a process $y_{t}$ such that $(1-L)^{d}\left(y_{t}-\mu\right)=u_{t}$, with $d$ nonintegral, $\mu$ a constant, and $u_{t}$ a white noise. The fractional difference operator is:

$$
(1-L)^{d}=\sum_{k=0}^{\infty} \frac{\Gamma(k-d) L^{k}}{\Gamma(-d) \Gamma(k+1)},
$$

where $\Gamma$ (.) denotes the generalized factorial, gamma function. More generally, a process $y_{t}$ is said to be fractionally integrated when $(1-L)^{d} y_{t}$ is stationary, with $d$ being between 0 and 1. An ARFIMA process would admit autoregressive and moving average components after having been applied the fractional difference $(1-L)^{d}$, with a white noise innovation (see Hosking, 1981; Granger and Joyeux, 1980; Mills, 1990; Geweke and Porter-Hudak, 1983). The process is stationary and ergodic for $-0.5<d<0.5$, with a bounded and positively valued spectrum at all frequencies. When $0<d<0.5$, the sum of the absolute values of the autocorrelation coefficients goes to infinity and the $\operatorname{ARIMA}(0, d, 0)$ process is said to have a long memory. When $-0.5<d<0$, all auto and partial correlations are negative and the absolute values of
the autocorrelations have a finite sum, so that the process does not have long-term persistence. All of its autocorrelations, excluding lag zero, are then negative and decay hyperbolically to zero. The process is said to be antipersistent or to have intermediate memory. When $|d| \geq 0.5$, the variance of $y_{t}$ is infinite, and hence the process is nonstationary. When $d=0.5, y_{t}$ is invertible but not stationary, while when $d=-0.5, y_{t}$ is stationary, but it is not invertible.

Although they eventually die away, shocks to long-memory processes persist for a long time, which may allow for forecasting improvement at very far time horizons, as it has been shown in Barkoulas and Baum (1997) precisely for the case of Euromarket interest rates. That would again be a violation of market efficiency. A survey discussion of alternative estimators of the difference parameter and their properties can be found in Baillie (1996).

Our two tests are complementary: with the first one we search for short-time dependence, while with the second we consider the possibility of very long-run dependence patterns, both against market efficiency.

Columns 5 and 6 in Table 4 contain statistics to test for the existence of information in past term structures, additional to that contained in lagged forward rates, useful to explain future interest rates. Column 5 presents the $t$-test for an additional lag of the forward rate. For this test, the 1-month equation explains $r_{t}^{1}$ using $f_{t-1, t}^{2}$ and $f_{t-2, t-1}^{2}$ as regressors. The second equation explains $r_{t}^{3}$ using $f_{t-3, t}^{3}$ and $f_{t-4, t-1}^{3}$ as regressors, while the third equation explains $r_{t}^{6}$ using $f_{t-6, t}^{6}$ and $f_{t-7, t-1}^{6}$. At the $99 \%$ confidence level, the additional lag of the forward rate is only significant for the 1 month rate on the lira, and the 6 -month rates on the US dollar and the Swiss franc. Column 6 adds two lags of the interest rate being explained. The 1-month equation explains uses $f_{t-1, t}^{2}$ and $r_{t-1}^{1}, r_{t-2}^{1}$ as regressors, the second equation uses $f_{t-3, t}^{3}$ and $r_{t-3}^{3}, r_{t-4}^{3}$ as regressors, and the third equation uses $f_{t-6, t}^{6}$ and $r_{t-6}^{6}, r_{t-7}^{6}$ as explanatory variables. Here, the evidence is overwhelming in favor of explanatory power in past interest rates, against the Expectations Hypothesis and violating market efficiency. In particular, there seems to exist information content in past interest rates to predict 1 -month rates in all currencies. Besides, each lagged interest rate is often individually significant, suggesting that the dynamics of the relationship between forward rates and future short-term rates is much richer than it is captured by Eq. (7). In particular, efficiency fails due to too much short-term persistence at the three maturities for the yen, Deutsche mark and Swiss franc. This evidence should, however, be qualified by the fact that the $t$-ratios of these least-squares regressions do not have a standard $t$-distribution, but attempts to estimate by maximum-likelihood a multivariate system for the interest rates involved failed, due to high multicollinearity.

Regarding the second test, the last panel in Table 4 shows our estimates of the $d$ parameter using the Geweke and Porter-Hudak (1983) method (GPH), for the residual of the regression (left column), as well as for the differenced residual (right column). A process $y_{t}$ with the representation $(1-L)^{d} y_{t}=u_{t}$, where $u_{t} \sim I(0)$, can be written as,

$$
\begin{equation*}
f(\omega)_{y}=\left|1-e^{-i \omega \mid}\right|^{-2 d} f(\omega)_{u}, \tag{8}
\end{equation*}
$$

where $f(\omega)_{y}$ and $f(\omega)_{u}$ are the spectral densities of $y_{t}$ and $u_{t}$, respectively. But, since Eq. (8) can also be written as,

$$
\log \left[f_{y}\left(\omega_{j}\right)\right]=\log \left[f_{u}(0)\right]-d \log \left[4 \sin ^{2}\left(\omega_{j} / 2\right)\right]+\log \left[f_{u}\left(\omega_{j}\right) / f_{u}(0)\right] .
$$

GPH suggest estimating $d$ from an OLS regression using spectral ordinates $\omega_{1}, \omega_{2}, \ldots, \omega_{m}$, from the periodogram of $y_{t}, I_{y}\left(\omega_{j}\right)$ :

$$
\log \left[I_{y}\left(\omega_{j}\right)\right]=a+b \log \left[4 \sin ^{2}\left(\omega_{j} / 2\right)\right]+v_{j} j=1,2, \ldots, m
$$

where: $v_{j}=\log \left[f_{u}\left(\omega_{j}\right) / f_{u}(0)\right]$, which is assumed to be $i$., i.d., and has zero mean and variance $\pi^{2} / 6$. If the number of ordinates $m$ is chosen such that: $m=g(T)$, where $T$ is sample size, and

$$
\lim _{T \rightarrow \infty} g(T)=\infty, \lim _{T \rightarrow \infty}[g(T) / T]=0, \lim _{T \rightarrow \infty}\left[(\log T)^{2} / g(T)\right]=0,
$$

then $\left(\hat{d}_{G P H}-d\right) / \sqrt{ } \operatorname{var}\left(\hat{d}_{\mathrm{GPH}}\right) \rightarrow N(0,1)$, where $\operatorname{var}\left(\hat{d}_{\mathrm{GPH}}\right)$ is obtained from the usual OLS regression formula, either using the residual variance, or alternatively, setting it as $\pi^{2} / 6$.

There is evidence of long-memory structures in the residuals of the regressions for the 6-month rate on the US dollar, and the 1-month rates on the French franc and the Swiss franc. There is mixed evidence, of being either long-memory or nonstationary processes, for the residuals of the regressions of the 3-month rate on the yen, the 6 -month rate on the British pound, and the 3 - and 6 -month rates on the Italian lira. The residuals of the regressions for the 6-month rate on the yen and the 3-month rates on the British pound, French franc and Swiss franc seem to be nonstationary, while those from the regressions for the 1- and 3-month rates on the Deutsche mark, the three interest rates on the peseta and the 6 -month rate on the Swiss franc are stationary. In none of the four cases in which the $d$-estimate suggests lack of stationarity, the ADF test would have rejected the null hypothesis of a unit root at the $99 \%$ confidence level. However, there is no need to search for a close correspondence between both tests, since the ADF considers just the integer values $d=1$ versus $d=0$, while we have considered all $d \geq-0.50$ values when computing GPH-estimates.

In the remaining cases, evidence is inconclusive and, given the overwhelming evidence in favor of stationary residuals emerging from the ADF tests, we maintain the stationarity hypothesis in all these cases. In 11 of the 24 regressions, residuals are either long-memory processes or non-stationary. This is contrary to the EH and market efficiency, since it suggests that the component of future interest rates which cannot be explained from current forward rates, could be explained from interest rate observations very distant in the past.

Summarizing, we have found strong evidence against efficiency emerging from two tests: on the one hand, lagged interest rates tend to be significant in projections of future short-term rates on current information that already includes the appropriately lagged forward rate as a regressor. This shows that there is more short-term persistence in interest rates than it is consistent with the rational expectations version
of the EH. When searching for long-memory processes in the residuals of these regressions, we have found conclusive evidence on the residuals showing persistent temporal dependence even between very distant observations for almost half of the interest rate comparisons. Both of these results are contrary to market efficiency.

## 5. Conclusions

Analyzing the long-term relationships among interest rates at different maturities we have found strong evidence in favor of the Expectations Hypothesis (EH) as an adequate representation of the term structure in the market for Eurodeposits. First, working with monthly data on 1-, 3-, 6- and 12-month interest rates on deposits denominated in US dollar, Japanese yen, German mark, French and Swiss francs, British pound, Italian lira and Spanish peseta over the 1978-1998 period, we have found interest rates offered on a currency at a given time to be cointegrated over the term structure. Considering the four rates of return at the different horizons, we have provided evidence in favor of either two or three cointegrating vectors in all currencies but the Swiss franc, in consistency with the EH. This is stronger evidence than was found in previous papers using similar methods and data sets, although with an older sample.

Besides, a joint test of the full set of restrictions implied by the EH on the cointegrating vectors gets support in 6 of the 8 currencies considered, or in all of them if we exclude the restrictions affecting the return on 12-month deposits. The lower volume of transactions at this maturity may explain this difference. In addition, all of the 24 spreads between interest rates on successive maturities seem to be stationary, as the EH would imply.

As a third test of the EH, we have also shown that implicit forward rates contain explanatory power for future short term interest rates in all currencies and maturities. We have found evidence of cointegration between both rates and we have tested for a unit slope, with general support for that hypothesis. More specifically, we have found forward rates to be unbiased predictors of future rates for most currencies, with some exceptions arising mainly at the 6 - versus 12 -month interest rate comparison. We have not detected any evidence suggesting constant risk/term premia.

A fourth test has produced more damaging results for the EH, since we have found current and past interest rates to have explanatory power, additional to that in forward rates, in projections of future short-term rates on current information. This contradicts the rational expectations version of the EH, as well as market efficiency. According to the latter, current interest rates should contain all information regarding future interest rates, while the EH implies that forward rates summarize all relevant information contained in the current term structure. Finally, our estimates of the difference parameter for the residuals in the mentioned projections has produced evidence of either long-memory or non-stationary processes for almost half of the interest rates. That, again, goes against market efficiency, since observations very distant in the past would contain information on future interest rates, additionally to that in current forward rates.

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[^1]:    ${ }^{\text {a }}$ Augmented Dickey-Fuller (left) and Phillips-Perron (right) statistics for testing the null hypothesis of a unit root in the spreads. Sample: 1979:1-1998:12. The number of lags used in the model in first differences of the spread is shown in brackets. $c$ and $t$ denote whether a constant or trend were included. Critical values when no constant or trend are included are $-1.62,-1.94,-2.57$ at the $90 \%, 95 \%$ and $99 \%$ confidence levels, respectively, for both tests. If a constant is included, critical values are -2.57 , -2.87 and -3.46 , for both tests. If a constant and a trend are included, critical values are $-3.14,-3.43$ and -4.00 , for both tests. An (two) asterisk denotes rejection of non-stationarity at the $95 \%$ ( $99 \%$ ) confidence level.

[^2]:    ${ }^{\text {a }}$ Standard errors in brackets are robust to heteroskedasticity and autocorrelation, as in Newey and West (1987)
    ${ }^{\text {b }}$ Augmented Dickey-Fuller tests for stationarity of the residuals. No constant or trend were included in the model in first differences of the residuals. Critical values at the
    $1 \%, 5 \%$ and $10 \%$ significance level are $-3.90,-3.34$ and -3.04 , respectively, as shown in MacKinnon (1991). The number of lags used is shown in brackets.
    c The left column (A) contains the $t$-test for an additional lag of the forward rate included as a regressor. The right column (B) contains the $F$-test for joint significance of two lags of the interest rate, included as additional regressors. two lags of the interest rate, included as additional regressors.
    ${ }^{\text {d }}$ GPH estimates (Geweke and Porter-Hudak, 1983) of the difference parameter for the residuals. Left column $(C)$ contains estimates of the differencing parameter for the
    original series. Figures in the right column $(D)$ are equal to one plus the estimate of the differencing parameter for the differenced series. Superindices $o$, $a$ denote significance,
    relative to the OLS or the asymptotic variance, respectively. relative to the OLS or the asymptotic variance, respectively.

[^3]:    ${ }^{\text {a }}$ Maximum eigenvalue and Trace statistics. Their critical values when testing the existence of zero cointegrating relationship, at the $10 \%$, $5 \%$ and $1 \%$ significance levels, are 13.8, 15.7 and 20.2 for the Maximum eigenvalue, and 17.8, 20.0 and 24.7, for the Trace statistic (Osterwald-Lenum, 1992)
    c Number of lags used in the VAR in first differences.
    ${ }^{d}$ Likelihood ratio statistic to test the null hypothesis that the cointegrating vector is $(1,-1)$.
    in brackets. Critical values for both tests at the $10 \%, 5 \%$ and $1 \%$ significance levels are $-1.62,-1.94$ and -2.57 when no constant is included in the vector autoregression, and $-2.57,-2.87$ and -3.46 when a constant is included.

