

## Ejercicios propuestos C. Alexander IV.2 – Parametric VaR

1. Suppose that a portfolio's daily log returns are normally distributed with a standard deviation of 1% and a mean of 0.01% above the discount rate. Calculate a) the portfolio volatility and b) the 1% 10-day Normal linear VaR of the portfolio under the assumption of i.i.d. daily log returns and under the assumption that daily log returns are autocorrelated with first order autocorrelation  $\rho=0.2$ . R: a) 15.81%, b) 7.26%, 8.72%.
2. [EIV.2.2] (Sin Excel) Considere dos tipos de interés con una correlación de 0.9. Uno de ellos está en el 10% con volatilidad de 30%, y el otro está en 8%, con volatilidad de 25%. ¿Cuál es la matriz de varianzas y covarianzas diaria en términos de puntos básicos? R: 360; 216; 160.
3. [EIV.2.3] Consider a cash flow of \$1 million in 1 year and of \$1.5 million in 2 year's time. Calculate the volatility of the discounted P&L of the cash flows, given that the 1-year interest rate is 4% and the 2-year rate is 5%, the volatility of the 1-year rate is 100 basis points, and the volatility of the 2-year rate is 75 basis points, and their correlation is 0.9. Calcular también el VaR Normal lineal al 5% a 1 día y al 1% a 10 días. R: \$28.052; \$2.918; \$13.052. *Sugerencia: Utilizar los PV01*
4. [EIV.2.4] Consider a cash flow with sensitivities: PV01(1-year)=\$1000, PV01(2-year)=\$1500, PV01(3-year)=\$2000. Suppose the interest rates at maturities 1, 2 and 3 years have daily volatilities of 75, 60 and 50 basis points, and correlations of 0.95 (1yr,2yr), 0.9 (1yr,3yr) and 0.975 (2yr,3yr). Find the 1% 10-day normal linear VaR. Now assume that interest rates are 4.0%, 4.5% and 5.0% at the 1-year, 2-year and 3-year vertices and suppose that a trader considers entering into a swap with the following cash flow: (\$3million at 1-year, -\$3million at 2-year, -\$0.25 at 3-year). What is the incremental VaR of the trade? R: \$120.970; -\$6.693. *Sugerencia: Utilizar los PV01*
5. [EIV.2.5] The first table shows the PV01 vectors, both in US dollars, for a portfolio with exposures to the UK and US government yield curves. For simplicity, we assume the portfolio has been mapped to only the 1-year, 2-year and 3-year interest rates in each country, and the basis point volatilities for these interest rates are given below each PV01. The correlation matrix of daily interest rates is given in Table 2. Calculate the 1% 10-day normal linear interest rate VaR, the stand-alone VaR due to the US and UK yield curve risk factors, and the marginal VaRs of these risk factors.  
R:\$87.668; \$54.673 (US), \$40.931 (UK); \$51.335 (US marginal), \$36.353 (UK marginal).  
*Sugerencia: Utilizar los PV01*

Interest rates		US			UK	
Maturity (years)	1	2	3	1	2	3
PV01(\$)	1000	-1500	2000	800	900	-750
Volatility (bps.)	100	80	70	85	75	65

*Correlation coefficients*

			US			UK	
		1	2	3	1	2	3
	1	1	.95	.90	.70	.67	.62
US	2	.95	1	.97	.65	.75	.75
	3	.90	.97	1	.60	.79	.80
	1	.70	.65	.60	1	.98	.95
UK	2	.67	.75	.79	.98	1	.99
	3	.62	.75	.80	.95	.99	1

6. [EIV.2.6] A portfolio of A-rated corporate bonds and swaps has its cash flows mapped to vertices at 1-year, 2-year, 3-year, 4-year and 5-years. The volatilities of the LIBOR rates (in basis points per annum) and PV01 vector of the portfolio are shown in Table 1. The correlations are given in Table 2. The 1-year and 5-year A-rated credit spreads are, like the LIBOR parameters, assumed to have been estimated from a historical sample. Suppose the 1-year spread has volatility 80 basis points per annum and the 5-year spread has volatility 70 basis points per annum, and their correlation is 0.9. Suppose the cross-correlations

Maturity (years)	1	2	3	4	5
PV01(\$)	750	1000	500	250	500
Volatility (bps.)	100	95	85	75	60

Correlations		1	2	3	4	5
	1	1	.95	.92	.90	.88
	2	.95	1	.97	.65	.75
	3	.92	.97	1	.60	.79
	4	.90	.65	.60	1	.98
	5	.88	.75	.79	.98	1

between these credit spreads and the LIBOR rates of different maturities are as shown in Table 3. Estimate the 1% 10-day total interest rate VaR and decompose the total VaR into the VaR due to LIBOR rate uncertainty and the VaR due to credit spread uncertainty. Then estimate the marginal VaR of the LIBRO and credit spread components.

Correlations		LIBOR				
		1yr	2yr	3yr	4yr	5yr
Credit spread	1yr	-0.25	-0.020	-0.18	-0.15	-0.10
	5yr	-0.20	-0.21	-0.23	-0.24	-0.25

R: \$137.889; \$115.943 (LIBOR), \$104.301 (credit); \$78.242 (LIBOR), \$59.647 (credit).

*Sugerencia: La cartera consta de bonos corporativos. No tenemos esa curva cupón cero, sino la curva LIBOR y los credit spread por separado. Esto equivale a tener posiciones de igual cuantía en ambos activos (tipos LIBOR y credit spread). Además, solo disponemos de volatilidades para credit spreads a dos vencimientos. Para obtener las volatilidades a los demás vencimientos (2, 3 y 4 años), hay que interpolar entre las volatilidades a 1 y 5 años. Lo mismo sucede con el cálculo de correlaciones entre credit spread y tipos LIBOR.*

7. [EIV.2.7] (Case Study PC VaR) Consider a portfolio of UK bonds and swaps with PV01 vector  $\theta$  given in the Spreadsheet. Find an approximation to the change in the portfolio's value given that UK interest rates change as follows:

- the UK yield curve moves upward with a parallel shift of 10 basis points at all maturities
- there is a tilt in the UK yield curve where the 1-month rate increases by 35 basis points, the 2-month rate by 34 basis points, the 3-month rate by 33 basis points and so on up to the 59-month rate decreasing by 23 basis points and the 60-month rate decreasing by 24 basis points.

R:  $\Delta PV = \text{£}9.518$ ;  $\Delta PV = \text{£}396.478$ ; *Sugerencia: nótese que los PV01 están en  $10^3 \text{£}$ .*

8. [EIV.2.8] Use the 1-day covariance matrix given in the spreadsheet to find the volatility of the discounted P&L of the portfolio with PV01 vector shown in the **Figure**. Assuming that each interest rate change is i. i. d. normally distributed, calculate the 1% 10-day VaR on 31 December 2007. R: P&L volatility:  $\text{£}379.455$ ; VaR  $\text{£}176.549$ . *Sugerencia: La hoja de cálculo contiene un gráfico de la estructura temporal de volatilidades, que no es precisa para el ejercicio. Resolver el ejercicio utilizando los PV01.*

9. [EIV.2.9] Suppose that the cash-flow representation of the bond portfolio whose PV01 vector is shown in Figure IV.2.2 was taken on 31 December 2007. Also, suppose that we base our daily interest rate covariance matrix  $\Omega$  on daily changes in the UK spot curve for maturities measured at monthly intervals up to 5 years, using the data period from 2 January to 31 December 2007. Find a principal component representation based on  $\Omega$  with three principal components, and specify the diagonal matrix D that has their standard deviation along its diagonal. Then use this principal component representation to calculate the UK bond portfolio's sensitivities to the three principal component risk factors.

R: A PC analysis on the 60x60 covariance matrix is given in the Excel workbook for this case study. The sum of all the eigenvalues is 917.27, and the first three eigenvalues are 856.82, 45.30 and 9.15 so, they explain over 99% of the total variation. Sensitivities: ( $\text{£}428$ ; - $\text{£}2.975$ ;  $\text{£}1.041$ )

10. [EIV.2.10] Estimate the VaR of the portfolio based on the mapping to the first three principal components of the previous exercise, and compare this with the full evaluation interest rate VaR from [EIV.2.8]. R:  $\text{£}175.457$ . *Sugerencia: En este ejercicio, la volatilidad de la cartera se estima utilizando la matriz W de componentes principales (autovectores de la*

matriz de covarianzas de los tipos de interés) y los PV01. El producto nos da las Sensibilidades de la cartera respecto de los CPs. La varianza de la cartera se obtiene mediante el producto escalar del vector de Sensibilidades al cuadrado, por el vector de las varianzas de los CPs (los autovalores que los definen).

11. [EIV.2.11] Calculate the 1% 10-day parametric linear VaR for a portfolio with the characteristics in the Table, discounting using a risk-free rate of 5%. How much is the VaR reduced by the mean adjustment? Repeat your calculations for a risk horizon of 1 year.

	Position	Volatility	Expected return	Correlation	
Stock 1	€4m	20%	10%	Stock1– Stock2	0.8
Stock 2	-€5m	10%	2%	Stock1– Stock3	0.5
Stock 3	€1m	15%	5%	Stock2 - Stock3	0.3

*Sugerencia: Tiene sentido descontar porque se trata del VaR nominal, no en términos de rentabilidades.*

12. [IV.2.12] A linear model with two risk factors indicates that a stock portfolio has net betas of 0.8 and 1.2 with respect to these factors. Their volatility is 15% and 20% respectively, and they have a correlation of -0.5. If the portfolio is expected to return the risk free rate over the next month, calculate the 5% 1-month systematic VaR on an investment of \$20 million on the portfolio. R: \$1.973.824. *Sugerencia: Nótese que el exceso de rentabilidad es igual a cero.*
13. [IV.2.13] Suppose that the volatility of the portfolio returns in [EIV.2.12] is 25%. Find the 1% 1-month total VaR and the 1% 1-month specific VaR using the normal linear model, based on method 2 described in the classnotes. R: \$2.374.142 y \$1.319.305.
14. [EIV.2.14] A US investor buys \$2 million of shares in a portfolio of UK (FTSE100) stocks and the portfolio beta is 1.5. Suppose the FTSE and \$/£ volatilities are 15% and 20%, respectively, and their correlation is 0.3. What is the 1% 10-day systematic VaR in US dollars? Decompose the systematic VaR into a) stand-alone and b) marginal equity and forex components. R: VaR = \$319.142; (\$209.371 (E), \$186.108 (F); \$173.985 (E), \$145.157 (F).
15. [EIV.2.15] Consider a US dollar investment in a large international stock portfolio with the characteristics in the table. Suppose that the correlation between all equity risk factors is 0.75, the correlation between the two forex risk factors is 0.5, and the quanto correlations are each 0.2. Find the 1% 10-day systematic VaR of the portfolio and decompose it into a) stand-alone components and b) marginal equity and forex components. R:\$1.490.889; \$1.333.847(E), \$413.541(F); \$1.284.765(E), \$206.125(F).

Index	Local equity					Forex		
	Nominal	Beta	Return	Volatility	Net dollar $\beta$	Return	Volatility	Nominal
S&P500	\$2m	0.9	X1	20%	\$1.8m			
FTSE100	\$2m	1.1	X2	22%	\$2.2m	X5(\$/£)	15%	\$2m
CAC40	\$3m	1.2	X3	25%	\$3.6m	X6(\$/€)	10%	\$7m
DAX30	\$4m	1.3	X4	27%	\$5.2m			

16. [EIV.2.16] A US investor buys \$2 million of sterling 10 days forward, when the 10-day Treasury bill rate is 5% and the 10-day spot rate is 4.5% in the UK. If the interest rates have volatilities of 100 basis points for the Treasury bill and 80 basis points for the UK rate, and a correlation of 0.9, calculate the 1% 10-day interest rate VaR. R: \$114. *Sugerencia: utilizar 365 días al capitalizar tipos de interés. Utilizar PV01 para estimar varianzas.*
17. [EIV.2.17] A European investor has \$5 million invested in a portfolio of volatile S&P500 stocks, with an S&P500 market beta of 1.5. The volatilities of the S&P500 and €/ \$ rate are 20% and 15% respectively, and their correlation is -0.5
- Find the 1% 1-day total systematic VaR and the VaR due to each risk factor
  - He now hedges the portfolio's equity exposure by selling a 3-month future on the S&P500 index and further hedges the currency exposure with a short position on US dollars, 3 months forward. The 3-month US dollar and euro interest rates are 4% and 3.5% respectively, and the dividend yield on the S&P500 is 3%. The volatilities are: 80 bps for the US 3-month rate, 100 bps for the Euro 3-month rate, 20 bps for the S&P500 dividend yield. The correlations are: 0.5 (US rate-euro rate), 0.3 (US rate-dividend yield), and .0 (euro rate-dividend yield. Find the 1% 1-day VaR due to each of these risk factors.
  - If the portfolio volatility is 35%, calculate the hedged portfolio's 1% 1-day specific VaR.
- R: a) \$191.129, \$220.697(E), \$110.348 (F); b) \$2.978, \$3.606 (US), \$1.807 (Euro-rate), \$544 (US dividend yield); c) \$132.622. *Sugerencia: Recuerde que la varianza específica es igual a la varianza total menos la varianza sistemática.*

*Sugerencia: El inversor ha vendido euros por un importe de US\$ 5 millones. Está comprado en dividend yield por 7,5 millones de US\$ (nominal \*beta), y en US\$ 5 millones de US\$-rates a 3-meses. Para cobertura debe tomar posiciones contrarias, por lo que se colocará largo por US\$ 5 millones en eurorates a 3 meses, corto en US-rates a 3-meses por US\$ 5 millones y corto en US\$ 7,5 millones de dólares, que es la cantidad que debe vender del futuro S&P500.*

18. [EIV.2.18] Using the daily FTSE 100 data from 4 January 2005 to 7 April 2008, estimate the degrees of freedom parameter for a generalized Student t distribution representation of the daily returns, using a) the method of moments and b) MLE. Then compute the 1% 1-day Student-t VaR, as a percentage of portfolio value, using both estimates for the degrees of freedom parameter. R: 2.81%, 2.94%.
19. [EIV.2.19] For the same data on FTSE 100, compare the Student-t linear VaR with the Normal linear VaR over a 1-day horizon, at the 0.1%, 1% and 10% significance levels. Express your results as a percentage of the portfolio value.
20. [EIV.2.20] Use the parameters estimated for a Normal mixture of two distributions for FTSE 100, S&P 500 and FX Rate [see Table] to estimate the 100 $\alpha$ % 10-day Normal mixture VaR for a US investment in the FTSE 100 and S&P 500 indices. Report your results as a percentage of the local currency exposure to each risk factor and compare them with the normal estimate of VaR. Use significance levels of  $\alpha=10\%$ , 5%, 1%, and 0.1%.
21. [EIV.2.21] Using the daily FTSE 100 data from 4 January 2005 to 7 April 2008, apply the method of moments to estimate the parameters for a mixture of two normal distributions representations of the daily returns. Then, using both the maximum likelihood and the method of moments estimate of the degrees of freedom for the Student-t density

representation of the FTSE 100 index returns from [EIV.2.18], compare the Student-t linear VaR with the Normal mixture linear VaR over a 10-day horizon, at the 0.1%, 1% and 5% significance levels. Express your results as a percentage of the portfolio value.

22. [EIV.2.22] For  $\alpha=0.1\%$ , 1%, 5% and 10%, compute the  $100\alpha\%$  10-day VaR of mixture of two distributions, the first with mean 0 and volatility 20% and the second with (annualized) mean -10% and volatility 40%. The probability weight associated with the first distribution is 75% and the daily returns are assumed to be i., i.d.. Compare two cases: in the first case the two component distributions are assumed to be Normal, and in the second case the first component distribution is a Student t with 10 degrees of freedom and the second is a Student-t with 5 degrees of freedom.
23. [EIV.2.23] Recalculate the Normal mixture and Student-t mixture 1% 10-day VaR estimates from the previous example when daily returns are assumed to have autocorrelation +0.25, and when they are assumed to have autocorrelation -0.25. R: 3.97%, 2.51%.
24. [EIV.2.24] A portfolio has two risk factors with percentage sensitivities to these risk factors of 0.8 and 1, respectively, The risk factors have a bivariate Normal mixture distribution  

$$\pi_1 = 0.02, \mu_{11} = -300\%, \mu_{12} = 2\%, \sigma_{11}^2 = 75\%, \sigma_{12}^2 = 15\%,$$
with  $\pi_1 = 0.03, \mu_{21} = -200\%, \mu_{22} = 2.5\%, \sigma_{11}^2 = 65\%, \sigma_{12}^2 = 18\%$   

$$\rho = (\rho_1, \rho_2, \rho_3, \rho_4) = (0.3, 0, 0, 0.8)$$
Calculate the 1% 10-day VaR of the portfolio.
25. [EIV.2.25] Utilice una serie temporal de volatilidades EWMA para estimar el VaR Normal, lineal, 100p% a 10 días sobre una posición en FTSE100 a 18 de abril de 2008 ¿Cómo afecta la elección de constante de suavizado al resultado?
26. [EIV.2.26] Consider a large portfolio of US stocks having a percentage beta with respect to the S&P 500 index of 1.1 and a percentage beta with respect to the NASDAQ 100 index of 0.85. Assume that \$3 million is invested in the S&P 500 stocks and \$1 million is invested in the NASDAQ 100 stocks. Compare the 1% 10-day Normal VaR of this portfolio on 18 April 2008 based on the RiskMetrics regulatory matrix and based on the daily matrix, and in each case disaggregate the VaR into S&P 500 and NASDAQ 100 stand-alone VaR.
27. [EIV.2.27] Suppose a portfolio is expected to return the risk free rate with a volatility of 30%. Assuming the returns are i., i.d., find the 1% 10-day parametric linear VaR and ETL as a percentage of the portfolio's value. R:13.96%; 16.0%.
28. [EIV.2.28] Suppose a portfolio is expected to return the risk free rate with a volatility of 30%. Assuming the returns are i., i.d., with a Student t distribution with  $v$  degrees of freedom. Find the 1% 10-day Student t VaR and ETL, as a percentage of the portfolio's value, for  $v=5, 10, 15, 20$  and 25. R: VaR=(15.64%, 14.83%, 14.54%, 14.39% 14.30% 13.96%); ETL=(30.26%, 20.00%, 18.12%, 17.40%, 17.03%, 15.99%).
29. [EIV.2.29] Suppose a portfolio is expected to return the risk free rate with a volatility of 30%. Assuming the returns are i., i.d., with a Normal mixture distribution with discounted mean returns of zero, but with the two Normal components having different volatilities: with probability 0.2 the volatility is 60% and with probability 0.8, the volatility is 15%. Find the 1% 10-day Normal mixture VaR and ETL as a percentage of the portfolio's value. R: 19.74%; 25%.
30. [EIV.2.30] Suppose a portfolio is expected to return the risk free rate with a volatility of 30%. Assuming the returns are i., i.d., with a Student t mixture distribution. Both Student t distributed components have a discounted mean returns of zero, but different volatilities

and degrees of freedom: with probability 0.2, the distribution has 5 degree of freedom and a volatility of 45%, and with probability 0.8, the distribution has 10 degrees of freedom and volatility of 25%. Find the  $100p\%$   $h$ -day VaR and ETL as a percentage of the portfolio's value for  $p=0.1\%$  and  $1\%$  and  $h=1$  and  $10$ . Compare your results with those obtained in the previous exercises, using a Normal, Normal mixture and individual Student t distributions.