Ejercicios propuestos C. Alexander II.3 – Volatility

1. Suponga que las rentabilidades de un activo se distribuyen i., i.d.. La varianza muestral de las rentabilidades diarias es 0,001. Considerando 250 dias de mercado en el año, ¿Cuál es su volatilidad? Si la volatilidad fuese 36 ¿cuánto sería la desviación típica de las rentabilidades semanales? R: 50%; 0,05.

2. Monthly returns of a hedge fund over the last three years have a standard deviation of 5%. Assume the returns are i., i.d.. What is your volatility estimate? Now suppose you discover that the returns have been smoothed before reporting them to the investors. In fact, the returns are autocorrelated with autocorrelation 0,25. What is your volatility estimate now? R: 17.32%; 21.86%.

3. A portfolio has $1 million invested in asset 1, $2 million invested in asset 2, and $3 million invested in asset 3. The volatilities of the asset returns, in annualized percentages, are 20%, 10% and 5%, respectively and the correlations are: 0.8 (assets 1 and 2), 0.5 (assets 1 and 3), and 0.3 (assets 2 and 3). Find the portfolio volatility.

4. With the volatilities and correlations of the previous exercise, calculate the annual covariance matrix. Then, assuming the returns are multivariate normal i., i.d., and assuming 250 trading days per year, derive from this the 10-day covariance matrix, i.e., the matrix of variances and covariances of 10-day returns.

5. Daily closing values on the FTSE 100 index between Friday 10 August and Friday 24 august 2007 were: 6038.3, 6219.0, 6143.5, 6109.3, 5858.9, 6064.2, 6078.7, 6086.1, 6196.0, 6196.9, 6220.1. Use this data to estimate the volatility of FTSE 100 index at the close of the market on 24 August 2007. R: 32.9%.

6. Re-estimate the FTSE100 volatility without assuming this time the expected return is zero (or assuming the mean return is zero, if you used a non-zero mean return in the previous exercise). R: 34.32%.

7. Over the same period as in the previous exercises, the S&P500 values were: 1453.64, 1452.92, 1426.54, 1406.70, 1411.27, 1445.94, 1445.55, 1447.12, 1464.07, 1462.50, 1479.37. Estimate the correlation between FTSE100 and S&P 500. R: 0.5032.

8. Assuming the daily log returns on the FTSE 100 are normally distributed, use the sample given in exercise 5 to construct a 95% confidence interval for the variance of the returns. R: [0.000211; 0.001333].

9. An equally weighted volatility estimate based on 30 observations is 20%. Find a two-sided confidence interval for this estimate. R: [16.0%; 26.7%].

10. An equally weighted volatility estimate is 20%, based on a sample of 100 observations. Estimate the standard error of the estimator and find an interval for the estimate based on one-standard error bounds. R: [18.59%; 21.41%]

11. A historical correlation of 0.2 is obtained using 36 observations. Is this significantly greater than zero? R: No.

12. The spreadsheet provides daily closing prices for the Italian MIB 30 stock index between the beginning of January 2000 and the end of December 2007, and compares these with the S&P 500 index prices over the same period. Calculate the 30-day, 60-day and 90-day historical volatilities of these two stock indices and compare them graphically. Then, compute the EWMA volatilities with $\lambda=0.95$.