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# An error correction factor model of term structure slopes in international swap markets

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**Abstract**

The first two principal components in a vector of term structure slopes from IRS markets in eight major currencies explain above 90% of the fluctuations in the vector of slopes, and each of the eight slopes considered is cointegrated with these two factors. The implied error correction models are shown to be accurate for short-0 and medium-term slope forecasting for the eight currencies, as compared to univariate models, which allows for a drastic reduction of dimensionality, since we just need to use univariate forecasts for the two factors. Adding more factors to the model does not lead to a significant improvement in forecasting performance, while forecasts obtained using just one factor are not as good as those from two-factor error correction models.


*JEL classification:* E37; E43

*Keywords:* Factor models; Term structure of interest rates; Principal components; Swap markets; IRS

## 1. Introduction

Characterizing the main properties of term structure slopes has quickly become a major focal point in the analysis of fixed income markets. The main reason for this increased attention is that a long line of research has accumulated robust evidence on the fact that

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changes in term structure slope can anticipate turning points in the business cycle. Estrella and Hardouvelis (1991) were pioneers in showing that for the US the slope of the term structure is positively related to future economic activity, as well as with a wide variety of leading indicators. These results have been, by and large, confirmed for different countries. Using GNP data, Harvey (1991) showed a similar result for Germany, Davis and Henry (1994) for the UK, Clinton (1994) for Canada, Davis and Fagan (1994) for EU countries, Estrella and Mishkin (1996) for the US, and Hu (1993) for the G-7 countries.

It is important to bear in mind that in most of this work, term structure slopes, used by them, have been shown to anticipate future changes in economic activity. In further research [Hardouvelis (1988), Moersch (1996) and Sauer and Scheide (1995), among others], short-term interest rates and money supply growth have sometimes been included to show that the information content in term structure slopes is not just due to monetary policy interventions. Similar lines of research have paid attention to the role of the term structure slope as a leading indicator of future inflation and future stock market activity. From this research, emerges the idea that fluctuations in term structure slopes contain information relative to future economic activity and future inflation that is different from that contained in more standard indicators of production activity or demand. As a consequence, the slope of the term structure is currently considered as a potentially very important indicator to anticipate future real developments in actual economies.

We do not enter in this paper on that discussion but rather, we take the relevance of term structure slopes in different countries as a starting point, to analyze the extent to which changes in term structure slopes are related across countries. Together with their potential role, anticipating turning points, common fluctuations in slopes would suggest the possibility of using changes in the term structure in one or two economic areas to predict business cycle fluctuations in a large number of economies. There is also an evidence that the relationship between the slope of the term structure and economic activity has drastically changed after the 1980s in most countries. Fig. 1 suggests that the positive correlation in the high inflation 1970–1985 period seems to have changed sign afterwards. Whether the slope is still able to anticipate future economic activity or, on the contrary, that ability is currently non-existent should be a matter of further research. However, the presumption that the relationship might exist for high inflation periods, say, easily justifies analyzing the extent to which the information incorporated by term structure slopes in different currencies is common.

Interest rates should indeed be expected to be correlated across currencies, since it is widely believed that monetary policy interventions in different countries are not independent from each other. Furthermore, increased monetary policy coordination, as it was the case in Europe prior to the constitution of the Euro area, led to common interest rate fluctuations among countries in the European Union. However, since monetary authorities determine very short-term interest rates, we should expect to see high correlations in the shorter end of the term structure, but not necessarily among longer-term rates. The Expectations Hypothesis of interest rates suggests that interest rates at longer maturities are the average of current and expected short-maturity rates, so that fluctuations in short-term rates translate less than perfectly into fluctuations in interest rates at longer-maturities.[1] Hence, even if

[1] An exception is the case when the change in short-term rates is perceived as being permanent, and no further changes are expected over the period covered by the longer maturity.

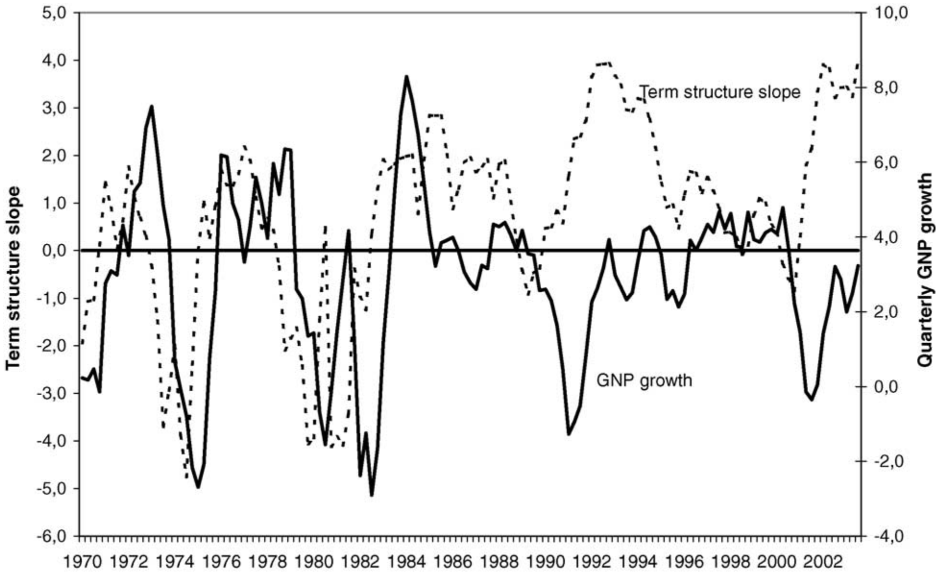


Fig. 1. US growth and term structure slope: 1970–2003.

short-term rates display high correlations that might not be the case for the longer maturities. On the other hand, while there is consistent evidence in favor of the Expectations Hypothesis at maturities up to 1 year, the evidence is much less clear for the longer maturities.[2]

To characterize co-movements among slopes in different countries, two different lines have been followed. Working with monthly interest rate data on Eurodeposits on the British pound, French franc, Deutsche mark, Swiss franc, Japanese yen and US dollar over 1979–1998, Domínguez and Novales (2000b) use linear regression models to estimate causal relationships among slopes from markets on Eurodeposits for a variety of currencies, showing that the US and Deutsche mark slopes help predict future slope fluctuations in other countries.

A second approach has used factor analysis, generally in the form of principal components, to summarize such co-movements.[3] As an example, Domínguez and Novales (2002) used data from markets in Eurodeposits and a factor model approach to show that one or two factors are enough to produce forecasts for term structure slopes, which are at least as good as those obtained from univariate models. It is quite striking in that result that a regression projection on just the first factor can compete with dynamic, univariate models in terms of

[2] The literature on tests of the Expectations Hypothesis of the term structure is too long to be surveyed here. For tests of the hypothesis at different maturities, see Bekaert et al. (1997), Bekaert and Hodrick (2001), Domínguez and Novales (2000a), Engsted and Nyholm (2000), Engsted and Tanggaard (1994), Longstaff (2000), Sutton (2000), among many others.

[3] Using principal components to summarize the evidence provided in a vector of interest rates is a long tradition in the study of fixed income markets [see Litterman and Scheinkman (1991), Steeley (1990) and Knez et al. (1994) among others].

the forecasting performance of term structure slopes, and that adding more factors does not generally lead to a significant gain in forecasting performance.

In this paper, we use the principal components technique to search for factors among a wide set of international term structure slopes from Interest Rate Swap (IRS) markets. At a difference of markets in Eurodeposits, used in previous research, which include maturities up to 1-year, the IRS term structure goes up to 10-year maturities, allowing for the possibility of a richer variety of changes in interest rates. After showing that fluctuations in the vector of slopes in eight different currencies can be summarized by changes in a few factors, we test for the quality of slope forecasts obtained from factor models, as compared with those obtained from univariate slope models. Using the former strategy would greatly simplify the problem of producing slope forecasts when searching for changes in economic activity across countries, since it would only be necessary to forecast a small number of factors.

Slopes are computed as the difference between 10- and 2-year rates in interest rate swap (IRS) markets in each currency. Once the most relevant factors have been characterized, we estimate projections of each slope on the common factors. Since IRS slopes turn out to be I(1) variables, these projections take the form of error correction models (ECM). Forecasts from these ECM models are then compared to slope forecasts from univariate models.

Section 2 describes the data and the methodology used to characterize the common factors among the set of international term structure slopes. In Section 3, we report estimates for univariate as well as for error correction models for each slope. In Section 4, we describe the forecasting exercise, and present the obtained results. The paper closes with some conclusions.

## 2. Factor analysis

### 2.1. *The data*

Data for interest rate swap (IRS) rates at 2, 3, 4, 5, 7 and 10 year maturities for the Deutsche mark, US dollar, Japanese yen, British pound, Italian lira, Swiss franc, French franc and Spanish peseta were obtained from *DataStream*™, between June 26, 1991 and December 12, 1998. In this database, IRS daily data are collected at 18:00 h GTM. They are the average of bid and ask quotes, as provided by Dark limited, from Intercapital Brokers Limited. We use data from each Wednesday as weekly data. When a Wednesday fell on a holiday, we take data from the previous market day. A term structure of zero coupon rates were then derived from swap rates by the *bootstrapping* method (described in Appendix A). We used the spread between 10- and 2-year zero coupon rates as term structure slope.

### 2.2. *First results*

Fig. 2a shows the time behavior of term structure slopes in IRS markets for the eight currencies considered. Table 1 (panel a) presents Augmented Dickey–Fuller (ADF) and Phillips–Perron (PP) unit root tests, suggesting that slopes follow I(1) processes except for the Japanese yen, for which the evidence is inconclusive, the ADF and PP statistics falling between the critical values corresponding to 1 and 5% significance levels. However, the slope from Japan does not cross its mean value over the sample period, and its autocorrelation function dies away very slowly, suggesting non-stationarity. Non-stationarity could give rise to spurious correlations among slopes, seriously biasing the results of our proposed analysis, so that it needs to be explored in detail. Since Fig. 2a shows evidence of a possible break in 1992 in European countries, as well as a break around 1994 in the US slope, we performed an intervention analysis [see Box and Tiao (1975)] for each time series of slopes except for Japan, where evidence of such a break does not exist. Fig. 2b shows the time evolution of slopes after correcting for the break. Panel b in Table 1 shows unit root statistics for the corrected series, again showing conclusive evidence of non-stationarity, so the evidence in panel a of the table was not just a spurious implication of possible breaks in the series. Hence, we will use in what follows appropriate econometric methods to deal with non-stationary slopes for all countries. For consistency, we solve the ambiguity in the case of Japan by treating its slope as an I(1) process as well.

Figures above the main diagonal in Table 2 are sample correlation coefficients among slopes. Slopes in European currencies display high correlations, being significant smaller for the British pound. Correlations between European slopes and that of the Japanese yen are also large and positive. Correlations between European slopes and the US slope are

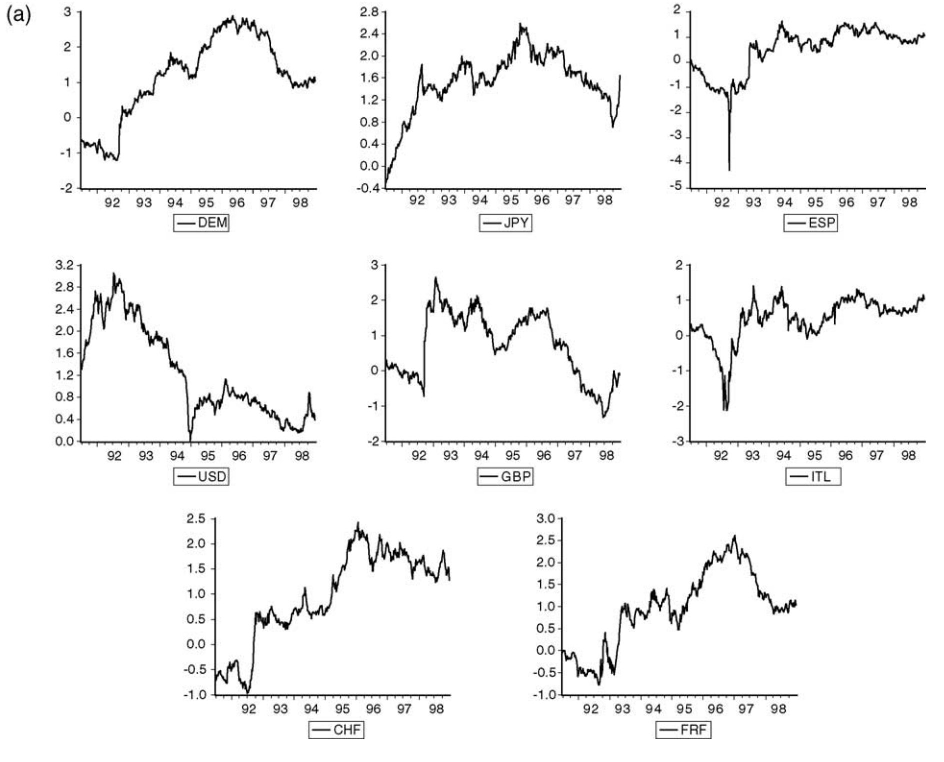


Fig. 2. Time behavior of term structure slopes in IRS markets: (a) before intervention analysis; (b) after intervention analysis.

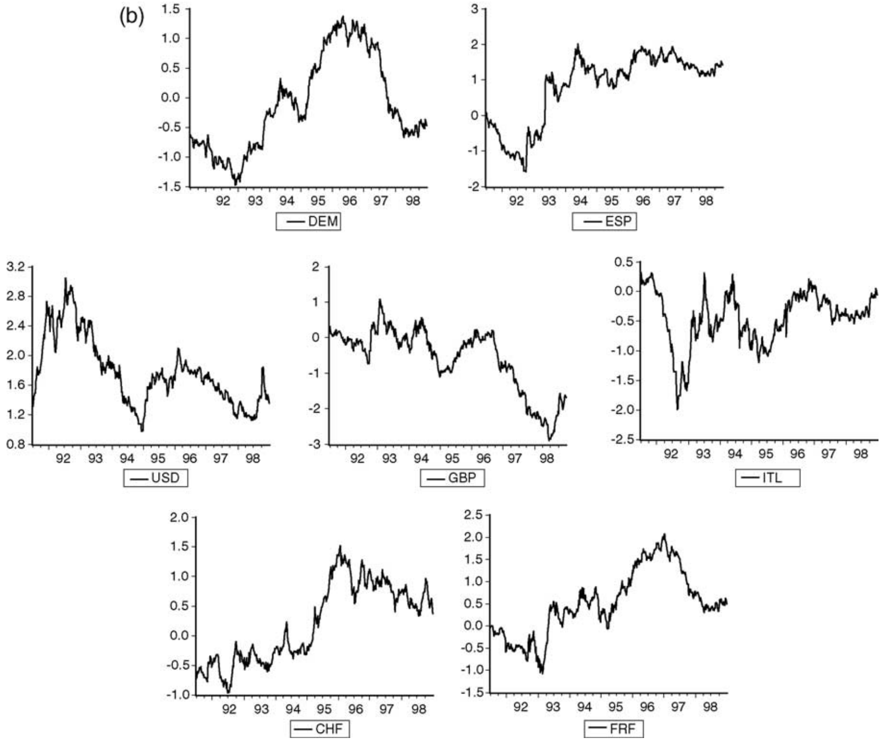


Fig. 2. (*Continued*).

large, but negative. Whether the high correlations are purely spurious, due to the presence of unit roots, will be discussed later. As a first check, we present below the diagonal in Table 2 correlations between differenced slopes. Slopes in the group of European countries show again significant correlations among themselves except for Spain, but there are no other noticeable correlations. The correlations across continents that were present in level slopes all but disappear in first differences, suggesting that they were spuriously produced by the presence of trend with either the same or opposite sign in both slopes.

### *2.3. Principal components among international slopes*

Principal components is a particular form of factor analysis,[4] aimed at producing a few linear combinations of a set of variables explaining as much as possible of the fluctuations in the whole vector of variables. The principal components analysis starts from the variance-covariance matrix of the standardized variables. This is a semi-positive definite matrix, with non-negative eigenvalues. The eigenvector associated to the largest eigenvalue defines the linear combination of variables that explains the largest percentage of the variance in the

[4] Some analytical details of the method are described in Appendix B.

Table 1
Unit root tests

| | Slopes | | | | | | | | Principal components | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DEM | JPY | ESP | USD | GBP | ITL | CHF | FRF | First | Second | Third | Fourth |
| Panel a: before intervention analysis | | | | | | | | | | | | |
| Augmented Dickey–Fuller | | | | | | | | | | | | |
| Level | −1.596 | −3.026 | −1.431 | −0.846 | −1.798 | −1.778 | −1.876 | −1.456 | −1.847 | −0.571 | −2.167 | −2.656 |
| Differences | −7.734 | −8.128 | −10.695 | −8.399 | −8.137 | −8.431 | −7.678 | −7.798 | −8.191 | −7.989 | −7.699 | −8.772 |
| Phillips–Perron statistic | | | | | | | | | | | | |
| Level | −1.544 | −3.399 | −1.853 | −0.806 | −1.484 | −2.012 | −1.870 | −1.439 | −1.843 | −0.540 | −1.803 | −2.851 |
| Differences | −20.246 | −17.846 | −28.492 | −18.195 | −19.620 | −20.719 | −19.327 | −20.216 | −18.168 | −23.056 | −19.243 | −22.056 |
| Panel b: after intervention analysis | | | | | | | | | | | | |
| Augmented Dickey–Fuller | | | | | | | | | | | | |
| Level | −1.038 | – | −1.352 | −1.600 | −1.276 | −2.526 | −1.707 | −1.450 | | | | |
| Differences | −8.820 | – | −9.131 | −8.792 | −8.927 | −8.711 | −8.488 | −7.953 | | | | |
| Phillips–Perron statistic | | | | | | | | | | | | |
| Level | −1.020 | – | −1.304 | −1.715 | −1.245 | −2.669 | −1.651 | −1.400 | | | | |
| Differences | −19.855 | – | −18.733 | −19.049 | −18.925 | −20.762 | −19.470 | −20.633 | | | | |

*Note:* Augmented Dickey–Fuller and Phillips–Perron unit root statistics. Critical values for both statistics: −3.45 (1%), −2.87 (5%).

Table 2
Contemporaneous correlation coefficients between slopes

| | DEM | JPY | ESP | USD | GBP | ITL | CHF | FRF |
|---|---|---|---|---|---|---|---|---|
| DEM | 1.000 | 0.799 | 0.833 | −0.703 | 0.344 | 0.687 | 0.914 | 0.939 |
| JPY | 0.134 | 1.000 | 0.495 | −0.393 | 0.422 | 0.279 | 0.701 | 0.664 |
| ESP | 0.117 | 0.071 | 1.000 | −0.813 | 0.059 | 0.814 | 0.769 | 0.887 |
| USD | 0.169 | 0.092 | 0.063 | 1.000 | 0.304 | −0.601 | −0.780 | −0.728 |
| GBP | 0.286 | 0.003 | −0.250 | 0.127 | 1.000 | 0.210 | 0.099 | 0.180 |
| ITL | 0.201 | −0.038 | −0.098 | 0.051 | 0.345 | 1.000 | 0.651 | 0.755 |
| CHF | 0.423 | −0.043 | −0.004 | 0.133 | 0.140 | 0.040 | 1.000 | 0.877 |
| FRF | 0.637 | −0.010 | 0.041 | 0.178 | 0.272 | 0.317 | 0.231 | 1.000 |

*Note:* The upper triangular matrix contains correlation coefficients between level slopes. The lower triangular matrix contains correlation coefficients between first differenced slopes.

vector of original variables. The ratio between the largest eigenvalue and the sum of all of them is the percentage of variance being explained by the first principal component. Since eigenvectors corresponding to different eigenvalues are orthogonal to each other, the correlation between linear combinations defined by eigenvectors associated to successive (if different) eigenvalues display zero correlation.

Table 3 contains the eigenvalues of the covariance matrix of slopes, as well as the percentage of the variance in the vector of slopes that is explained by each principal component. The first principal component is defined by the linear combination of slopes that uses as coefficients the components of the eigenvector associated to the largest eigenvalue, and it explains 73.5% of the fluctuation in the vector of eight term structure slopes. The second principal component is defined through the eigenvector associated to the second largest eigenvalue of the variance-covariance matrix. It adds information that is all new, not overlapping with that contained in the fluctuations of the first principal component. The two together explain 92.0% of the fluctuation in the original variables. Adding a third component raises the percentage of explained variance to 96.4%, while a fourth one would take us to 98.4%.

The entries of a given eigenvector are not readily interpretable as the relative importance of the different slopes in each principal component, since slope levels in different currencies may be quite different from each other. In spite of that, panel a in Table 4 shows that the larger coefficients in the first eigenvector correspond to three European countries: Germany, France and Switzerland, as well as to Japan, being negative in al cases. The largest coefficient in the second eigenvector, which is positive, is the one associated to the US, while the largest coefficient in the third eigenvalue is the one associated to the UK, being negative. Principal components are defined up to a scale factor, so that the same linear combinations, multiplied by a given real number, or changed in sign, would contain the same information. However,

Table 3
Principal components in IRS slopes

| | First | Second | Third | Fourth | Fifth | Sixth | Seventh | Eighth |
|---|---|---|---|---|---|---|---|---|
| Eigenvalues | 4026.4 | 1012.5 | 238.7 | 110.3 | 44.5 | 25.2 | 13.0 | 5.8 |
| Variance explained (%) | 73.5 | 18.5 | 4.4 | 2.0 | 0.8 | 0.5 | 0.2 | 0.1S |

Table 4
Principal components in IRS slopes

| | DEM | JPY | ESP | USD | GBP | ITL | CHF | FRF |
|---|---|---|---|---|---|---|---|---|
| Panel a: coefficients defining the principal components | | | | | | | | |
| First eigenvector | −0.4922 | −0.4811 | −0.2456 | −0.2577 | −0.2632 | −0.1936 | −0.3819 | −0.3835 |
| Second eigenvector | −0.2510 | 0.2312 | −0.3445 | 0.7604 | 0.2992 | −0.1390 | −0.2032 | −0.191 |
| Third eigenvector | −0.1378 | 0.312 | −0.0322 | 0.2451 | −0.8708 | −0.1113 | 0.2113 | 0.0848 |
| Fourth eigenvector | 0.2668 | 0.2721 | −0.4833 | −0.3403 | 0.1062 | −0.6208 | 0.2762 | −0.1802 |
| Panel b: *R*-squared coefficients from regressions of slopes on individual principal components | | | | | | | | |
| On first component | 0.9644 | 0.674 | 0.6526 | 0.3554 | 0.2113 | 0.5032 | 0.7796 | 0.8604 |
| On second component | 0.6257 | 0.1911 | 0.8163 | 0.9349 | 0.0530 | 0.5141 | 0.7223 | 0.7216 |
| On third component | 0.1370 | 0.0977 | 0.0328 | 0.0329 | 0.9295 | 0.1287 | 0.0113 | 0.0515 |
| On fourth component | 0.0530 | 0.3246 | 0.0525 | 2.87E-03 | 0.0209 | 0.2105 | 0.0801 | 9.31E-05 |
| Panel c: *R*-squared coefficients from regressions of slopes on subsets of principal components | | | | | | | | |
| On first component | 0.9644 | 0.674 | 0.6526 | 0.3554 | 0.2113 | 0.5032 | 0.7796 | 0.8604 |
| On first two component | 0.9823 | 0.7164 | 0.8724 | 0.9511 | 0.8339 | 0.5952 | 0.8803 | 0.9343 |
| On first three component | 0.9824 | 0.8257 | 0.9000 | 0.9537 | 0.9957 | 0.7242 | 0.9096 | 0.9374 |
| On first four component | 0.9915 | 0.8932 | 0.9492 | 0.9904 | 0.9984 | 0.9008 | 0.931 | 0.9574 |

the sign of the coefficients is important for interpreting the estimation results of factor models in the next section. It is also useful to examine scatter diagrams in Fig. 3, which suggest a similar interpretation, with the first principal component more closely related to the Deutsche mark slope than with those of any other European currency although with a negative sign, the second principal component being essentially the US dollar slope, the third component being the British pound slope again with a negative sign, and the fourth one being the slope for the Japanese yen or Italian lira.

To further identify the components, we use linear projections of each slope on a given component. Panel b in Table 4 shows $R$-squared coefficients from linear regressions of each slope on individual components, suggesting that, indeed, the first principal component is most closely associated with European slopes, especially in the three countries mentioned above. The second component is essentially the US dollar slope, while the third principal component is essentially the slope for the British pound. This interpretation may not be independent from the fact that the euro and US dollar are the more heavily traded currencies

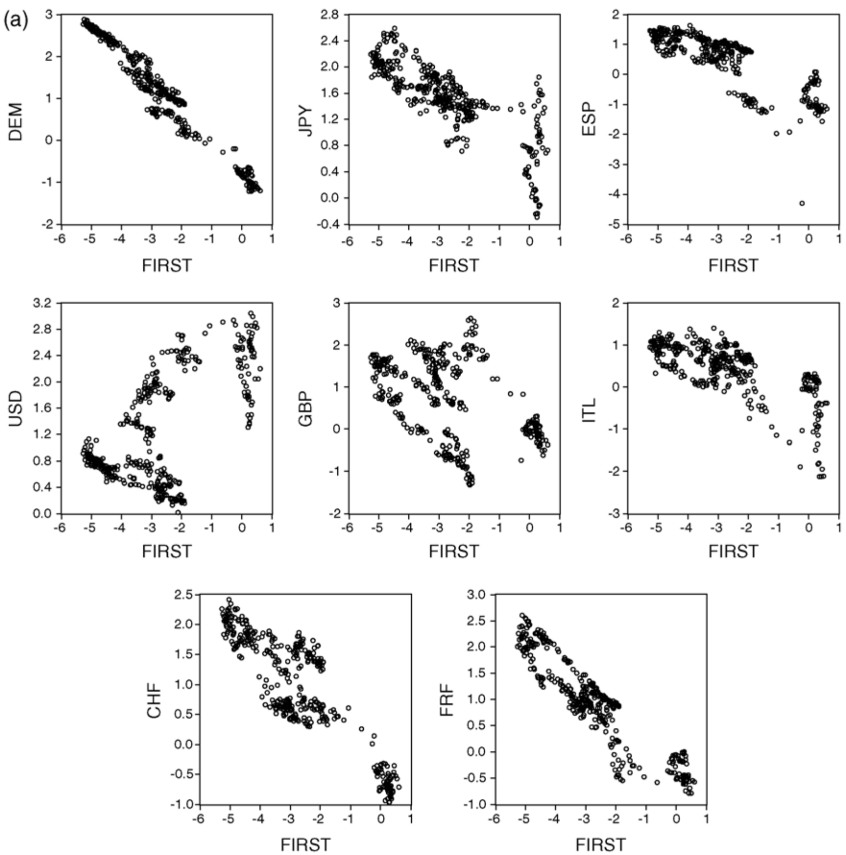


Fig. 3. Projections on principal components: (a) on first component; (b) on second component; (c) on third component; (d) on fourth component.

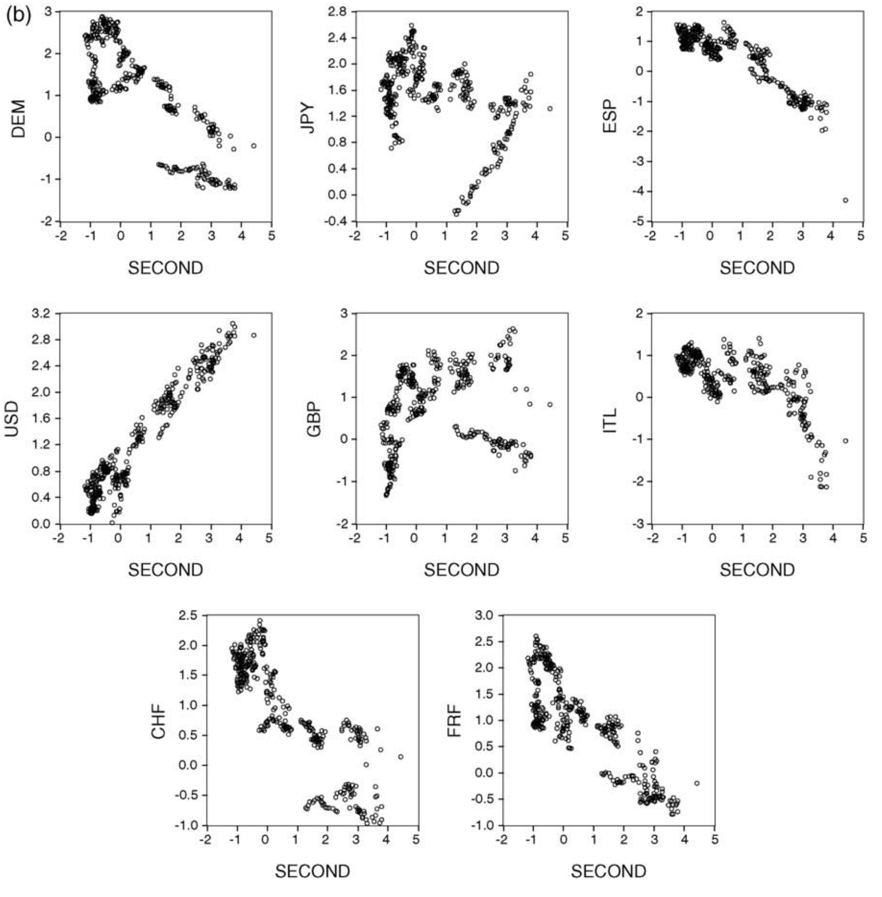


Fig. 3. (*Continued*).

in IRS markets. At the end of 1998, the Bank for International Settlements reported that 34.7 and 23.7%, respectively, of trades in IRS markets were made in these two currencies. This evidence is further corroborated by panel c in the table, which presents a sequence of non-decreasing $R$-squared values, obtained from regressions of each slope on the first, the two and the three first principal components as explanatory variables.

As a last identification strategy, we computed the changes induced on each slope by a change in a given principal component. Fig. 4 represents the variation induced on each slope by a two standard deviation change in a given component. The implied changes in slope have been normalized by their standard deviation, so as to make them comparable across currencies. A change in the first component induces noticeable changes in the slopes in European currencies, as it should be expected from a change in the slope of the term structure in Deutsche marks. Changes in the second and third components affect mainly slopes for the US dollar and British pound, respectively. A change in the fourth component implies a change in slope for the Italian lira and Japanese yen.

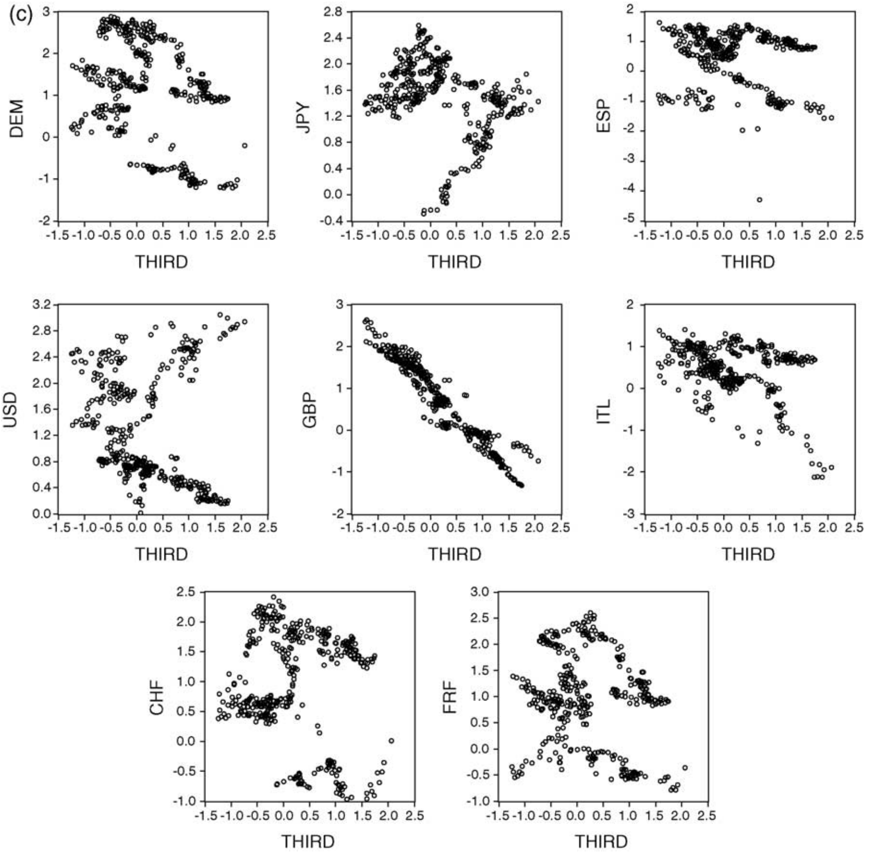


Fig. 3. (*Continued*).

Having a reasonable interpretation for each factor, the right panel in Table 1 shows them to be I(1) variables, not surprisingly, since we have just seen how they can be interpreted as specific slopes in most cases.

Working with data from Eurodeposits for a similar set of currencies, Domínguez and Novales (2000b) find that a factor linked to European currencies explains 61% of the fluctuation in a vector of term structure slopes in a variety of currencies. However, in their analysis, the US slope plays a minor role, revealing a significant difference between cross-currency correlations among term structures from Eurocurrency and IRS markets. This difference might be explained by a higher relative volume traded in US dollars in the IRS than in the Eurocurrency market. On the other hand, it might also reflect the fact that it is fluctuations in long maturity interest rates for the US dollar that play a leading role in influencing fluctuations in similar rates in other currencies. That would barely show in Eurocurrency markets, where just interest rates up to 1-year maturity are negotiated. Analyzing in detail these two alternative interpretations remains as an interesting issue for further research.

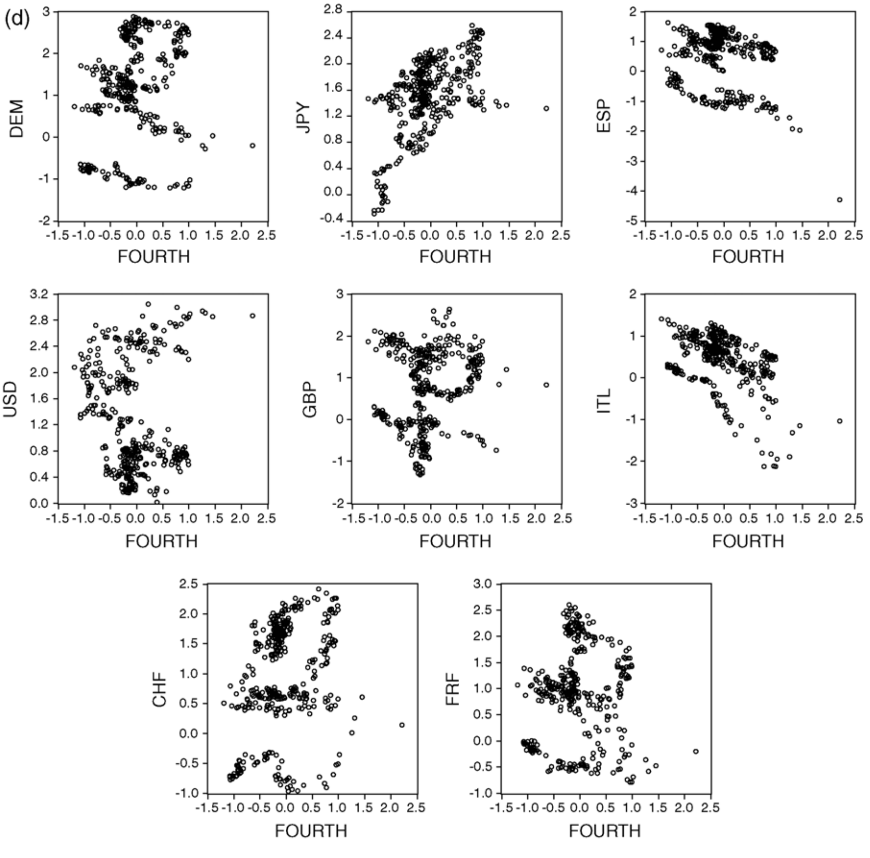


Fig. 3. (*Continued*).

We have shown that the information in term structure slopes in a set of eight major currencies can be safely summarized by three factors that can be interpreted as representing term structure slopes in major economic areas. Four of these currencies no longer exist, having been substituted by the euro, which could raise the question of whether our result is specific of the currencies in the paper. However, that is not the case, and the existence of extensive correlations that allow for characterizing a reduced number of factors can be extended to a wider set of currencies. Working with weekly data for the 2/26/1997–11/6/2002 period for the New Zealand Dollar, Australian Dollar, Canadian Dollar, Danish Krone, Norwegian Kroner, Swedish Krona, US dollar, Japanese yen and British pound, we have found that the first principal component explains 78.18% of the fluctuation in the vector of slopes, with the first two factors explaining 89.32%, the percentage increasing to 94.93% for the first three components. The ability to summarize the information contained in fluctuations in a wide vector of slopes seems to be robust to the choice of currencies and time periods. We, therefore, proceed to analyze here the characteristics of the relationships between term structures for old European currencies and those of Japan, UK and the US, while leaving

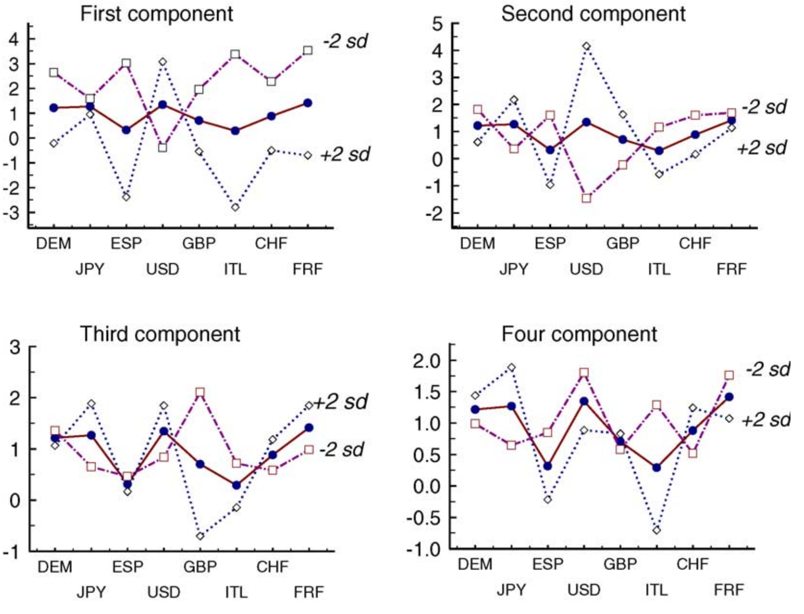


Fig. 4. Changes induced on each slope by a change in a given principal component.

the analysis of a wider, current set of currencies in a more recent time period for further research.

## 3. Models for term structure slopes

Given the evidence on non-stationarity of IRS slopes, a clear choice would be to estimate univariate models in first differences. As an alternative, an autoregression of up to third order in level slopes produces stationary residuals while leaving no significant evidence of autocorrelation, so it could also be considered acceptable for forecasting purposes. For some currencies, a first order autoregression [AR(1)] with a coefficient close to one for the term structure slope leaves stationary residuals and no autocorrelation. For other currencies, a third order autoregression [AR(3)] is needed. In that case, the sum of estimated coefficients for its own lags is very close to one, reflecting the unit root in slopes. This is a simple model, the characteristic equation of the associated autoregressive polynomial having three roots, which may capture a possible cycle through two complex roots, plus a possible unit root. Even though a second order autoregression in first differences should be seen as equivalent to a third order autoregression in levels, both models do not always perform equally well in practice. In fact, it is often the case working with interest rate data that level models produce better forecasts than models in differences [Abad and Novales (2002)].

Table 5 compares the performance of alternative univariate models to produce *static* and *dynamic* forecasts over the last 6 months in our sample, July to December 1998. Six months should be a long enough period so that results are not contaminated by any particular event.

Table 5
A forecasting comparison among univariate models for slopes

| | Panel a: static forecasts | | | | | | | | Panel b: dynamic forecasts | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DEM | JPY | ESP | USD | GBP | ITL | CHF | FRF | DEM | JPY | ESP | USD | GBP | ITL | CHF | FRF |
| Sample mean | 1.0034 | 1.1044 | 0.9304 | 0.4476 | 0.6012 | 0.8745 | 1.4941 | 1.0013 | 1.0034 | 1.1044 | 0.9304 | 0.4476 | 0.6012 | 0.8745 | 1.4941 | 1.0013 |
| AR(1) model | | | | | | | | | | | | | | | | |
| MAE | **0.0502** | 0.0659 | **0.0511** | **0.0624** | 0.0868 | **0.0510** | 0.0678 | **0.0505** | 0.0629 | **0.2953** | 0.2579 | 0.3037 | 0.5845 | 0.2810 | **0.1514** | 0.0888 |
| Median | 0.0383 | 0.0394 | 0.0494 | 0.0344 | 0.0633 | 0.0455 | 0.0648 | 0.0400 | 0.0543 | **0.2252** | 0.2424 | 0.3129 | 0.6077 | 0.2382 | **0.1042** | 0.0736 |
| RMSE | **0.0631** | 0.0963 | **0.0696** | 0.0952 | 0.1214 | **0.0667** | 0.0864 | **0.0635** | 0.0783 | 0.3672 | 0.3007 | 0.3659 | 0.6945 | 0.3306 | **0.1940** | 0.1058 |
| *U*-Theil | **0.0313** | 0.0428 | **0.0375** | 0.0980 | 0.0806 | **0.0381** | 0.0287 | **0.0318** | 0.0398 | **0.1467** | 0.1865 | 0.5776 | 0.3620 | 0.2231 | **0.0666** | 0.0549 |
| AR(2) model | | | | | | | | | | | | | | | | |
| MAE | **0.0502** | 0.0651 | 0.0557 | 0.0631 | 0.0869 | 0.0513 | 0.0677 | 0.0507 | 0.0625 | 0.2963 | 0.2016 | 0.2930 | 0.5856 | 0.2778 | 0.1516 | 0.0889 |
| Median | **0.0377** | 0.0346 | **0.0368** | **0.0329** | 0.0634 | 0.0447 | 0.0645 | 0.0397 | 0.0529 | 0.2270 | 0.1933 | 0.2943 | 0.6090 | 0.2350 | 0.1050 | 0.0741 |
| RMSE | 0.0635 | 0.0950 | 0.0748 | 0.0945 | 0.1215 | 0.0673 | 0.0863 | 0.0640 | 0.0778 | 0.3681 | 0.2439 | 0.3543 | 0.6958 | 0.3275 | 0.1943 | 0.1060 |
| *U*-Theil | 0.0315 | 0.0422 | 0.0403 | 0.0970 | 0.0806 | 0.0385 | 0.0287 | 0.0319 | 0.0395 | 0.1472 | 0.1462 | 0.5504 | 0.3624 | 0.2205 | 0.0668 | 0.0550 |
| AR(3) model | | | | | | | | | | | | | | | | |
| MAE | 0.0508 | 0.0677 | 0.0553 | 0.0631 | **0.0857** | 0.0511 | **0.0668** | 0.0516 | **0.0611** | 0.2954 | 0.2047 | **0.2745** | **0.5453** | **0.2730** | 0.1526 | **0.0881** |
| Median | 0.0387 | **0.0319** | 0.0387 | 0.0507 | **0.0613** | **0.0428** | **0.0638** | 0.0418 | **0.0522** | **0.2252** | 0.1959 | **0.2789** | **0.5648** | **0.2312** | 0.1093 | **0.0728** |
| RMSE | 0.0640 | 0.0989 | 0.0745 | **0.0941** | **0.1202** | 0.0671 | **0.0857** | 0.0648 | **0.0761** | **0.3671** | 0.2474 | **0.3359** | **0.6502** | **0.3234** | 0.1957 | **0.1051** |
| *U*-Theil | 0.0318 | 0.0440 | 0.0402 | **0.0963** | **0.0799** | 0.0383 | **0.0284** | 0.0323 | **0.0386** | **0.1467** | 0.1486 | **0.5071** | **0.3457** | **0.2164** | 0.0673 | **0.0546** |
| ARMA(1,1) model | | | | | | | | | | | | | | | | |
| MAE | **0.0502** | **0.0649** | 0.0548 | 0.0627 | 0.0869 | 0.0512 | 0.0677 | 0.0507 | 0.0626 | 0.2967 | **0.1946** | 0.2973 | 0.5859 | 0.2756 | 0.1516 | 0.0884 |
| Median | **0.0377** | 0.0337 | 0.0394 | 0.0330 | 0.0634 | 0.0443 | 0.0645 | **0.0396** | 0.0530 | 0.2277 | **0.1877** | 0.3015 | 0.6092 | 0.2333 | 0.1049 | 0.0731 |
| RMSE | 0.0634 | **0.0947** | 0.0731 | 0.0947 | 0.1215 | 0.0673 | 0.0863 | 0.0640 | 0.0778 | 0.3685 | **0.2360** | 0.3589 | 0.6961 | 0.3252 | 0.1943 | 0.1054 |
| *U*-Theil | 0.0315 | **0.0421** | 0.0394 | 0.0972 | 0.0806 | 0.0384 | 0.0384 | 0.0319 | 0.0396 | 0.1472 | **0.1409** | 0.5611 | 0.3625 | 0.2187 | 0.0667 | 0.0547 |
| Selected model | | | | | | | | | | | | | | | | |
| | AR(1) | ARMA | AR(1) | AR(3) | AR(3) | AR(1) | AR(3) | AR(1) | AR(3) | AR(3) | ARMA | AR(3) | AR(3) | AR(3) | AR(1) | AR(3) |

*Note:* The Sample mean row contains sample absolute mean values for slopes over the forecasting period, 7/1/1998–12/30/1998. The last row shows the univariate model displaying the best forecasting performance for each currency (ARMA refers to an ARMA(1,1) model). Mean and median are the mean and median absolute values of the forecasting errors. RMSE denotes the root mean square error, while *U*-Theil denotes Theil's statistic. Boldface figures denote the best model according to each statistic for forecast performance.

*Dynamic forecasts* are obtained with models estimated using data until the end of June1998. These are once-and-for all predictions over the 27 weeks of the second semester of 1998, so previously obtained forecasts need to be used recursively, to proceed along the forecasting period. For *static forecasts*, we estimated the previous models with data until the end of June 1998, to obtain forecasts for the first week in July 1998. We then repeatedly estimated the models adding one data point at a time, producing forecasts for the next week each time. That way, we have a sequence of 27 one step ahead forecasts for each slope. Except for the first forecasting week, dynamic forecasting always produces larger forecast errors than static forecasting, since the former exercise projects over the whole forecasting period using only data up to the last week of June 1998. Short-order moving average models MA(1) and MA(2) performed badly, and are not included in the table. ARMA models of higher order do not add much forecast gain to those included in the table.

Table 6 displays mean and median absolute errors for level slopes, the root mean square error and Theil's *U*-statistic. Boldface figures denote the lowest value of each forecasting statistic for each currency, over the set of models considered. There is a fair amount of consistency among the evidence provided by the four statistics considered, and the last row in the table displays the selected univariate model. In most cases, an AR(3) is best for dynamic forecasting,[5] while a shorter model may be preferred for static forecasting. For the US dollar and British pound, an AR(3) model is best for static as well as for dynamic forecasting.

Table 6 presents estimates for the best univariate models, as selected in Table 5. In spite of the apparent differences among estimated models for level slopes in Table 6, their stochastic properties are very similar. The characteristic equation of each autoregression has a root close to one: 1.006, 1.025, 1.018, . . ., as corresponds to non-stationary processes. Additionally, implied impulse response functions show high persistence starting from an initial response close to one in all cases, and suggesting that the effects of unexpected shocks in any slope take a long time to die away. There is not evidence of residual autocorrelation, except for the Spanish peseta. Autocorrelation in this currency may be reflecting some kind of non-linearity, since fitting higher order models did not lead to white noise residuals.

For each currency, Table 7 displays two sets of estimates: the upper panel presents estimations of a long-run relationship between each slope and the corresponding first two principal components,

$$S_t = \hat{\alpha}_0 + \hat{\alpha}_1 F_{1,t} + \hat{\alpha}_2 F_{2,t} + \hat{u}_t \qquad (1)$$

Being a static model, residuals display extensive autocorrelation, but they turn out to be stationary in all cases, according to the Augmented Dickey–Fuller and Phillips–Perron included in the table at 95% confidence. That suggests that this equation can be seen as a cointegrating relationship between each slope and the first two common factors, which we associated in the previous section to the European and US term structure slopes. This relationship can be interpreted as a long-term equilibrium relationship between the three variables. The choice of just two factors for this model is based on our intention to try to obtain a sharp reduction in the dimensionality of the forecasting problem. The test

[5] In the case of the Japanese yen, AR(1) and AR(3) models perform equally as well for dynamic forecasting, but the latter was used for consistency with other currencies.

Table 6

Estimated autoregressive univariate models

| Model | DEM | | JPY | | ESP | | USD | GBP | ITL | | CHF | | FRF | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | AR(1) | AR(3) | ARMA | AR(3) | AR(1) | ARMA | AR(3) | AR(3) | AR(1) | AR(3) | AR(3) | AR(1) | AR(1) | AR(3) |
| AR lag | | | | | | | | | | | | | | |
| $i=1$ | 0.994 (0.004) | 0.967 (0.051) | 0.975 (0.008) | 1.059 (0.051) | 0.970 (0.012) | 0.987 (0.008) | 1.076 (0.051) | 1.002 (0.051) | 0.976 (0.011) | 0.942 (0.051) | 1.009 (0.051) | 0.991 (0.005) | 0.992 (0.006) | 0.970 (0.051) |
| $i=2$ | | 0.049 (0.071) | | −0.153 (0.074) | | | 0.034 (0.074) | 0.044 (0.072) | | 0.007 (0.070) | 0.040 (0.073) | | | 0.062 (0.071) |
| $i=3$ | | −0.022 (0.051) | | 0.070 (0.051) | | | −0.114 (0.051) | −0.058 (0.051) | | 0.028 (0.051) | −0.057 (0.051) | | | −0.040 (0.051) |
| MA lag | | | | | | | | | | | | | | |
| $j=1$ | | | 0.091 (0.052) | | | −0.306 (0.050) | | | | | | | | |
| *R*-squared | 0.994 | 0.994 | 0.981 | 0.980 | 0.940 | 0.946 | 0.991 | 0.978 | 0.951 | 0.950 | 0.991 | 0.991 | 0.987 | 0.987 |
| S.E.E. | 0.090 | 0.091 | 0.075 | 0.075 | 0.218 | 0.208 | 0.077 | 0.139 | 0.136 | 0.137 | 0.085 | 0.084 | 0.099 | 0.099 |
| *Q*(3) | 4.10 [0.25] | 6.16 [0.29] | 1.76 [0.42] | 0.96 [0.97] | 55.60 [0.00] | 12.44 [0.00] | 0.96 [0.97] | 8.94 [0.11] | 0.82 [0.85] | 11.21 [0.05] | 11.06 [0.05] | 5.75 [0.13] | 1.17 [0.76] | 2.69 [0.75] |
| *Q*(10) | 9.51 [0.48] | 8.50 [0.58] | 4.67 [0.86] | 2.71 [0.99] | 64.00 [0.00] | 19.81 [0.02] | 8.19 [0.61] | 16.88 [0.08] | 29.16 [0.00] | 27.61 [0.00] | 16.91 [0.08] | 18.10 [0.05] | 7.81 [0.65] | 7.12 [0.71] |
| LM(1) | 0.29 [0.59] | 2.27 [0.13] | 1.26 [0.26] | 0.44 [0.51] | 45.38 [0.00] | 1.85 [0.17] | 0.38 [0.54] | 1.21 [0.27] | 0.46 [0.50] | 0.23 [0.63] | 4.56 [0.03] | 0.13 [0.72] | 0.20 [0.65] | 0.34 [0.56] |
| LM(4) | 1.10 [0.35] | 3.89 [0.42] | 0.62 [0.65] | 4.40 [0.36] | 15.33 [0.00] | 4.79 [0.00] | 1.74 [0.78] | 7.68 [0.10] | 0.98 [0.42] | 3.82 [0.43] | 11.02 [0.03] | 2.76 [0.03] | 0.34 [0.85] | 0.80 [0.94] |
| ADF | −7.76 | −7.82 | −8.39 | −8.10 | −9.84 | −9.48 | −8.92 | −8.17 | −8.11 | −7.96 | −7.76 | −7.67 | −7.74 | −7.87 |
| PP | −20.25 | −19.70 | −19.58 | −19.47 | −27.36 | −20.14 | −19.73 | −19.76 | −20.41 | −19.71 | −19.67 | −19.33 | −20.16 | −19.68 |

*Note:* Models estimated using levels of term structure slopes. In each currency, the left column corresponds to the best model for static forecasting, while the right column presents estimates of the best model for dynamic forecasting. ARMA refers to an ARMA(1,1) model. A (generally non-significant) constant was included in all models. Standard deviations (S.D.) in parentheses. Statistics shown for each regression include: adjusted *R*-squared, standard error of estimate (S.E.E.), Ljung–Box autocorrelation statistics of orders 3 and 10 (*Q*(3), *Q*(10)), Breusch–Godfrey autocorrelation statistics of orders 1 and 4 (LM(1), LM(4)), and Augmented Dickey–Fuller and Phillips–Perron statistics to test for the presence of a unit root in the residuals. Critical values for both statistics: 3.45 (1%), −2.87 (5%); *p*-values are included in square brackets.

Table 7
Estimated error correction models

| | DEM | JPY | ESP | USD | GBP | ITL | CHF | FRF |
|---|---|---|---|---|---|---|---|---|
| Cointegration relationship | | | | | | | | |
| | (0.024) | (0.045) | (0.049) | (0.028) | (0.058) | (0.059) | (0.047) | (0.034) |
| First component | −0.604 (0.007) | −0.344 (0.013) | −0.183 (0.014) | −0.091 (0.008) | −0.720 (0.017) | −0.151 (0.017) | −0.307 (0.014) | −0.346 (0.010) |
| Second component | −0.150 (0.008) | 0.108 (0.014) | −0.401 (0.015) | 0.610 (0.009) | 0.711 (0.019) | −0.178 (0.019) | −0.271 (0.015) | −0.225 (0.011) |
| ADF | −3.80 | −3.67 | −3.42 | −3.22 | −3.66 | −2.45 | −3.08 | −3.03 |
| PP | −3.94 | −3.59 | −5.41 | −3.65 | −3.82 | −3.03 | −3.00 | −3.17 |
| Error correction model | | | | | | | | |
| Error correction term (*t*-1) | 0.066 (0.031) | −0.031 (0.013) | −0.152 (0.033) | −0.058 (0.022) | −0.058 (0.018) | −0.054 (0.017) | −0.008 (0.014) | −0.074 (0.022) |
| Slope (*t*-1) | −0.117 (0.085) | 0.115 (0.056) | −0.096 (0.071) | 0.118 (0.068) | −0.067 (0.068) | −0.093 (0.054) | −0.032 (0.058) | −0.131 (0.068) |
| First component (*t*-1) | −0.037 (0.053) | 0.025 (0.030) | 0.030 (0.078) | 0.026 (0.034) | −0.212 (0.063) | −0.153 (0.053) | −0.064 (0.035) | −0.085 (0.048) |
| Second component (*t*-1) | −0.071 (0.042) | −0.013 (0.036) | 0.459 (0.127) | 0.001 (0.045) | −0.064 (0.077) | −0.170 (0.060) | −0.036 (0.039) | −0.133 (0.044) |
| *R*-squared | 0.008 | 0.011 | 0.177 | 0.019 | 0.072 | 0.077 | 0.003 | 0.066 |
| S.E.E. | 0.090 | 0.076 | 0.199 | 0.077 | 0.134 | 0.131 | 0.084 | 0.096 |
| *Q*(3) | 3.02 [0.39] | 1.23 [0.75] | 7.48 [0.06] | 5.88 [0.12] | 0.81 [0.85] | 1.17 [0.76] | 5.30 [0.15] | 1.40 [0.71] |
| *Q*(10) | 7.74 [0.65] | 4.86 [0.90] | 14.92 [0.14] | 14.89 [0.14] | 20.22 [0.03] | 24.30 [0.01] | 17.17 [0.07] | 8.60 [0.57] |
| LM(1) | 3.99 [0.05] | 0.30 [0.59] | 4.79 [0.03] | 4.23 [0.04] | 0.33 [0.57] | 2.49 [0.11] | 4.83 [0.03] | 0.00 [1.00] |
| LM(4) | 7.49 [0.11] | 1.76 [0.78] | 9.12 [0.06] | 6.55 [0.16] | 7.25 [0.12] | 6.64 [0.16] | 19.55 [0.00] | 1.76 [0.78] |
| ADF | −7.98 | −8.03 | −8.41 | −8.33 | −8.39 | −8.12 | −7.74 | −7.83 |
| PP | −19.52 | −19.38 | −19.01 | −19.99 | −19.89 | −20.09 | −19.41 | −19.65 |

*Note:* In each currency, the upper panel displays estimates of the cointegration relationship between each slope and the corresponding first two principal components. The ADF and PP statistics in that panel The lower panel presents estimates of the implied error correction model. Standard deviations (S.D.) in parentheses. Statistics shown for each regression include: adjusted *R*-squared, standard error of estimate (S.E.E.), Ljung–Box autocorrelation statistics of orders 3 and 10 (*Q*(3), *Q*(10)), Breusch–Godfrey autocorrelation statistics of orders 1 and 4 (LM(1), LM(4)), and Augmented Dickey–Fuller and Phillips–Perron statistics to test for the presence of a unit root in the residuals. Critical values for both statistics: −3.45 (1%), −2.87 (5%); *p*-values are included in square brackets.

proposed by Bartlett (1951) and applied in Fase (1976), among others, to reduce the dimensionality of a large vector of interest rates for different countries, would suggest using up to six principal components when working at 95% confidence level. That would be contrary to our goal of proposing a relatively simple forecasting model for slopes in the set of currencies considered. So, we proceed by using just the first two components, which explain 92.0% of the fluctuations in the set of slopes, as mentioned above, and hope that forecasts based on the information provided by these two components might be accurate.

The lower panel in Table 7 contains estimates of the error correction model (ECM) for each slope. In this equation, the differenced slope is projected on one lag of itself, the first lagged difference of the two principal components, and the lagged error-correction term $\hat{u}_t$, the residual from cointegrating relationship (1),

$$\Delta S_t = \hat{\beta}_1 \hat{u}_{t-1} + \hat{\beta}_2 \Delta S_{t-1} + \hat{\beta}_3 \Delta F_{1,t-1} + \hat{\beta}_4 \Delta F_{2,t-1} + \hat{\varepsilon}_t \quad (2)$$

Even though this is a single-equation exercise, it should be interpreted as the corresponding equation from the vector ECM. All statistics in the lower panel of Table 7 correspond to ECM Eq. (2).

It is interesting to see the important consistencies across currencies. The S.E.E. is essentially the same for models in Tables 6 and 7. Estimated coefficients are statistically significant in the cointegrating relationship (1). The first factor, which is negatively related to the slope in European markets, enters with a negative sign in the long-run relationships for those countries, as expected. The second factor, which is positively related to the US slope, enters with a positive sign in the cointegrating equation for that country. In ECM estimation, the error correction term is statistically significant except for the Swiss franc, and the associated coefficient has the right sign in all cases except for the Deutsche mark. There is no significant evidence of residual autocorrelation, so the model seems to appropriate capture the dynamics in slope fluctuations.

## 4. Forecasting with slope models

As mentioned in Section 1, our final goal is to analyze the extent to which a few common factors for a set of international term structure slopes are able to provide good predictions of future fluctuations in the set of eight slopes considered. By good forecasts we understand forecasts at least as good as those that could be obtained from univariate models for each slope.

If our search is successful, we could reduce the problem of forecasting a potentially large number of slopes to that of forecasting a reduced number of factors. This possibility is far from trivial, since principal components are designed to fit the data, but not to capture the autocorrelation in the data, and a better fit does not necessarily come together with an improvement in forecasting ability. Hence, it is important that we discuss next the extent to which the in-sample explanatory power of the principal components for term structure slopes across countries can actually be used to improve upon more complex forecasting models.

### 4.1. The forecasting exercise

As a first option, we computed static and dynamic forecasts from univariate models for slopes. As discussed above, level slopes produce forecasts at least as good as differenced slopes, in spite of the general presence of a unit root. Univariate models estimated in Table 6 produce stationary residuals in spite of being specified for level slopes, so they can be safely used for forecasting purposes. As an alternative, we obtained forecasts from estimated ECM in Table 7, and we devote this section to comparing the forecasting performance of both sets of models.

As we ran out of actual data for the lagged slope in dynamic forecasting from univariate models, we use previously obtained forecasts. The same applies to the use of ECM, in which forecasts for the principal components need to be obtained previously to computing slope forecasts. To do so, we need to start by computing forecasts for the principal components. Since we do not want that a possible positive evidence in favor of the use of factor models might be purely spurious, we did not search for the best forecasting models for the two principal components, contrary to what we did with the slopes themselves. We rather used AR(3) models in levels for both factors, from which forecasts were then readily obtained. In addition to being satisfactory from the point of view of estimation and forecasting, the choice of a AR(3) univariate structure for each principal component avoids the possibility that model searching could bias the forecasting results in favor of factor models.

In each case, we computed four indicators of forecasting performance: mean absolute error, median absolute error, root mean squared error, and the *U*-statistic proposed by Theil. Percent root mean square errors are not advisable in this forecasting exercise, since the slope often becomes small in absolute value, to the point that even acceptable forecast errors might produce huge percent errors for a single period, dominating the value of any time aggregate forecasting performance indicator. Hence, we will use their versions in absolute terms. Since we have four forecasting error criteria, and compute static as well as dynamic forecasts for each of the eight currencies in our data set, we have 64 comparisons in total.

### 4.2. Forecasting results

Below each currency code in Table 8, we show the sample average absolute value of each slope over the forecasting horizon, the reference against which forecast statistics should be compared to evaluate forecasting performance. Bold figures highlight cases in which the principal components model outperforms the univariate slope model in forecasting.

Statistics in Table 8 show that:

(1) Median one-step-ahead errors from univariate models in static forecasting oscillate between 3 and 5.3% of the sample mean absolute slope for all currencies except the US dollar and British pound, for which they reach levels of 11.3 and 10.2%, respectively. Hence, univariate slope models in levels produce acceptable one step ahead forecasts in most currencies.

(2) Quite strikingly, in spite of the need to predict the principal components, ECM with factors produce slope forecasts that are in some cases even better than those

Table 8
Forecasting performance indicators[a]

| | Forecasting model | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | DEM 1.0034 | | JPY 1.1044 | | ESP 0.9304 | | USD 0.4476 | | GBP 0.6012 | | ITL 0.8745 | | CHF 1.4941 | | FRF 1.0013 | |
| | Best AR model | Factor models | Best AR model | Factor models | Best AR model | Factor models | Best AR model | Factor models | Best AR model | Factor models | Best AR model | Factor models | Best AR model | Factor models | Best AR model | Factor models |
| Static forecasts | | | | | | | | | | | | | | | | |
| Mean | 0.0502 | **0.0483** | 0.0649 | 0.0658 | 0.0511 | 0.0593 | 0.0631 | **0.0617** | 0.0857 | **0.0823** | 0.0510 | 0.0553 | 0.0668 | 0.0700 | 0.0505 | 0.0525 |
| Median | 0.0383 | **0.0376** | 0.0337 | 0.0450 | 0.0494 | **0.0323** | 0.0507 | **0.0384** | 0.0613 | **0.0542** | 0.0455 | **0.0433** | 0.0638 | 0.0685 | 0.0400 | 0.0463 |
| RMSE | 0.0631 | **0.0630** | 0.0947 | 0.0973 | 0.0696 | 0.0812 | 0.0941 | 0.0960 | 0.1202 | **0.1142** | 0.0667 | 0.0676 | 0.0857 | 0.0868 | 0.0635 | **0.0627** |
| *U*-Theil | 0.0313 | **0.0312** | 0.0421 | 0.0435 | 0.0375 | 0.0432 | 0.0963 | 0.0995 | 0.0799 | **0.0760** | 0.0381 | 0.0387 | 0.0284 | 0.0288 | 0.0318 | **0.0311** |
| Dynamic forecasts | | | | | | | | | | | | | | | | |
| Mean | 0.0611 | 0.2756 | 0.2954 | **0.1853** | 0.1946 | **0.0923** | 0.2745 | **0.2721** | 0.5453 | **0.4716** | 0.273 | **0.2549** | 0.1514 | 0.1698 | 0.0881 | **0.0544** |
| Median | 0.0522 | 0.2284 | 0.2252 | **0.1090** | 0.1877 | **0.0906** | 0.2789 | **0.2677** | 0.5648 | **0.4852** | 0.2312 | **0.2187** | 0.1042 | 0.1352 | 0.0728 | **0.0509** |
| RMSE | 0.0761 | 0.3403 | 0.3671 | **0.2393** | 0.2360 | **0.1096** | 0.3359 | **0.3343** | 0.6502 | **0.5664** | 0.3234 | **0.3023** | 0.1940 | 0.2165 | 0.1051 | **0.0660** |
| *U*-Theil | 0.0386 | 0.1948 | 0.1467 | **0.1026** | 0.1409 | **0.0587** | 0.5071 | **0.5006** | 0.3457 | **0.3129** | 0.2164 | **0.2005** | 0.0666 | 0.0753 | 0.0546 | **0.0326** |

*Note:* The Sample mean row contains sample absolute mean values for slopes over the forecasting period, 7/1/1998–12/30/1998. Forecasts obtained from estimated models in Table 5. Mean and median are the mean and median absolute values of the forecasting errors. *RMSE* denotes the root mean square error, while *U-Theil* denotes Theil's statistic. Boldface figures denote cases when factor models forecast better than univariate models.

[a] Sample absolute mean values: 7/1/1998–12/30/1998

from univariate models. In 38 out of the 64 forecasting performance indicators, the use of principal components leads to better slope forecasts than univariate models. The long-run cointegrating relationship between slope and principal components by itself does not perform as well, beating univariate models in just eight of the 64 comparisons.[6] This is clear evidence that the dynamics embedded into the error correction model is crucial for a good forecasting performance. The distribution of positive results is not symmetric, with 14 of the 38 cases corresponding to static forecasts and the remaining 24 cases being situations of dynamic forecasting.

(3) The Swiss franc is the only currency for which univariate models produce better forecasts than the ECM model according to all the forecast indicators used. The fact that it is also the only currency for which the error correction term turns out not to be statistically significant reinforces our conclusion in the previous point on the relevance that the dynamics incorporated by the combination of short- and long-term relationships in the ECM model has for forecasting purposes. Furthermore, The Deutsche mark is the only currency for which the error correction term has a sign opposite to that suggested by theory. It is also the only case, together with the Swiss franc, for which the factor model does not beat univariate models in dynamic forecasting. Once again, this result suggests that the dynamics in the ECM model are central to a good forecasting performance for term structure slopes.

(4) Just two principal components are enough to produce this forecasting performance. In fact, even though there is additional explanatory power in further components (as shown in Table 3), adding them to the least-squares projections does not significantly improve forecasting performance. So, our decision in Section 3 to proceed with this short number of factors is validated by the quality of the resulting forecasts. On the contrary, we could not proceed along with just one factor, factor ECM models then leading to better forecasts than univariate models in just 14 of the 64 comparisons.[7]

(5) Even though the first two components reflect fluctuations in Deutsche mark and US dollar slopes, the proposed error correction factor model can satisfactorily predict future slope values for the Japanese yen and British pound as well. Even though the third and fourth principal components can be approximately interpreted as slopes in these two currencies, the explanatory power of the first two components seems to be important enough so that the information in the next two factors can be safely ignored for forecasting purposes. Hence, the choice of the first two principal components leads to the simplest model producing a significant forecasting gain for the set of slopes in the currencies considered.

These results are quite striking because slope forecasts from ECM rest on forecasts from autoregressive models for the factors, so the sampling error in estimating these models is compounded with that in estimating the ECM. Yet, in spite of this double estimation process, factor models often predict better than univariate models for level slopes. We

[6] Results are not included in the paper, but they are available from the authors.

[7] Detailed results available from the authors.

should also remember that, contrary to our strategy with univariate models, and in order to design a relatively simple forecasting mechanism for slopes, we did not search for the best forecasting models for the factors. Rather, we used AR(3) models for both factors because of the interesting characteristics of these models. These models are nevertheless, reasonably good, with mean absolute errors in static forecasting of 4.3 and 7.7% of the sample mean absolute values of the first and second factors, respectively.

The practical implication is that to forecast the set of eight international IRS market slopes, we only need to forecast the two common factors. Applying estimates from cointegrating relationships (1) to new slope data will provide us with the error correction residual. Adding univariate forecasts for the factors, we can easily compute forecasts for the vector of eight slopes from estimates of ECM models (2).

## 5. Conclusions

The first two principal components in a vector of term structure slopes from IRS markets in eight major currencies explain above 90% of the fluctuations in the vector of slopes. Each of the eight slopes considered is cointegrated with these two factors, which can be shown to be closely related to the slopes for the Deutsche mark and US dollar. We have also found strong evidence that reducing the dimensionality of the vector of slopes by using these two factors can be very fruitful for short- and medium-term slope forecasting in all these currencies. The reduction in dimensionality leads to a very simple forecasting scheme for term structure slopes, in the form of an error correction model between each slope and two common factors. Adding more factors to the model does not lead to a significant improvement in forecasting performance, while forecasts obtained using just one factor are not as good as those from two-factor error correction models.

To obtain our results, we have compared forecasts from error correction models to the best univariate model for each currency slope, selected from a variety of univariate models according to several statistics of forecasting performance. On the contrary, in the case of ECM models, we did not perform any search for the best forecasting model for the factors, so our results cannot be spurious. Furthermore, we have obtained evidence that it is in the case of the two currencies for which the ECM model does not fit the data too well, that its forecasting performance deteriorates. The combination of short-term dynamics with a long-term relationship between each slope and the two common factors embedded in ECM models seems to be crucial to improve upon the forecasts provided by univariate models for term structure slopes.

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## Appendix A. The bootstrapping method

The bootstrapping method is applied to obtain an estimate of the term structure from interest rates corresponding to the fixed branch of an IRS. We use interest rates traded at fair value in each currency for generic *swaps* at 2, 3, 4, 5, 7 and 10-year maturities, and the 1 year money market interest rate. Interest rates for 6- and 8-year swaps were first obtained from the other maturities by interpolation.

Since the 1 year money market rate is a zero coupon rate, we can evaluate the discount function at that maturity by,

$$d_{t,1} = \frac{1}{(1 + R_{t,1}\alpha_{0,1})} \tag{A.1}$$

where $\alpha_{0,1}$ is the fraction of year corresponding to the (0,1) time interval according to the computational convention established in each currency (ACT/365, ACT/360, ...).

To obtain zero coupon rates as well as the value of the discount function at maturities above one year, we exploit the fact that the net present value (NPV) for a pair swap must be zero. From the observed price, $C_n$, for a *swap* maturing at $n$, and its principal $P$, the net value is the difference between the present value of the stream of payments from the fixed branch and the stream of variable payments:

$$\begin{aligned}\mathrm{VAN}(\mathrm{IRS}_n) = (&PC_n\alpha_{0,1}d_{t,1} + PC_n\alpha_{1,2}d_{t,2} \\ &+ \cdots + PC_n\alpha_{(n-1),n}d_{t,n}) - (P - Pd_{t,n}) = 0\end{aligned} \tag{A.2}$$

where we have taken into account that the stream of variable payments of a swap is financially equivalent to two payments of opposite sign and an amount equal to the principal, one at time t and the other at maturity time.

Applying this property sequentially to *swaps* with successive maturities, we obtain the discount function for $n = 2, \ldots, 9$ and 10 years:

$$d_{t,n} = \frac{1 - C_n[\alpha_{0,1}d_{t,1} + \cdots + \alpha_{(n-2),(n-1)}d_{t,(n-1)}]}{1 + C_n\alpha_{(n-1),n}} \tag{A.3}$$

Finally, we recover term structure interest rates for $\alpha_{0,n}$ from the estimated discount function defined on a 30/360 basis:

$$R_{t,n} = \left[\frac{1}{d_{t,n}}\right]^{1/\alpha_{0,n}} \tag{A.4}$$

## Appendix B. The principal components methodology

Let $z_{it}$ denote the standardized slope in currency $i$ at time $t$, $i = 1, 2, \ldots, n$, $t = 1, 2, \ldots, T$. If all slopes move proportionally to each other, we would have,

$$z_{it} = \alpha_{i1} f_{1t} \text{ for all } i, t \tag{B.1}$$

where $\alpha_{i1}$ would be a set of constants to be determined from actual data, and $f_{1t}$ would be the first principal component. In general, (B.1) would only hold as an approximation, and we will be interested in finding those $\alpha_{i1}$, $f_{1t}$ minimizing

$$S_1 = \sum_i \sum_t (z_{it} - \alpha_{i1} f_{1t})^2 \tag{B.2}$$

Since $f_{1t}$ can only be determined up to a scalar factor, some normalization is needed, like $\sum_t f_{1t}^2 = 1$. It can be shown that $S_1$ is minimized when

$$f_{1t} = \frac{1}{\lambda_1} \sum_i z_{it} \alpha_{i1} \tag{B.3}$$

where $\lambda_1$ denotes the largest eigenvalue of the *nxn* sample symmetric cross matrix $\boldsymbol{M} = (m_{ij})$, $m_{ij} = \sum_t z_{it} z_{jt}$, while $\alpha_{i1}$ are derived from the elements of the eigenvector associated $\lambda_1$, multiplied by $\sqrt{\lambda_1}$. Hence, the first principal component $f_1$ is a linear combination of the observed slopes, with coefficients proportional to the elements of the eigenvector associated to the largest eigenvalue of $\boldsymbol{M}$. Furthermore, it can be shown that

$$\lambda_1 = \sum_i \alpha_{i1}^2 \tag{B.4}$$

Following a similar argument, the second principal component may be taken from the resulting residuals. In general, the *k*-th principal component can be obtained as,

$$f_{kt} = \frac{1}{\lambda_k} \sum_i z_{it} \alpha_{ik} \tag{B.5}$$

where $\lambda_k$, $k \leq n$ denotes the *k*-th largest eigenvalue of $\boldsymbol{M}$, with $\lambda_k = \sum_i \alpha_{ik}^2$. Since we are working with standardized variables, $\boldsymbol{M}$ is a matrix of correlation coefficients, and the factor loadings $\alpha_{ij}$ are correlation coefficients between each slope and the the *j*-th principal component.

Each principal component is orthogonal to all the others. Furthermore, the sum of the eigenvalues of $\boldsymbol{M}$ is equal to its trace, which is equal to *n* for standardized variables. Therefore, each principal component accounts for a proportion $\lambda_j / \sum_{i=1}^{n} \lambda_i$ of the total variation in the vector of slopes $z = (z_{1t}, z_{2t}, \ldots, z_{nt})$.

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