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A factor analysis of volatility across the term structure: the Spanish case

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Abstract

We show that the term structure of volatilities for zero-coupon interest rates from the Spanish secondary debt market can be explained by a reduced number of factors. This factor representation can be used to produce volatility time series across the whole term structure. As an alternative, volatilities can also be derived from a factor model for interest rates themselves. We find evidence contrary to the hypothesis that these two procedures lead to statistically equivalent time series, so that choosing the right model to estimate volatility is far from trivial. However, observed differences seem to be of little consequence for VaR estimation on zero coupon bonds.

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1. Introduction

Searching for a sensible factor representation of the term structure of interest rates has been object of study for some time. If interest rates at any given maturity could be written to a reasonable approximation as a linear combination of a small number of factors, then fluctuations of the yield curve could be characterized by just analyzing the behaviour of the chosen factors. These could either be rates of return for specific maturities, like the one month rate, simple linear combinations of them, like the spread between a long- and a short-term rate, or more complicated linear combinations of interest rates at different maturities. In particular, interest rate forecasts for every maturity could be derived from forecasts for the factors.

With some differences across a variety of international fixed income markets, this type of analysis concludes in a positive note, by characterizing a small number of factors able to represent, to a large extent, the behaviour of the term structure of interest rates [Stock and Watson (1988), Elton, Gruber and Michaely (1990), Litterman and Scheinkman(1990), Hall, Anderson and Granger(1992), Zhang (1993), Engsted and Tanggaard(1994), Navarro and Nave(1997), Domínguez and Novales(2000)]. This line of research was originally proposed to reduce the dimensionality of a usually large vector of interest rates by obtaining a simple linear representation of the term structure. However there is some sense in which representing interest rates levels by a small number of factors also leads to a simple representation of interest rates fluctuations. This is why sometimes a reference is made to the fact that the factor representation is a representation of interest rates as well as a representation of volatilities across the term structure.

On the other hand, if we have a set of time series for estimated volatilities for each of a large set of maturities across the term structure, we can directly search for a factor representation of the set of volatility time series. We show in this paper that, maybe contrary to a simple intuition, the volatility series estimated from a factor model for interest rates are not equal to those obtained from a factor model for volatilities. This observation may have significant consequences for many issues related to risk management in fixed income markets, like Value at Risk (VaR) analysis, in which a numerical estimate of the future evolution of risk over the term structure is needed, to be compared with that obtained from similar markets.

The rest of the paper is organized as follows. In section 2 we describe the data used in the paper and present univariate estimates of conditional volatilities across the term structure. A factor analysis approach is used in section 3 to reduce the dimensionality of the vector of volatilities across the term structure. As an alternative, in section 4 we use a factor model for daily interest rates changes to estimate volatility at the specific maturities we consider, and evaluate the ability of the volatility factor model to account for volatility across the term structure of interest rates. The two approaches we have used to produce a factor representation for volatilities across the term structure are compared in section 5. The differences between them in estimating VaR are presented in section 6. We close with the main conclusions in section 7.

2. The data

We use closing daily prices from the secondary market for Spanish government debt to estimate the Nelson-Siegel model every day, from which zero-coupon rates can be inferred for any maturity. We focus on 1-, 3-, 6-, 8-, and 10-month rates, together with 1-, 3-, 5-, 6-, 7-, 8-, 9-, and 10 year rates. Our sample runs from September 1st, 1995 to December 31st, 2002.

Since January 1999, when the European Monetary Union was created, the European Central Bank, together with the individual central banks, have been in charge of implementing monetary policy in all country members, among them Spain. Before that, Banco de España was the single official organism in charge of monetary police in Spain. Over the sample period considered, not only the institution in charge of monetary police, but also the way how policy is implemented, have changed. It is then almost mandatory to perform the common factor study in two different subsamples. The first sample covers from September 1st, 1995 to December 31st, 1998, the pre-monetary union period, while the second sample runs from January 4th, 1999 to December 31st, 2002.

An EGARCH(1,1) model can be shown to adequately represent the conditional volatility in both subsamples, with parameter estimates being shown in Table 1. Table 2 presents sample correlations between any two volatilities. Correlations among the conditional volatility of short term interest rates were higher in the second than in the first subsample. On the contrary, correlations among the conditional volatility of the longer term interest rates were higher in the first than in the second subsample. The conditional volatility of the one month rate shows a high correlation with the volatilities of the 3-, 6-, 8-, 10-month and 1-year interest rates, while the conditional volatility of the 10-year rate displays a large correlation with the volatility of the 3-, 5-, 6-, 7-, 8-, 9-year rates of interest. It looks as if there is substantial volatility transmission across adjacent maturities, whereas transmission of volatility between the two extremes of the term structure is much less obvious. In addition, the central region, represented by the one year maturity, seems to display some specific properties. This preliminary evidence suggests that it might be hard to obtain a good representation of volatility across the term structure with just two factors, and that al least three factors might be needed. Exploring that possibility is the object of the next sections.

3. A principal component analysis of volatilities along the term structure

In an attempt to reduce the dimensionality of the vector of 13 time series of conditional volatilities, we compute their principal components. The first five eigenvalues of the variance-covariance matrix of conditional volatilities are 24.85, 10.73, 2.29, 0.41 and 0.30 in the first sample, with a percent cumulative explained variance of 63.53%, 90.98%, 96.84%, 97.89% and 99.35%. In consistency with observations in the previous section, three principal components would be enough to capture 95% of the time variation in the conditional volatilities, while up to five principal components would be needed to capture 99% of the time variation. The explanatory ability increases somewhat in the second sample, in which cumulative explained percent variance is: 82.99%, 92.86%, 97.46%, 98.98 and

99.54%. In this case, the first four factors capture 99% of the time variation in volatility, although again, three of them would be enough to capture 95% of the variation in the whole set of volatility time series.

Table 3 shows that, for the first sample, the coefficients defining the first principal component are quite similar over the whole term structure, so that this component can be interpreted as the general level of volatility. The second component is represented with coefficients of the same sign over the short-end of the term structure (1-month to 1-year), and coefficients of opposite sign over the 3- to 10- year maturity range. Even though the coefficients change somewhat for the different maturities, this component can be interpreted as representing the difference between the levels of volatility between the two ends of the term structure. In that respect, it is worthwhile noting that the volatility of the 1-year interest rate does not have any presence in this second component.

The loadings of the long term volatilities in the composition of the third principal component are almost zero, so that this component is represented as a linear combination of volatilities in the shorter end of the term structure. Because of the signs of the different coefficients, changes in this third component would imply changes of different sign in the volatilities of the 1-, 3-, 6- month rates, relative to changes in the volatility of the 8-, 10-, month and 1 year rates. This third component could be interpreted as representing changes in the curvature at the short end of the term structure of volatilities.

Results in the second subsample are similar regarding the third component, while there are significant differences for the first and second principal components. The loadings of long term volatilities in the composition of the first principal component are now almost zero so that, in the second subsample, this component can be seen to represent the general level of volatility in the short end of the term structure, since all coefficients there share the same sign. The loadings of short term volatilities in the composition of the second principal component are almost zero while the longer maturities enter with the same sign, so that this second component can now be seen as representing the general level of volatility in the long end of the term structure. The third component captures again changes in the curvature of volatility at the short end of the term structure.

To evaluate the ability of the first three principal components to account for the conditional volatility at each of the 13 maturities considered, we use the three components as explanatory variables in a system of regression equations having alternatively the volatility at each maturity as the dependent variable. We will refer to this system as the *factor model for interest rate volatilities*.

Figures 1(a) to 12(a) present the conditional volatility obtained from an univariate EGARCH(1,1) model estimated for each of the 13 maturities considered in the first sample (except for the 10-month maturity)[1], together with the volatility obtained for each maturity from the estimated factor model. The conditional volatility obtained from the factor model seems to exhibit a very similar behaviour to the volatility estimated with the univariate EGARCH(1,1) model. The major differences between both series are observed in the 1-, 3- and 10-year interest rates. This is best seen in figures 1(a,b) to 12(a,b), were we present a

[1] Which we have excluded for reasons of space.

scatter graph of both series at each maturity. The same comment can be made about results for the second subsample (see figures 13(a,b) to 24(a,b)).

In the first subsample, the regression R-square is very high in all cases, being above 95% for most maturities (table 4). The ability of the first three components to explain the volatility of the 10 month, 1- and 3 years interest rate is a little lower. The fit in the second subsample is very similar, although the explanatory power for the 3-, 9- and 10 year interest rates is now somewhat lower.

Mean Absolute Errors for the linear projections of volatility on the first three components is very low in each of the two subsamples and for each of the 13 maturities considered, being below 0.3 basis points in all cases (table 4). With only a few exceptions, Root Mean Square Errors (RMSE) in table 4 are below 5% in the first sample, reflecting the fact that the three first principal components explain, on average, 95% of the fluctuation in volatility over the term structure. RMSE values increase up to almost 10% for the 10 month, 1- and 3 year maturities. RMSE values are a bit higher in the second sample, but they remain below 10% in all cases.

In summary, we have shown that a relatively simple representation can account for the time behaviour of volatility over the term structure of interest rates. As a consequence, we can obtain volatility forecasts for a large set of interest rates at different maturities by forecasting just three variables, the first three principal components. Volatility forecasts are central for many applications in risk management, so the relevance of our analysis is that it allows us to measure portfolio risk with a minimum effort.

4. A principal component analysis of interest rate changes

If we have a good model to account for the term structure of interest rates, it is natural to think that this model should also be able to account for the behaviour of interest rate volatility. Following this view, we have used a factor model created to explain interest rate fluctuations, to estimate the variance and covariance matrix of a large set of interest rates.

Alexander (2000) obtained the variance-covariance matrix of a large set of interest rates by just estimating the variance of the first three principal components of interest rate changes. Gento (2001) estimated the variance-covariance matrix of a large set of interest rates from the secondary Spanish public debt market by just estimating the variance of two variables: the 4-month rate and the spread between the 7-year and the 4-month rate. Abad and Benito (2005) use the Nelson and Siegel model, which represents the zero-coupon curve through four parameters, to generate the variance-covariance matrix for a large set of interest rates by just estimating the variance of daily time series of estimated parameters.

An alternative way to estimate the volatility of all interest rates in the term structure with a minimum cost is to use the volatility factor model we describe in the previous section. But then, the question is whether the volatility representation that emerges from a factor model for interest rate changes will be the same as the one we get from a factor model for volatility. To provide an answer to this question we have computed the first three principal components of daily interest rate changes.

The percentage variance explained by the first principal component in the first subsample is of 51.04%. The second and third components explain, respectively, a 40.54% and 5.80% of the variance, so that the first three components together explain more than 95% of daily changes in the variance along the term structure of interest rates. At a difference of results obtained with the vector of univariate conditional volatilities in the previous section, now both subsamples produce very similar results. In the second sample the percent variance explained by the first component is 58.8%, while the percentage of variance explained by the second and third components is 35.2% and 4.16%, respectively. The percent cumulative variance explained by the first three components is in this case of 98.1%.

Table 5 shows that, for the first sample, the coefficients defining the first principal component are again quite similar over the whole term structure, so that this component can be seen to represent daily global shifts across the whole term structure of interest rates. The second component is characterized by coefficients of opposite sign at both ends of the term structure, so that this component can be interpreted as a slope component of interest rate changes. Finally, the third component can be interpreted as a curvature component. These results are similar to those presented in Section 3. Results for the second sample are also quite similar to those in Section 3 and admit the same interpretation as in the first subsample, with only some minor differences. These results are fully in line with similar ones obtained for different international fixed income markets in previous work referred to in the Introduction.

The ability of the first three principal components to explain daily changes in interest rates can be examined by estimating:

$$dR_{j,t} = \sum_{i=1}^{3} \phi^{j}{}_{i} df_{i,t} + \varepsilon_{j,t} \qquad (1)$$

where $dR_{j,t}$ represents daily changes in interest rate at the j –th maturity, for j = 1, 3-, 6-, 12-month, 1-, 3-, 5-, 6- and 10-years, and $\boldsymbol{df_{i,t}}$, i =1, 2, 3, represents the first three principal components of daily changes in interest rates. The R-squared of the regression is quite high in all cases, being generally above 95% in both subsamples. The Mean Absolute Error is below 1 basis point for all maturities in each of the two samples considered (Table 6).

Following Alexander (2000), we estimate the variance-covariance matrix of the vector of interest rates by:

$$Var(dr_t) = A\,Var(df_t)A^T \qquad (2)$$

where $\boldsymbol{Var(df_t)}$ is a diagonal matrix with the conditional variance of the first three principal components along the diagonal, and $\boldsymbol{A}$ is a 13 by 3 matrix having in each row the estimated coefficients from each individual regression in (1). $Var(dr_t)$ is a 13 by 13 matrix representing the conditional variance-covariance matrix of interest rates.

Coefficients in matrix $\boldsymbol{A}$ are the loadings of each interest rate in each principal component. So, in fact, it is not necessary to estimate equation (1) to get the variance-covariance matrix of a large set of interest rates, once the principal component analysis has been done. Alternatively an EGARCH(1,1) model can be used to represent the conditional volatility of the first three components in each subsample. Once we have the conditional volatility of the first three components, we use expression (2) to get the conditional volatility of the 13 interest rates considered.

As we did in the previous section, we compare the conditional variance estimates obtained from this procedure with those obtained from a univariate EGARCH(1,1) specification for each interest rate. Time fluctuations in the conditional variances obtained from the factor model for interest rate changes are similar to those experienced by univariate EGARCH conditional variances. However, the conditional variances obtained from a factor model for interest rate volatilities fit the univariate EGARCH conditional variances significantly better than the volatility obtained from a factor model for interest rate changes, as can be seen by comparing Figures 1(a,b)-12(a,b) with Figures 25(a,b)-36(a,b), and Figures 13(a,b)-24(a,b) with Figures 37(a,b)-48(a,b).

5. Do factor models for interest rate volatilities and for interest rate changes lead to the same volatility estimates?

Table 7 presents Root Mean Square Errors for both factor models: the one for interest rate volatilities, and the one for interest rate changes. In the first subsample, the Root Mean Square Error for the factor model of volatilities remains below 5% for most maturities. The Root Mean Square Error for the factor model for interest rate changes is above 16% in all cases. In the second subsample, the Root Mean Square Error for the factor model of volatilities is a bit higher in almost all maturities although for the interest rate to 8 months, 10 months, 1 year and 3 years is lower. For most maturities, the Root Mean Square Error for the factor model for interest rate changes is more than three times the Root Mean Square Error for the factor model for volatilities. Table 8 presents the Mean Absolute Error for both models. In each of the two samples and for all maturities the Mean Absolute Error is higher for the factor model for changes in interest rates. These results suggest that the volatility representation obtained from a factor model for interest rate volatilities is different from the one obtained from a factor model for interest rate changes. We now proceed to formally test that hypothesis.

Mann-Whitney and Kruskal-Wallis statistics to test for whether both volatility series have the same mean are shown in Table 9. In the first subsample, both statistics offer evidence against the null hypothesis of homogeneity for the shorter maturities, up to 3-years. For the longer maturities, we do not find evidence against such hypothesis. In the second subsample, we find evidence against the null hypothesis of equal means for most maturities. The Siegel-Tuckey statistic to test for whether the conditional volatilities produced by the two models have the samevariance is presented in the same table. For most maturities, this statistic offers strong evidence against the null hypothesis.

Finally, we test for whether the probability distributions for the volatility representation we get from the volatility factor model and for the volatility that can be obtained from the factor model for interest rate changes are the same. Table 10 contains the values of the Wilcoxon and the Kolmogorov-Smirnov statistics. In both subsamples and for all maturities, the two statistics offer evidence against the null hypothesis of homogeneity of the probability distribution for the volatility series generated by the two factor models.

Summarizing, the analysis in this section has shown that the conditional volatility estimates produced by the factor model for interest rate volatilities and the factor model for interest rate changes display statistically significant differences being the first approach, as it should be expected, the one that fits better the volatilities that emerge from a univariate specification. This evidence might be taken to suggest that in order to estimate risk over the term structure, we might be better off by using a volatility factor model than an interest rate factor model. Unfortunately, it is unclear that the conditional volatility series we obtain through univariate specifications constitute a better estimate of risk than the volatility series we get from an interest rate factor model. We now turn in the next section to analyzing the ability of both factor models for risk evaluation of fixed-income assets.

6. Value at Risk under alternative volatility estimates

In general, a risk manager will be directly interested on the performance of a given volatility model in estimating a risk indicator like Value at Risk (VaR). So as an extension of the analysis in the previous sections, it would be interesting to compare the ability of both models to estimate the VaR of a given fixed-income portfolio. However, that is a somewhat complex exercise that requires specification and estimation of the conditional variance-covariance matrix for the vector of interest rates, so we focus in this section on estimating VaR for a set of individual zero coupon bonds, paying attention to the performance of both factor models at short-, medium- and long-term maturities.

Using continuous discount factors, the theoretical price for the zero coupon bonds can be written,

$$p_i(i) = N\exp(-t_i r(i)) \qquad (4)$$

for i = 1-, 3-, 6-, 8-, 10-months, 1-, 3-, 5-, 6-, 7-, 8-, 9-, 10-years, where N denotes the face value of the bond, which we take to be one, t_i denotes time to maturity for the i-th bond, and $r_t(i)$ is the zero coupon rate of interest at maturity i at time t. From (4), we can approximate price changes through,

$$dp_t(i) \approx -D\, d(r_t(i)) \qquad (5)$$

where D denotes duration: $D = t_i \exp^{-t_i r(i)}$. This expression can be used to approximate the standard deviation of price changes by,

$$\sigma_{dp(i)} = D\sigma_{dr(i)} \qquad (6)$$

where $\sigma_{dp(i)}$, $\sigma_{dr(i)}$ denote conditional standard deviations for changes in the price of the *i*-th bond and in interest rates at maturity of *i* years.

Once we have the conditional mean and standard deviation for bond price changes, the VaR can be obtained,

$$VaR(\alpha\%) = \mu_{dp(i)} + \sigma_{dp(i)}\, k_{\alpha\%} \qquad (7)$$

In previous sections we have used two different methods to estimate the conditional variance of interest rate changes: a factor model for the conditional variances of a vector of interest rates, and a factor model for interest rate changes themselves. Here, we use both approaches to produce two different approximations to the variance of price changes in zero coupon bonds. Each approximation can be used in (7), in turn, to compute the VaR for each zero coupon bond under consideration.

We perform this exercise at a 5% and 1% confidence level and a one-day horizon, for each of the zero coupon bonds considered. We then examine actual daily price changes in the theoretical zero coupon bonds, as implied by daily fluctuations in zero coupon interest rates, and compare them with the 5% and 1% VaR. If the estimation of the theoretical VaR is appropriate, we should expect about 5% and 1% of daily price changes to be below these thresholds. Our first sample being of size 813 data points, that amounts to 41 daily price changes below the 5% VaR and 8 daily price change below 1% VaR. The size of the second sample considered is 992, suggesting that about 50 daily price changes should be below the 5% VaR and 10 daily price change below 1% VaR.

The results for the first subsample are shown in table 11. The absolute and relative frequencies of price changes below the 5% and 1% VaR are shown for each bond. For short-term bonds, between 1- and 10-month maturity, we observe between 14 and 24 daily price changes below 5% VaR with the factor model for volatilities, and between 20 and 27 daily price changes for the factor model for interest rate changes. This amounts to a percentage between 1.7% and 2.9% under the volatility factor model, and between 2.5% and 3.3% under the factor model for interest rate changes. The 5% VaR obtained from both methods is overstimating risk, since the number of days that the price changes by less than the 5% VaR is below its theoretical level. The 1% VaR estimate seems to perform somewhat better, approching the 1% theorical confidence level with both models. None of them seems to perform better than the other in estimating VaR.

For medium- and longer-term bonds both, the 5% and the 1% VaR estimates are relatively accurate, the simulation results approaching the 5% and 1% theoretical confidence level. Again, no model seems to do better than the other in estimating VaR. Nevertheless, the relative frequency of daily price changes below the 5% VaR is for almost all bonds lower than the theoretical level, so both models seem to overestimate the level of risk at this confidence level. On the other hand, the relative frequency of daily price changes below the

1% VaR is for almost all bonds above the theoretical level, so both models seem to underestimate the level of risk at this lower confidence level.

Results for the second sample are displayed in table 12. The number of daily price changes below the 5% VaR for short-term bonds now falls between 45 and 51, with a relative frequency between 4.5% and 5.1%. Relative frequencies obtained under the factor model for interest rates fall between 3.7% for the 3-month bond and 5.2% for the 10-month bond. Differences between both models are minor, although the factor model for volatilities seem to produce relative frequencies closer to the 5% level. In medium and long-term bonds, we have a relative frequency between 4.0% and 4.9% of daily price changes below the 5% VaR under the factor model for volatilities, and between 4.7% and 5.5% under the factor model for interest rate changes. In this second sample, the factor model for interest rate changes seems to produce a slight underestimation of risk.

In order to formally test wheter VaR estimate are accurate, we have used the perfomance test propoused by Kupiec(1995). Following this author, we define a random variable x. Taking the value 1 if the portfolio value changes below VaR(α %) and 0 in other case. In a sample of n data, the number of exceptions, that is, the number of days that the portfolio value change falls below VaR(α %) is distributed as a binomial (n, p) with $p = \alpha$. After building a confidence interval using this distribution, we reject the null hypothesis that $p=\alpha$ if the number of exceptions is out of the confidence interval. In this case, we will then say that the VaR estimate is not accurate. The opposite will happen if the number of exceptions is inside the interval.

For medium and long-term bonds, the number of exceptions we found in both samples and with both volatility models falls inside the interval for 5% and 1% VaR. So, VaR estimates seem accurate for these bonds. For short-term bonds results are not so good. In the first sample, the number of exceptions falls outside the interval for the 5% VaR. However, for the 1% VaR, the number of exceptions is inside the interval. The opposite happen in the second sample. These results are similar for both volatility models[2].

Summarizing, both models produce reasonable VaR estimates for medium- and long-term bonds. VaR estimates are not very accurate for short-term bonds. For them, both models overestimate risk at a 5% confidence level, underestimating risk at the 1% confidence level. This analysis sugfest again that no model seems to produce a better VaR estimate than the other. Differences in VaR estimation between both factor models are very small, so both should be considered essentially equivalent from the point of view of risk evaluation.

7. Conclusions

Searching for a sensible factor representation of the term structure of interest rates has been object of study for some time. If interest rates at any given maturity could be

[2] At a 5% confidence level and a sample of size 812, as our first subsample, the confidence interval for the 5% VaR is [29, 53]. For the 1% VaR, the confidence interval is [3, 14]. Confidence intervals for the second subsample are: [37, 63] for the 5% VaR and [4, 16] for the 1% VaR.

written, to a reasonable approximation, as a linear combination of a small number of factors, then fluctuations of the yield curve could be characterized by just analyzing the behaviour of the chosen factors. The linear factor representation for the term structure leads to a natural representation for volatility across the term structure as a linear transformation of factor volatilities.

An alternative way to represent volatility across the Term Structure of Interest Rates is by means of a factor model for interest rate volatilities. Even though it might seem as if volatility estimates for a given maturity obtained from a volatility factor model ought to be similar to those obtained from an interest rate factor model, that proposition should be tested, specially because estimating volatilities through a factor model for interest rate changes is a standard procedure. The purpose of this paper has been, in fact, to test whether both volatility representations are statistically equivalent. For several reasons, including the nonstationarity of interest rates at all maturities, we work with daily changes, comparing the volatilities that can be obtained from the factor model for interest rate changes, with those that emerge from the factor model of volatilities in daily changes in interest rates.

We have used zero-coupon rates from the Spanish secondary public debt market for 1-, 3-, 6-, 8-, and 10-month, 1-, 3-, 5-, 6-, 7-, 8-, 9-, 10- year maturities. To check the robustness of our results, we have split the sample in two. The first sub sample runs from September 1st, 1995 to December 31st, 1998, (the period before the European Monetary Union), while the second covers the January 4th, 1999 to December 31st, 2002 period.

As a first, more standard approach, we have constructed the first three principal components of daily interest rate changes, which explain more than 95% of the variability in the term structure, and we have used an EGARCH(1,1) model to estimate their conditional variance. The projection of daily changes in each individual interest rate on the three principal components is used to estimate the conditional volatility for each of the 13 interest rates considered from the univariate conditional variance time series for the components.

An alternative approach uses an EGARCH(1,1) specification to estimate the conditional volatility of each single interest rate considered. The first three principal components of the set of conditional variances explain more than 95% of the variability in the term structure of volatilities. We then estimate the volatility along the term structure using the linear projections of volatility at each of the 13 maturities considered on the three principal components for volatility.

To test if the volatility series estimated by both models are statistically equivalent we implement a variety of formal non-parametric tests. By and large, the evidence is contrary to such hypothesis, so that the election of the model used to estimate conditional volatilities across the term structure of interest rates is, in general, far from irrelevant. So, this analysis suggests that there is not the same information regarding volatility across the term structure in the volatility of a vector of interest rates than in interest rates themselves. It might also be the case that some information on second order moments is lost when computing a small set of principal components for interest rate changes.

Quite naturally, the factor model for interest rate volatilities fits the set of univariate EGARCH volatilities across the term structure much better than the volatilities obtained from the factor model for daily interest rate changes. One might be tempted to conclude that the volatility factor model should be preferred over the interest rate factor model when estimating risk across the term structure. However, it is unclear that the volatility series we obtain through univariate modelling constitutes an appropriate estimation of risk, so the preference of the volatility factor model over the interest rate factor model is not fully warranted.

Precisely because of that, we have also examined whether statistically significant differences in volatility estimation are relevant for risk estimation. As a first analysis, we have just considered individual zero coupon bonds of different maturities, leaving the analysis of portfolio risk for further research, since it requires a more laborious specification of conditional covariances over the term structure. Our results suggest that, at least for this specific set of assets, differences in estimated volatilities do not lead to noticeable differences in Value at Risk estimation, so a risk manager might be indifferent between the two factor model approaches, in spite of their statistical differences.

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Table 1 (a): Estimates of the univariate model (conditional mean and variance equations)

Sample: September 1995 to December 1998

	Autorregresive model								EGARCH(1,1) model			
	α	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	σ	ω	θ	β	γ
1 m.	0.003	-0.184	-0.059	-0.065	-0.053			0.106	-0.573	0.230	-0.045	0.909
	(0.003)	(0.036)	(0.043)	(0.045)	(0.040)				(0.081)	(0.026)	(0.018)	(0.014)
3 m.	-0.001	-0.204	-0.047					0.082	-0.521	0.236	-0.021	0.929
	(0.002)	(0.034)	(0.045)						(0.057)	(0.022)	(0.017)	(0.009)
6 m.	-0.004	-0.189	-0.092	0.057	0.044	0.140	0.045	0.057	-0.268	0.216	-0.047	0.980
	(0.001)	(0.031)	(0.037)	(0.044)	(0.041)	(0.042)	(0.039)		(0.024)	(0.018)	(0.017)	(0.003)
8 m.	-0.006	-0.169	-0.036	0.062	0.106	0.091	0.075	0.049	-0.292	0.2050	-0.033	0.975
	(0.001)	(0.033)	(0.037)	(0.040)	(0.040)	(0.037)	(0.036)		(0.024)	(0.024)	(0.016)	(0.003)
10 m.	-0.008	-0.124	-0.025	0.051	0.069	0.039		0.044	-0.723	0.242	-0.046	0.911
	(0.001)	(0.036)	(0.039)	(0.038)	(0.047)	(0.044)			(0.086)	(0.030)	(0.021)	(0.012)
1 y.	-0.009	-0.084	-0.028	0.042				0.045	-3.424	0.373	-0.011	0.494
	(0.001)	(0.043)	(0.039)	(0.032)					(0.662)	(0.055)	(0.036)	(0.101)
3 y.	-0.007	0.022	-0.009					0.054	-0.171	0.132	-0.011	0.988
	(0.001)	(0.035)	(0.037)						(0.043)	(0.025)	(0.012)	(0.004)
5 y.	-0.007	0.029	-0.054					0.058	-0.161	0.113	-0.028	0.987
	(0.002)	(0.037)	(0.036)						(0.031)	(0.018)	(0.010)	(0.003)
6 y.	-0.007	0.030	-0.075					0.059	-0.161	0.102	-0.028	0.985
	(0.002)	(0.037)	(0.036)						(0.034)	(0.018)	(0.009)	(0.004)
7 y.	-0.007	0.023	-0.081					0.059	-0.169	0.102	-0.028	0.984
	(0.002)	(0.037)	(0.036)						(0.038)	(0.019)	(0.010)	(0.005)
8 y.	-0.008	0.011	-0.081					0.059	-0.169	0.103	-0.028	0.983
	(0.002)	(0.038)	(0.037)						(0.038)	(0.019)	(0.010)	(0.005)
9 y.	-0.008	-0.005	-0.081					0.059	-0.204	0.116	-0.031	0.979
	(0.002)	(0.040)	(0.037)						(0.046)	(0.021)	(0.012)	(0.006)
10 y.	-0.007	-0.016	-0.088					0.059	-0.204	0.116	-0.031	0.979
	(0.002)	(0.040)	(0.038)						(0.046)	(0.021)	(0.012)	(0.006)

The models used to estimate the conditional mean and variance are:

$$\nabla r_t = \alpha + \sum_{i=1}^{p} \delta_i \nabla r_{t-i} + \varepsilon_t \qquad (1)$$

$$\log h_t^2 = \omega + \theta\left(\frac{\varepsilon_t}{h_t}\right) + \beta\left(\left|\frac{\varepsilon_t}{h_t}\right| - \left(\frac{2}{\pi}\right)^{1/2}\right) + \gamma \log h_{t-1}^2 \qquad (2)$$

σ is the standard deviation of regression.

Table 1 (b): Estimates of the univariate model (conditional mean and variance equations)

Sample: January 1999 to December 2002

	Autorregresive model							EGARCH(1,1) model			
	α	δ_1	δ_2	δ_3	δ_4	δ_5	σ	ω	θ	β	γ
1 m.	-0.004	-0.168	-0.115	0.073	-0.050		0.079	-1.202	0.429	-0.179	0.833
	(0.001)	(0.036)	(0.036)	(0.031)	(0.024)			(0.076)	(0.027)	(0.019)	(0.011)
3 m.	-0.003	-0.127	-0.044	0.074	-0.055	-0.014	0.071	-1.102	0.527	-0.113	0.870
	(0.001)	(0.035)	(0.033)	(0.035)	(0.031)	(0.022)		(0.064)	(0.028)	(0.021)	(0.009)
6 m.	-0.001	-0.043	-0.059	0.044			0.064	-1.001	0.449	-0.111	0.882
	(0.001)	(0.037)	(0.037)	(0.034)				(0.054)	(0.023)	(0.021)	(0.008)
8 m.	0.000	-0.096	-0.051	0.051			0.059	-1.133	0.399	-0.122	0.855
	(0.001)	(0.037)	(0.036)	(0.037)				(0.069)	(0.020)	(0.019)	(0.011)
10 m.	0.000	-0.109	-0.056				0.057	-1.395	0.385	-0.137	0.810
	(0.001)	(0.038)	(0.036)					(0.095)	(0.019)	(0.018)	(0.015)
1 y.	-0.001	-0.113	0.000				0.055	-1.756	0.376	-0.147	0.749
	(0.001)	(0.042)	(0.035)					(0.132)	(0.018)	(0.017)	(0.021)
3 y.	-0.001						0.053	-0.301	0.139	-0.036	0.966
	(0.001)							(0.050)	(0.017)	(0.011)	(0.008)
5 y.	-0.001						0.054	-0.333	0.157	-0.038	0.963
	(0.002)							(0.067)	(0.021)	(0.014)	(0.010)
6 y.	-0.000	-0.031					0.052	-0.359	0.164	-0.034	0.961
	(0.001)	(0.034)						(0.085)	(0.025)	(0.016)	(0.012)
7 y.	-0.000	-0.032					0.050	-0.366	0.164	-0.030	0.960
	(0.001)	(0.033)						(0.101)	(0.027)	(0.016)	(0.015)
8 y.	-0.000	-0.029					0.047	-0.352	0.158	-0.023	0.963
	(0.001)	(0.032)						(0.011)	(0.028)	(0.017)	(0.016)
9 y.	-0.000						0.046	-0.349	0.151	-0.018	0.962
	(0.001)							(0.104)	(0.028)	(0.015)	(0.015)
10 y.	-0.000						0.044	-0.303	0.137	-0.014	0.969
	(0.001)							(0.095)	(0.027)	(0.014)	(0.013)

Note: see note to Table 1(a).

Table 2: Sample correlations

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
1 m.	1.00	0.94	0.81	0.69	0.60	0.61	0.46	0.45	0.43	0.42	0.41	0.40	0.39
3 m.		1.00	0.89	0.77	0.68	0.67	0.39	0.37	0.35	0.34	0.33	0.33	0.32
6 m.			1.00	0.96	0.83	0.80	0.29	0.27	0.25	0.23	0.22	0.21	0.18
8 m.				1.00	0.90	0.83	0.26	0.22	0.20	0.19	0.18	0.16	0.13
10 m.					1.00	0.82	0.31	0.26	0.25	0.23	0.22	0.20	0.19
1 y.						1.00	0.50	0.49	0.48	0.46	0.45	0.43	0.40
3 y.							1.00	0.96	0.94	0.92	0.91	0.89	0.88
5 y.								1.00	1.00	0.99	0.98	0.97	0.95
6 y.									1.00	1.00	0.99	0.98	0.97
7 y.										1.00	1.00	0.99	0.98
8 y.											1.00	1.00	0.99
9 y.												1.00	1.00
10 y.													1.00

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
1 m.	1.00	0.95	0.88	0.85	0.81	0.74	0.13	0.16	0.21	0.25	0.26	0.25	0.22
3 m.		1.00	0.97	0.93	0.87	0.80	0.09	0.12	0.18	0.24	0.26	0.26	0.23
6 m.			1.00	0.99	0.94	0.88	0.14	0.15	0.22	0.28	0.30	0.29	0.25
8 m.				1.00	0.98	0.94	0.20	0.20	0.26	0.31	0.32	0.30	0.25
10 m.					1.00	0.98	0.25	0.24	0.29	0.32	0.32	0.30	0.24
1 y.						1.00	0.28	0.26	0.30	0.32	0.32	0.29	0.23
3 y.							1.00	0.94	0.91	0.86	0.80	0.74	0.69
5 y.								1.00	0.99	0.95	0.88	0.81	0.73
6 y.									1.00	0.98	0.94	0.88	0.80
7 y.										1.00	0.98	0.94	0.87
8 y.											1.00	0.98	0.94
9 y.												1.00	0.98
10 y.													1.00

Table 3: Representation of the principal components

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
First principal component	0.52	0.44	0.38	0.27	0.17	0.15	0.22	0.22	0.20	0.18	0.18	0.18	0.17
Second principal component	-0.20	-0.25	-0.32	-0.25	-0.13	-0.03	0.32	0.34	0.32	0.31	0.31	0.31	0.32
Third principal component	0.58	0.28	-0.32	-0.49	-0.37	-0.30	-0.08	-0.06	-0.05	-0.05	-0.04	-0.02	0.00

Note: the table shows the coefficients of each volatility in each principal component.

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
First principal component	0.52	0.55	0.43	0.33	0.27	0.22	0.03	0.03	0.04	0.04	0.04	0.04	0.03
Second principal component	-0.12	-0.14	-0.02	0.04	0.08	0.09	0.43	0.46	0.41	0.36	0.32	0.29	0.27
Third principal component	0.64	0.16	-0.29	-0.36	-0.40	-0.41	0.03	0.08	0.08	0.07	0.07	0.07	0.08

Note: the table shows the coefficients of each volatility in each principal component.

Table 4: Goodness of fit for the principal component model for univariate conditional volatilities

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Coefficient of determination (R^2)	0.99	0.97	0.98	0.99	0.86	0.84	0.90	0.99	0.99	0.99	0.99	0.98	0.96
Mean Absolute Error	0.17	0.17	0.23	0.13	0.31	0.28	0.37	0.13	0.09	0.09	0.12	0.16	0.20
Root Mean Square Error	4.7	4.1	6.3	4.9	8.8	9.7	10.3	3.1	2.1	2.2	2.9	3.7	4.7

Note: the table shows the coefficient of each volatility in the definition of each principal component.

(*) basis point

(++) percentage

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Coefficient of determination (R^2)	0.99	0.99	0.98	1.00	0.98	0.93	0.87	0.95	0.98	0.97	0.93	0.85	0.75
Mean Absolute Error *	0.27	0.28	0.24	0.09	0.18	0.27	0.31	0.18	0.11	0.11	0.17	0.22	0.29
Root Mean Square Error $^{++}$	5.7	8.1	7.0	2.3	4.4	6.7	7.4	4.2	2.7	2.9	4.6	6.8	8.7

Note: the table shows the coefficient of each volatility in the definition of each principal component.

(*) basis point

(++) percentage

Table 5: Characterization of principal components
(daily changes in interest rate)

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
First principal component	0.44	0.37	0.29	0.25	0.21	0.17	0.21	0.25	0.26	0.26	0.26	0.26	0.26
Second principal component	-0.52	-0.39	-0.22	-0.13	-0.04	0.03	0.25	0.27	0.27	0.27	0.27	0.27	0.27
Third principal component	0.40	0.09	-0.25	-0.39	-0.46	-0.50	-0.25	0.06	0.12	0.14	0.15	0.14	0.12

Note: the table shows the weight of each interest rate in the composition of the first three principal component

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
First principal component	0.46	0.43	0.40	0.37	0.35	0.33	0.17	0.11	0.09	0.08	0.08	0.07	0.07
Second principal component	-0.20	-0.17	-0.11	-0.07	-0.03	0.01	0.32	0.40	0.40	0.38	0.36	0.34	0.32
Third principal component	0.55	0.26	-0.05	-0.21	-0.33	-0.42	-0.40	-0.05	0.06	0.13	0.18	0.20	0.22

Note: the table shows the weight of each interest rate in the composition of the first three principal component

Table 6: Goodness of fit for the principal component model for daily interest rate changes

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Coefficient of determination (R^2)	0.99	0.98	0.99	0.99	0.95	0.88	0.91	0.97	0.99	1.00	0.99	0.98	0.96
Mean Absolute Error *	0.40	0.17	0.30	0.31	0.30	0.39	1.10	0.69	0.37	0.16	0.32	0.57	0.81

(*) MAE in basis points

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Coefficient of determination (R^2)	0.99	1.00	1.00	0.99	0.99	0.99	0.95	0.96	0.98	1.00	0.99	0.95	0.89
Mean Absolute Error *	0.49	0.13	0.18	0.28	0.31	0.30	0.64	0.65	0.40	0.10	0.27	0.59	0.88

(*) MAE in basis points

Table 7: Root Mean Square Error for each volatility

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Volatility factor model	4.7	4.1	6.3	4.9	8.8	9.7	10.3	3.1	2.1	2.2	2.9	3.7	4.7
Interest rate factor model	17.7	20.0	34.2	30.4	18.0	16.5	21.0	18.8	18.4	18.5	18.6	18.8	19.4

Note: RMSE as a percentage

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Volatility factor model	5.7	8.1	7.0	2.3	4.4	6.7	7.4	4.2	2.7	2.9	4.6	6.8	8.7
Interest rate factor model	31.2	34.2	15.9	11.6	15.1	19.0	22.4	15.2	12.3	10.6	10.7	12.2	14.9

Note: RMSE as a percentage

Table 8: Mean Absolute Error

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Volatility factor model	0.17	0.17	0.23	0.13	0.31	0.28	0.37	0.13	0.09	0.09	0.12	0.16	0.20
Interest rate factor model	1.32	1.15	1.22	0.98	0.59	0.49	0.80	0.82	0.78	0.79	0.79	0.81	0.85

Note: MAE in basis points

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Volatility factor model	0.49	0.13	0.18	0.28	0.31	0.30	0.64	0.65	0.40	0.10	0.27	0.59	0.88
Interest rate factor model	1.37	1.13	0.53	0.48	0.63	0.80	0.84	0.57	0.43	0.35	0.36	0.42	0.53

Note: MAE in basis points

Table 9. Test for equal mean and variance

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Mann-Whitney	2.56	2.54	5.31	5.91	1.99	4.14	2.03	0.52	0.06	0.28	0.06	0.55	1.48
	(0.01)	(0.01)	(0.00)	(0.00)	(0.05)	(0.00)	(0.04)	(0.60)	(0.95)	(0.78)	(0.95)	(0.58)	(0.14)
Kruskal-Wallis	6.53	6.43	28.16	34.88	3.97	17.15	4.12	0.27	0.00	0.08	0.00	0.31	2.18
	(0.01)	(0.01)	(0.00)	(0.00)	(0.05)	(0.00)	(0.04)	(0.60)	(0.95)	(0.78)	(0.95)	(0.58)	(0.14)
Siegel-Tukey	0.81	5.01	82.86	107.2	25.18	11.64	60.98	30.94	10.27	5.06	4.00	4.67	7.11
	(0.37)	(0.03)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.02)	(0.05)	(0.03)	(0.01)

Note: p-values in parenthesis

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Mann-Whitney	9.90	9.83	4.65	0.60	0.60	3.07	6.39	3.74	2.75	2.49	3.33	5.57	9.23
	(0.00)	(0.00)	(0.00)	(0.55)	(0.55)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)
Kruskal-Wallis	97.95	96.53	21.63	0.36	9.48	40.84	46.84	13.97	7.58	6.19	11.09	31.01	85.21
	(0.00)	(0.00)	(0.00)	(0.55)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)
Siegel-Tukey	43.43	53.82	1.36	11.74	47.92	114.02	28.61	17.63	27.18	40.30	60.26	82.58	104.92
	(0.00)	(0.00)	(0.24)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Note: p-values in parenthesis

Table 10: Sample homogenity test

Sample: September 1995 to December 1998

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Wilcoxon	3.2	1.6	5.7	6.0	0.1	-11.6	-5.3	-3.7	-3.3	-3.2	-3.8	-5.0	-6.7
Kolmogorov-Smirnov(*)	0.072	0.083	0.201	0.207	0.100	0.100	0.116	0.081	0.075	0.072	0.065	0.072	0.083

(*) Critical values for the Kolmogorov test are 0.055, 0.067 and 0.081 at confidence levels of 90%, 95% and 99%.

Critical values for Wilcoxon test are 1.29, 1.65 and 2.3 at a confidence level of 90%, 95% and 99%.

Sample: January 1999 to December 2002

	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Wilcoxon	17.3	19.3	16.9	6.0	-5.7	-10.3	-14.0	-13.5	-11.5	-10.1	-11.3	-15.3	-19.7
Kolmogorov-Smirnov(**)	0.246	0.236	0.100	0.047	0.112	0.206	0.152	0.103	0.091	0.102	0.119	0.156	0.239

(**) Critical values for the Kolmogorov test are 0.050, 0.061 and 0.073 at confidence levels of 90%, 95% and 99%.

Critical values for Wilcoxon test are 1.29, 1.65 and 2.3 at a confidence level of 90%, 95% and 99%.

Tabla 11. September 1995 to December 1998

	Volatility factor model												
	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Observations below:													
VaR(5%)	24	20	17	14	24	38	38	35	37	36	37	36	35
VaR(1%)	9	8	7	4	6	11	10	11	13	14	14	14	13
Relative frequencies:													
VaR(5%)	2.9	2.5	2.1	1.7	2.9	4.7	4.7	4.3	4.5	4.4	4.5	4.4	4.3
VaR(1%)	1.1	1.0	0.9	0.5	0.7	1.3	1.2	1.3	1.6	1.7	1.7	1.7	1.6

	Interest rate factor model												
	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Observations below:													
VaR(5%)	21	20	21	21	27	45	42	38	34	35	38	40	42
VaR(1%)	10	8	8	8	11	19	9	11	13	13	13	11	11
Relative frequencies:													
VaR(5%)	2.6	2.5	2.6	2.6	3.3	5.5	5.1	4.7	4.2	4.3	4.7	4.9	5.1
VaR(1%)	1.2	1.0	1.0	1.0	1.3	2.3	1.1	1.3	1.6	1.6	1.6	1.3	1.3

Table 12: January 1999 to December 2002

	Volatility factor model												
	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Observations below:													
VaR(5%)	49	47	51	50	45	40	46	48	47	49	47	48	48
VaR(1%)	22	26	23	20	18	17	15	11	8	8	6	8	10
Relative frequencies:													
VaR(5%)	4.9	4.7	5.1	5.0	4.5	4.0	4.6	4.8	4.7	4.9	4.7	4.8	4.8
VaR(1%)	2.2	2.6	2.3	2.0	1.8	1.7	1.5	1.1	0.8	0.8	0.6	0.8	1.0

	Interest rate factor model												
	1 m.	3 m.	6 m.	8 m.	10 m.	1 y.	3 y.	5 y.	6 y.	7 y.	8 y.	9 y.	10 y.
Observations below:													
VaR(5%)	3.9	37	42	51	52	52	55	51	50	50	47	52	55
VaR(1%)	20	19	17	20	20	20	19	12	9	8	7	9	16
Relative frequencies:													
VaR(5%)	3.9	3.7	4.2	5.1	5.2	5.2	5.5	5.1	5.0	5.0	4.7	5.2	5.5
VaR(1%)	2.0	1.9	1.7	2.0	2.0	2.0	1.9	1.2	0.9	0.8	0.7	0.9	1.6

September 1, 1995 to December 31, 1998. Figures 1 to 6.

Figure 1(a). Conditional Standard Deviation
1 Month Interest Rate

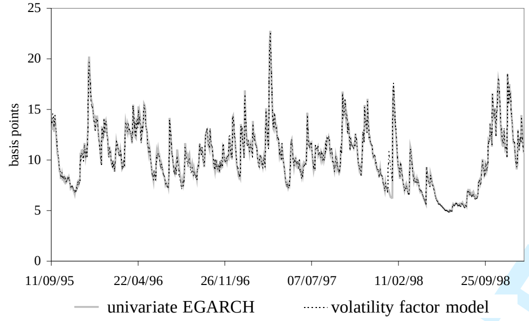


Figura 1(b). 1 month

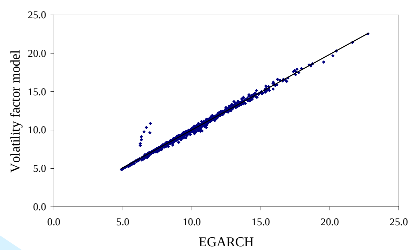


Figure 2(a). Conditional Standard Deviation
3 Month Interest Rate

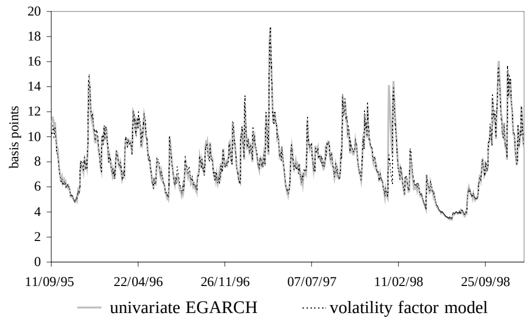


Figure 2(b). 3 month

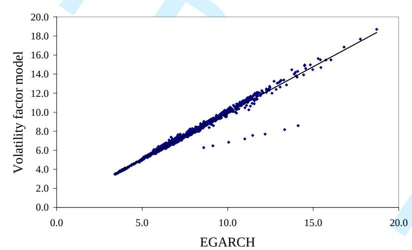


Figure 3(a). Conditional Standard Deviation
6 Month Interest Rate

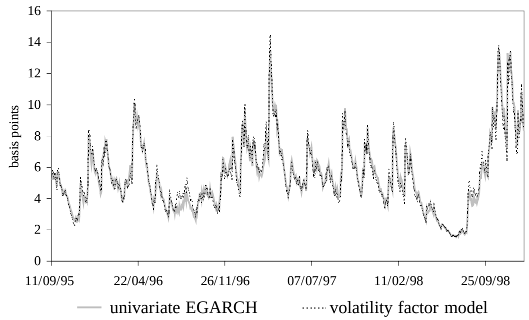


Figure 3(b). 6 month

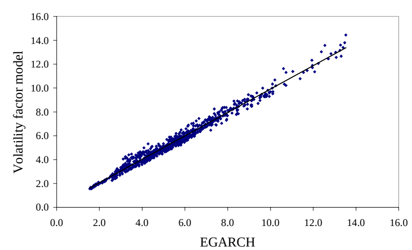


Figure 4(a). Conditional Standard Deviation
8 Month Interest Rate

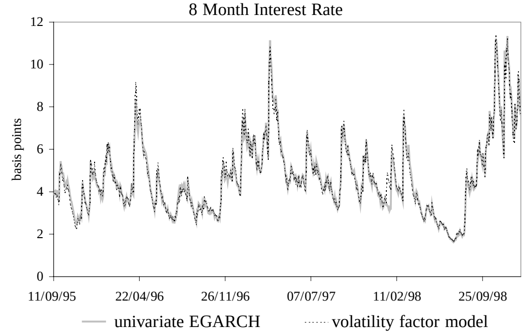


Figure 4(b). 8 month

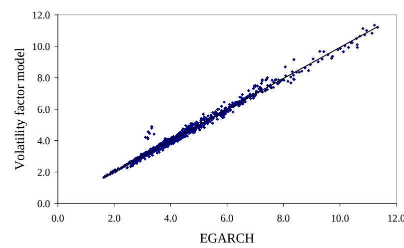


Figure 5(a). Conditional Standard Deviation
1 Year Interest Rate

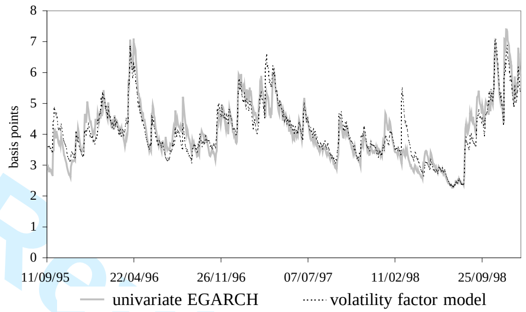


Figure 5(b). 1 year

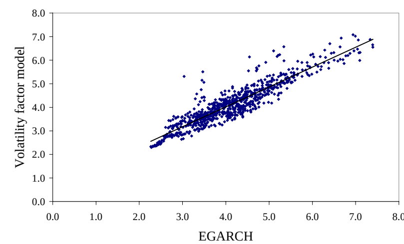


Figure 6(a). Conditional Standard Deviation
3 Year Interest Rate

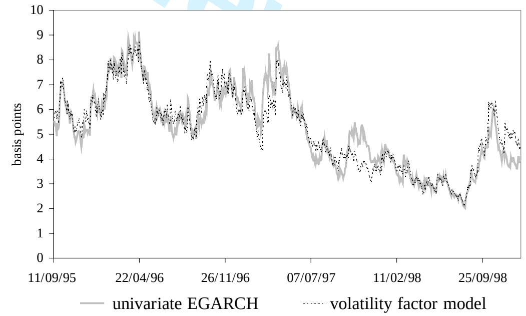
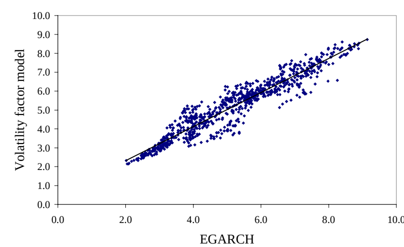


Figure 6(b). 3 year



September 1, 1995 to December 31, 1998. Figures 7 to 12.

Figure 7(a). Conditional Standard Deviation
5 Year Interest Rate

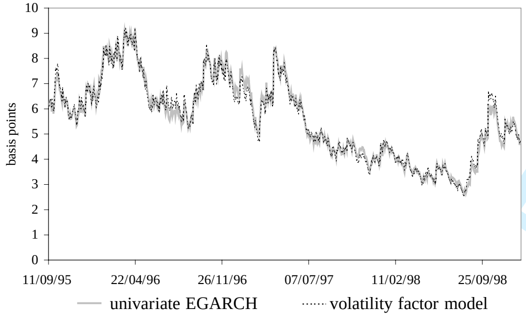


Figure 7(b). 5 year

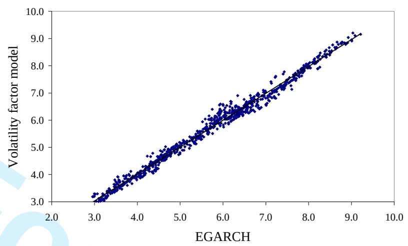


Figure 8(a). Conditional Standard Deviation
6 Year Interest Rate

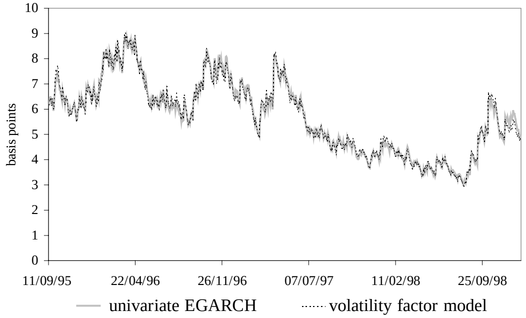


Figure 8(b). 6 year

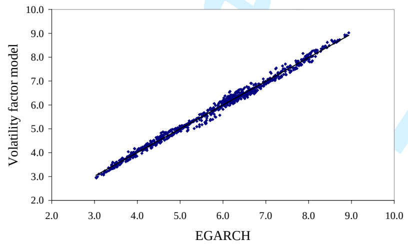


Figure 9(a). Conditional Standard Deviation
7 Year Interest Rate

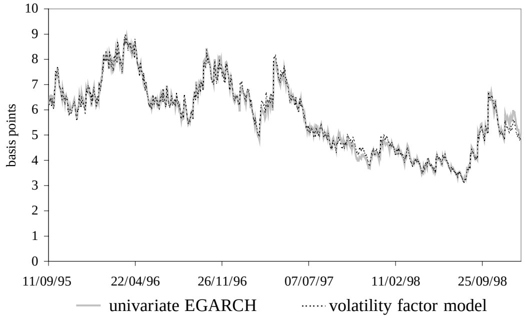


Figure 9(b). 7 year

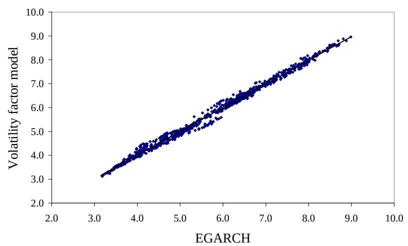


Figure 10(a). Conditional Standard Deviation
8 Year Interest Rate

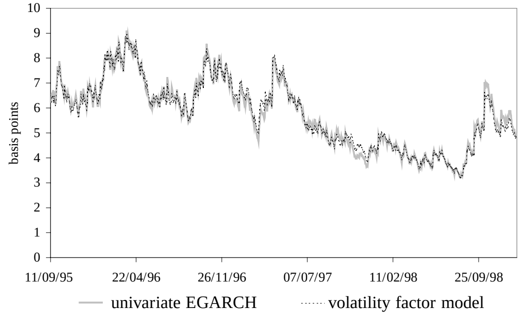


Figure 10(b). 8 year

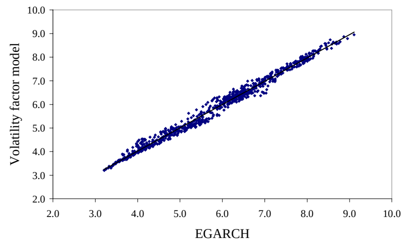


Figure 11(a). Conditional Standard Deviation
9 Year Interest Rate

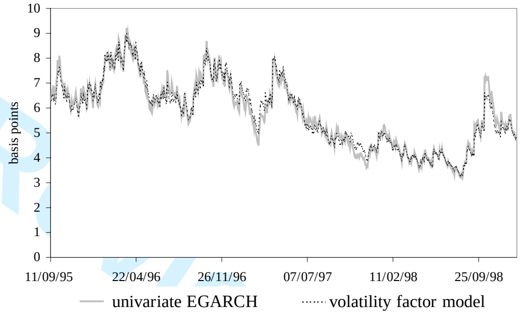


Figure 11(b). 9 year

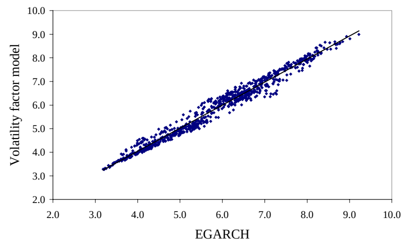


Figure 12(a). Conditional Standard Deviation
10 Year Interest Rate

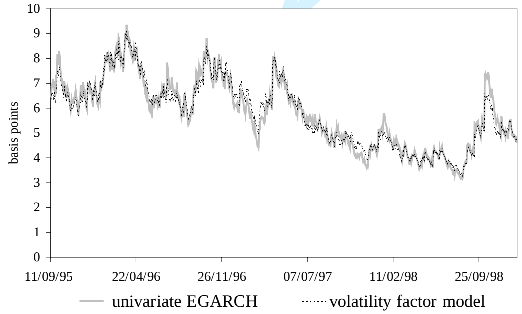
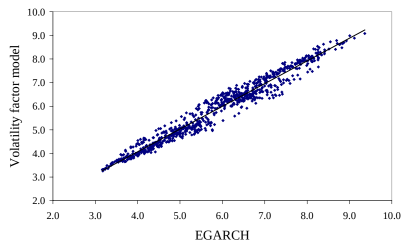


Figure 12(b). 10 year



January 4, 1999 to December 31, 2002. Figures 13 to 18.

Figure 13(a). Conditional Standard Deviation
1 Month Interest Rate

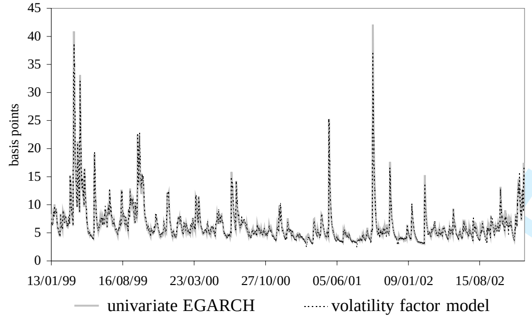


Figura 13(b). 1 month

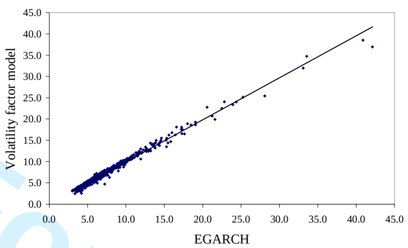


Figure 16(a). Conditional Standard Deviation
8 Month Interest Rate

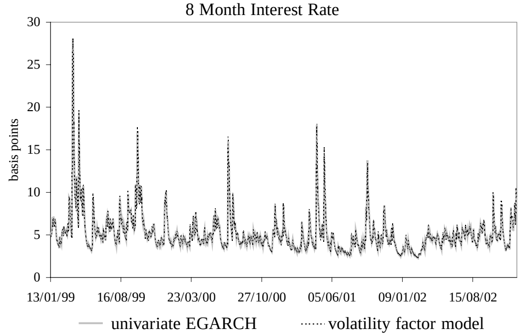


Figure 16(b). 8 month

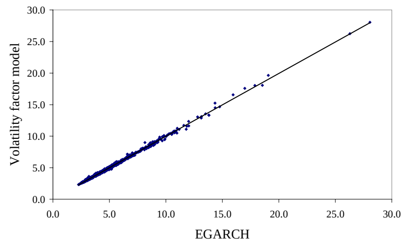


Figure 14(a). Conditional Standard Deviation
3 Month Interest Rate

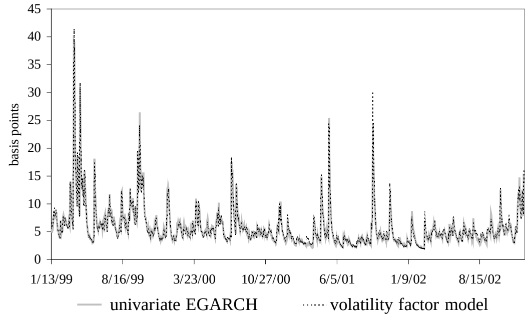


Figure 14(b). 3 month

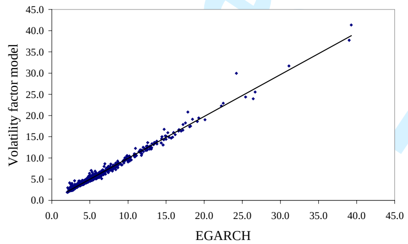


Figure 17(a). Conditional Standard Deviation
1 Year Interest Rate

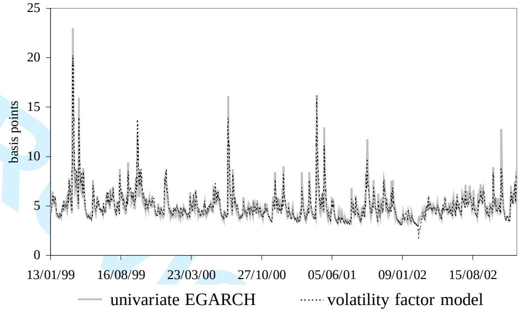


Figure 17(b). 1 year

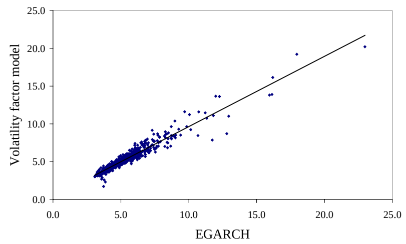


Figure 15(a). Conditional Standard Deviation
6 Month Interest Rate

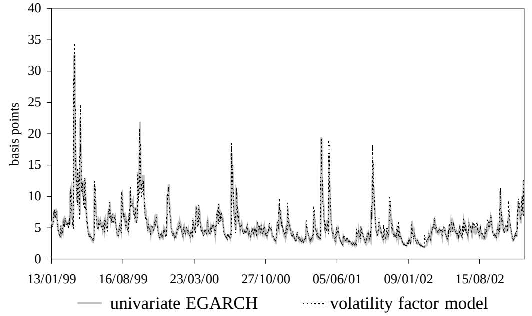


Figure 15(b). 6 month

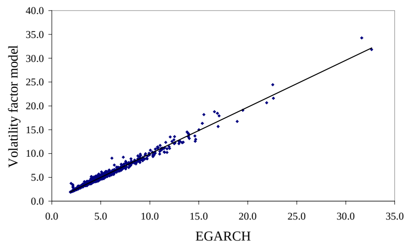


Figure 18(a). Conditional Standard Deviation
3 Year Interest Rate

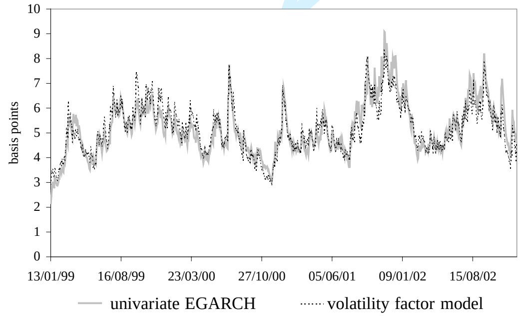
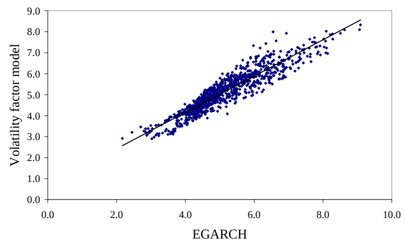


Figure 18(b). 3 year



January 4, 1999 to December 31, 2002. Figures 19 to 24.

Figure 19(a). Conditional Standard Deviation
5 Year Interest Rate

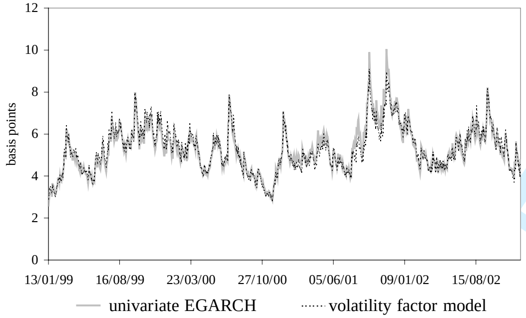


Figure 19(b). 5 year

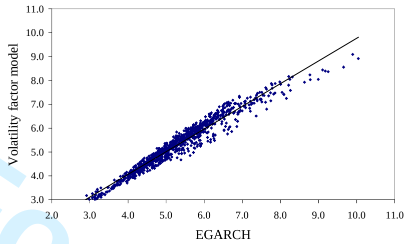


Figure 20(a). Conditional Standard Deviation
6 Year Interest Rate

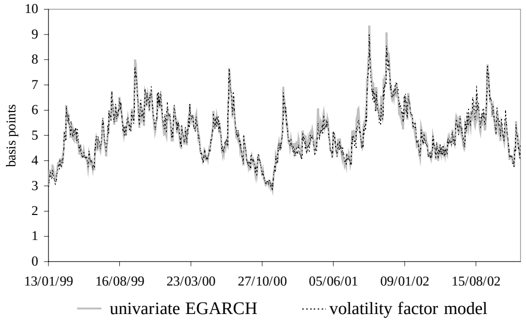


Figure 20(b). 6 year

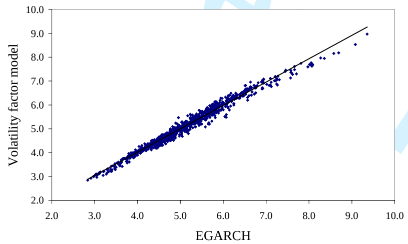


Figure 21(a). Conditional Standard Deviation
7 Year Interest Rate

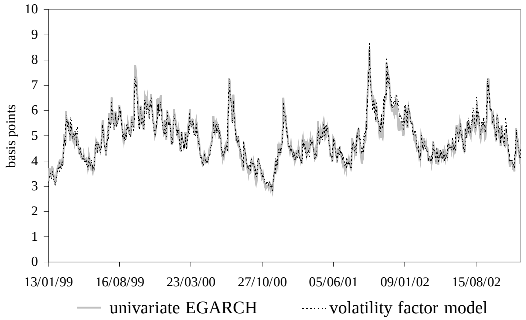


Figure 21(b). 7 year

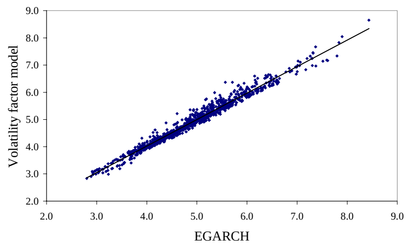


Figure 22(a). Conditional Standard Deviation
8 Year Interest Rate

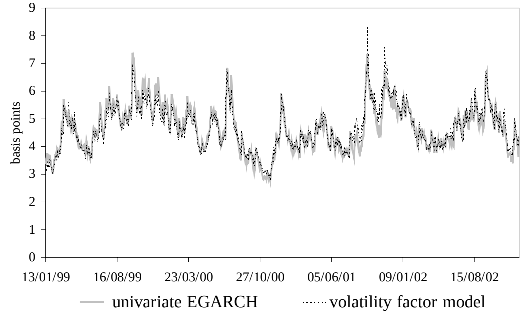


Figure 22(b). 8 year

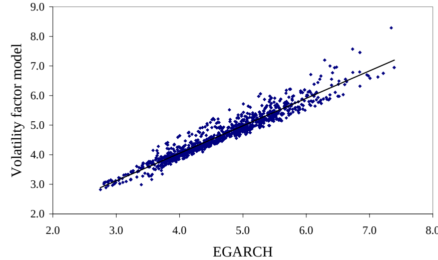


Figure 23(a). Conditional Standard Deviation
9 Year Interest Rate

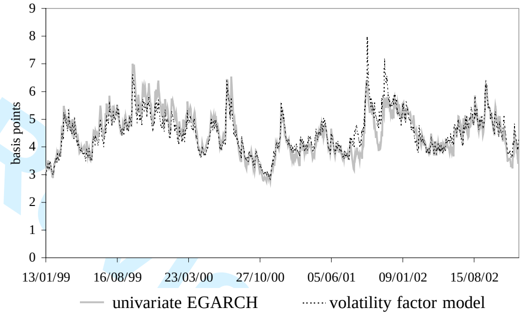


Figure 23(b). 9 year

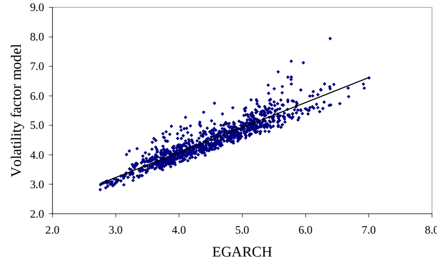


Figure 24(a).Conditional Standard Deviation
10 Year Interest Rate

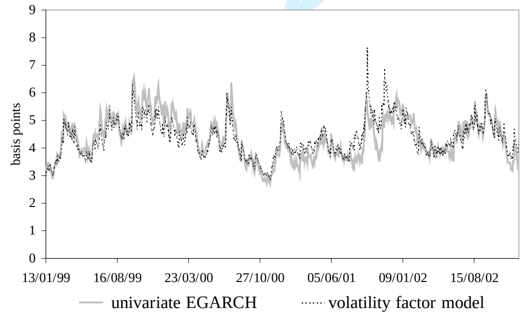
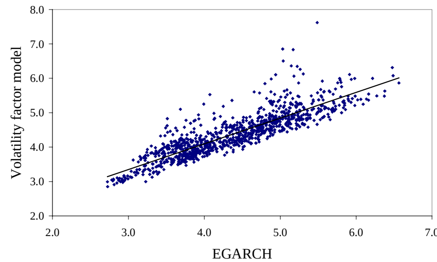


Figure 24(b). 10 year



September 1, 1995 to December 31, 1998. Figures 25 to 30.

Figure 25(a). Conditional Standard Deviation
1 Month Interest Rate

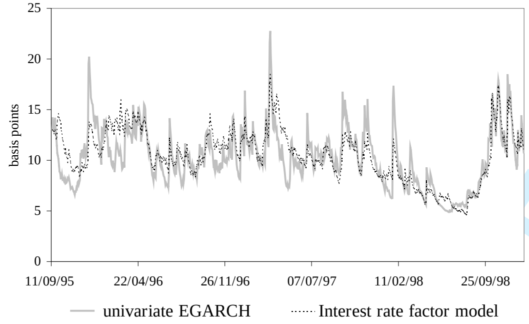


Figura 25(b). 1 month

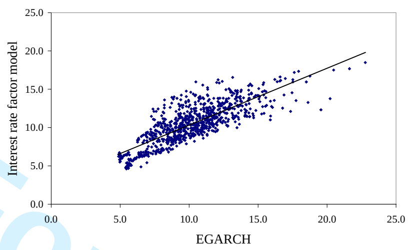


Figure 28(a). Conditional Standard Deviation
8 Month Interest Rate

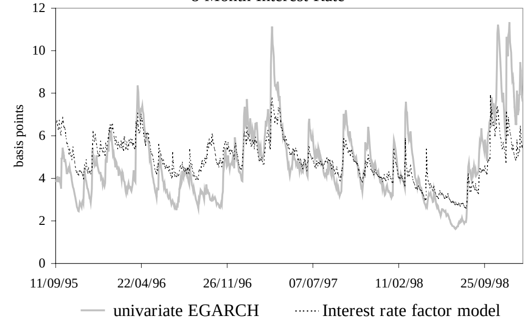


Figure 28(b). 8 month

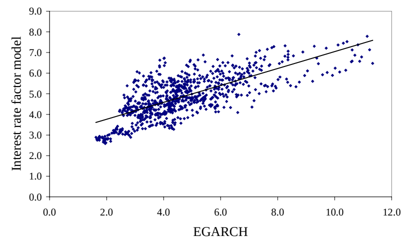


Figure 26(a). Conditional Standard Deviation
3 Month Interest Rate

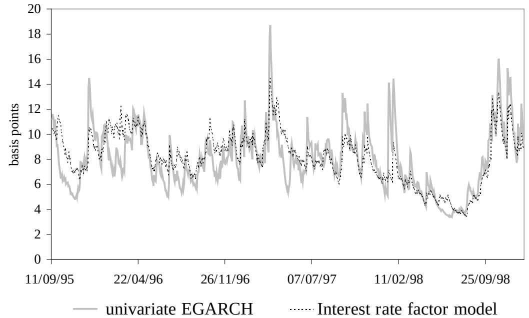


Figure 26(b). 3 month

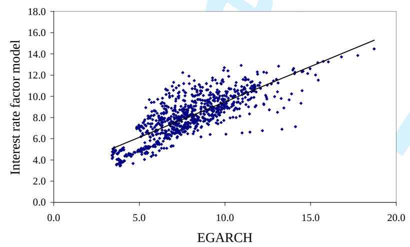


Figure 29(a). Conditional Standard Deviation
1 Year Interest Rate

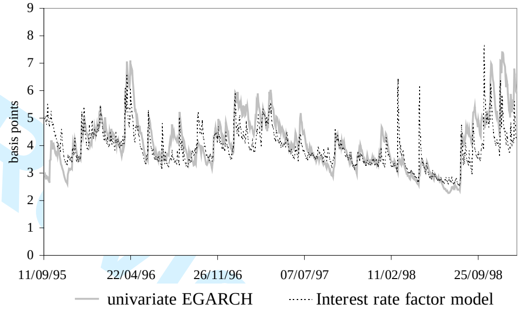


Figure 29(b). 1 year

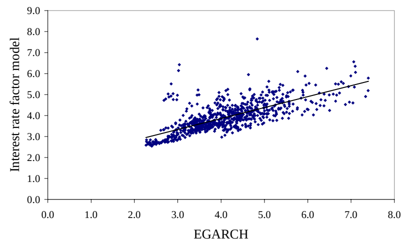


Figure 27(a). Conditional Standard Deviation
6 Month Interest Rate

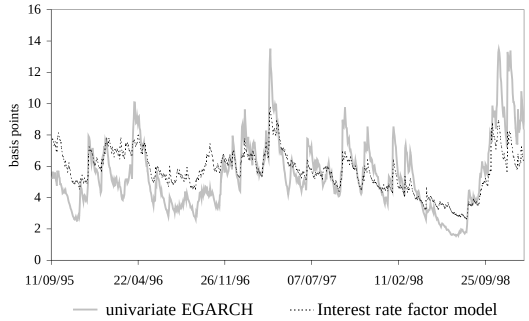


Figure 27(b). 6 month

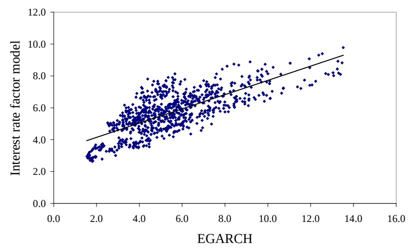


Figure 30(a). Conditional Standard Deviation
3 Year Interest Rate

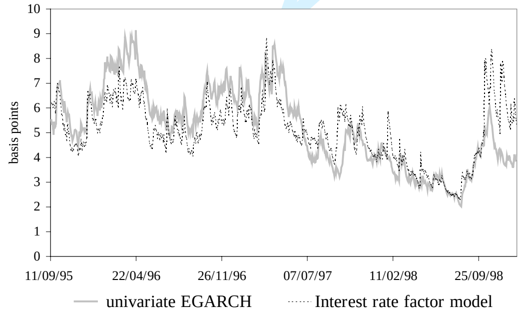
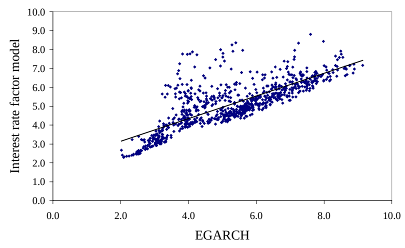


Figure 30(b). 3 year



September 1, 1995 to december 31, 1998. Figures 31 to 36.

Figure 31(a). Conditional Standard Deviation
5 Year Interest Rate

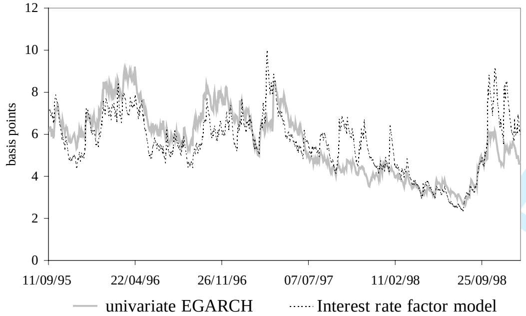


Figure 31(b). 5 year

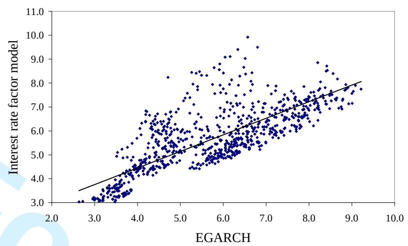


Figure 34(a). Conditional Standard Deviation
8 Year Interest Rate

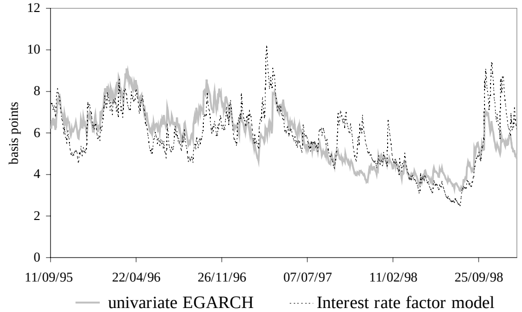


Figure 34(b). 8 year

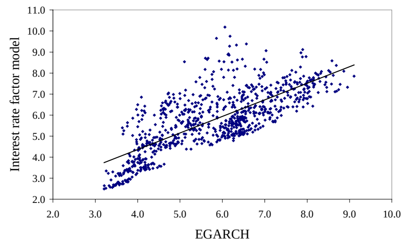


Figure 32(a). Conditional Standard Deviation
6 Year Interest Rate

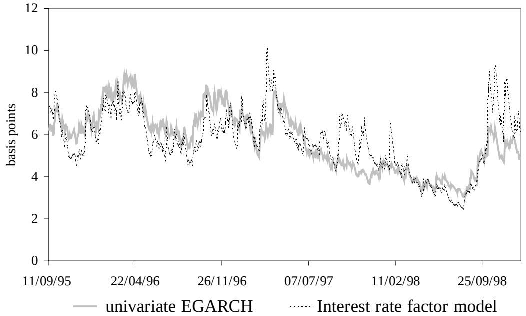


Figure 32(b). 6 year

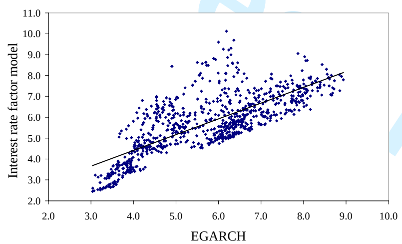


Figure 35(a). Conditional Standard Deviation
9 Year Interest Rate

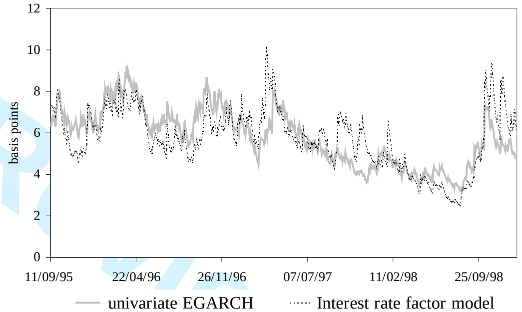


Figure 35(b). 9 year

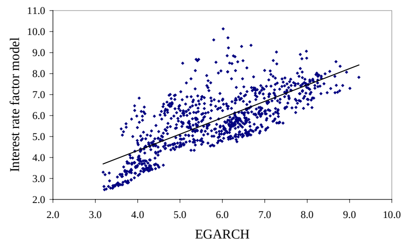


Figure 33(a). Conditional Standard Deviation
7 Year Interest Rate

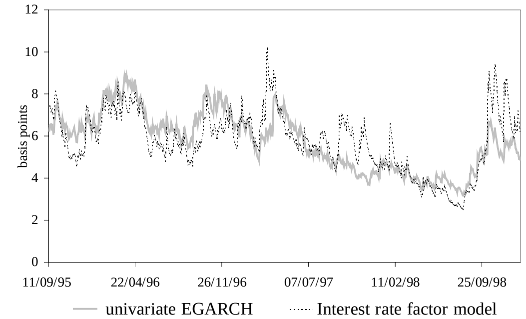


Figure 33(b). 7 year

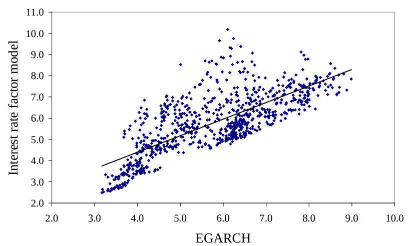


Figure 36(a). Conditional Standard Deviation
10 Year Interest Rate

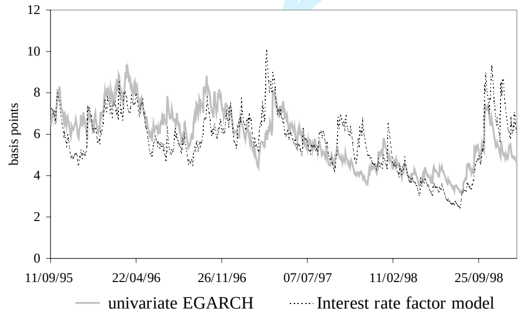
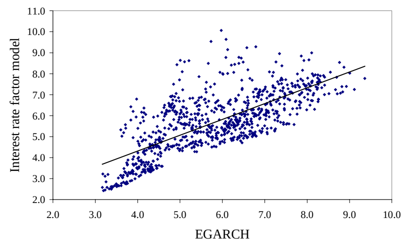


Figure 36(b). 10 year



January 4, 1999 to December 31, 2002. Figures 37 to 42.

Figure 37(a). Conditional Standard Deviation
1 Month Interest Rate

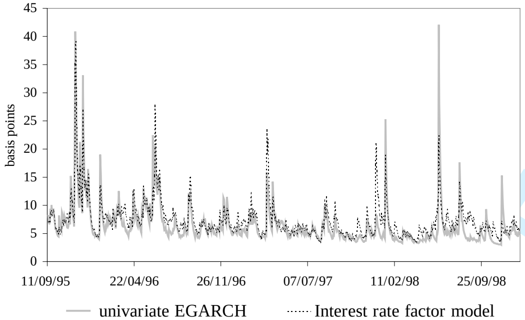


Figura 37(b). 1 month

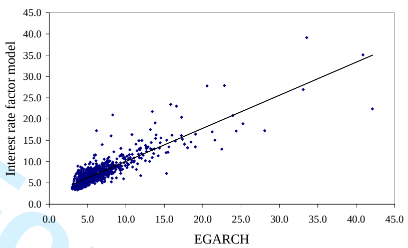


Figure 40(a). Conditional Standard Deviation
8 Month Interest Rate

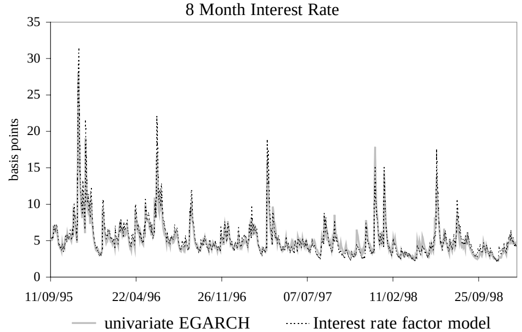


Figure 40(b). 8 month

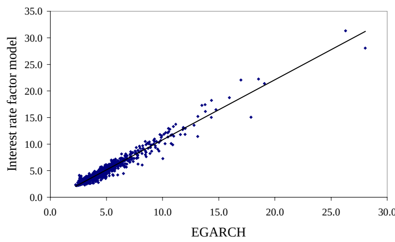


Figure 38(a). Conditional Standard Deviation
3 Month Interest Rate

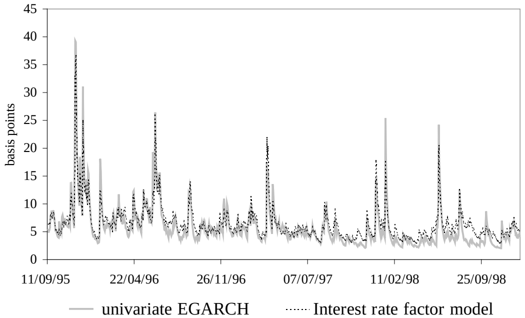


Figure 38(b). 3 month

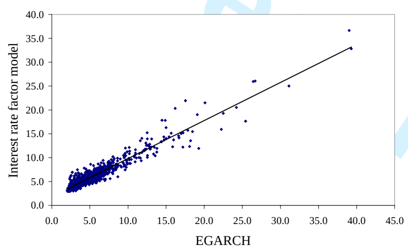


Figure 41(a). Conditional Standard Deviation
1 Year Interest Rate

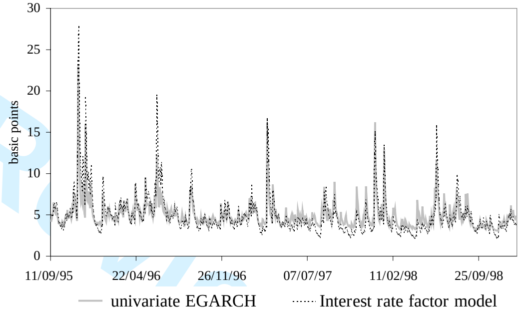


Figure 41(b). 1 year

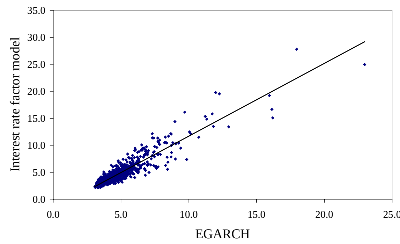


Figure 39(a). Conditional Standard Deviation
6 Month Interest Rate

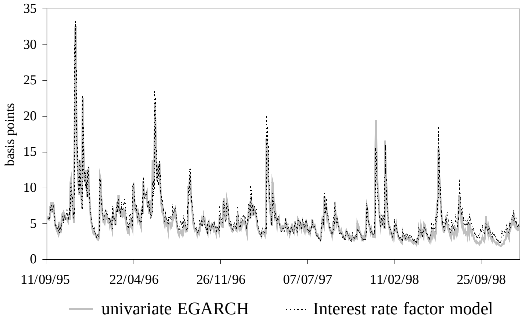


Figure 39(b). 6 month

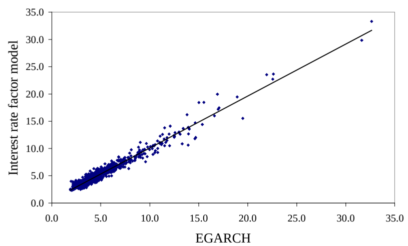


Figure 42(a). Conditional Standard Deviation
3 Year Interest Rate

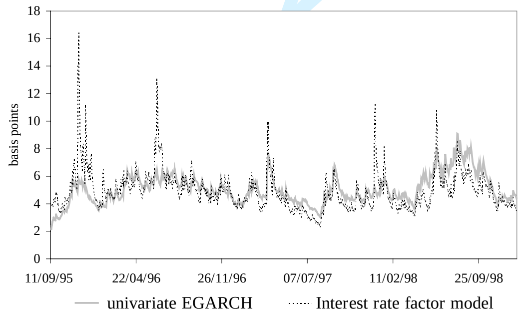
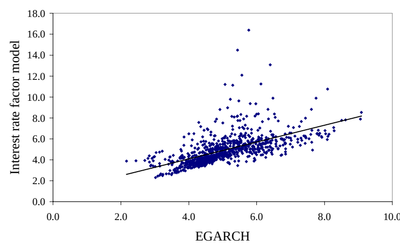


Figure 42(b). 3 year



January 4, 1999 to December 31, 2002. Figures 43 to 48.

Figure 43(a). Conditional Standard Deviation
5 Year Interest Rate

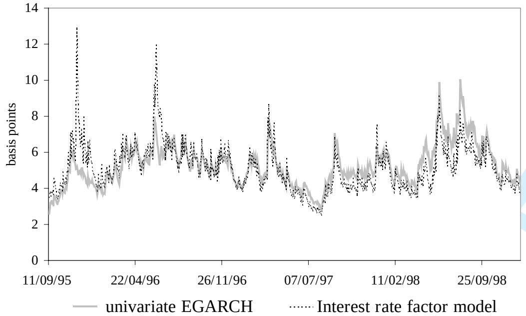


Figure 43(b). 5 year

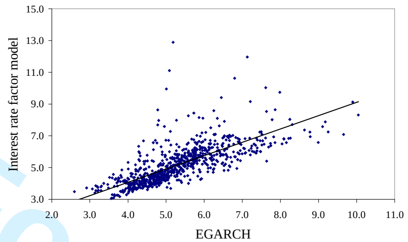


Figure 46(a). Conditional Standard Deviation
8 Year Interest Rate

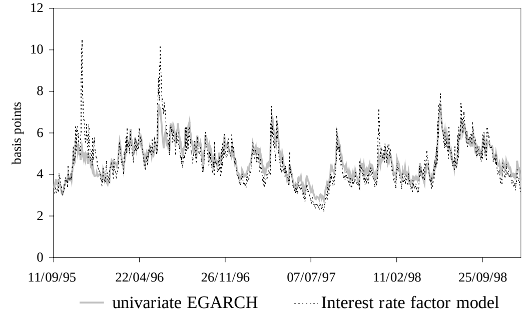


Figure 46(b). 8 year

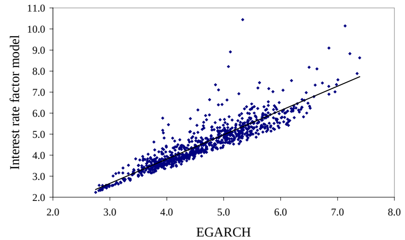


Figure 44(a). Conditional Standard Deviation
6 Year Interest Rate

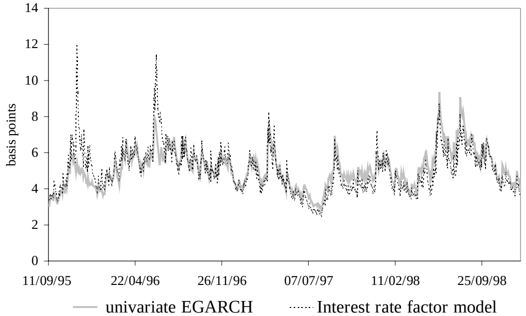


Figure 44(b). 6 year

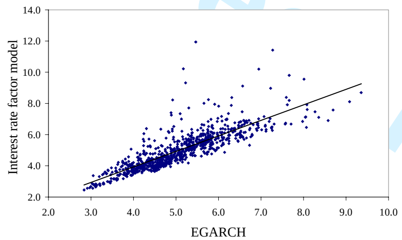


Figure 47(a). Conditional Standard Deviation
9 Year Interest Rate

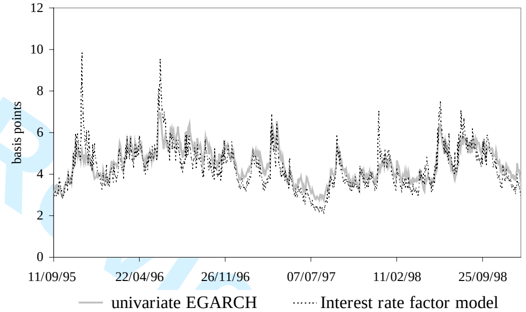


Figure 47(b). 9 year

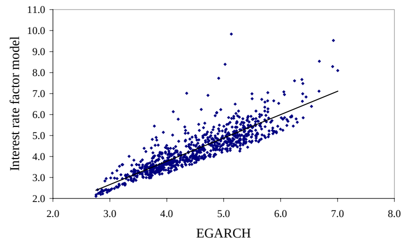


Figure 45(a). Conditional Standard Deviation
7 Year Interest Rate

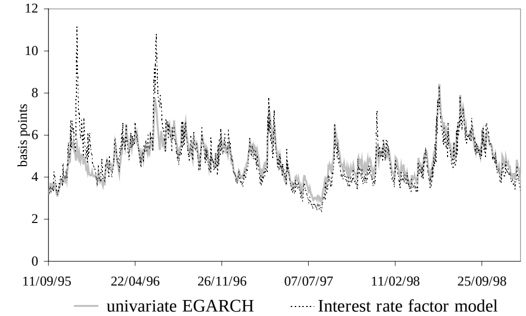


Figure 45(b). 7 year

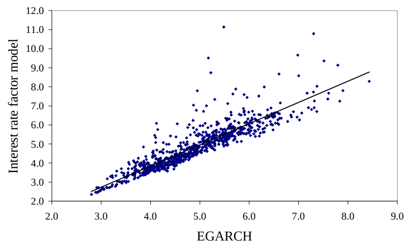


Figure 48(a). Conditional Standard Deviation
10 Year Interest Rate

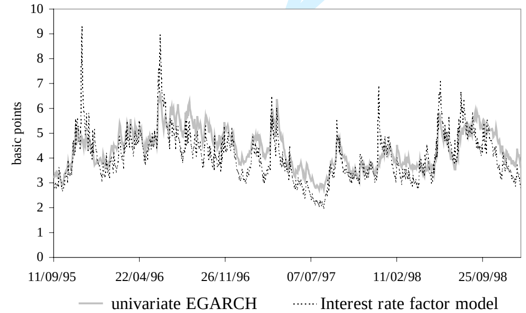


Figure 48(b). 10 year