Estimación de curva cupon cero con función descuento polínómica

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Noviembre 2012

0.1 Zero coupon curve estimation

Before describing the use of the Principal Component technique for risk management in fixed income markets, let us remember the main idea behind zero coupon curve estimation.

Note: Zero coupon curves are estimated using market prices for bonds that pay coupon. As illustration for those of you interested, I leave the 'polynomial zero coupon curve.xls' file, that solves the following exercise. A .zip file named 'nelson_siegel' will also be made available for those of yo interested in estimating Nelson-Siegel and Svensson models of zero coupon curves using Matlab.

Consider the following exercise. Today is November 5, 2011. The first column of file 'polynomial zero coupon curve.xls' contains the coupon of each bond traded in the secondary market for Government debt. The second column contains the maturity date, the third column the date the bond was first issued, which is assumed to be the same for all bonds, 15/08/2011. Each bond is assumed to have a nominal of 100 monetary units. This is just for simplification, and it cold be changed without any difficulty. Finally, we see the (average) market price for each bond.

We assume a polynomial discount function,

$$d(t) = a + bt + ct^2 + dt^3 + et^4$$

to be applied to each cash flow.

Hence, the price of a bond can be represented:

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$$P_{it} = \sum_{j=1}^{n_i} c_{ij} d_j(t) = \sum_{j=1}^{n_i} c_{ij} \left(a + bt_{ij} + ct_{ij}^2 + dt_{ij}^3 + et_{ij}^4 \right)$$

where n_i denotes the number of cash-flows to be paid by the i-th bond before maturity. We assume that all bonds pay coupon each semester (half of the annual amount).

For each vector of parameter values (a, b, c, d) we have a theoretical price for each bond. We want to find the parameter values so that

$$\underset{(a,b,c,d)}{Min} \sum_{i=1}^{N} (P_{it}^{M} - P_{it}^{T})^2$$

where P_{it}^{M} denotes the market price for each bond, and P_{it}^{T} denotes the theoretical price for that parameter vector.

The market price is 'ex coupon', meaning that we need to add to it the part of the coupon which would correspond to the current holder since the last date that a coupon was paid. To calculate that amount, we multiply the size of the next coupon payment by the proportion of the 2-month interval that has already gone by. Adding that to the 'ex coupon' market price, we get the true traded price.

The polynomial function $d_j(t)$ is the discount function, giving us the price of a bond that would mature at any future date, with a single payment, to be effective at maturity. This would be a zero coupon bond maturing t periods from now.

Estimate a discount function using a polynomial of degree 2, and another one using a polynomial of degree 4, and represent both discount functions. Draw a bar diagram with the market and the theoretical prices for each bond under each specification of the discount function.

The zero coupon curve itself, that represents zero coupon interest rates as a function of maturity, is obtained from:

$$r_t = 100\left(\left(\frac{1}{d_t}\right)^{1/t} - 1\right)$$

Draw a diagram with the zero coupon curves that obtain from the two discount functions you have estimated. In view of the results do you consider a second degree polynomial to be adequate for this market?