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Further evidence on alternative continuous time models of the short-term interest rate

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Abstract

This paper examines the stochastic behavior of the 1-month interbank rate in ten countries. Various one-factor models are estimated using an exact maximum likelihood estimator, which is based on the recently introduced Gaussian methodology. Interest rate volatility is found to be less sensitive to interest rate levels than stated in the literature. In addition, the constant elasticity variance (CEV) model is superior to other formulations in terms of data fit. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Starting with the paper of Chan et al. (1992), hereafter CKLS, the literature has addressed two related issues about the specification of interest rate dynamics: selecting a standard model best suited to capture the short-term interest rate movements and determining the sensitivity of interest rate volatility to interest rate levels. These issues are important, affecting several areas in finance such as the pricing of fixed income securities and derivatives.

CKLS examine a class of one-factor, continuous time stochastic models, which they estimate via the generalized method of moments (GMM). The main conclusion

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in CKLS is that interest rate volatility is very sensitive to interest rate levels, with an elasticity of about 1.5 for US Treasury bill rates.

Other studies look at more flexible models, which either nest or extend the CKLS formulation, and incorporate additional factors such as interest rate volatility (Longstaff and Schwartz, 1992) or expected inflation (Chen and Scott, 1992) in the general equilibrium framework of Cox et al. (1985). Bakshi and Chen (1997) study the equilibrium valuation of foreign exchange contingent claims within a two-country model, where volatility is endogenously determined. Buhler et al. (1999) seek one- and two-factor spot and forward models that are best suited for interest rate risk management, while Andersen and Lund (1997), Brenner et al. (1996) and Koedijk et al. (1997) focus on the volatility dynamics of interest rates.

Unfortunately, the existing evidence in the literature is confined mainly to the USA or the UK, with some exceptions such as Tse (1995), Dahlquist (1996) and Koutmos (1998), for instance. Thus, the need to have further international evidence on the subject is compelling.

This paper contributes to the literature by concentrating on single factor, continuous time models based on ten country series of interbank rates. The objective is to examine the performance of alternative models on data from various countries. In contrast to Tse (1995), where the CKLS methodology is employed, this paper rests on the Gaussian estimation developed by Bergstrom (1983, 1985, 1986) and applied by Nowman (1997, 1998) in the CKLS setting. A robust alternative to GMM, the Gaussian technique has the advantage that it produces an exact maximum likelihood estimator. However, any new technique should pass through empirical testing to establish its usefulness. The present paper makes a side contribution to this end by demonstrating the Gaussian estimation on a large international sample.

The paper's main conclusion is that the constant elasticity variance (CEV) model outperforms other competing models and this result is new in the literature. The second result is that the observed sensitivity of interest rate volatility to interest rates appears to be much lower than the CKLS study suggests.

The remainder of the paper is organized as follows: Section 2 describes the methodology and data used. Section 3 discusses the Gaussian estimation results and Section 4 concludes.

2. Methodology and data

Following CKLS, let us assume that the risk free interest rate obeys the stochastic process

$$\mathrm{d}r_t = (\alpha + \beta r_t)\mathrm{d}t + \sigma r_t^{\gamma}\mathrm{d}W_t \tag{1}$$

where r_t is the risk-free interest rate in month t, and $\mathrm{d}W_t$ is the increment of the standard Weiner process. Parameters α and β specify the drift and mean reversion of the process, respectively, while σ is a volatility coefficient. Parameter γ shows the degree to which the standard deviation, σr_t^{γ}, depends on the interest rate level. In other words, γ is the elasticity of volatility with respect to the interest rate. The question of the size of γ, largely an empirical issue, has been the object of ongoing debate in the literature.

The setup in Eq. (1) is quite broad, encompassing several standard models if the parameters are constrained at specific values. For instance, setting $\gamma = 0$ yields the process used in the discount bond prices model of Vasicek (1977), while setting $\gamma = \frac{1}{2}$ produces the process in the Cox et al. (1985) model[1], hereafter CIR SR. Table 1 shows some alternative formulations for given parameter restrictions.

In order to estimate Eq. (1), the model has to be in discrete form. Based on the Gaussian estimation methodology, Nowman (1997) computes the exact discrete model derived from the solution of Eq. (1) as:

$$r_t = e^{\beta} r_{t-1} + \frac{\alpha}{\beta}(e^{\beta} - 1) + \varepsilon_t \tag{2}$$

where the errors ε_t are assumed normal, are uncorrelated and have a conditional variance v_t^2 obeying

$$v_t^2 = \left(\frac{\sigma^2}{2\beta}\right)(e^{2\beta} - 1) r_{t-1}^{2\gamma} \tag{3}$$

with t running from 2 to N, the total number of observations. Maximizing the log-likelihood function

$$\operatorname{Log} L = \sum_{t=1}^{N} \left\{ -\frac{1}{2} \ln v_t^2 - \frac{1}{2}\left(\frac{\varepsilon_t}{v_t}\right)^2 \right\} \tag{4}$$

gives estimates for the four parameters, i.e. α, β, σ and γ.

Table 1
Parameter restrictions of alternative interest rate models[a]

Model	α	β	γ
Vasicek			0
CIR SR[b]			1/2
BR-SC[c]			1
CIR VR[d]	0	0	3/2
CEV	0		

[a] Several models of the short-term interest rate can be nested within the unrestricted model $dr_t = (\alpha + \beta r_t)dt + \sigma r_t^{\gamma} dW_t$.

[b] CIR SR, Cox et al. (1985).

[c] BR-SC, Brennan and Schwartz (1980).

[d] CIR VR, Cox et al. (1980).

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[1] Additional models that can be nested in Eq. (1) are Merton (1973) with $\beta = \gamma = 0$; Dothan (1978) with $\alpha = \beta = 0$, $\gamma = 1$; and Geometric Brownian Motion (GBM) with $\alpha = 0$, $\gamma = 1$. Examples of other one-factor models, which however cannot be nested in Eq. (1), are Marsh and Rosenfeld (1983), Longstaff (1989), and Constantinides (1992).

One advantage of the Gaussian estimation over other techniques such as GMM, for instance, is that it provides an exact maximum likelihood estimator. However, the technique makes the important assumption that the variance of the stochastic variables remains constant between discrete observations. Although this assumption probably does not hold, in practice it does not seem to affect the estimation of Eq. (1), as shown in the paper by Nowman (1997), at least for the case of monthly USA and UK interest rate data.

In the present study, monthly observations of the 1-month interbank rate in ten countries are extracted from the 'Datastream' database. The interbank rate is the interest rate at which banks borrow from and lend money to each other. This rate is sometimes affected by structure considerations in the banking sector of a country but is mainly related to the daily money supply steered by the monetary authorities.

Table 2 shows the basic statistics. Mean interest rates range from 3.8 (Japan) to 10.9% (New Zealand), while their standard deviations range from 0.014 (Singapore) to 0.044 (New Zealand). The first six autocorrelations of the levels decay slowly for most countries while those of 1-month differences are small and do not follow a consistent sign pattern. This provides some evidence for stationarity in the interest rates, although the augmented Dickey–Fuller (ADF) tests fail to reject a unit root in the levels series. Therefore, large differences are observed from country to country and stationarity cannot be guaranteed on the basis of ADF tests.

3. Empirical results

Before maximizing Eq. (4) to arrive at the Gaussian estimates, the results of Nowman (1997) on the UK interbank rate were adapted to calibrate the model and verify its integrity. A variety of reasonable initial parameter values was tried but did not affect the convergence of the maximum likelihood algorithm. However, the estimates were affected by shifts in the sampling period (see also Section 3.3 below).

3.1. Unrestricted models

In Table 3, the first two lines of each country section show the results for the unrestricted models. We find that the important parameter γ, which measures the degree of volatility dependence on interest rate levels, is statistically significant. However, γ differs widely from country to country, ranging from 0.20 to 1.56, although in seven out of ten countries it is less than unity. The result is in contrast to CKLS, where γ is about 1.5 for the USA. The γ estimates are much different also from those in Tse (1995), especially with respect to the USA, Australia and Belgium (where γ is found to be 1.7, 0.7 and 0.8, respectively, compared with 0.4, 1.5 and 1.6 in this paper). The discrepancy may be due to the different periods covered or the different estimation methods. However, our findings regarding γ are much in line with the stationarity restriction derived in Broze et al. (1995), namely that γ should be between zero and unity.

Table 2
Sample statistics of interbank rates and their first differences[a]

Country	Range	N	Mean	Standard deviation	ρ_1	ρ_2	ρ_3	ρ_4	ρ_5	ρ_6	ADF
Australia[1]	3/86–4/98	146	0.0969	0.0417	0.98	0.95	0.93	0.91	0.89	0.85	−1.6319
		145	−0.0007	0.0067	−0.08	0.14	−0.13	0.10	0.22	0.15	−3.9932
Belgium[2]	10/89–4/98	103	0.0677	0.0276	0.97	0.93	0.90	0.87	0.85	0.83	−1.6784
		102	−0.0006	0.0055	0.17	−0.24	−0.25	−0.12	0.12	0.06	−5.1053
Germany[2]	11/90–4/98	90	0.0602	0.0250	0.98	0.96	0.95	0.92	0.90	0.88	−1.4794
		89	−0.0006	0.0018	0.24	−0.01	0.31	0.26	0.11	0.15	−2.4717
Japan[2]	12/85–4/98	149	0.0381	0.0239	0.98	0.96	0.94	0.92	0.91	0.89	−1.8005
		148	−0.0005	0.0033	−0.14	0.08	0.38	−0.07	0.04	0.27	−4.6618
Netherlands[1]	1/79–4/98	232	0.0676	0.0254	0.97	0.94	0.92	0.88	0.85	0.82	−2.2452
		231	−0.0002	0.0060	−0.10	−0.06	0.25	−0.10	−0.06	0.19	−6.6117
New Zealand[1]	4/86–4/98	145	0.1088	0.0436	0.95	0.92	0.89	0.85	0.81	0.79	−2.6103
		144	−0.0007	0.0115	0.01	−0.09	0.08	0.09	−0.25	−0.09	−6.3356
Singapore[1]	4/86–4/98	145	0.0387	0.0144	0.90	0.81	0.76	0.69	0.62	0.56	−2.1228
		144	0.0001	0.0063	−0.06	−0.20	0.04	0.05	−0.04	−0.16	−5.4862
Switzerland[2]	1/86–4/98	148	0.0480	0.0257	0.97	0.95	0.93	0.8	0.87	0.85	−1.2138
		147	−0.0002	0.0053	−0.04	−0.16	0.17	−0.04	−0.09	0.18	−5.6833
UK[1]	1/75–4/98	280	0.1039	0.0331	0.97	0.93	0.90	0.86	0.83	0.79	−2.6234
		279	−0.0002	0.0077	0.14	0.07	−0.05	−0.05	0.02	0.04	−7.2472
USA[2]	1/86–4/98	148	0.0614	0.0179	0.98	0.96	0.94	0.92	0.89	0.87	−1.5161
		147	−0.0002	0.0035	−0.13	0.03	0.26	−0.09	0.07	0.20	−4.3947

[a] The first and second row for each country contains statistics for the 1-month interbank rate r_t and for the first difference $r_t - r_{t-1}$, respectively. Superscripts 1, 2 denote middle and offered rate, respectively. N is the number of monthly observations. ρ_i is the correlation coefficient of order i, $i = 1, 2, \ldots, 6$. ADF is the augmented Dickey–Fuller test. Source for data: Datastream.

Table 3
Gaussian estimates[a]

Country	Model[b]	α	β	σ^2	γ	Avg. Log L	χ^2-test[c]	df	S-test
Australia	Unrestricted	0.0008 (1.1774)[c]	−0.0170 (−1.7768)	0.0354 (1.4193)	1.5174 (10.6698)	4.8563			36.04
	Vasicek	0.0008 (0.5645)	−0.0158 (−1.1886)	0.0000[d] (8.6940)	0	4.5212	97.1916 (0.0000)	1	18.08
	CIR SR	0.0006 (1.1670)	−0.014 (−3.2712)	0.0004 (18.0767)	0.5	4.6923	47.5873 (0.0000)	1	21.06
	BR-SC	0.0006 (0.7127)	−0.0143 (−1.1884)	0.0031 (14.3482)	1	4.8112	13.0997 (0.0003)	1	26.92
	CIR VR	0	0	0.0325 (12.8704)	1.5	4.8483	2.3296 (0.5069)	3	40.67
	CEV	0	−0.0051 (−1.3370)	0.0333 (1.4874)	1.5059 (10.9783)	4.8532	0.91 (0.3401)	1	36.25
Belgium	Unrestricted	0.0007 (0.8434)	−0.0192 (−1.0321)	0.1147 (0.9298)	1.5617 (7.8914)	4.9362			9.03
	Vasicek	0.0008 (0.6658)	−0.0204 (−1.3262)	0.0000 (7.4045)	0	4.7156	45.0076 (0.0000)	1	27.24
	CIR SR	0.0006 (0.5797)	−0.0173 (−1.1224)	0.0004 (14.8371)	0.5	4.8259	22.5112 (0.0000)	1	22.28
	BR-SC	0.0006 (0.6899)	−0.0168 (−1.0024)	0.0054 (7.2833)	1	4.9026	6.8344 (0.0089)	1	15.87
	CIR VR	0	0	0.0808 (22.5978)	1.5	4.9303	1.2079 (0.7511)	3	9.35
	CEV	0	−0.0043 (−0.6738)	0.1039 (0.9047)	1.5452 (7.8098)	4.9327	0.6995 (0.4029)	1	9.21
Germany	Unrestricted	0.0002 (0.3783)	−0.0133 (−1.6556)	0.0001 (0.8922)	0.5501 (2.8852)	5.8864			20.94
	Vasicek	−0.0002 (−0.4964)	−0.0065 (−0.8377)	0.0000 (6.1584)	0	5.8391	8.4286 (0.0037)	1	25.7
	CIR SR	0.0001 (0.9511)	−0.0128 (−4.3133)	0.0001 (7.6746)	0.5	5.8861	0.072 (0.7885)	1	21.33
	BR-SC	0.0005 (1.1893)	−0.0198 (−2.2536)	0.0010 (6.6957)	1	5.8549	5.6101 (0.0179)	1	18.87
	CIR VR	0	0	0.0242 (10.3226)	1.5	5.7035	32.5716 (0.0000)	3	45.22

Table 3 (Continued)

Country	Model[b]	α	β	σ^2	γ	Avg. Log L	χ^2-test[c]	df	S-test
	CEV	0	−0.0105	0.0001	0.5286	5.8857	0.1365	1	21.28
			(−3.3145)	(1.2486)	(3.8629)		(0.7118)		
Japan	Unrestricted	0.0001	−0.0148	0.0002	0.4143	5.3305			46.57
		(0.3751)	(−1.4464)	(2.0512)	(6.2722)				
	Vasicek	0	−0.0111	0.0000	0	5.2244	31.4249	1	52.44
		(−0.1297)	(−2.2490)	(9.2282)			(0.0000)		
	CIR SR	0.0002	−0.0161	0.0003	0.5	5.3249	1.6717	1	45.28
		(0.5727)	(−1.5894)	(9.2652)			(0.1960)		
	BR-SC	0.0004	−0.0287	0.0192	1	5.0535	82.0189	1	41.34
		(5.5189)	(−1.9273)	(20.7972)			(0.0000)		
	CIR VR	0	0	2.2182	1.5	4.4469	261.5630	3	37.65
				(32.8881)			(0.0000)		
	CEV	0	−0.0121	0.0002	0.411	5.3301	0.1399	1	45.95
			(−1.7014)	(2.0391)	(6.1506)		(0.7084)		
Netherlands	Unrestricted	0.0007	−0.0126	0.0072	1.0245	4.8055			20.44
		(0.9240)	(−0.9144)	(1.7297)	(9.8679)				
	Vasicek	0.0015	−0.0248	0.0000	0	4.6294	81.3598	1	47.4
		(1.3259)	(−1.5881)	(11.1623)			(0.0000)		
	CIR SR	0.0009	−0.0159	0.0004	0.5	4.7561	22.8211	1	32.85
		(2.8353)	(−4.0081)	(33.5623)			(0.0000)		
	BR-SC	0.0007	−0.0126	0.0063	1	4.8054	0.0517	1	20.89
		(2.4576)	(−2.6947)	(33.2518)			(0.8201)		
	CIR VR	0	0	0.1077	1.5	4.7617	20.2282	3	14.62
				(40.615)			(0.0002)		
	CEV	0	−0.0007	0.0071	1.023	4.8036	0.8758	1	20.13
			(−0.1521)	(1.6677)	(9.5340)		(0.3494)		
New Zealand	Unrestricted	0.0045	−0.048	0.0034	0.7815	4.1567			11.46
		(2.1021)	(−2.1597)	(1.9698)	(7.2141)				
	Vasicek	0.0046	−0.0487	0.0001	0	3.9817	50.4024	1	20.54
		(1.8959)	(−2.2584)	(8.3736)			(0.0000)		
	CIR SR	0.0041	−0.0447	0.0010	0.5	4.1332	6.7797	1	11.81
		(2.026)	(−2.2176)	(8.5626)			(0.0092)		
	BR-SC	0.005	−0.0536	0.0096	1	4.1431	3.9217	1	13.41
		(8.3841)	(−6.9255)	(13.5641)			(0.0477)		

Table 3 (Continued)

Country	Model[b]	α	β	σ^2	γ	Avg. Log L	χ^2-test[c]	df	S-test
	CIR VR	0	0	0.1215	1.5	3.9904	47.903	3	22.84
				(20.8112)			(0.0000)		
	CEV	0	−0.0038	0.0030	0.7567	4.1412	4.4701	1	11.39
			(−0.4781)	(2.0131)	(7.1676)		(0.0345)		
Singapore	Unrestricted	0.0043	−0.109	0.0002	0.1976	4.5980			17.15
		(2.7148)	(−2.7100)	(1.2311)	(1.6673)				
	Vasicek	0.0038	−0.0984	0.0000	0	4.5884	2.7522	1	14.76
		(2.4245)	(−2.5002)	(8.4926)			(0.0971)		
	CIR SR	0.0056	−0.1437	0.0013	0.5	4.5751	6.6035	1	21.12
		(3.733)	(−3.4554)	(8.2625)			(0.0102)		
	BR-SC	0.0096	−0.2632	0.0507	1	4.4453	43.9594	1	32.45
		(6.5883)	(−5.6486)	(8.0158)			(0.0000)		
	CIR VR	0	0	3.6293	1.5	3.8481	215.9670	3	19.19
				(34.7029)			(0.0000)		
	CEV	0	−0.008	0.0001	0.1221	4.5717	7.5694	1	18.77
			(−0.6648)	(12.0543)	(9.3581)		(0.0059)		
Switzerland	Unrestricted	0.0007	−0.019	0.0001	0.2064	4.7509			29.58
		(0.8710)	(−1.0546)	(1.4345)	(1.9206)				
	Vasicek	0.0006	−0.0163	0.0000	0	4.7388	3.5473	1	27.28
		(0.6753)	(−0.9841)	(8.6933)			(0.0596)		
	CIR SR	0.0011	−0.0258	0.0007	0.5	4.7251	7.5777	1	31.24
		(1.4185)	(−1.3288)	(11.7970)			(0.0059)		
	BR-SC	0.0018	−0.0493	0.0253	1	4.5596	56.2412	1	28.96
		(3.0506)	(−2.2083)	(8.6830)			(0.0000)		
	CIR VR	0	0	1.2897	1.5	4.1686	171.1842	3	27.08
				(20.8448)			(0.0000)		
	CEV	0	−0.0058	0.0001	0.1921	4.7484	0.752	1	29.19
			(−0.7265)	(1.5240)	(1.9141)		(0.3858)		
UK	Unrestricted	0.0023	−0.0238	0.0008	0.5663	4.4015			21.78
		(1.7231)	(−1.7213)	(1.4984)	(3.9745)				
	Vasicek	0.0026	−0.0263	0.0001	0	4.3731	15.8911	1	22.33
		(5.5357)	(−6.8499)	(22.2201)			(0.0001)		
	CIR SR	0.0023	−0.024	0.0006	0.5	4.4011	0.234	1	21.77
		(1.7132)	(−1.7422)	(11.9782)			(0.6286)		

Table 3 (Continued)

Country	Model[b]	α	β	σ^2	γ	Avg. Log L	χ^2-test[c]	df	S-test
	BR-SC	0.0023	−0.0237	0.0061	1	4.3824	10.6625	1	22.13
		(1.9612)	(−1.7192)	(13.2670)			(0.0011)		
	CIR VR	0	0	0.0711	1.5	4.2984	57.554	3	21.71
				(34.6877)			(0.0000)		
	CEV	0	−0.0013	0.0008	0.5605	4.3962	3.0149	1	21.13
			(−0.2985)	(1.7082)	(4.5247)		(0.0825)		
USA	Unrestricted	0.0013	−0.0234	0.0001	0.4239	5.1768			60.51
		(1.4696)	(−1.5710)	(1.0309)	(2.5099)				
	Vasicek	0.0013	−0.0235	0.0000	0	5.1569	5.8655	1	55.37
		(4.3499)	(−5.6112)	(16.1077)			(0.0154)		
	CIR SR	0.0013	−0.0241	0.0002	0.5	5.1761	0.201	1	61.74
		(4.6916)	(−5.8893)	(13.0558)			(0.6539)		
	BR-SC	0.0014	−0.0255	0.0038	1	5.1365	11.8748	1	67.81
		(6.1214)	(−6.1221)	(14.9738)			(0.0006)		
	CIR VR	0	0	0.0794	1.5	5.0220	45.529	3	72.29
				(21.0933)			(0.0000)		
	CEV	0	−0.003	0.0001	0.4063	5.1705	1.8477	1	59.12
			(−0.6401)	(24.7981)	(31.0657)		(0.1740)		

[a] The general model to be estimated is given by: $\mathrm{d}r_t = (\alpha + \beta r_t)\mathrm{d}t + \sigma r_t^{\gamma} \mathrm{d}W_t$; where r_t is the 1-month interbank rate, $\mathrm{d}W_t$ is the increment of the Weiner process, α and β are the drift and mean reversion parameters, and σ and γ specify the conditional variance of r_t. Maximizing the log of the Gaussian likelihood function yields estimates for the four parameters α, β, σ^2 and γ.

[b] CIR SR, CIR VR: Cox et al. (1980, 1985), respectively. BR-SC: Brennan and Schwartz (1980). Avg. Log$L$: the average of the estimated maximum of the log-likelihood function for each country. χ^2-test: the likelihood ratio test for each restricted model. The test statistic is computed as $2[L(b) - L(b^*)]$, where b is the parameter vector (α, β, σ^2, γ), b^* is the restricted vector and $L(b)$ the log-likelihood function evaluated at the maximum. The statistic follows a χ^2 distribution with degrees of freedom (df) equal to the number of restrictions in each model. S-test: the Bergstrom (1990) dynamic specification test for white noise residuals. The statistic is given by

$$S = \frac{1}{n(N-l)} \sum_{k=1}^{l} \sum_{t=l+1}^{N} (z_t z_{t-k})^2,$$

where $n = 1$, $l = 12$, N is the number of observations and z are standardized residuals and follows a χ^2 with 12 degrees of freedom. The critical value at the 5% significance level is 21.03.

[c] T-statistics in parentheses except for the χ^2 Test column, which shows P-values in parentheses.

[d] 0.0000 denotes numbers less than 10^{-4}.

With regard to mean reversion, in all unrestricted models the reversion parameter β has the 'correct' negative sign but it is not significant (except for Singapore and New Zealand). This means that interest rates display little tendency to return to their average trend level. Similarly, the drift parameter and the volatility coefficient σ^2 are statistically weak for most countries.

With respect to model specification, the portmanteau test statistic proposed by Bergstrom (1990) is applied on the standardized residuals, z_t, to check for white noise. The test is given by the formula

$$S = \frac{1}{n(N-1)} \sum_{k=1}^{l} \sum_{t=l+1}^{N} (z_t z_{t-k})^2 \tag{5}$$

where $n = 1$, $l = 12$ and N is the number of observations for each country. S follows asymptotically the χ^2 distribution with l degrees of freedom. As shown in the last column of Table 3, the test fails to reject the null hypothesis of random walk in the standardized residuals for the case of Australia, Japan, Switzerland and USA. This indicates the existence of some misspecification due perhaps to the presence of non-linearity in the data.

3.2. Restricted models

Turning to the various nested models, two issues are now addressed: Parameter estimation and model selection based on a statistical criterion.

The models are estimated by maximizing Eq. (4) while keeping the respective parameters fixed. The results are also shown in Table 3. For instance, the third and fourth lines in each country section show the Vasicek model with α, β, and σ^2 free and γ set at 0. We observe first the high significance of γ in the CEV model. Second, whenever γ is constrained, the t-statistics of σ^2 are generally high by comparison to those of α and β. Interpreted very loosely, this result implies that the volatility coefficient is the second most important parameter after γ. Third, parameter β has a negative sign for all models and is sometimes significant.

In order to select the 'best' model out of the nested ones, the criterion used is the likelihood ratio test of the restrictions for each model. The test is performed as follows: let b denote the parameter vector (α, β, σ^2, γ) and $L(b)$ the log-likelihood function evaluated at the maximum. If b^* is the restricted vector, then the quantity $2[L(b) - L(b^*)]$ follows a χ^2 distribution with degrees of freedom equal to the number of restrictions. For instance, the Vasicek model has one degree of freedom compared to the general case. The calculated tests are reported in the eighth column of Table 3.

Based on the P-values of the χ^2 statistics, the CEV model is accepted for all countries except New Zealand and Singapore[2]. The dominance of CEV is understandable because this model includes all the features of the unrestricted model. In particular, it includes the crucial element, γ, which is highly significant and, in a way, has a heavy statistical influence.

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[2] The Merton (1973) model is rejected for all countries, while the Dothan (1978) and GBM models are rejected for all countries but Netherlands. For space reasons, results are not shown.

Table 4
Tests for structural change

Country	Period 1	Period 2	W^a	P-value
Australia	3/86–3/92	4/92–4/98	1.0989	0.8944
Belgium	10/89–12/93	1/94–4/98	17.6667	0.0014
Germany	11/90–7/94	8/94–4/98	33.5901	0.0000
Japan	12/85–1/92	2/92–4/98	7.2111	0.1251
Netherlands	1/79–8/98	9/88–4/98	3.2060	0.5240
New Zealand	4/86–3/92	4/92–4/98	55.6073	0.0000
Singapore	4/86–3/92	4/92–4/98	4.9106	0.2966
Switzerland	1/86–2/92	3/92–4/98	9.8778	0.0425
UK	1/75–8/86	9/86–4/98	14.4058	0.0061
USA	1/86–2/92	3/92–4/98	1.8986	0.7544

[a] W is a Wald statistic equal to $(b_1-b_2)'(V_1+V_2)(b_1-b_2)$, where $b_i=(\alpha_i, \beta_i, \sigma_i^2, \gamma_i)$ is the parameter vector over period $i, i=1,2$ and V_i is the variance-covariance matrix of b_i. Under the null hypothesis that $b_1=b_2$, W follows χ^2 with degrees of freedom equal to four, the number of parameters.

Some additional comments are worth noting about the selection process. Models other than CEV have a chance to pass the χ^2-test, as a second choice, if the restriction they impose on γ is close to the unrestricted γ estimate. More specifically, from Table 3, we have: For Belgium, CIR-VR model passes the test 'because' $\gamma=1.56\cong 1.5$. For Germany, the UK and USA, the CIR-SR model passes the test 'because' $\gamma=0.55\cong 0.5$, $\gamma=0.57\cong 0.5$ and $\gamma=0.42\cong 0.5$, respectively. Finally, for Netherlands, $\gamma=1.01\cong 1$ suits the BR-SC model that can be accepted via the likelihood ratio test. These observations are again attributed to the dominant nature of γ in terms of statistical significance.

Interestingly, the selection of CEV as the 'best' model, is at variance with other studies. Based on likelihood ratio tests, CEV is widely rejected in Tse (1995), while in CKLS and Nowman (1997) CEV is accepted only marginally and after first accepting other models such as the Generalized Brownian Motion or the CIR SR.

3.3. Structural change tests and multicollinearity

To investigate the issue of parameter stability, structural change tests are conducted. The samples are broken in two adjacent periods and the parameters are re-estimated. Let $b_i=(\alpha_i, \beta_i, \sigma_i^2, \gamma_i)$ be the parameter vector over period i, $i=1, 2$, and let V_i be the variance-covariance matrix of b_i. Then, the quadratic $W=(b_1-b_2)'(V_1+V_2)(b_1-b_2)$ is a Wald statistic that follows a χ^2 distribution with four degrees of freedom. The intuition behind the test is that if the null hypothesis of no difference in the parameters is correct, then b_1-b_2 will have elements close to zero and W will be small (Andrews and Fair, 1988). The converse holds if the null hypothesis is not correct. As shown in Table 4, there is structural change for six countries: Belgium, Germany, Japan, New Zealand, Switzerland, and UK. There is no structural change for the USA and this is in line with the findings of CKLS. However, it must be emphasized that the number of observations in the sub-samples is often small. Thus, a country may be in distinct phases of its economic cycle from sample to sample. Germany, for instance, moved gradually from a regime of high interest rates (above 9%) in the 1990–1992 period to a regime of very low interest rates (below 4%) in the post-1996 period, as a result of the process of convergence to the Economic and Monetary Union (EMU). Therefore, the tests may simply be picking up incidental differences in the parameters.

These results indicate a sensitivity of the estimates to the sampling period for some countries, as reported also in Section 2. In addition, the correlation coefficient between the estimators of σ^2 and γ for the full samples is very high, ranging from 97.0 (Japan) to 99.3% (USA). Naturally, these findings raise the question of whether multicollinearity interferes with the convergence of the optimization algorithm. To check the robustness of the models to multicollinearity, a variant of a practical approach appearing in Koutmos (1998) is used as follows: We let γ take values in the set $\{0.0, 0.1, 0.2, \ldots, 2.0\}$ and re-estimate the remaining parameters by maximizing the log-likelihood function with γ fixed. Then select the model with the highest log-likelihood[3]. Interestingly, the new point estimates are almost identical to those of the unrestricted models of Table 2, except that the t-statistics for σ^2 are much larger due to the lower standard errors, as expected. Therefore, it appears that multicollinearity is not responsible for the observed sensitivity of the parameters to the sampling period for the data at hand. Thus, the structural change tests presented must indeed reveal the existence of a true change in the stochastic process of the interest rate in the specific countries.

4. Concluding remarks

The paper has provided international evidence on the stochastic behavior of short-term interest rates and particularly the 1-month interbank rates. The main result is that interest rate volatility is not as sensitive to the interest rate level as stated in the literature. In particular, parameter γ in the general specification is less than unity in most countries of the sample. In addition, comparison of the various one-factor models leads to the conclusion that the CEV model is superior in terms of data fit. These results are in contrast to existing studies in the literature. Clearly, further work on the international markets is necessary to uncover the stochastic nature of interest rates.

Demonstrating the use of Gaussian estimation in a finance problem is a secondary objective of the paper, which has been met. Some words of caution, however, about the analysis are due. Interest rate data may exhibit structural breaks (due perhaps to changing central bank policies), which can severely alter the data generating process. In that case, approaches such as the self-exciting threshold autoregressive (SETAR) model used in Pfann et al. (1996), for example, are ways to handle this issue. In addition, the Gaussian approach itself rests on various

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[3] The results are available from the author upon request.

assumptions, such as the constancy of volatility in each observation period, which do not hold in practice. A systematic comparison of the Gaussian estimation and other techniques is therefore needed but is outside the scope of the paper. Thus, the estimation results should be interpreted carefully as patterns or tendencies of the short-term interest rate in the countries under study.

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