

## Ejercicios propuestos C. Alexander – II.4. Univariate GARCH models

1. Estimate the parameters of a symmetric normal GARCH model for the FTSE 100 using a sample of daily data from 3 January 1995 to 29 August 2007.
2. What would the GARCH term structure for the FTSE 100 on 29 August 2007 look like if you believed that the long term average volatility were 10% instead of its estimated value? Estimate the GARCH lag and error parameters that are consistent with this view, using the sample period from 2 January 2003 to 29 August 2007.
3. Estimate the parameters of the asymmetric normal A-GARCH model for the FTSE 100 using a sample of daily data from 3 January 1995 to 29 August 2007. Also estimate the long term volatility of the FTSE 100 index based on this model.
4. Estimate the parameters of the normal EGARCH model for the FTSE 100 using a sample of daily data from 3 January 1995 to 29 August 2007.
5. Consider again the 1995-2007 FTSE 100 data set, and set the values for the symmetric GARCH model parameters equal to their values in exercise 1. Starting with an initial value of  $\nu=15$  for the degrees of freedom in Student t GARCH, use Excel Solver to find the value of  $\nu$  that maximizes the likelihood function. Compare the result with the symmetric Student t GARCH parameters that are estimated using Matlab.
6. Compare the time series of covariances between the daily sterling-dollar and euro-dollar exchange rates between 3 January 2003 and 27 June 2006, using a) the constant correlation GARCH model, b) the DCC model based on EWMA correlations with a smoothing constant of 0.94.
7. In the Case Study II.1.4. we estimated several fundamental factor models for Vodafone and Nokia, and in the single-factor model we used the NYSE index as the broad market factor. The equity betas were estimated as 1.325 for Vodafone and 1.777 for Nokia. Using these factor sensitivities and based on historic daily returns on the NYSE from 2 January 2000 to 20 April 2006, estimate the F-GARCH volatilities and covariances for these two stocks, based on the factor model.
8. An European average rate call option has pay-off  $\max(A_T - K, 0)$ , where  $K$  is the strike of the option and  $A_T = n^{-1} \sum_{i=0}^{n-1} S_{T-ik}$  is an average of the underlying price, taken at  $n$  equally spaced dates  $k$  days apart, on and before the expiry date of the option. Use risk neutral valuation to price a European average rate call option with strike 95 and maturity 360 days when the spot price of the underlying is 100, the spot volatility is 25%, the risk-free interest rate is 5%, the underlying pays no dividends and the averaging is over the prices on days 300, 310, 320, 330, 340, 350 and 360. Assume that the underlying returns a) are i.i.d., and b) follow a symmetric normal GARCH process with parameters  $\omega = 10^{-6}; \alpha = 0.05; \beta = 0.94$ .
9. A European down and out barrier put option has pay-off  $\max(K - S_T, 0)$  provided that  $S_t > B$  for all  $t=1,2,\dots,T$ , otherwise the pay-off is 0. Here  $K$  is the strike,  $B$  is the barrier and  $T$  is the maturity date of the option. Use risk neutral valuation to price a European down and out barrier put with strike 95, barrier 75 and maturity 360 days. As before, assume the spot price of the underlying is 100, the spot volatility is 25%, the risk-free interest rate is 5%, the underlying pays no dividends. Also as in the previous example, Assume that the

underlying returns a) are i. i.d., and b) follow a symmetric normal GARCH process with parameters  $\omega = 10^{-6}$ ;  $\alpha = 0.05$ ;  $\beta = 0.94$ .

10. Three assets X, Y and Z have annual volatilities 25%, 20% and 30% respectively when estimated using an equally weighted average of square monthly returns over long time period. Their monthly returns correlations are 0.6 for X and Y, -0.5 for X and Z, and -0.6 for Y and Z. Their expected returns over the next month are 5%, 4% and -1% respectively. A multivariate normal A-GARCH model is estimated using daily returns on these assets and, when the long term covariance matrix is constrained to take the values above, the GARCH parameter estimates are shown in the spreadsheet. The long term correlations are identical to the correlations given above. The current GARCH estimates for the asset's volatilities are, in annualized terms, 32%, 26% and 35%, respectively, and the GARCH correlation estimates are 0.75 for X and Y, -0.6 for X and Z and -0.7 for Y and Z. Find a portfolio that is expected to return at least 2.5% over the next month, with the minimum possible variance, based on a) the equally weighted average covariance matrix, and b) the GARCH model.