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Nonlinear Ionic Excitations, Dynamic Bound States, and Nonlinear Currents in a One-dimensional Plasma

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We study the role of nonlinear effects in a classical one-dimensional model of a conducting electron–ion system. In particular we investigate the excitations of strongly nonlinear deformed phonons (cnoidal waves, solitons) on electric currents. We show that in a nonlinear lattice a new type of dynamic bound states of solitons and electrons ("solectrons") may be formed. In our simulations we use Langevin dynamics with $N = 10$ ions and periodic boundary conditions. The electron-ion interaction is modelled by screened Coulomb forces with appropriate cut-off at small distance; the ion-ion interaction is approximated by an exponential repulsion. Under the influence of a weak external electrical field, the charged particles and "solectrons" yield a stochastic current in the direction of the field. We study several mechanisms to generate and maintain the "solectrons". Then we show how the system develops driven ionic solitons moving opposite to the field. Since the extra current driven by the solitons is (nearly) independent on the external field we find a strongly nonlinear current field characteristics corresponding for small fields to a highly conducting state.

1 Introduction

We study here the influence of nonlinear ionic excitations on the electric conductivity of dense plasmas. For simplicity we restrict our investigation to one-dimensional (1D) systems. The first theory of conductivity in electron-ion systems is due to Drude (1900). His "ansatz" is based on the assumption that the electrons, as well as the ions, are subject to friction forces and form in external fields a stationary current. For a survey of the recent state of art in the transport theory in dense Coulombic systems as e.g. solids and dense plasmas we refer to [1–3]. It has been shown that different elementary excitations like plasmons, phonons, polarons, and excitons, determine in one way ore another electric conduction [1]. Of special interest is the influence of bound states on conductance [2, 3]. Less studied are the effects of nonlinear excitations, in particular soliton effects. Recall that the soliton concept, and the coinage of the word soliton, originates from plasma physics and in particular from the pioneering work of Zabusky and Kruskal (1965) on solitons in a collisionless plasma [4]. Far more recent studies are devoted to the one-dimensional case (see Refs. [5–9]). Here we are interested in analyzing dynamic bound states, created by deep local potential minima due non-linear excitations. We study the role of dynamic bound states between electrons and solitons (called "solectrons") [10, 11]. These bound states are metastable in equilibrium systems and may lead to nonlinear conductance effects if appropriately maintained under nonequilibrium conditions. The 1D system studied here consists of two types of charged masses (caricatures of "ions" and "electrons") which are moving on a line (it could be considered as a wire) with periodic boundary conditions (alternatively we can speak about a ring) according to classical Langevin dynamics. The 1D-lattice bearing the charges is imbedded into a 3D-medium which creates dissipative and screening effects and is the carrier of the field lines. Therefore the interaction of the charges is 3D-Coulombic. Note the similarity with a polymer strand carrying charges imbedded into a dielectric medium [9, 12]. Following the pioneering work of Toda [13], studies

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of one-dimensional nonlinear lattices have significantly contributed to the understanding of nonlinear excitations in various physical systems [14, 15]. Of special interest is the coupling of finite size nonlinear rings to a heat bath and properties of the resulting excitation spectra [16–18]. The thermal excitations in ring chains with Morse potentials (akin to the Lennard-Jones) and small particle number N were investigated in [19]. Here we report on new results on electrical transport under the influence of external fields [10, 11] and hence situations far from equilibrium. As shown earlier, we may drive a nonlinear lattice away from equilibrium by applying dissipative forces [20–22] and excite ”dissipative solitons” [23–27]. Another concept which plays an important role in our problem, is that of active Brownian motion [28, 29].

2 Interactions and electron-ion dynamics

Let us stay on the same classical level as the Drude theory and start with Langevin equations for N electrons (mass m_e, charge $-e$) and N ions (mass m, charge $+e$) moving on a chain of length $L = N\sigma$ with $m_e \ll m$ with periodic boundary conditions. We assume that the lattice is imbedded into a surrounding reservoir which creates screening, friction and noise. Take the N electrons located at the positions y_j moving in the nonuniform, and, in general, time-dependent electric field generated by the positive ions located at x_k and by the other electrons. The effective interaction between two electrons at y_i and y_j (distance: $r_{ij} = |y_i - y_j|$) is modelled by the quantum-statistical (screened) pseudo-potential of Kelbg-Deutsch type [30]

$$U_{ee}(r_{ij}) = \frac{e^2}{r_{ij}} \left[\exp[-\kappa r_{ij} - \exp[-\alpha r_{ij}]\right] \tag{1}$$

The quantum effects are expressed by one constant α which is in a first-order approximation given by [30]

$$\alpha = (\sqrt{\pi}/\lambda) + (kT/e^2)\ln 2, \qquad \lambda = \hbar/\sqrt{m_e kT} \tag{2}$$

Further κ is the screening parameter, which is in our model a constant defined by the medium in which the lattice is imbedded. We assume here $\kappa \simeq 1/\sigma$, where σ is the average equilibrium distance between the electrons. To simplify the simulations we linearize the exponential factors

$$U_{ee}(r_{ij}) = e^2 \left[(\alpha - \kappa) - \frac{r}{2}(\alpha^2 - \kappa^2)\right] \qquad if \qquad r_{ij} < r_e \tag{3}$$

$$U_{ee}(r_{ij}) = 0 \qquad if \qquad r_{ij} > r_e = 2/(\alpha + \kappa) \tag{4}$$

Due to the quantum effects (Heisenberg- and Pauli-effects), the electron-electron interaction is rather weak [30] and may be neglected in many cases. The electron-ion interaction is described by an appropriate pseudo-potential $V(r_{jk})$ with finite value at small distance [31] including also screening:

$$U_{ei}(r_{jk}) = V(r_{jk}) - \frac{ee_k}{r_{jk}} \left(\exp[-\kappa r_{jk}] - 1\right), \qquad r_{jk} = |y_j - x_k| \tag{5}$$

Again we use a linear approximation leading to a shifted bare potential

$$U_{ei}(r_{jk}) = -\frac{ee_k}{\sqrt{r_{jk}^2 + h^2}} + ee_k\kappa \qquad if \qquad r_{jk} < r_0, \tag{6}$$

$$U_{ei} = 0 \qquad if \qquad r_{jk} > r_0 = \frac{3}{2}\sigma, \qquad \kappa^{-2} = \frac{9}{4}\sigma^2 + h^2. \tag{7}$$

This potential is of finite range and finite strength and has a minimum value $U_{min} = ee_k(\kappa - 1/h)$, where $h \simeq \sigma/2$ is a free parameter which determines the short-range cut-off. Accordingly, the electrons would be able to transit from one to the other side of an ion and yield an electron current opposite to the ion current. Let us consider an electron between 2 ions e_k at distance R. If the electron is shifted by the distance x from the center, it feels the potential ($\kappa = 0$)

$$U(x) = \frac{-2ee_k}{R}\left[\frac{1}{\sqrt{(1+2x/R)^2+4(h/R)^2}}+\frac{1}{\sqrt{(1-2x/R)^2+4(h/R)^2}}\right] \qquad (8)$$

If the ions are far apart, the local minimum generated by a pair of nearby ions is at the position of the nearest ion (Fig. 1). However if the electron is somewhere between a pair of ions with much reduced distance, the local potential created by the ions may offer a minimum, depending on the distance of the two ions, and the electrons may feel the local compression of the ions (Fig. 1). The depth of the minimum is

$$U(0) = -\frac{4ee_k}{\sqrt{R^2+4h^2}} \qquad (9)$$

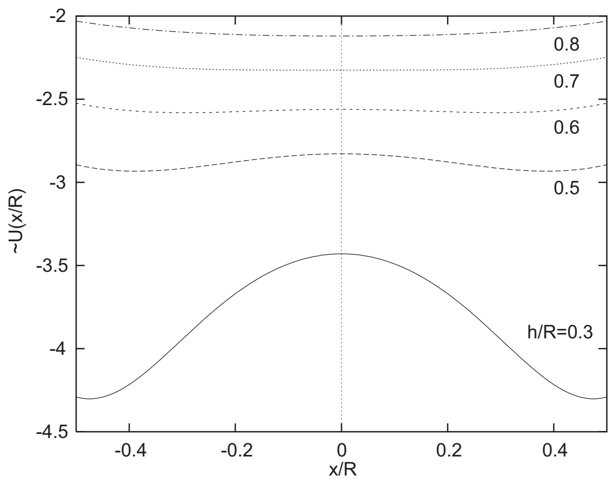

Fig. 1 Effective potential $U(x)/(e^2/R)$ acting on an electron placed between two nearby lattice particles as a function of the lattice spacing R, the relative deviation from the center x/R and the parameter $H = h/R$. For small H the minima are near to the lattice particles. As H grows, for example because R is small due to a local compression, the maximum in the center yields to a minimum midway between the two lattice particles, which may give rise to a local bound state.

For the electron dynamics we assume

$$\frac{dv_j}{dt} + \sum_k \frac{\partial(U_{ei}(y_j,x_k)+U_{ee}(y_j,y_k))}{m_e\partial y_j} = -\frac{eE}{m_e} - \gamma_{e0}v_j + \sqrt{2D_e}\,\xi_k(t) \qquad (10)$$

The evolution of the electrons is considered passive ($\gamma_{e0} > 0$, positive damping for all velocities) and $\xi(t)$ denotes a Gaussian white noise. The corresponding stochastic forces $\sqrt{2D_e}\,\xi_j(t)$, model a surrounding heat bath obeying a fluctuation-dissipation theorem. Due to the large difference of the masses ($m \simeq 10^3 m_e$), the friction acting on the electron is small relative to the friction acting on the ions $m_e\gamma_e \ll m\gamma_0$. The character of the dynamics of an electron strongly depends on h and on the positions of the nearest ions. For instance: (i) If $h \simeq 0.5\sigma$, then if ions are at equilibrium distances, the resulting potential is nearly flat. If the ions are compressed to about 20 percent of the previous distance, the resulting potential shows a trap in the center between the two ions, (ii) If $h \simeq 0.3\sigma$, then if the ions are at equilibrium positions, the potential minima are shifted to their location. Therefore by varying the compression and the parameter h we are able to shift the potential minimum (the favorite place for the electron) either to the place of the lattice centers ($h \simeq 0.3\sigma$) or we can create a flat profile, which generates minima only for strong (soliton-like) compressions (Fig. 1).
The heavy ions have their own evolution independent of that of the light electrons. We use adiabatic approximation. Then for the lattice we consider N identical Brownian point-particles with masses m and charges $+e$ located on the line with periodic boundary conditions. The particles are described by coordinates $x_k(t)$ and velocities $v_k(t)$, $k = 1,\ldots,N$.

$$x_{k+N} = x_k + L. \qquad (11)$$

The interaction between two neighbour lattice particles located at x_i and x_{i+1} is given by

$$U_{ii}(r_i) = U'_{ii}(r_i) + \frac{e^2}{|r_i|}\exp[-\kappa|r_i|] \qquad (12)$$

with $r_i = x_{i+1} - x_i$. Here $U^{'}$ models a short range repulsion of the ion cores and κ is the Debye screening parameter. Due to the repulsive forces, the ions form in equilibrium an equidistant lattice, the equilibrium distance σ will serve as the unit length in our analysis. The motion of the lattice particles is essentially a nonlinear oscillation around the equilibrium distances. Thus we can approximate the lattice interaction (12) by a two-parameter exponential approximation

$$U_{ii}^{T}(r_i) = \frac{a}{b}\,\exp(-b(r_i - \sigma)) \tag{13}$$

This exponential pair potential was studied in detail by Toda [13]. The parameters of this potential are the mean distance $\sigma > 0$, the stiffness b of the lattice repulsion and the amplitude a. Toda's potential yields an exactly solvable model, in two extreme limits it reduces to the hard sphere interaction and to harmonic forces, respectively. We see from Taylor expansions of eq.(13) and from Fig. 2 that $\omega_0^2 = ab/m$ corresponds to the basic oscillation frequency, and that ab^2 controls the anharmonicity of the forces. The lattice dynamics is in first approximation harmonic (Fig. 2), corresponding to oscillations with the frequency $\omega_0 = \sqrt{ab/m}$. In the harmonic aproximation, the characteristic excitations are phonons. Note that in the present context, the deviations from the harmonicity are very essential. Toda studied the nonlinear dynamics of the exponential lattice in detail and provided analytical solutions which generalize the phonon concept. Todas solutions are expressed by elliptic functions (cnoidal waves) which replace the trigonometric functions (phonons) known from linear theory [13].

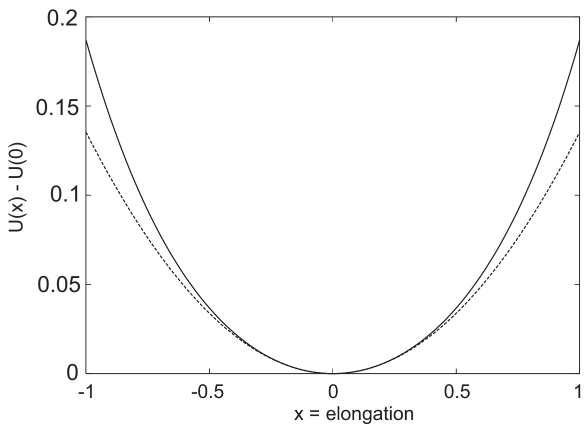

Fig. 2 The effective potential acting on an ion oscillating between two fixed neighbours at distance 2σ in comparison with the harmonic approximation. The minimum corresponds to the equilibrium configuration of the central ion.

Beside the earlier described interaction forces we introduce an external electric force eE acting on the ion charges $e_i = +e$. Further - this is an essential feature of our model - we introduce a velocity-dependent force $F(v_k)$ which describes dissipative effects, passive as well as active dissipative interactions with the reservoir, and corresponding stochastic sources. Then the evolution of the ions is given by the Langevin equations

$$\frac{d}{dt}v_k + \sum_j \frac{\partial U_{ii}}{m\partial x_k} + \sum_j \frac{\partial U_{ie}}{m\partial x_k} = \frac{1}{m}(eE + F(v_k)) + \sqrt{2D}\,\xi_k(t), \tag{14}$$

governing the stochastic motion of the kth ion on the ring. The stochastic forces $\sqrt{2D}\,\xi_k(t)$ also model a surrounding heat bath (Gaussian white noise). In the passive case the force $F(v) = -m\gamma_0 v$ describes the standard friction acting on the ions from the side of the surrounding heat bath. We assume again the validity of the Einstein relation $D = k_B T\gamma_0/m$ [17], where T is the temperature of the heat bath. For the dissipative forces we shall follow Rayleigh and use a model which admits input of energy into the system:

$$F(v_k) = m\gamma_0(\mu - v_k^2/v_d^2)\,v_k. \tag{15}$$

In the case $\mu = -1$ and $v_d \to \infty$ we come back to the passive Stokes case. With $\mu > 0$ there is energy input from the surrounding medium. The quantity μ is the bifurcation parameter of our problem ($\mu = 0$ means no friction, $\mu > 0$ models negative friction). The balance for the total energy of the ionic subsystem reads

$$\frac{dE_{ion}}{dt} = m\gamma_0 \sum_k (\mu - v_k^2/v_d^2)v_k^2 + m\sqrt{2D}\sum_k v_k\xi. \tag{16}$$

The sign of the first term on the r.h.s. is crucial for the energy balance as for $\mu > 0$ the energy input may drive the system far away from equilibrium [21, 29]. The effects of active friction force were investigated in [11, 21, 29] to model active Brownian particles that carry refillable energy depots (internal degrees of freedom). For the passive regime $\mu < 0$ the deterministic dynamics has a single attractor at $v = 0$. Without noise all particles come to rest at $v = 0$. For $\mu > 0$ the point $v = 0$ becomes unstable but we have now two additional zeros at

$$v = \pm v_0 = v_d \sqrt{\mu} \tag{17}$$

These two velocities are the new attractors of the free deterministic motion if $\mu > 0$.

3 Results of computer simulations

The differential equations (7) and (14) have been integrated by means of a fourth-order Runge-Kutta algorithm adapted for solving stochastic problems [32]. We used σ as the length unit and $1/\omega_0 = (m/ab)^{1/2}$ as the time unit. The electric field strength is measured in units $E_0 = v_0 m_e \gamma_{e0}/e$ and the energy (temperature) in units $U_0 = ab\sigma^2 = m\omega_0^2\sigma^2$. In all simulations we set $e^2/(m\sigma) = 0.2U_0$. All computations start with the initial state of equal distances between ions. The initial velocities of the ions were randomly taken from a normal distribution with amplitude v_{in}. This corresponds to an initial temperature $T_{in} = v^2$ and a Maxwell distribution of the velocities. We may assume that the initial temperature was reached by a heat shock applied to the lattice. Each electron was placed midway between two ions. The motions of ions and electrons occur in different time scales. Heavy ions are not affected practically by light electrons and electrons move on the background of the Coulomb potential landscape created by the ions. The dynamics of the ion ring with Toda interactions leads to soliton-like excitations. Typical solitonic excitations correspond to local compressions moving along the ring. Fig. 3 shows a characteristic landscape of the electric field created by the ion configuration at certain time instant. We see a rather deep potential well moving right to left or left to right along the lattice. The light electron may be captured in this dynamic potential well and may follow the soliton dynamics.

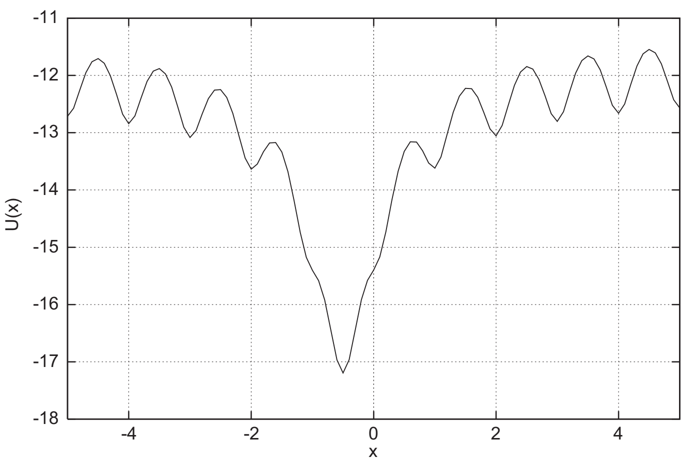

Fig. 3 Typical landscape of the local electric field created by the solitonic excitation. The minimum corresponds to a local compression of ions which means an enhanced charge density.

In our simulations the integration step is chosen to describe correctly the fastest component of the process, the oscillations of the electrons in the potential well.

We are interested here in regimes where solitonic excitations are generated in the ion plasma. As we have shown in earlier work, most solitons are generated around some characteristic temperature defining the soliton-generating region [18]. In this region besides other excitations many solitons are generated. In order to investigate this effect we start with the case without electric field $E = 0$, and no friction forces $F(v) = 0$. In order to generate many solitons we applied sudden heating and quenching (Fig. 4). Hence we used a Gaussian distribution of the ion velocities corresponding to a high-temperature Maxwellian as initial condition of the order of $k_B T_{in} \simeq 0.1$ (in units of the energy of harmonic oscillations with amplitude σ). This is near to the critical temperature $k_B T_{cr} \simeq 0.16$, where we are in the soliton-generating region [18]. Then we quench to a low temperature. Some solitons survive since they have a higher lifetime than most other excitations. Looking at the trajectories we observe the expected nonlinear soliton-like excitations that decay after a time of the order $t_{rel} \simeq 1/\gamma_0$ (Figs. 4 and 5).

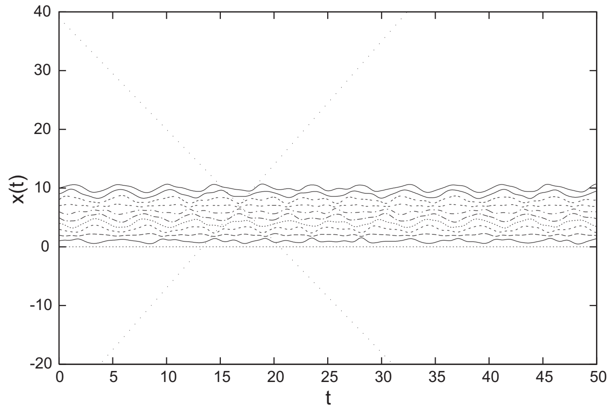

Fig. 4 Metastable soliton excitations in the 1D- ion system. As a result of quenching of an initial state with $T(0) \simeq 0.1 \simeq T_{tr}$ the trajectories of 10 ions generate left to right and right to left running solitons. A soliton is a local excitation which is detected by the slope of the wavy trajectories of the ions representing subsequent excitations of the ions. The local compression is running from one ion pair to the next. Note that there is no mean ion motion ($E = 0$). (Parameter values: $\gamma_0 = \gamma_{e0} = 0.00045/\omega_0$, unit of time on the abscissa, $1/(\sqrt{5}\omega_0)$.).

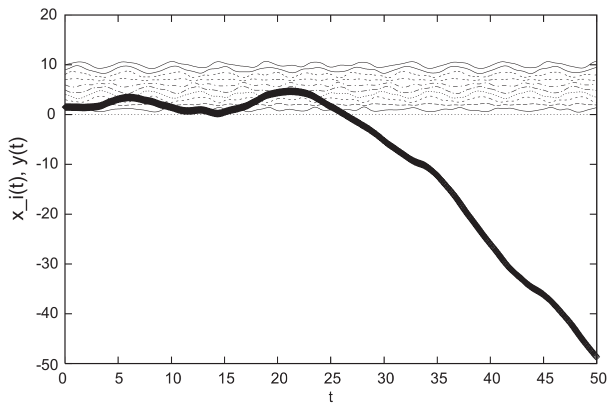

Fig. 5 Electron-ion 1D lattice system without external field. As a result of quenching of an initial state with $T(0) \simeq 0.1$ the trajectories of 10 ions generate solitons. As in Fig. 4 the soliton is a running local excitation which is detected by the slope of the wavy trajectories of the ions. The local compression is running from one ion pair to the next. Here an electron has been captured by the soliton and forms with it a dynamic bound state (solectron). During this time interval the electronic trajectory is parallel to the "tangent" also providing the solitonic velocity. (Parameter values: $\gamma_0 = \gamma_{e0} = 0.00045/\omega_0$, unit of time on the abscissa, $1/(\sqrt{5}\omega_0)$, unit of length on the ordinate, σ).

Metastable nonlinear excitations of the lattice exist also under equilibrium conditions [18].
In order to maintain the solitonic excitations for a longer time interval we applied the active Rayleigh friction in the period after heating and quenching. Then the soliton regime becomes a stable attractor of motion [19,20,22]. The driven ionic solitons appear moving opposite to the field. The electrons which are coupled to the ions form dynamic bound states with the solitons ("solectrons").
Recall that for the passive regime $\mu < 0$ the deterministic dynamics has a single attractor at $v = 0$. Without noise all particles come to rest at $v = 0$. Recall that for $\mu > 0$ the state $v = 0$ becomes unstable and we get the new stable states

$$v = \pm v_0 = v_1\sqrt{\mu} \tag{18}$$

The simulations presented below correspond to the Rayleigh approximation with $\mu = 0.25, v_d = 1, m/m_e = 1000$, and $\gamma_0 = \gamma_{e0} = 0.5/\omega_0$.
Since the electrons search for the deepest nearby minimum of the potential, they will be most of the time located near to local ion clouds. This is a dynamic phenomenon, the lattice particles participating in the local compression are changing all the time. Hence, the electrons have always new partners for forming the "solectron".
Three stages appear: In the first one the initial state tends to one of $N + 1$ attractors [20]. The maximal average velocity among the running waves corresponds to the excitation with one local compression on the ring. The attractor, reached by the system without noise and external field, is defined mainly by the value v_{in} given by the initial conditions. With increasing v_{in}, k-solitonic waves may be excited with increasing k. There exists always a favourite attractor for a given value v_{in}. For our case the initial conditions leads preferentially to the one-soliton attractor. In the absence of the field both directions have equal probability, the field breaks the symmetry. The value of the cut-off distance in Eq. (5) is $h = 0.3\sigma$. In this case the difference between the maximum in the electron-ion interaction force and the corresponding value for an electron and an ion being away from the electron more than 1.5σ is less than a factor of 10. To simplify, the parameters of both Toda and Coulomb potentials, of the Rayleigh force, the friction coefficients, both masses and charges of particles were held fixed. The initial

velocities v_{in}, the values of the external field and the electronic temperature T_e are varied in different runs.
In Fig.6 we show a computer simulation of the evolution of 10 lattice particles creating one dissipative soliton which moves in opposite direction. After a transient time interval, most of the electrons are coupled to the soliton and move approximately with the soliton velocity in the direction opposite to the ions. In the driven case ($\mu > 0$)

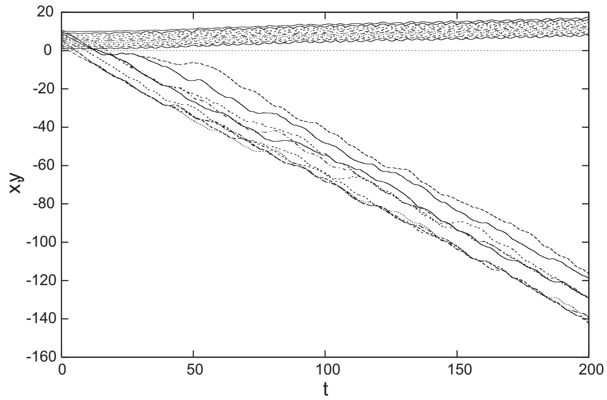

Fig. 6 Electron-ion 1D-lattice including an extrernal field. Trajectories of 10 lattice particles moving left to right creating one fast dissipative soliton moving in opposite direction and trajectories of 10 electrons captured in part by the soliton which is maintained due to the energy input ($\mu = 0.25$).

the ions make a drift following the direction of the field. Again after a transitory regime, solitonic excitations of the ions are formed moving with velocity v_{sol} opposite to the mean motion of the ions. As earlier indicated, the electrons "like" the deep potential well formed by the local compression connected with the soliton. After a while the electron is captured by the local compressions and moves with the soliton velocity opposite to the ion drift.

4 Influence of nonlinear exciations on the currents

The currents on the ion-electron line are determined by the electrons. The electron current density of the electrons is given by averages of the electronic velocities, $v_j^e = \dot{y}_j$ taken over long trajectories

$$j_e = -n_e e \sum_j \langle v_j^e \rangle \tag{19}$$

Note that the currents do not depend practically on the value of the field in a wide range of values, limited both from below and above (Fig. 7). We introduce a characteristic value of the field E_0, which corresponds to that field which would drive a free electron with friction γ_{e0} to the velocity v_0. We see a strongly nonlinear current-field characteristics with a region of constant current (corresponding to zero differential conductivity). At very small field values there is a gap in the current as a function of the external field. In the narrow region around zero field we could not find reliable data from the simulations. The existence of a current gap may be considered as a hint for the existence of rather high conductivity. For very low external fields we cannot specify in our computations the direction of soliton propagation, they may travel in either direction. On the other hand at very high external fields, the external forces do not allow electrons to be trapped by the dynamic potential well. In Fig. 7 the current curves for negative external fields were constructed from symmetry considerations, we confirmed however several points on this branch by extra calculations.

5 Discussion

By computer simulations of the evolution of the electron-ion system described by Eqs. (5) and (11) we have shown an enhancement of the currents in comparison to the Ohm-Drude current at weak electric fields. This enhancement is due to the fact that some of the charge carriers, the "solectrons" move with soliton velocity [13]

$$v_{sol} = \omega_0 \sigma \frac{\sinh \chi}{\chi} \tag{20}$$

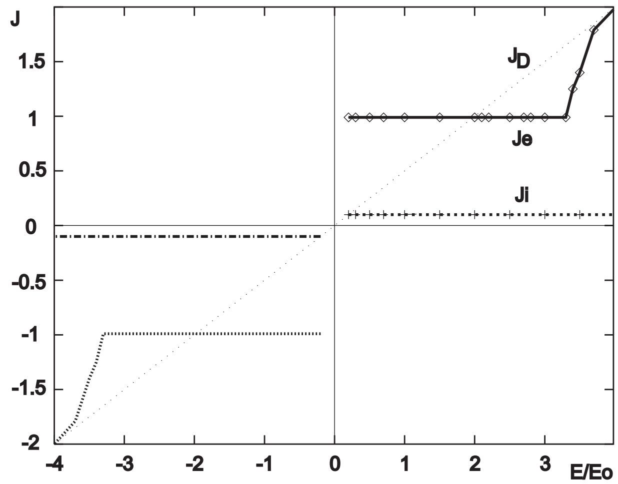

Fig. 7 Electron-ion stationary currents corresponding to the case of dissipative solitons stabilized by negative friction $\mu = 0.25$, as functions of the field strength. The dashed straight line gives for comparison the linear Ohm-Drude current.

where χ is the width of the solitons, which depends on the stiffnes and the soliton energy. The stiffness parameter χ is given by the nonlinear equation

$$\sinh\chi\cosh\chi - \chi = \frac{1}{2}b^2\epsilon_{sol} \tag{21}$$

Solitons move supersonically and v_{sol} may be rather high for large stiffness of the lattice. The formula (20) is exact only for the conservative case and gives still a good approximation for small μ. With increasing μ the soliton velocity approaches $v_0 = v_1\sqrt{\mu}$, which depends only on the driving strength and not on the stiffness. Therefore the region of interest (in the context of soliton enhancement effects) are large b- and small μ- values. This underlines that the active friction is of limited relevance in this context, it is just an auxiliary factor which indeed serves to maintain the solitons in the spirit of Lord Rayleigh's approach.

A most interesting property of the soliton effects studied here is that the currents switch on at rather small fields and do not depend on the electrical field in a wide region. Relevant is only the symmetry breaking by the field. This reminds magnetization phenomena, charge transport in DNA [12] and high-T superconductivity [33]. Let us conclude with a brief discussion about the possible influence of quantum effects on our results: The "dynamic bound states" described here are classical phenomena. A complete quantum theory of these effects seems to be out of range at this moment, due to the transient character (the running) of the solitonic minima. One possible approximation is the tight-binding model [12]. A simple estimate of quantum effects is provided by the adiabatic approximation. This is based on the assumption that the forming of the wave functions occurs in the running minima on a shorter time period than the shift of the minimum in time $x_{min}(t) \simeq v_{sol}t$. In the adiabatic approach the quantum states may be estimated by a parabolic approximation of the solitonic minima shown e.g. in Fig. 3:

$$U_{sol}(t) = \frac{m}{2}\,\omega_{min}^2(x - x_{min}(t))^2 \tag{22}$$

We find in this approximation the harmonic oscillator levels

$$\epsilon_n = \hbar\omega_{min}\left(n + \frac{1}{2}\right) \tag{23}$$

which may be filled by electrons according to the Pauli principle. In the simplest case of a flat solitonic minimum or low electronic density, just one pair of electrons with opposite spins will occupy the lowest bound state. This way we may find a specific electron pairing effect caused by the solitonic excitations. A more detailed study of quantum effects as well as the investigation of band structures, we leave to future investigations.

In conclusion, we have shown the significance of nonlinear collective excitations in one-dimensional plasmas and, in particular, the possible role of soliton - electron bound states called here "solectrons". The authors thank M. Alonso, D. Hennig, G. Nicolis, G. Röpke, and A.C. Scott for advice and fruitful discussions.

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