Investment, technological progress and energy efficiency*

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Abstract

In this paper we propose a theory to study how the aggregate demand of energy responds to energy prices and technical innovations that affect the price of energy services. In our theory, energy use is determined by the interaction of the choice of Energy Saving Technical Change with energy prices and Investment Specific Technical Change (ISTC). The key mechanism is that the energy saving technology is embodied in capital vintages as a requirement that determines their energy intensity. We show that higher ISTC that *increases the quality of capital goods* is an energy saving device and, therefore, a substitute for Energy Saving Technical Change (ESTC). However, higher ISTC that *rises the efficiency in producing capital goods* is energy consuming and fosters ESTC to compensate for the amount of energy required by the new investment. A higher price of energy also induces a higher level of ESTC, but the aggregate amount of energy used may not be affected as investment does not change. These effects are amplified with rising prices of energy. Thus, our theory can be used to test when and how we should see a rebound effect in energy use at the aggregate level and to evaluate the aggregate effect of any policy aiming to reduce energy use.

Keywords: Energy use, energy saving technical change, vintage capital, investment specific technical change, rebound effect
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1 Introduction

In this paper we study how the aggregate demand of fossil fuel-based energy responds to changes in energy prices and technical innovations that affect the price of energy services. The answer matters for various reasons: First, there is the question whether the limited supply of fossil fuel may limit economic growth (see, among others, Krautkraemer, 1998). Second, policies designed to effectively control emissions need understanding of the behavior of the aggregate demand of energy to assess, in particular, if there are macro rebound effects, as pointed by Frondel et al. (2012) and Gillingham et al. (2016), i.e., do innovations that are potentially energy saving trigger a rise in energy demand? Finally, there is an ongoing discussion about moving from “dirty” to “clean” technologies where the key issue is when turning to the clean technology is profitable, (see, for instance, Acemoglu et al., 2012). To understand and estimate switching times we need to take into account, and model, the fact that our economies are becoming more efficient in the use of fossil fuel based-energy over time.

The first key element of our theory is that energy intensity is a feature of technology and is embodied in capital as a fixed requirement. Capital is putty-clay as in Atkeson and Kehoe (1999): once the requirement is fixed it cannot be changed over the lifetime of capital. Agents, though, can choose to invest in less (or more) energy intensive capital and that choice responds to economic conditions. The second key element in our theory is that we assume that there is Investment Specific Technical Change (ISTC hereafter). That is, there are innovations that either rise the quality of capital (*intensive margin*) and/or increase the efficiency in the production of capital goods (*extensive margin*). The combination of a choice of the energy requirement of the latest vintage with the existence of ISTC provides a description of the endogenous process of Energy Saving Technical Change (ESTC hereafter), and therefore, of the path of aggregate energy use and energy intensity in our economy. Additionally, there is an energy price (this is a small open economy) that may exhibit trend growth responding to the possible scarcity of the resource (although there is no uncertainty in our framework). Thus, aggregate energy demand depends on the choice of ESTC and its interaction with ISTC and the energy price. Moreover, ISTC induces economic depreciation that combined with the energy requirement make optimal to scrap capital in finite time.[1]

[1] Micro evidence points to the role of ISTC in determining the choice of energy intensity of capital. For instance, Newell et al. (1999) by looking at energy-using consumer durables, and Popp (2002) based on energy patent data, find that energy prices have a strong positive effect on adopting energy efficient technologies. Other authors such as Steinbuks and Neuhoff (2014) by analysing manufacturing input use, or Knittel (2011) for the automobile industry, point out that adoption of energy efficiency may be delayed by exogenous technical progress. The latter estimates that if car quality would have not improved from 1980 to 2006, energy efficiency of both passenger cars and light

We start by organizing the empirical evidence at the macro level about the trends in energy use per unit of GDP, energy use per unit of capital, and the evolution of the energy share in GDP. We use the US economy as the illustrative case. The data shows, as expected, that the US grows along a balanced growth path, but it uncovers facts related to energy. For instance, after the 1970s, aggregate (fossil fuel based) energy use declines at the annual rate 0.26 percent, whereas energy use per unit of value added declines at the rate 1.54. The other more salient feature of the data is that the relative price of capital has been declining at 2.58 percent annually since 1982. The energy share is, on average, 4.74 for the period 1960-2010. We will use these data to discipline our theory when we turn to our quantitative counterfactual exercises.

It is in light of this evidence that we present our physical environment and market arrangements. In our theory, final output is produced with labor and capital services. The interaction of ISTC and ESTC make capital heterogeneous in two dimensions: vintage and energy intensity. New vintages, due to the existence of ISTC, yield more services than older vintages, regardless of their energy intensity. More energy intensive capital yield more services but entail higher energy costs (leading to lower value added). While the passage of time rises ISTC, the many varieties of Energy Saving Technologies are always available at a cost in terms of a total factor productivity loss. The adoption of a particular energy-saving technology depends, therefore, on ISTC and the energy price. Ours is a theory of energy demand, and we incorporate the eventual scarcity of the resource by assuming different constant rates of growth of the energy price.

We show theoretically that, in the long run, energy intensity of new capital (our index of ESTC) falls at the rate of growth of the energy price and the extensive margin of ISTC. While the first effect seems natural and understood in the literature (ESTC rises to offset the effect of the energy price growth on the aggregate energy bill) the second is novel: it is due to the fact that, after a rise in the extensive margin of ISTC, agents invest more. Since energy intensity is a feature of capital, higher investment implies a higher energy bill unless new capital is less energy intensive than old capital. Therefore, there is adoption of energy-saving technology even if energy prices do not rise over time, and this is due to the existence of ISTC along the extensive margin. This result is in line with Linn (2008), who finds evidence that new plants respond to higher prices by adopting less energy intensive technologies. Interestingly, the intensive margin of ISTC (i.e., which determines services of capital) does not affect the growth rate of ESTC, but it rises the (detrended)

trucks could have increased by nearly 60 percent from 1980 to 2006, instead of the observed meager 15 percent. Our focus is on the production side of the economy, and we revise this part of the empirical evidence in Section2.

energy intensity level of investment. This is so because higher ISTC at the intensive margin rises obsolescence of capital and shortens its service life, which rises the profitability of relatively high energy intensity capital. Thus, the effect of ISTC on Energy Saving Technical Change is complex and, quantitatively, it also depends on the growth rate of the energy price.

To study further our economy we need to resort to quantitative tools. We calibrate our model economy to reproduce salient features of the US economy for the period 1960-2010. We study the effect of permanent changes in ISTC growth rates and the energy price from two different points of view. First we study the effect on relevant aggregates from the economic point of view: aggregate energy intensity (i.e., energy use per unit of value added) and the energy share. Next, we study the key variable for climate change concerns: aggregate energy use. We do so in two different extreme scenarios: Scenario 1 is one in which the energy is abundant and its price is constant. In Scenario 2 energy is scarce and its price grows over time. We show that a permanent change in the growth rate of ISTC has mild effects on aggregate energy intensity and the energy share (deviations are below 1.30 percent of their benchmark values) but that it has a significant growth effect on aggregate energy use, regardless of the scarcity scenario considered: its growth rate rises in a full percentage point. The scarcity scenario matters for the economic variables: A 50 percent permanent change in ISTC growth produces a rise the energy share of 1.20 percent in Scenario 1, whereas it produces a fall in the energy share of 0.66 percent in Scenario 2. The difference is due to the quantitative interaction of investment and ESTC in each scenario.

We also show that permanent changes in the level of energy price do not affect the macroeconomy in either scenario: neither energy use, energy use per unit of value added nor the energy share are affected. The growth rate of Energy Saving Technical Change is not affected either. There is only a level effect: the average energy intensity of capital falls, leaving the economy unaffected. Hence, indirect taxes on energy use do not affect the long run evolution of the economy. Finally, we assess whether increased energy scarcity will harm the economy. We compute the effect of a permanent 10 percent increase in the growth rate of the energy price. Acceleration in ESTC and the fall in energy use shield the economy against the scarcity of the resource: The effect on the growth rate of output is negligible even though the energy share rises 0.66 percent. This is so because the gains in energy efficiency implied by investing in less intensive capital reduce value added per unit of energy.

Finally, we show that our economy can be represented in terms of aggregate capital and aggregate energy use. The aggregate representation is a version of Greenwood et al. (1997) where there is a law of motion for energy use. The energy requirement of capital and its irreversibility imply that aggregate energy use behaves as a stock whose depreciation is given by the physical decay of capital and economic obsolescence (older capital is more energy intensive), although the latter for energy corresponds to capital retirements. Depreciation of the capital stock, though, depends on physical decay and obsolescence due to ISTC and Energy Saving Technical Change. Aggregate output is produced with a technology with unitary elasticity between capital and labor and zero elasticity in the short run between the capital-labor composite and energy. The endogenous choice of ESTC and investment allow to rise the substitution of the capital-labor composite and energy over time. This representation illustrates on the short-run dynamics of our economy. For instance, changes in ISTC produce a non-monotone response of energy use per unit of value added and the energy share do to the fact that investment changes slowly the average energy intensity of the economy.

We view ours as a theory of investment where we make explicit the determinants of energy use and Energy Saving Technical Change. We are not the first in studying the macroeconomic implications of the complementarity of capital and energy at the micro level. An early attempt to reconcile the short-run response to energy price changes with a long-run adjustment of energy aggregates is in Atkeson and Kehoe (1999). In their model, higher efficiency comes at the cost of less capital services and, therefore, less output. Building upon the idea of heterogeneous capital, Díaz et al. (2004) discuss a model economy where capital is putty-putty but there are adjustment costs in investment at the aggregate level, so that energy efficiency comes at an output cost. These authors, however, abstracted from ISTC which is key to study the determinants of rebound effects in aggregate energy use. On the other hand, our theory complements a theory of directed technical change, as Acemoglu (2002), and Acemoglu et al. (2012), where the former studies the determinants of skill biased technical change and the latter focus on the substitution from environmental damaging to friendly technologies. A related work to ours is that by Hassler et al. (2016) who build on the tradition of Pindyck and Rotemberg (1983) and use an aggregate production function to study the aggregate effects of resource scarcity in the presence of technical change. Alternatively, Fiori and Traum (2016) try to account for vintage effects on emission rates (for the main pollutants) in a generalized (S,s) framework with energy efficiency tied to technical progress. A paper close to ours is Rausch and Schwerin (2017) who also study a version of Atkeson and Kehoe (1999) to study the efficiency paradox. They, however, do not explore the interaction of the two margins of Investment Specific Technical Change and Energy Saving Technical Change.

The organization of the paper is as follows. Section 2 discusses the basic evidence at the macro level about the patterns of energy use. Section 3 presents our environment. In Section 4 we discuss the importance of assuming that energy intensity is embodied in capital by means of a counterfactual economy where there is not such a requirement. Section 5 studies theoretically the interaction of ISTC and energy prices to determine the growth rate (and level) of our notion of Energy Saving Technical Change: the choice of energy intensity of new capital. In Section 6 we turn to describe our quantitative exercises. Section 7 concludes.

2 Energy use and energy intensity at the aggregate level: evidence from aggregate US data

To fix ideas we present some facts about energy use and energy intensity at the aggregate level. We use the US economy as an illustrative case and we construct our own database to summarize those aggregate facts. The data are also used to calibrate the models in the quantitative exercises. A detailed explanation of the sources and methods used in our data construction is given in the Appendix A.

Figure 1 shows some aggregate statistics for the US economy. Figure 1(a) shows the evolution of aggregate Value Added, VA, net of value added of energy producing sectors, and aggregate energy consumption, E, per worker. The aggregate energy measure, E, is a composite of total consumption of oil, coal, natural gas and electricity measured in British Termal Units. Renewable energy is not included.[2] Value Added has grown at an average rate of 1.27 annually, although the growth rate has changed markedly. During the period 1960-73, the average annual growth rate of Value Added was 3 percent, slowing down to 0.68 percent during the latter period. The evolution of energy use is even more striking. It was growing at 2.38 up to 1973 and since then, it declines at the rate of 1.2 percent. In spite of those very stark movements in energy use, the ratio of aggregate Value Added to energy use VA/E, shown in Figure 1(b) has grown at an average annual rate of 1.54 percent during the entire period.[3]

It may be asked how much of the trend in Value Added per unit of energy is due to changes

[2] We focus our analysis on exhaustible resources. We view renewable energy as a reproducible factor.

[3] The ratio VA/E is the inverse of a standard measure of *aggregate energy intensity* and it is closely related to the index of energy efficiency reported by the World Bank, which measures the amount energy consumed in millions of Joules per unit of GDP.

in aggregate activity, or sectoral composition, and how much is due to improvements in energy efficiency. Metcalf (2008) is an early attempt to address this issue. He uses a US state-level analysis and, in particular, shows with a Fisher decomposition that about three quarters of the observed decline in aggregate energy intensity results from efficiency improvements (measured by changes in sector specific energy intensity) and the other quarter comes from changes in sectoral activity. The analysis of state-level data determines that efficiency improvements are found to be higher in states where the investment rate is higher than the average for the US, the drivers for improvements being higher energy prices and rising per capita income. This is consistent with the fact that newer capital tends to be more energy efficient.

We have to emphasize the fact that the steady fall in energy intensity, measured as E/VA, has occurred even at periods where energy prices were falling. Figure 1(c) shows the relative price of energy, p^e. It is the weighted average of the relative price of a BTU in units of non durable goods and services, in logs. We have added a linear trend growing at 1.54 percent. It is apparent that the energy price has substantial fluctuations. For the period 1960-2010, the average growth rate of the price was 2.10 percent. Since the price fluctuates very much whereas energy intensity, E/VA, does not, the energy share, $p^e\, E/VA$ (not in logs) takes all the fluctuations: the average of this statistic for the entire period is 4.74 percent, it reaches a maximum of 8.86 percent in 1981 and a minimum of 3.19 in 1998. This figure suggests that the fall of energy intensity has to be driven by other forces aside from energy prices. We have added to this picture the average growth trend of Value Added per unit of energy, VA/E. The determinants of the evolution of energy prices, specially that of oil, is a matter of much controversy (see, for instance, Alquist et al., 2013); in particular, there is a lively debate about the existence of a unit root in energy prices (see, for instance, Zaklan et al., 2016). We do not aim to enter this debate and we just want to point out that there is a high energy price volatility but that the energy share, in spite of its volatility, has never been larger than 9 percent of aggregate Value Added.

Some micro literature suggests that the fall in aggregate energy intensity, E/VA, is related to capital accumulation and energy efficiency improvements. For instance, Steinbuks and Neuhoff (2014) study how capital stocks adapt to energy price changes for 19 OECD countries and five manufacturing industries. Their results indicate that the impact of energy prices on energy efficiency improvements depends significantly on the vintage of capital. The reason is that energy efficiency varies across vintages of capital depending on the interaction of energy prices and *exogenous tech-*

nological change (sic). Aggregate data seems consistent with this finding. Figure 1(d) shows the evolution of the ratio of capital to energy, K/E, that we use as an aggregate measure of the *energy efficiency of capital*. The statistic K is the value of the stock of capital in units of final output. We have also added a linear trend which again grows at the mean rate of the ratio VA/E. During the period 1960-2010 the growth rate of VA/E was 1.54 percent, whereas that of K/E was 1.60 percent. We also show the capital to value added ratio, K/VA, which is not logged since it has no trend. Both pictures together suggest that the US economy grows along a balanced growth path and that aggregate energy efficiency is growing over time.

Finally, the evidence of Steinbuks and Neuhoff (2014), among others, suggest that the observed trend in energy intensity hides a substantial amount of heterogeneity at the micro level. For instance, Boyd and Lee (2016) use plant level data for five US manufacturing industries and estimate the evolution of energy intensity in each plant. They decompose improvements in energy intensity into changes in energy efficiency and technical change. In so doing they stress the vintage nature of the capital stock. They find that the effect of technical change is sizable although it varies over the years. We interpret these latter findings as suggestive of the time variation in Investment Specific Technical Change. Figure 1(e) depicts the inverse of the relative price of investment goods taken from Rodríguez-López and Torres (2012) who update the work by Cummins and Violante (2002), so it is adjusted by quality. The growth rate of the inverse of the price is a measure of technical progress. Notice that there is an acceleration since 1982, period after which the relative price falls at the annual rate of 2.58. Prior to that date, the rate was 0.79 percent.

Summarizing, the US economy seems to grow along a balanced growth path, it is becoming more efficient in terms of its capital to energy ratio, and its energy intensity (per unit of value added) is falling, even during the periods in which the fall in the energy price is very intense, as during the period 1981-1998. As a result, the capital to value added ratio, K/VA, and the energy share, $p^e\, E/VA$, have no trend, and the relative price of investment goods falls at a constant rate. This evidence shown will provide us the quantitative discipline to build and take to the data our theory model.

3 A putty-clay vintage model of energy use

Inspired by the evidence shown, we have built a model economy based on Atkeson and Kehoe (1999) in which we formalize the idea of Energy Saving Technical Change embodied in capital. We model energy intensity (i.e., the amount of energy needed for the unit to capital to yield services) as a fixed energy requirement for capital to yield services. We add to Atkeson and Kehoe (1999) the explicit consideration of Investment Specific Technical Change. That is, we assume that there are two types of technical change: Energy Saving Technical Change, which is endogenous to the model and responds to economic conditions, and a standard notion of Investment Specific Technical Change (innovations that increase quality of capital and increase efficiency of producing capital goods). Hence, in our framework, capital is heterogeneous in two dimensions: vintage (given by ISTC) and energy intensity (governed by the choice of ESTC). Energy Saving Technical Change interacts with ISTC to govern the evolution of energy intensity and aggregate energy use in the economy.

3.1 The environment

This is is an infinite horizon economy populated by infinitely lived agents. Time is discrete.

3.1.1 Preferences

There is a continuum of households that seek to maximize expected discounted lifetime utility,

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}, \; \beta \in (0,1)\,, \; \sigma > 0, \tag{3.1}$$

where c_t is consumption. Households are endowed with one unit of time every period.

3.1.2 Technology and the physical environment

The production of the unique final good needs of capital services, $\mathcal{K}$ and labor, h, according to the production function

$$y_t = A_t \, \mathcal{K}_t^{\alpha} h_t^{1-\alpha}, \; \alpha \in (0,1). \tag{3.2}$$

A_t is the neutral technical change factor, which grows at the constant rate γ_a. Capital is heterogeneous in two dimensions: vintage, denoted by z, and the energy required for the unit of capital to yield services, denoted by e. Thus, e is an energy requirement that, as we shall see, acts as a fixed cost of operating capital. We denote as $k_t(z,e)$ the amount of capital of class (z,e) at time t. Formally, one unit of capital of class (z,e) yields

$$\kappa\,(z,e) = (1+\lambda)^{z\,(1-\phi)} e^{\phi},\ z \leq t,\ e \in \mathbf{R}_{++} \tag{3.3}$$

where $\phi < 1$. Thus, e is the index of energy intensity of the unit of capital. $(1+\lambda)^t$ measures the change in quality of capital goods over time and is an index of Investment Specific Technological Change. Energy intensity type, which is our notion of Energy Saving Technology, and ISTC are meant to refer to different factors. We may think of type e as engine power of a car, whereas λ refers to technological improvements that increase capital services which are not directed to saving energy. For instance, any improvement in the ergonomics of car seats, so that driving is less tiring, would be a change in the vintage of capital.[4] Notice that more energy intensive capital yield more services and, thus, is more productive but, on the other hand, it is more expensive due to energy related costs. Finally, aggregate capital services satisfy

$$\mathcal{K}_t = \sum_{z=-\infty}^{t} \int_{e\in\mathbf{R}_{++}} \kappa\,(z,e)\ k_t(z,e)\,de. \tag{3.4}$$

where $\mathcal{K}_t$ are aggregate capital services over all units of capital of class (z,e), $k_t(z,e)$, *utilized*. It is important to notice that not all classes (z,e) will be used in equilibrium as we shall see.

In addition, there is a technology that allows agents to transform final good of period t into $(1+\theta)^t$ units of capital of class $(t+1,e)$. As it is standard in the literature, we take the view that ISTC not only brings higher quality, given by λ, but also the production of capital goods becomes increasingly efficient with the passage of time (see Greenwood et al., 1997). For convenience, we will refer to λ as the *intensive margin* of ISTC and to θ as the *extensive margin* of ISTC.

Notice that the passage of time rises the Investment Specific Technology level in both terms, intensive and extensive margin. This implies that a time t the vintage $t+i$, $i \geq 1$, is not available for production because the technology has not been created yet. Differently from ISTC, agents can always invest in any type of Energy Saving Technologies, $e \in \mathbf{R}{++}$ to be combined with ISTC to

[4] See Knittel (2011) for particular examples of embodied technical innovations.

produce new classes of capital. Agents, though, have to trade-off the higher services yielded by energy intensive capital with the high energy expenditure needed to use it. We will denote as $x_t(e)$ the amount of final good invested in capital of vintage $t+1$ that will be operated with intensity type $e \in \mathbf{R}_{++}$ at period $t+1$.

Capital is irreversible; that is, capital of class (z, e) cannot be converted in capital of class (z', e'). Capital, though, can be left idle. Finally, once production has taken place, capital depreciates at the rate $\varpi \in [0, 1]$. Therefore,

$$k_t(z, e) = (1 - \varpi)^{t-z}\, k_z(z, e),\ z \leq t, e \in \mathbf{R}_{++}. \tag{3.5}$$

3.2 Market arrangements

We assume that energy is entirely bought in an international market at an exogenously given price p_t^e, which grows at the constant rate γ_p. There is no international borrowing and lending. In absence of an international credit market we can think of the price of energy as given by nature. There is no uncertainty in this economy. The sources of growth are neutral progress, ISTC, and Energy Saving Technical Change.

Households are the owners of all inputs. There is a secondary market for capital that opens at the end of the period, once its return has been realized. We further assume that all households start out with the same amount of capital. They can trade a one period risk free asset which is in zero net supply.

3.2.1 The firm's problem

The firm producing the final good has to decide how much capital of each class wants to use, $k_t(z, e)$. The rental price differs across classes, $r_t(z, e)$, since they yield different services:

$$\begin{aligned} \pi_t = & \max_{\substack{h_t \geq 0, \\ k_t(z,e) \geq 0}} && A_t\, \mathcal{K}_t^{\alpha} h_t^{1-\alpha} - w_t\, h_t - \sum_{-\infty}^{t} \int_{e \in \mathbf{R}_{++}} [r_t(z, e) + p_t^e\, e]\ k_t(z, e)\, de \\ \text{s. t.} & && \mathcal{K}_t = \sum_{z=-\infty}^{t} \int_{e \in \mathbf{R}_{++}} (1+\lambda)^{z\,(1-\phi)} e^{\phi}\, k_t(z, e)\, de. \end{aligned} \tag{3.6}$$

This problem shows that new vintages yield higher services, regardless its energy type, whereas more intensive types yield more services but at the cost of higher energy expenditures. Again, notice that capital of any class (z, e) can be left idle at no cost.

3.2.2 The production of new capital

The production of capital uses a linear technology in final good:

$$\begin{aligned} \pi_t^x = & \max_{x_t(e) \geq 0} \quad \int_0^\infty \left[p_t\,(t+1, e)\,(1+\theta)^t - 1 \right] x_t(e)\, de, \\ \text{s. t.} & \qquad x_t(e) \geq 0,\; e \in \mathbf{R}_+. \end{aligned} \tag{3.7}$$

3.2.3 The household's problem

Capital can be traded at the individual level. New investment, however, comes in new vintage—it is a technological restriction. We denote household's holdings of capital of class (z, e) as $m_t(z, e)$ to differentiate them from the demand of capital undertaken by the firm, $k_t(z, e)$. It is implicit in the problem that agents decide how much they want to invest in the new classes of capital, $m_{t+1}(t+1, e)$. The household's problem can be written in the following way:

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \\ \text{s. t.} \quad & c_t + \sum_{z=-\infty}^{t+1} \int_{e \in \mathbf{R}_{++}} p_t(z, e)\, m_{t+1}(z, e)\, de + a_{t+1} \leq w_t\, \hbar + (1 + r_t^a)\, a_t + \\ & \quad + \sum_{z=-\infty}^{t} \int_{e \in \mathbf{R}_{++}} \left[(1 - \varpi)\, p_t(z, e) + r_t\,(z, e) \right] m_t(z, e)\, de + \pi_t^x, \\ & m_{t+1}(t+1, e) \geq 0,\; e \in \mathbf{R}_{++}, \\ & a_{t+1} \geq \underline{a}, \\ & m_0(z, e),\; z \leq 0,\; e \in \mathbf{R}_{++},\; a_0, \text{ and energy prices given.} \end{aligned} \tag{3.8}$$

The market return to each class of capital, $r_t(z, e)$ fully captures the trade-offs of investing in more or less energy intensive capital. Given a particular vintage, more intensive capital (larger e) yields a higher return in the short run but that returns declines rapidly over time if the energy price rises over time and the new vintages are more productive.

3.2.4 Definition of equilibrium

An equilibrium for this economy given the sequence of energy prices, $\{p_t^e\}_{t=0}^{\infty}$, is a sequence of prices $\left\{\{p_t(z,.), r_t(z,.)\}_{z=-\infty}^{t+1}, w_t, r_t^a\right\}_{t=0}^{\infty}$, an allocation $\left\{c_t, \{m_{t+1}(z)\}_{z=-\infty}^{t+1}, a_{t+1}\right\}$ for each consumer, an allocation for the firm, $\{h_t, k_t(z,.)\}_{z=-\infty}^{t}$, and an investment plan, $x_t(.)$, such that:

1. $\left\{c_t, \{m_t(z,.)\}_{z=-\infty}^{t+1}, a_{t+1}\right\}$ solves the household's problem shown in (3.8) given the sequence of prices,
2. $\{h_t, k_t(z,.)\}_{z=-\infty}^{t}$, solves the plant's problem given the sequence of prices,
3. the price of investment satisfies $p_t(t+1,e) \leq (1+\theta)^{-t}$, for all $e \in \mathbf{R}_{++}$,
4. markets clear,
   (a) the bond is in zero net supply, $a_{t+1} = 0$,
   (b) the labor market clears, $1 = h_t$,
   (c) the market of capital services clear, $k_t(z,e) = m_t(z,e)$, for all $z \leq t$, $e \in \mathbf{R}_{++}$,
   (d) the market of new capital clears, $m_{t+1}(t+1,e) = (1+\theta)^t x_t(e)$, $e \in \mathbf{R}_{++}$,
   (e) the final good market satisfies $c_t + x_t = y_t - p_t^e \sum_{z=-\infty}^{t} \int_{e\in\mathbf{R}_{++}} k_t(z,e)\, dxe$,
5. and the law of motion of capital of class (z,e) is $m_t(z,e) = (1-\varpi)^{t-z}(1+\theta)^{z-1} x_{z-1}(e)$, for all $t \geq z$, for all t, $e \in \mathbf{R}_{++}$.

4 On the role of embodied energy intensity

In this section we study the importance of modeling Energy Saving Technical Change embodied in capital. The fact that energy intensity is a feature of capital implies that, in the short run, there is no substitution between capital and energy. Only by investing in less energy intensive capital we can substitute, in aggregate terms, energy for capital. To assess the implications of our modeling choice we examine first the energy and capital allocation in an economy in which energy intensity *is not* a feature of capital. Hereafter we refer to this economy as the *putty-putty vintage economy.*

Assume, for the sake of the argument, that capital is heterogeneous in only one dimension, its vintage. Energy intensity of all vintages can be chosen period by period, $e_t(z)$, $z \leq t$, responding to market conditions. That is, energy intensity is not a technical requirement. The firm's problem becomes:

$$
\begin{aligned}
\pi_t = & \max_{\substack{\mathcal{K}_t \geq 0,\, h_t \geq 0,\\ k_t(z)\geq 0,\, e_t(z)\geq 0}} && A_t\, \mathcal{K}_t^{\alpha} h_t^{1-\alpha} - w_t\, h_t - \sum_{-\infty}^{t} \left[r_t(z) + p_t^e\, e_t(z) \right]\, k_t(z) \\
\text{s. t.} & && \mathcal{K}_t = \sum_{z=-\infty}^{t} (1+\lambda)^{z\,(1-\phi)} e_t(z)^{\phi}\, k_t(z),
\end{aligned}
\tag{4.1}
$$

where $e_t(z)$ denotes the optimal use of energy of vintage z. The solution to the firm's problem yields some key implications about the use of energy. It is important to distinguish between *gross return* to capital of vintage z, which is given by the sum of the rental price and the associated cost of energy, $r_t(z) + p_t^e\, e_t(z)$, and the *net return*, $r_t(z)$.

Result 1. *All capital vintages are always utilized in equilibrium.*

Capital is always utilized because the net return to capital is always positive:

$$
r_t(z) = (1-\phi)\alpha\, \frac{y_t}{\mathcal{K}_t}\, (1+\lambda)^{z\,(1-\phi)} e_t(z)^{\phi}, \tag{4.2}
$$

This result is due to the fact that energy is not a fixed requirement of capital in this economy and its demand responds to economic conditions. As we shall see, this is the opposite of our benchmark model, where capital has a finite service life.

Result 2. *Older capital is allocated less energy than new capital. The distribution of energy use across vintages is independent of time and the energy price.*

This counterfactual result is due to the fact that higher ISTC rises the productivity of energy. By inspecting the first order conditions of the firm's problem with respect to $k_t(z)$ and $e_t(z)$,

$$
p_t^e = \phi\, \alpha\, \frac{y_t}{\mathcal{K}_t}\, (1+\lambda)^{z\,(1-\phi)} e_t(z)^{\phi-1}. \tag{4.3}
$$

Aggregate energy use is given by the sum of energy used by all vintages:

$$
\mathcal{E}_t = \sum_{z=-\infty}^{t} e_t(z)\, k_t(z). \tag{4.4}
$$

Aggregating using (4.3) we find that the share of energy in aggregate (gross) output and aggregate value added is always constant. This implies that the value added to energy ratio, $\left(y_t - p_t^e\, \mathcal{E}_t\right)/\mathcal{E}_t$, must always follow the price of energy:

Proposition 1. *The value added to energy ratio is equal to* $\frac{y_t - p_t^e \mathcal{E}_t}{\mathcal{E}_t} = \frac{(1-\phi\,\alpha)\,p_t^e}{\phi\,\alpha}$.

The proof of this Proposition is very easy and only needs of manipulating the first order conditions of the firm's problem. Notice that this technology cannot deliver a time varying share of energy, which is clearly at odds with the data (recall Figure 1(c)), but it also implies that any type of technical innovation has no effect on the aggregate use of energy per unit of aggregate value added, which also seems to be at odds with the evidence. The reason why this economy delivers counterfactual predictions is that the short-run elasticity between capital and energy is very large (in fact, it is equal to 1). This is even the case when we assume a lower elasticity by using CES production function as Atkeson and Kehoe (1999) show. This is why we assume next, as Atkeson and Kehoe (1999) do, that energy is a fixed requirement of capital. In the aggregate, however, energy is not a fixed requirement due to Energy Saving Technical Change, which we turn to analyze now.

5 Energy intensity of new capital and ISTC in the long run

We turn to analyze the main properties of our *putty-clay vintage capital economy* in the long run. Before discussing the evolution of the value added to energy use ratio we need to analyze some properties of the equilibrium of this economy. In this section we focus on the determinants of energy intensity of investment, that is, of the new capital, in the long run. We do so because, along the balanced growth path, the energy intensity of the capital stock is determined by the choice of energy intensity of the latest vintage and, in its turn, it determines the evolution of energy use per unit of vale added.

5.1 Energy intensity of new capital

Proposition 2. *Investment at any time* t *only takes place on one energy intensity type,* e_{t+1}.

Proof: See Appendix B. This result implies that all capital of the same vintage has the same energy intensity. Therefore, we are going to denote as e_z the energy intensity of vintage z. The fact that there is no uncertainty in the energy price, and the concavity of the net return to capital, $r_t(z, e)$, with respect to the energy intensity of capital, e, ensure that there is a unique type, e_{t+1}, that maximises the present value of all future returns. This simplifies the analysis of the putty-clay vintage economy very much.[5] The corollary of this Proposition is that the average energy intensity in the economy depends on the size of investment. Now we turn to discuss properties of the balanced growth path of this economy, for which we need a technical condition: The ISTC growth rate is sufficiently large compared to that of the energy price:

Assumption 1. $[(1+\lambda)(1+\theta)]^{1-\phi}(1+\gamma_p)^{-\phi} > 1.$

5.2 The balanced growth path

The heterogeneity of capital makes the analysis of the balanced growth path a bit complex. In order to know the growth rate of energy intensity of investment at time t, e_{t+1}, we need to know the growth rate of gross output. Thus, we need to proceed step by step.

Proposition 3. *The length of service life of capital is constant, T.*

Proof: See Appendix B. This occurs because the net return to any class of capital, $r_t(z,e)$, falls over time and, eventually, becomes zero, because of two reasons: First, as we will show, the output to capital services ratio, $y_t/\mathcal{K}_t$ falls at a constant rate because of the existence of ISTC. Second, the energy price grows over time. Solving the firm's problem we see that the net return, $r_t(z,e)$, is given by the difference between marginal productivity of capital in production and the associated energy expenditure:

$$r_t(z,e_z) = \alpha\,\frac{y_t}{\mathcal{K}_t}\,(1+\lambda)^{z(1-\phi)}e_z^{\phi} - p_t^e\,e_z,\; z \leq t. \tag{5.1}$$

Hence, as the ratio $y_t/\mathcal{K}_t$ falls and/or the energy price rises, the net return falls. T satisfies $r_{z+T-1}(z,e_z) \geq 0$. This result stand in stark contrast with Result 2 above, which states that the length of the service life of capital is infinite the putty-putty economy. This Proposition has a straightforward Corollary about the number of vintages used in production at the balanced growth path:

Corollary 1. *At the balanced growth path, the number of vintages used in equilibrium is T. The*

[5] The absence of uncertainty in the energy price is a sufficient condition to obtain this result. Atkeson and Kehoe (1999) find the same result when price fluctuations are not large. Something that we can also exploit in stochastic applications of our theory.

oldest vintage used at time t, $\underline{z}_t$, satisfies

$$\underline{z}_t = t - T + 1. \tag{5.2}$$

The fact that capital has a finite service life implies that, at the aggregate, the utilization rate of capital is endogenous as in Gilchrist and Williams (2000) and Wei (2003). This capital replacement mechanism is also common to the Schumpeterian growth literature where it helps to reconcile steady long-run growth with large movements in energy prices as in Ferraro and Peretto (2017).

Now we can turn to study the trend of the energy intensity of investment, which will determine the evolution of energy use in the long run and the growth rate of our measure of Energy Saving Technical Change.

Proposition 4. *Energy Saving Technical Change, measured by energy intensity of new capital, e_{t+1}, grows over time at the growth rate of the extensive margin of ISTC and the growth rate of the energy price, $(1+\gamma_p)(1+\theta)$,*

$$e_{t+1} = \widehat{e}\left[(1+\gamma_p)(1+\theta)\right]^{-t}. \tag{5.3}$$

Proof: See Appendix B. The intuition is as follows: The fact that energy intensity is built in capital and that capital is irreversible, imply that energy use is a stock instead of a flow. The amount of energy used at time t is

$$\mathcal{E}_t = \sum_{z=\underline{z}_t}^{t} e_z\, k_t(z, e_z). \tag{5.4}$$

This stock $\mathcal{E}_t$ depreciates because of physical decay of capital, given by ϖ, and also because of economic obsolescence. This is due to the fact that every period a vintage is left idle and never used again. At the balanced growth path, the combined depreciation of energy use is a constant fraction of the stock, $\delta_{\mathcal{E}}$, which is larger than ϖ, and whose expression is given in Appendix B. At the balanced growth path, we can write

$$\mathcal{E}_{t+1} = e_{t+1}\,(1+\theta)^t\, x_t + (1-\delta_{\mathcal{E}})\,\mathcal{E}_t, \tag{5.5}$$

where $(1+\theta)^t x_t$ is the amount of capital of vintage $t+1$, $k_t(t+1, e_{t+1})$. Along the balanced growth path, aggregate energy use is governed by the energy intensity of new capital and the size of investment. The ratio $p_t^e\, e_{t+1}\,(1+\theta)^t\, x_t/y_t$ must be constant along the balanced growth path. It follows that energy intensity must fall at the combined growth rate of the energy price and the extensive margin of ISTC. This result is in line with the evidence presented by Linn (2008), who uses Census plant level data and finds that energy intensity of new entrants depends on energy prices and that technology adoption by incumbents accounts for a relatively small fraction of changes in aggregate energy demand in the 1970s and 1980s. This is a natural implication of this framework when we assume that incumbents use older vintages of capital than entrants.

Thus, agents invest in Energy Saving Technical Change (that is, energy intensity of capital falls over time) even if the energy price does not fall. This result is due to the extensive margin of ISTC. The larger θ, the more investment goods are created with one unit of final good. Since energy is embodied in each unit of capital, intensity must fall for energy expenditure not to rise. The fall in energy intensity implies that there is a loss in capital services (compared to more intensive capital), but that loss is compensated by the fall in energy expenditures. It is interesting to note that the growth rate of changes in quality of capital, λ, does not affect the growth rate of energy intensity of the latest vintage. It will affect, though, the detrended intensity level, $\widehat{e}$, as we will see next.

Proposition 5. *A larger ISTC growth rate implies a shorter service life of capital and a larger level of energy intensity,* $\widehat{e}$.

Proof: See Appendix B. The intuition is as follows: the gross return to capital, $r_t(z, e_z) + p_t^e\, e_z$, as shown in expression (5.1), depends on the evolution of the output to capital services ratio, $y_t/\mathcal{K}_t$. As shown in Appendix B, this ratio falls at the rate $[(1+\lambda)(1+\theta)]^{1-\phi}\,(1+\gamma_p)^{-\phi}$. The energy expenditure associated to any vintage, $p_t^e\, e_z$, grows at the rate $(1+\gamma_p)$. Thus, an acceleration of ISTC shortens the service life of capital, regardless of the level of energy intensity embodied in capital. As a result, the present value of the future return of investing in energy intensive capital rises (since their service life is shorter) and, therefore, $\widehat{e}$ rises.

The main lesson we extracted before, from analyzing the putty-putty vintage economy, was that when energy demand is elastic in the short run, the amount of energy used per unit of value added, in the aggregate economy, is not affected by technological change, which seems to be at odds with the microevidence. In the putty-clay vintage economy, though, the trend in energy per unit of value added is only determined by the energy price but the level does depend on technological change. However, the sign of the response of the level to changes in ISTC is a quantitative matter now. The reason is the following: Changes in ISTC affect not only the energy intensity of investment, e_{t+1}, but also the size of investment. The long run level of energy use and the ratio $y_t/\left(p_t^e\,\mathcal{E}_t\right)$ depend on the combined response of both and cannot be ascertained without quantitative tools. This is why we turn to calibrate and simulate our economy.

6 Energy use and ISTC in the long run

In this section we conduct some exercises to quantify the macroeconomic implications of modeling energy intensity as a feature of technology embodied in capital, as the micro evidence suggests. In order to take our model to the data we need first to give a notion of the value of the aggregate stock of capital in units of output since, so far, we have just analyzed the evolution of the stock of capital services. Thus, we turn now to discuss the mapping from capital services to the capital stock which, additionally, will give us insights about the connection of Total Factor Productivity and both ISTC and ESTC.

6.1 Aggregate capital and Total Factor Productivity

National Accounts obtain the value of the stock of capital using a perpetual inventory method where investment is valued using the relative price of investment discounted by physical depreciation and economic obsolescence. In our model economy that information is comprised by the relative price of capital, $p_t(z, e_z)$, for any vintage $z \leq t$. The expression of this price is given by equation (B.6) in Appendix B and it is obtained solving the household's problem. This price is the present value of all future returns of capital *net of energy expenditures*. It becomes zero after T periods; that is, when the vintage is no longer used in production. Thus, economic obsolescence does not fall at a constant rate over time. As a result, aggregate capital is a function of the entire distribution of capital across vintages and does not admit a representation as a function of past aggregate capital and current investment. This is why we proceed to aggregate capital in units of gross output. First, we need to find the prices of capital in units of gross output.

Lemma 1. *The price of one unit of capital of class* (z, e)*,* $z \leq t+1$*,* $e \in \mathbf{R}_{++}$*, in units of gross* *output at time t is equal to :*

$$q_t\left(z,e\right) \equiv (1+\lambda)^{(1-\phi)(z-t-1)} \left(\frac{e}{e_{t+1}}\right)^{\phi} (1+\theta)^{-t},\ z \leq t+1,\ e \in \mathbf{R}_{++}. \tag{6.1}$$

See Appendix B for a discussion of these prices. Notice that $q_t(z,e)$ depends on energy intensity. This is consistent with Gordon (1990, 1996), who argued that changes in energy intensity also affect the relative price of capital goods (see Gordon, 1996, p. 262). In Appendix B we discuss the mapping of the market price of capital to this price and argue that the difference between both prices is negligible in the quantitative version of our economy.

Next, we follow National Accounts and define the aggregate stock as the sum of economically active capital. Recall that we have denoted as $\underline{z}_t$ the last vintage used in production at time t. Then, the stock of capital is

$$\mathbf{k}_t = \sum_{z=\underline{z}_t}^{t} q_{t-1}\left(z,e_z\right) k_t\left(z,e_z\right). \tag{6.2}$$

which can be written recursively as

$$\mathbf{k}_t = x_{t-1} + \left(1-\delta_{\mathbf{k}_t}\right)\mathbf{k}_{t-1}, \tag{6.3}$$

The factor $\delta_{\mathbf{k}_t}$ is the depreciation rate of the total stock of capital $\mathbf{k}_t$ between period $t-1$ and t. Its expression is found in equation (B.25) in Appendix B and along the balanced growth path it is

$$1-\delta_{\mathbf{k}} = \frac{1-\varpi}{(1+\lambda)^{1-\phi}(1+\theta)} \left[(1+\theta)(1+\gamma_p)\right]^{\phi}(1-\psi). \tag{6.4}$$

The depreciation rate combines physical decay, measured by $1-\varpi$, economic obsolescence due to changes in the standard notion of ISTC, $(1+\lambda)^{1-\phi}(1+\theta)$, the effect of the services loss associated to Energy Saving Technical Change, $[(1+\theta)(1+\gamma_p)]^{\phi}$, and economic obsolescence arising from Energy Saving Technical Change, measured by $1-\psi$. The value of ψ is given by the weight in total capital services of the services of last vintage used in production at time t and no longer used at time $t+1$. Its expression is found in equation (B.14) in Appendix B. Notice that, contrary to ISTC, ESTC, as defined in Proposition 4, reduces the depreciation rate of capital. This is due to the fact that, *ceteris paribus*, less energy intensive capital yields lower services. Notice, also, that elasticity of capital services to energy, ϕ, affects the contribution of both types of technical change to the depreciation rate of capital.

Correspondingly, recall the expression of the stock of capital services, $\mathcal{K}_t$, shown in (3.4). It follows that the relative price of capital services in units of gross output is

$$q_t^{\mathcal{K}} = (1+\lambda)^{-(1-\phi)t} e_t^{-\phi} (1+\theta)^{-t+1}. \tag{6.5}$$

Hence, $\mathbf{k}_t = q_t^{\mathcal{K}} \mathcal{K}_t$. Consistently with Gordon (1996) and Cummins and Violante (2002), this price falls when ISTC rises, as measured by $(1+\lambda)(1+\theta)$, but it rises with Energy Saving Technical Change, measured by the growth rate of $e_{t+1}^{-\phi}$. The reason is that less intensive capital, yields less services which leads to less gross output. Final value added, however, depends on gross output minus energy expenditures. Thus, our theory says that we cannot attribute all changes in the relative price of investment to ISTC but also to changes in Energy Saving Technical Change, as it was suggested by Gordon (1996).

The expression of aggregate value added as a function of capital (instead of capital services) is

$$A_t \left(q_t^{\mathcal{K}}\right)^{-\alpha} \mathbf{k}_t^{\alpha} h_t^{1-\alpha} - p_t^e \, \mathcal{E}_t. \tag{6.6}$$

At the balanced growth path energy expenditures, $p_t^e \, \mathcal{E}_t$, are a constant fraction of aggregate value added. That means that the factor $A_t \left(q_t^{\mathcal{K}}\right)^{-\alpha}$ comprises Total Factor Productivity in the long run and can be used to decompose sources of long run growth. Inspecting this expression is clear that any factor leading to an acceleration in ESTC produces a productivity slowdown. This is so because if energy intensity of investment, e_{t+1}, falls at a faster rate, the growth rate of services of capital decreases. As a consequence, the growth rate of $q_t^{\mathcal{K}}$ decreases and there is a TFP slowdown. This feature of the model should play a significant role in computing the benefits of moving from "dirty" to "clean" technologies.

6.2 Calibration

We use the data collected for the US economy described in Section 2. Appendix A describes in detail the sources and the processing of the data, in particular, the construction of the series for energy use, E, and its relative price.[6] Table 1 shows the targets used in our calibration procedure, where we have also calibrated the *putty-putty* vintage economy discussed in Section 4 for illustrative purposes. Appendices B and C characterize the balanced growth path of the putty-clay and the putty-putty economies, respectively.

We consider two different scenarios. In *Scenario 1* we assume that the energy price has no trend growth. We could think of this scenario as one in which energy is not scarce. In *Scenario 2*, however, the energy price has a positive trend in growth, γ_p, due to foreseen energy scarcity. Instead of estimating a permanent trend in our series of energy price, for illustrative purposes, we set the growth rate equal to the mean growth rate of the ratio of aggregate value added to energy intensity, VA/E, which was 1.54 percent for the period 1960-2014.[7] Comparing both scenarios will be useful to understand the determinants of the ratio of value added to energy use.

The calibration of both economies is very similar. Long run growth rates of all aggregates is the same in both economies. For instance, aggregate value added grows at the rate

$$1 + g = \left[(1+\gamma_a)\,(1+\gamma_p)^{-\alpha\,\phi}\right]^{\frac{1}{1-\alpha}} \left((1+\lambda)(1+\theta)\right)^{\frac{\alpha(1-\phi)}{1-\alpha}}, \tag{6.7}$$

and the relative price of capital falls at the rate

$$\frac{q_t^{\mathcal{K}}}{q_{t+1}^{\mathcal{K}}} = \left[(1+\lambda)(1+\theta)\right]^{1-\phi} (1+\gamma_p)^{-\phi}. \tag{6.8}$$

In both economies, changes in energy prices affect the measurement of Investment Specific Technical Change. The key difference is the parameter ϕ, the elasticity of capital services with respect to energy. Not surprisingly, the calibrated value of ϕ is larger in the putty-putty economy. This, in its turn, implies that the ISTC growth rate, measured by $(1+\lambda)(1+\theta)$, is slightly lower in the putty-putty economy, 2.58 percent versus 3.11 percent in our benchmark economy. Different measurement of ISTC leads to different calibration of the growth rate of neutral progress, γ_a, -0.24 percent in the putty-putty economy versus -0.42 percent in the putty-clay economy.

We do not report the calibrated value of ψ, the weight of the oldest vintage used in production in total capital services in the putty-clay economy. We do not do it, because its value is never above

[6] We use the letter E to refer to the data and $\mathcal{E}$ to refer to the model statistic.

[7] The energy share must be constant for this economy to have a balanced growth path. In our data, the average growth rate of the energy price is 2.10, which is close to the growth rate of Value Added per unit of energy, VA/E. Thus, our assumption is not far from the data.

$1e^{-5}$, meaning that, by the time a vintage is no longer profitable, it has essentially disappeared due to physical decay. This implies that economic obsolescence due to the direct effect of Energy Saving Technical Change is almost negligible in our framework. Therefore, $\delta_{\mathbf{k}}$, whose expression is shown in (6.4), is the same in both economies.

Figure 2 shows the bias in which we incur by using prices in units of gross output, $q_t(z,e)$, to aggregate capital instead of market prices, $p_t(z,e)$, for all $z \leq t$ and $e \in \mathbf{R}_{++}$ in the putty-clay economy. Figure 2(a) shows the ratio $p_t(z,e_z)/q_t(z,e_z)$ for all active vintages in the two possible Scenarios. The bias rises substantially with the age of the vintage. In Figure 2(b) we show the bias weighted by the existing amount of capital of vintage z at time t, which is $(1-\varpi)^{t-z}$. Thus, the maximum weighted bias in the value of capital is about 2 percent. Hence, we conclude that our measure of capital (which is consistent with the National Accounts procedure) is a good proxy for the market value of the stock of capital.

Now we turn to explore the effect of ISTC on energy use in the long run. We study two cases: In the first one we assume that there is a permanent change in the growth rate of ISTC and in the second one we study the effect of a change in the energy price, namely, that of an indirect tax on energy consumption.

6.3 Long term consequences of changes in ISTC

Table 2 summarizes the long run effects of changes in ISTC and (after tax) energy prices in the four statistics considered in Section 2: the amount of value added per unit of energy used, VA/E, the energy share in aggregate value added, p^eE/VA, the ratio of capital to energy use, K/E, and the capital to aggregate value added ratio, K/VA, together with the investment rate, X/VA. We start by showing the effects of a permanent increase in the growth rate of ISTC of 50 percent.[8] We have denoted as $(1+\lambda)(1+\theta)$ the ISTC factor. Thus, we do not specify here the nature (i.e., intensive or extensive margin) of the change in ISTC.

The first row of Table 2 shows the effect of an increase in the growth rate of ISTC in the putty-putty economy. As we can see, its effect is limited to a reduction in the capital to value added

[8]Next we implement a 10 percent shock in the oil price and a 50 percent shock to ISTC growth. Given our calibration, the response to a 5 percent shock in ISTC growth (less than a 1 percent shock to the growth factor), is of the order of magnitude of the response to a 1 percent shock in the energy price (a shock which quantitatively we better know: a 1 percent shock to p^e is very small; the oil price shocks of the 70s and the 2005-2008 oil price boom were shocks of about a 20 percent size).

ratio. This is a standard result in a neoclassical growth model economy: whenever the growth rate of technical change increases, obsolescence of capital rises, investment rises as a share of value added, and the value of the stock in units of output decreases. Value added per unit of energy used, however, is not affected at all since it only depends on the energy price. As a result, the ratio K/E decreases, and the economy in the aggregate becomes more energy intensive: value added and energy both augment in the same proportion per unit of capital. The energy share, then, is unaffected, too. This is the case regardless of the scarcity scenario.

Let us turn now to the putty-clay economy in Scenario 1, where the energy price has no trend. Notice that in this economy, contrary to the evidence, the amount of value added per unit of energy used, VA/E, and the capital to energy ratio, K/E, are constant at the balanced growth path. Nevertheless, studying this economy is instructive. A permanent 50 percent increase in the growth rate of ISTC produces (1) a fall in the relative price of capital services, so that agents invest more (see Table 2) and (2) a reduction of the service life of capital (from 100 to 68 periods), as reported in Table 3. As a result, agents turn to invest in more energy intensive capital, in detrended terms (see the value of $\widehat{e}$ in Table 3, from 0.0133 to 0.144). The joint interaction of higher intensity and higher investment rate implies that energy use increases more than aggregate value added: the ratio VA/E falls in 1.19 percent. The energy share rises, accordingly, 1.20 percent. The effect is the same regardless of the nature (λ or θ margin) of the innovation in ISTC. Energy use per unit of value added rises in spite of energy intensity of investment falling at a faster rate (recall Proposition 4) along the balanced growth path. The change in the aggregate value added to energy use ratio can be decomposed in the capital to energy ratio (capital measured in units of output), K/E, and the capital to value added ratio, K/VA. Thus, mechanically we see that the ratio VA/E falls because K/E falls more than K/VA. That is, aggregate energy intensity of capital, the inverse of K/E, rises more than average capital productivity, VA/K.

Now we can move to discuss the effect of a permanent change in ISTC in Scenario 2, where the energy price grows at a constant rate (see row 3 in Table 2 and row 2 in bottom panel of Table 3). In this economy both the value added to energy used ratio, VA/E, and the capital to energy ratio, K/E, have a positive trend. Both ratios grow at the rate of γ_p, the growth rate of the energy price. The change of the main aggregate statistics is shown in the third row of Table 2. The effect on VA/E is the opposite to the one found in Scenario 1. Now, the amount of value added per unit of energy used rises to a higher path (with the same trend). In both Scenarios, the effect of

ISTC at the micro level is the same: the investment rate rises, the service life of capital is shortened and the detrended energy intensity of investment, $\widehat{e}$, rises. The key difference is the relative effect on aggregate energy intensity of capital, the inverse of K/E, compared to that on average capital productivity, VA/K.

Summarizing, the response of energy use per unit of value added when the relative price of capital falls (due to an acceleration in ISTC growth) depends on how the choice of investment and energy intensity (i.e, the choice of Energy Saving Technology) interact, and determine the aggregate intensity of capital and its productivity. This result is independent from the nature of the change in ISTC growth: either a change in the intensive margin, λ, or the extensive margin, θ. However, both margins have different implications on our notion of Energy Saving Technical Change. A change in λ rises the average intensity of capital, $\widehat{e}$, but it does not affect the growth rate at which energy intensity falls, $(1+\theta)(1+\gamma_p)$. This is so because an increase in λ rises services of capital regardless of its energy intensity. This, in its turn, rises gross output and value added. Thus, we could think of λ as an indirect energy saving technology. A change in θ, though, implies that agents have to sacrifice less consumption in order to produce new capital. Thus, a rise in θ makes agents to invest more and accelerate Energy Saving Technical Change.

6.4 Long term consequences of changes in the energy price

First, we want to consider the long run effect of a permanent increase in the level of the energy price due, possibly, to a rise in indirect taxes. It is very interesting to note that all economies behave in the same way in the long run (see again Table 2). An increase in 10 percent in the price produces a corresponding rise of 10 percent in the ratios VA/E and K/E, so as to keep constant the energy share. In the putty-clay vintage economy the energy intensity level falls (a permanent reduction in $\widehat{e}$) but the lifetime of capital as well as the depreciation rate of capital are unaffected (Table 3). Notice that the capital to value added ratio is unaffected in all economies, which also implies that the long run level of income is not affected by the level of the energy price. This also would be the long run effect of imposing taxes on the price of energy: a permanent reduction in the use of energy.

Secondly, we turn to study a change in the growth rate of the energy price. This would be the case if scarcity of the resource increases. The effect in the putty-putty vintage economy is similar to a fall in the ISTC growth rate: the (detrended) ratio of value added to energy used and the energy share are left unaffected, but the detrended capital to energy ratio, K/E, and the capital to value added ratio increase. More interesting is the effect in the putty-clay vintage economy. A permanent increase in the growth rate of the energy price is very similar to a permanent fall in the growth rate of the extensive margin of ISTC, θ. In the first place, energy intensity level, $\widehat{e}$, falls from 0.0110 to 0.0109 (see again Table 3). The service life of capital also falls, from 67 to 65 years. Back to Table 2, value added to energy used ratio falls, since a higher fraction of gross output is lost paying for the energy cost. This is why the energy share rises 0.67 percent points. The capital to energy ratio (detrended) falls. That is, the aggregate economy becomes less energy efficient. This is so because investment, which takes place in more energy efficient capital, falls. This case is interesting when we want to assess sustainability of growth when energy use comes from fossil fuel origin. For instance, suppose that the price rises 10 times, not far from what we saw in real terms from 1974 to 1981. This would be a 1000 percent change. Energy share rises 66 percent, which would mean an energy share going from 4.74 in our benchmark economy to a 7.87 percent, which is similar to the peak value of the energy share during the 1980s, 8.86 percent.

6.5 A rebound in energy use?

The previous experiments can be used to assess whether there is an energy "rebound" effect induced by energy innovations. According to Gillingham et al. (2013), who follow the seminal definition of Jevons (1865), a "direct rebound" effect exists if a drop in the price of some energy services causes a rise in demand of that service. Gillingham et al. (2016) differentiate between innovation induced changes in energy intensity and policy-induced changes. Thus, we could think of the effect of an acceleration in ISTC and of a rise in energy prices due to an increase in indirect taxation, as examples of each case, respectively. The first lesson that we extract from our previous analysis is that if we want to define a rebound effect in a growing economy at the macro level we need to differentiate between *level effects* and *growth effects*.

Proposition 6. *At the balanced growth path, the growth factor of value added,* $1+g$*, is shown in expression* (6.7)*, whereas energy use,* $\mathcal{E}_t$*, grows at the rate* $(1+g)/(1+\gamma_p)$*. The amount of energy used is equal to*

$$\mathcal{E}_t = \widehat{\mathcal{E}} \left(\frac{1+g}{1+\gamma_p} \right)^t, \; \widehat{\mathcal{E}} = \frac{\widehat{e}\,\widehat{x}}{(1+g-(1-\varpi)(1-\psi))}. \tag{6.9}$$

Proof: It follows from the features of the balanced growth path characterized in Appendix B. In Table 4 we show the changes in the detrended level of energy use, $\widehat{\mathcal{E}}$, and in the growth rate of energy use, $g_{\mathcal{E}}$. We also report the change in the growth rate of Value Added per worker, for comparison purposes.

Let us turn to examine the case of a permanent 50 percent increase in the growth rate of ISTC. In both Scenarios there is a significant increase in the growth rate of energy use: It goes from 1.28 to 2.28 percent in Scenario 1 and it rises from -0.26 to 0.78 percent in Scenario 2. Thus, the main effect is a growth effect. Mechanically, detrended level of energy use, however, falls about 10 percent with respect to its level in the benchmark calibration. Thus, there is a sizable rebound effect in the growth of energy use. This is the case in spite of the fact that Energy Saving Technical Change, measured as the fall in intensity in investment, e_t/e_{t+1}, has accelerated (if the change in ISTC is due to the extensive margin). This implies that the rebound effect is due to the fact that the growth rate of investment has increased substantially, from 1.27 percent to 2.33 percent. Summarizing, we conclude that permanent changes in ISTC imply meager rebound effects in aggregate energy intensity (if any), measured by the amount of energy use per unit of value added, but sizable effects in aggregate energy use.

It is interesting to note that a permanent change in the level of the energy price leaves energy use unaffected. New capital is less intensive and compensates for the extra energy cost. This result implies that *ad-valorem* taxes is not a useful policy to reduce aggregate energy use. A permanent increase in the growth rate of the energy price, though, reduces both the level and the growth rate of energy use. Energy Saving Technical Change accelerates and investment is reduced. The implication of this is that indirect taxes on energy expenditures do not affect aggregate energy use, unless the tax rate increases in consumption. The cost, however, would be a mild reduction in growth rate of income in Scenario 2: it goes from 1.27 percent in the Benchmark economy to 1.2636 percent in the alternative economy with higher price growth.

6.6 Dynamics of energy use and intensity

We have learnt in the previous discussion that assuming that energy intensity is a feature of capital at the micro level is key for energy intensity per unit of value added to respond to technological advances. In order to have a sense of the differences in short-run responses we need to study the transitional dynamics of our putty-clay economy. Even without uncertainty, tracking the dynamics of the putty-clay economy is complex since not only the distribution of capital across vintages changes along the transition path but the service life of capital changes, too. Hence, to analyze the dynamics of the putty-clay economy we are going to study an approximate economy where capital is always used until it disappears. We can safely do so because, in our calibration, economic obsolescence of capital due to Energy Saving Technical Change is very small, and therefore, by the time a vintage is left idle its size is negligible with respect to the size of total capital services. Thus, we can assume that the depreciation of energy (which in the putty-clay economy behaves as a stock) and that of capital services are just given by physical decay and not economic obsolescence due to Energy Saving Technical Change. We have labeled this approximate economy as the *proxy putty-clay vintage capital economy*, (Proxy P-C). The aggregate allocation of investment and energy intensity in this economy can be found by solving the following quasi-social planner's problem:

$$
\begin{aligned}
\max \quad & \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma} \\
\text{s. t.} \quad & c_t + x_t = A_t \, \mathcal{K}_t^{\alpha} - p_t^e \, \mathcal{E}_t, \\
& 0 \le \mathcal{E}_{t+1} \le e_{t+1} \, (1+\theta)^t \, x_t + (1-\varpi) \, \mathcal{E}_t, \\
& 0 \le \mathcal{K}_{t+1} = (1+\lambda)^{(1-\phi)(t+1)} e_{t+1}^{\phi} (1+\theta)^t \, x_t + (1-\varpi) \, \mathcal{K}_t, \\
& 0 \le x_t, \; 0 \le e_{t+1}.
\end{aligned}
\tag{6.10}
$$

This economy is a version of Greenwood et al. (1997) with the explicit consideration of the complementarity between capital and energy at the micro level and the corresponding aggregation. Its calibration is shown in the last column of Table 1. For completeness, the long run effects of changes in ISTC and the energy price are shown in Table 5. The only difference between the true putty-clay vintage capital economy and its proxy is that the response of energy use and intensity with respect to ISTC changes in Scenario 2 is smaller. This is due to the fact that we are forcing the economy to use capital that is no longer returnable (even if it size is negligible). Thus, we can think of the quantitative results of the proxy economy as a lower bound for the actual response of the putty-clay vintage capital economy.

We solve for the dynamics of the stationary representation of the aggregate economy in problem (6.10) with a Newton method using sparse matrices as implemented for deterministic Dynare (see Juillard, 1996), and we simulate the dynamics under perfect foresight. Figure 3 shows the dynamic response of the proxy putty-clay economy to a permanent 10 percent increase in the energy price (solid black line) and a permanent 50 percent increase in the growth rate of ISTC (dashed blue line) for Scenario 1, the case in which the energy price is constant over time.

Let us focus first on the energy price change. On impact, there is a drop in the ratio VA/E fully due to the fact that the rise in the energy bill reduces aggregate value added, since gross output is given. As a result, the investment rate falls before recovering again. The detrended energy intensity of investment, $\widehat{e}$, falls immediately. As time passes, aggregate energy intensity falls back to the initial steady state. The capital to value added ratio goes back to the initial steady state, which means that the increase in the value added to energy ratio is entirely due to the gain in aggregate energy intensity, as we can see by the change in the capital to energy ratio, K/E.

Now we can turn to focus on the dynamic response to a permanent change in the growth rate of ISTC. There is an interesting non-monotonic behavior of the ratio VA/E. Thus, there is a rebound effect in the long run, but an acceleration in ISTC growth is energy saving in the short run. The reason for the non monotonic behavior of VA/E is that the capital to energy ratio, K/E, falls slowly as old capital depreciates and is replaced by new capital. In the very short run this produces a boost in output (recall that more energy intensive capital yields more services than less intensive units), which is compensated later on by the rise in energy requirements of new capital.

7 Final comments

This paper proposes a theory of investment and energy use in which Energy Saving Technical Change (ESTC) is endogenous and responds to changes in energy prices and a standard notion of Investment Specific Technical Change (ISTC). We show that ESTC is positively related to the growth rate of ISTC and the energy price. An acceleration of ISTC produces and acceleration in ESTC but, at the same time, rises the growth rate of aggregate energy use. This is so because the rise in aggregate investment compensates for the reduction of energy intensity of new capital. We think that our theory can be used to test when and how we should see a rebound effect in energy use at the aggregate level and can be used to test the aggregate effect of any policy aiming to reduce energy use. Our analysis shows that if energy becomes more scarce (so that the energy price growth rate augments) the economy responds with an acceleration of ESTC, but the energy share rises.

We also show that aggregate capital admits a recursive representation as function of investment and past capital. The key for this representation is the depreciation rate of capital, which is endogenous since it depends on Energy Saving Technical Change. This result is very useful and allows us to study the dynamics of our economy. We find that the interaction of ISTC and energy prices has a non linear effect on the choice of energy intensity of new capital and on the size of investment in the short run. In particular, a permanent change in the growth rate of ISTC, has a non-monotone effect on Value Added per unit of energy use. Thus, this model economy seems a promising tool to analyze the quantitative effect of the interplay of ISTC innovations and energy price shocks in a stochastic framework in order to account for the the fluctuations of energy aggregates.

In this paper we have assumed that investment specific technological innovations arrive exogenously to focus our attention on how the interplay of ISTC and energy prices shape the energy intensity decision. A natural extension would be to make endogenous the size of ISTC innovations. This is a complex objective since differentiating the dual nature of ISTC (i.e., innovations at the extensive or the intensive margin) is important. We leave this for future research. We have also abstracted from climate change and the existence of clean (non fuel based) technologies. We have done so in order to isolate the effects on energy intensity of the interplay of ISTC and energy prices. This will be essential to study the effect of environmental policies and we also leave it for further research.

Appendix

A Data and Calibration

In this Appendix we document the construction of the data series we use in the empirical part of the paper. We obtain data from two sources: the Annual Energy Review (2000) and National Income and Product Accounts. The data we use can be accessed in the addresses: http://www.eia.gov/ and http://www.bea.gov. From now on we will refer to each source as AER, and NIPA, respectively. Our data set is available upon request.

A.1 Energy price, use, and expenditures series

Our energy data covers primary energy consumption of end-users and is obtained from the Annual Energy Review (AER, hereafter). We consider four forms of energy: coal, petroleum, natural gas and electricity. AER (Table 2.1a) gives data on total energy consumption by end users measured in British termal units (BTUs) disaggregated into the four forms of energy considered. We denote these data on energy consumption for each type of energy by Q_{it}, where the index i denotes the form of energy.

This measure Q_{it} is already net of energy consumption of the electricity sector. We subtract from total primary energy consumption of the industrial sector that of four energy sectors: oil and gas extraction, electricity and gas services, petroleum and coal production, and pipeline transportation. The BEA gives information on the net stock of Fixed Assets by industry and we assume that the amount of BTUs consumed by those four sectors, as a proportion of BTUs consumed by the industrial sector, is the same that the amount of capital in those sectors as a proportion of assets in the industrial sector. We define total energy use as $E_t = \sum_i Q_{it}$, where Q_{it} is the amount of BTUs yielded by each type of energy consumed. P_{it} is the price in dollars per BTUs of energy type, divided by the implicit price deflator of non durable consumption goods and services in NIPA (which is constructed as a weighted average of the two implicit price deflators). For coal, natural gas and petroleum we use the production price series (AER, Table 3.1). For electricity, we use the retail price of electricity sold by electric utilities (see AER, Table 8.10). In Table 8.10 the price for electricity is in cents per kilowatt-hour. We use AER Table A.6 to convert the price to cents per BTUs. All prices are in real terms; i.e., divided by the implicit price deflator of non durable consumption goods and services. We construct the energy price deflator as

$$p_t^e = \frac{\sum\limits_i Q_{it} P_{it}}{\sum\limits_i Q_{it}}. \tag{A.1}$$

Finally, energy expenditure is $p_t \cdot E_t = \sum_i Q_{it} P_{it}$.

A.2 Output, consumption, investment, and the capital stock

We follow the method described by Cooley and Prescott (1995) to construct broad measures of output, consumption, investment, and the capital stock. Specifically, our measure of capital includes private stock of capital, the stock of inventories, the stock of consumer durable goods and the government stock. Consequently, the measured value of GDP is augmented with the imputed flow of services from the stock of durable goods and the government stock. We subtract from each of the series of output, investment and capital the corresponding series for the energy producing sectors: oil and gas extraction, electricity and gas services, petroleum and coal production, and pipeline transportation, as our theory cannot account for the behavior of the energy-producing sectors. We have information on the three variables for the last two sectors but about the first two sectors we only have information about the net stock of capital, and we use it to impute estimates of output and investment. Gross output is the sum of value added and the final expenditure on energy. Real variables are obtained by dividing the nominal variables by the implicit price deflator of non durable consumption goods and services.

B The putty-clay economy

B.1 Proof of Proposition 2

We proceed in three steps:

Lemma 2. *Only capital that is not very intensive, $e \leq \overline{e}_t(z)$, is utilized, $k_t(z,e) \geq 0$, where $\overline{e}_t(z)$ is the maximum intensity type of utilized in equilibrium.*

Proof. The first order condition of the firm's problem with respect to $k_t(z)$ is

$$r_t(z,e) + p^e_t\, e \leq \alpha\, \frac{y_t}{\mathcal{K}_t}\, (1+\lambda)^{z\,(1-\phi)} e^{\phi}. \tag{B.1}$$

For a given vintage z, $\overline{e}_t(z)$ is defined as the energy type that satisfies

$$p^e_t = \alpha\, \frac{y_t}{\mathcal{K}_t}\, (1+\lambda)^{z\,(1-\phi)} \overline{e}_t(z)^{\phi-1}. \tag{B.2}$$

Since, in equilibrium, the return to capital must be non-negative, $r_t(z,e) \geq 0$, it follows that $k_t(z,e) = 0$ for any $e > \overline{e}_t(z)$. That is, any capital class for which $e > \overline{e}_t(z)$ is not utilized. $\square$

Lemma 3. *Across all units of capital of the same vintage, $z \leq t$, the net return, $r_t(z,e)$, is the highest for the type that satisfies*

$$e^*_t(z) = \phi^{\frac{1}{1-\phi}}\, \overline{e}_t(z), \tag{B.3}$$

where $\overline{e}_t(z)$ is the type for which return is zero at time t.

Proof. The net return at time t is

$$r_{t+i}(z,e) = \alpha\, \frac{y_t}{\mathcal{K}_t} (1+\lambda)^{z(1-\phi)}\, e^{\phi} - p^e_t\, e. \tag{B.4}$$

In Lemma 2 we defined $\overline{e}_t(z)$ as the efficiency type for which the net return is zero. Thus, capital of any type $e > \overline{e}_t(z)$ is not utilized. Conditional on utilizing capital, its net return is maximized at the value $e_t^*(z)$, which satisfies $e_t^*(z) = \phi^{\frac{1}{1-\phi}}\,\overline{e}_t(z)$. Since $\phi < 1$ we know that $e_t^*(z) < \overline{e}_t(z)$. Hence, return of vintage z has a unique maximum at $e_t^*(z)$. $\square$

Corollary 2. *The equilibrium net return of any capital not used is equal to* $r_t(z, e) = 0$.

Now we can turn to the proof of Proposition 2. Inspecting the problem solved by the firm producing new capita, (3.7), we see that new capital is produced whenever $p_t(t+1, e) \geq (1+\theta)^{-t}$, whereas equilibrium dictates that $p_t(t+1, e) \leq (1+\theta)^{-t}$. Typically, there will be investment in equilibrium, thus, $p_t(t+1, e) = (1+\theta)^{-t}$ for any type $e \in \mathbf{R}_+$ produced. Solving the household's problem we find that

$$p_t(z, e) = \frac{1}{1 + r^a_{t+1}}\,[(1-\varpi)\,p_{t+1}(z, e) + r_{t+1}(z, e)] + \xi_t(z, e), \tag{B.5}$$

where $\xi_t(z, e)$ is the Lagrange multiplier associated to the nonnegativity constraint on investment in the household's problem. Expression (B.5) is the first order condition with respect to $k_{t+1}(z, e)$ in the household's problem. If $\xi_t(z, e) > 0$, the household does not want to invest in class (z, e). Working backwards, it must be the case that

$$p_t(z, e) = \sum_{i=1}^{\infty} \frac{(1-\varpi)^{i-1}}{\prod\limits_{j=1}^{i}(1 + r^a_{t+j})}\, r_{t+i}(z, e). \tag{B.6}$$

Any new class of capital that is produced must satisfy

$$p_t(t+1, e) = (1+\theta)^{-t},\ e \in \mathbf{R}_+. \tag{B.7}$$

Thus,

$$(1+\theta)^{-t} = \sum_{i=1}^{\infty} \frac{(1-\varpi)^{i-1}}{\prod\limits_{j=1}^{i}(1 + r^a_{t+j})}\, r_{t+i}(t+1, e). \tag{B.8}$$

Expression (B.8) determines the set of intensity types $e \in \mathbf{R}_+$ for which investment will be positive. Since the present value of all future returns is a weighted average of strictly concave functions that have a unique maximum, it follows that the average also have a unique maximum, denoted as e_{t+1}. Moreover, it must be the case that $e_{t+1} < e^*_{t+1}(t+1)$, otherwise it would be returnable to reduce e to rise the net return after period $t+1$. Since agents have rational expectations, competition implies that households know that there is one intensity type that yields the highest present value of future net return. As a result, only one type has a positive price, which is the type that receives investment in equilibrium and, therefore, is produced. Thus, all units of capital of the same vintage have the same energy intensity.

B.2 Proofs of Proposition 3 and 4

We need to conjecture that the service life of capital is constant. Thus, $\underline{z}_t = t + 1 - T$, is the oldest vintage of capital utilized time t. Any $z < \underline{z}_t$ is left idle. The amount of energy used at time t is

$$\mathcal{E}_t = \sum_{z=\underline{z}_t}^{t} e_z \, k_t(z, e_z). \tag{B.9}$$

The evolution of this stock can be written as

$$\mathcal{E}_{t+1} = e_{t+1} k_{t+1}(t+1, e_{t+1}) + (1 - \varpi) \left[1 - \frac{e_{\underline{z}_t} k_t(\underline{z}_t, e_{\underline{z}_t})}{\mathcal{E}_t} \right] \mathcal{E}_t. \tag{B.10}$$

At the balanced growth path, it must be the case that energy grows at a constant rate. Thus, the distribution of energy used across utilized vintages must be constant. Thus, we can define the depreciation rate of energy use as $\delta_{\mathcal{E}}$ that satisfies

$$1 - \delta_{\mathcal{E}} = (1 - \varpi) \left[1 - \frac{e_{\underline{z}_t} k_t(\underline{z}_t, e_{\underline{z}_t})}{\mathcal{E}_t} \right]. \tag{B.11}$$

For the energy share in gross output, $p_t^e \, \mathcal{E}_t / y_t$, to be constant it must be the case that energy intensity of investment, e_{t+1} falls at the combined rate $(1 + \theta)(1 + \gamma_p)$.

In order to prove that the service life of capital is constant (and finite) we need to obtain the growth rate of the output to capital services ratio, $y_t / \mathcal{K}_t$. Recall the expression of capital services, shown in (3.4). The evolution of the stock can be written as

$$\mathcal{K}_{t+1} = \kappa(t+1, e_{t+1}) \, k_{t+1}(t+1, e_{t+1}) + (1 - \varpi) \left[1 - \frac{\kappa(\underline{z}_t, e_{\underline{z}_t}) \, k_t(\underline{z}_t, e_{\underline{z}_t})}{\mathcal{K}_t} \right] \mathcal{K}_t. \tag{B.12}$$

At the balanced growth path, it must be the case that capital services grow at a constant rate. Thus, the distribution of services across utilized vintages must be constant. Thus, we can define the depreciation rate of capital services as $\delta_{\mathcal{K}}$ that satisfies

$$1 - \delta_{\mathcal{K}} = (1 - \varpi)(1 - \psi). \tag{B.13}$$

where ψ measures the weight of services used in production in total capital services

$$\psi = \frac{\kappa(\underline{z}_t, e_{\underline{z}_t}) \, k_t(\underline{z}_t, e_{\underline{z}_t})}{\mathcal{K}_t}. \tag{B.14}$$

For capital services to grow at a constant rate, $g_{\mathcal{K}}$, it must be the case that $\kappa(t+1, e_{t+1}) \, k_{t+1}(t + 1, e_{t+1}) / \mathcal{K}_t$ is constant. Thus denoting a g the growth rate of gross output, y_t, it must be that

$$1 + g_{\mathcal{K}} = [(1 + \lambda)(1 + \theta)]^{1-\phi} \, (1 + \gamma_p)^{-\phi} \, (1 + g). \tag{B.15}$$

It follows that the output to capital services ratio, $y_t / \mathcal{K}_t$, falls at the rate $[(1 + \lambda)(1 + \theta)]^{1-\phi} \, (1 + \gamma_p)^{-\phi}$. Thus, recalling the expression of the net return of capital, shown in equation (5.1) it follows that the service life of capital is constant and finite along the balanced growth path.

B.3 Proof of Proposition 5

From the household problem we know that the amount of investment, x_t, satisfies:

$$(1+\theta)^t = \sum_{i=1}^{\infty} \frac{(1-\varpi)^{i-1}}{(1+r^a)^i}\, r_{t+i}(t+1, e_{t+1}). \tag{B.16}$$

The optimal intensity level, e_{t+1}, satisfies

$$0 = \sum_{i=1}^{\infty} \frac{(1-\varpi)^{i-1}}{(1+r^a)^i} \left[\phi\, r_{t+i}(t+1, e_{t+1}) - (1-\phi)\, p^e_{t+i}\, e_{t+1}\right], \tag{B.17}$$

That is, the intensity level e_{t+1} is chosen so that the present value of energy expenditures amount to the fraction ϕ of the present value of future gross returns. Finally, the service life of capital satisfies

$$r_{t+T}(t+1, e_{t+1}) \equiv \alpha\, \frac{y_{t+T}}{\mathcal{K}_{t+T}}\, (1+\lambda)^{(t+1)(1-\phi)} e^{\phi}_{t+1} - p^e_{t+T}\, e_{t+1} \geq 0. \tag{B.18}$$

These three equation together determine the investment level, the detrended intensity level of investment, $\widehat{e}$ and the service life of capital, T. Using the fact that the gross return to capital, $r_t(z, e_z) + p^e_t\, e_z$, falls at the same rate that $y_t/\mathcal{K}_t$, $(1+g_{\mathcal{K}})/(1+g)$, and combining the three equations, we find that the service life of capital, T, only depends on the discount factor, and the growth rate of income and capital services and not on the value of $\widehat{e}$ and investment,

$$\phi\, \left[(1+\lambda)(1+\theta)(1+\gamma_p)\right]^{(1-\phi)T} = \frac{\sum_{i=1}^{T} \left(\frac{\beta\,(1-\varpi)\,(1+\gamma_p)}{1+g}\right)^i}{\sum_{i=1}^{T} \left(\frac{\beta\,(1-\varpi)}{1+g_{\mathcal{K}}}\right)^i}. \tag{B.19}$$

It follows from (B.18) that an acceleration in ISTC growth reduces T. Now combining (B.16) and (B.17) we find that the value of $\widehat{e}$ is determined by the equation

$$(1+\theta)^{-t} = \frac{1-\phi}{\phi} \sum_{i=1}^{T} \frac{(1-\varpi)^{i-1}}{(1+r^a)^i}\, p^e_{t+1}\, e_{t+1}. \tag{B.20}$$

This equation, detrended, can be written as

$$1 + r^a = \frac{1-\phi}{\phi}\, p^e\, \widehat{e} \sum_{i=1}^{T} \left(\frac{1-\varpi}{1+r^a}\right)^{i-1} (1+\gamma_p)^i. \tag{B.21}$$

It follows from (B.21) that an acceleration in ISTC growth rises $\widehat{e}$.

B.4 The price of capital in units of gross output

In the Proof of Proposition 2 we have proved that the relative of investment in units of value added is $(1+\theta)^{-t}$. The price of investment in units of gross output is equal to

$$q_t(z,e) = \sum_{i=1}^{T} \frac{(1-\varpi)^{i-1}}{\prod_{j=1}^{i}(1+r^a_{t+j})} \left(r_{t+i}(z,e) + p^e_t\, e\right). \tag{B.22}$$

It is easy to check that $q_t(t+1, e_{t+1}) = (1+\theta)^{-t}/(1-\phi)$. The gross return to any class of capital (z,e) satisfies:

$$r_{t+i}(z,e) + p^e_t\, e = \frac{\kappa(z,e)}{\kappa\,(t+1, e_{t+1})} \left[r_{t+i}(z,e) + p^e_t\, e\right]. \tag{B.23}$$

The fact that capital services (in units of gross output) are perfect substitutes implies that the ratio of prices must be equal to the ratio of gross returns:

$$\frac{q_t(z,e)}{q_t\,(t+1, e_{t+1})} = \frac{\kappa(z,e)}{\kappa\,(t+1, e_{t+1})}. \tag{B.24}$$

B.5 The depreciation rate of the capital stock

From the expression of the aggregate stock of capital we find that the depreciation rate of capital may vary out of the balanced growth path. Its expression is

$$1 - \delta_{\mathbf{k}_t} = \frac{1-\varpi}{(1+\lambda)^{1-\phi}(1+\theta)} \left(\frac{e_{t-1}}{e_t}\right)^{\phi} (1-\psi), \tag{B.25}$$

where ψ is shown in equation (B.14). Along the balanced growth path, the depreciation rate is constant.

B.6 The market value of capital

We have aggregated capital using the price $q_t(z,e)$, shown in expression (6.1) from Lemma B.4. This is the price of capital in units of gross output. It is not the market price of capital (in units of value added), $p_t(z,e)$, which is given by the present value of all future returns, net of energy expenditures, as shown in expression (B.8). The market value of the total economically active stock is

$$\mathbf{k}^m_t = \sum_{z=\underline{z}_t}^{t} p_{t-1}\,(z, e_z)\, k_t\,(z, e_z). \tag{B.26}$$

The market price of capital of class (z, e_z) has a complicated expression. Nevertheless, at the balanced growth path, using Propositions 5.1 and 5, it can be written as

$$p_t(z, e_z) = q_t(z, e_z)\,\frac{\Upsilon_1(t,z)}{1-\phi} - (1+\theta)^{-(z-1)}(1+\gamma_p)^{t-z+1}\,\frac{\Upsilon_2(t,z)\,\phi}{1-\phi}, \tag{B.27}$$

where factors $\Upsilon_1(t,z)$ and $\Upsilon_2(t,z)$ are

$$\Upsilon_1(t,z) = \frac{1-\left(\frac{1-\delta_{\mathbf{k}}}{1+r}\right)^{T-t+z-2}}{1-\left(\frac{1-\delta_{\mathbf{k}}}{1+r}\right)^{T-1}},\; \Upsilon_2(t,z) = \frac{1-\left(\frac{1-\tilde{\varpi}}{1+r}\right)^{T-t+z-2}}{1-\left(\frac{1-\tilde{\varpi}}{1+r}\right)^{T-1}},\; 1-\delta_{\mathbf{k}} = \frac{1-\varpi}{[(1+\lambda)(1+\theta)]^{1-\phi}\,(1+\gamma_p)^{-\phi}}, \tag{B.28}$$

for all $z \le t+1$, and $T-(t+1-z) \ge 1$. Notice that this expression collapses to $(1+\theta)^{-t}$ for the latest vintage, $t+1$. The divergence between the market price and the value in units of gross value added increases with the age of capital. Thus, the stock of capital in units of gross output is larger than the value of capital in units of value added. Figure 2 show graphically the difference between the market price and the price in units of gross output in our benchmark calibration.

C The balanced growth path in the putty-putty economy

It is convenient to find the expression of the stock of capital in units of final good, $\mathbf{k}_t$:

$$\mathbf{k}_t = \sum_{z=-\infty}^{t} p_{t-1}(z)\,k_t(z). \tag{C.1}$$

Thus, we need to find the expression for the market price of capital. Here all capital is used in equilibrium. Expression (4.2) shows the net return to capital of vintage z in the putty-putty economy. Given the demand of energy shown in (4.3), it follows that

$$\frac{r_{t+i}(z)}{r_{t+i}(t+1)} = (1+\lambda)^{z-t-1}. \tag{C.2}$$

Since the price of new capital is equal to $(1+\theta)^{-t}$, it follows that the relative price of capital of vintage z at time t is

$$p_t(z) = (1+\lambda)^{z-t-1}(1+\theta)^{-t}. \tag{C.3}$$

It is easy to check that the aggregate stock of capital satisfies

$$\mathbf{k}_t = (1+\lambda)^{-t}(1+\theta)^{-t+1}\left[\sum_{z=-\infty}^{t}(1+\lambda)^{z}\,k_t(z)\right]. \tag{C.4}$$

Expression (4.3) shows the energy demand associated to utilization of capital of vintage z. Thus, aggregate energy use can be written as

$$\mathcal{E}_t = \sum_{z=-\infty}^{t} e_t(z)\, k_t(z) = \Psi_t\,(1+\lambda)^t(1+\theta)^{t-1}\mathbf{k}_t,\ \Psi_t = \left(\frac{\phi\,\alpha\,y_t}{p_t^e\,\mathcal{K}_t}\right)^{\frac{1}{1-\phi}}. \tag{C.5}$$

Using the expression for aggregate capital services shown in (3.4) we can express

$$\mathcal{K}_t = \Psi_t^{1-\phi}\,(1+\lambda)^t(1+\theta)^{t-1}\mathbf{k}_t, \tag{C.6}$$

which implies that we can express aggregate capital services as a function of aggregate energy use:

$$\mathcal{K}_t = \mathcal{E}_t^{\phi}\,\left[(1+\lambda)^t(1+\theta)^{t-1}\mathbf{k}_t\right]^{1-\phi}. \tag{C.7}$$

Using the expression for energy demand, (C.5), we find that aggregate value added can be expressed as

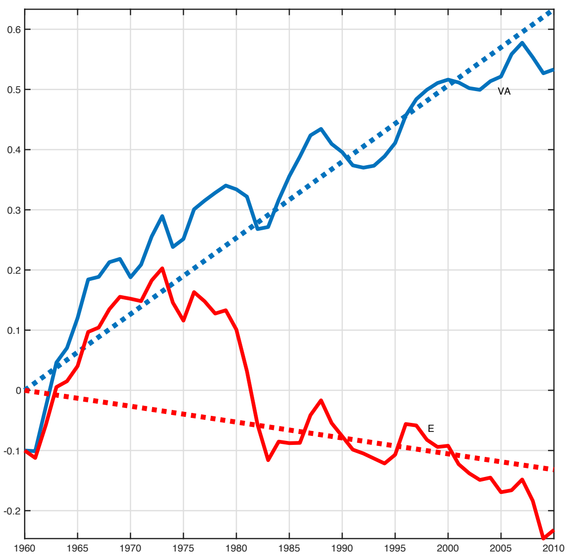
$$va_t \equiv y_t - p_t^e\mathcal{E}_t = (1-\phi\,\alpha)\left(\frac{\phi\,\alpha\,A_t}{(p_t^e)^{\phi\,\alpha}}\right)^{\frac{1}{1-\phi\,\alpha}}\left[(1+\lambda)^t(1+\theta)^{t-1}\mathbf{k}_t\right]^{\frac{\alpha(1-\phi)}{1-\phi\,\alpha}}\,h_t^{\frac{1-\alpha}{1-\phi\,\alpha}}. \tag{C.8}$$

It follows that the growth rate of value added, gross output and capital in units of final good is equal to

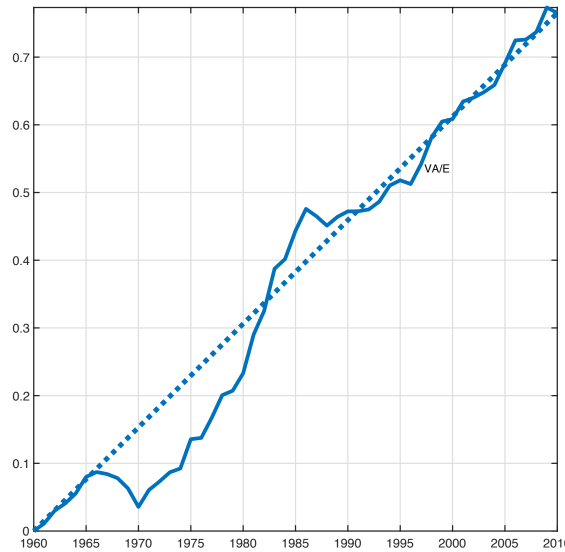
$$1+g = \left[(1+\gamma_a)\,(1+\gamma_p)^{-\alpha\,\phi}\right]^{\frac{1}{1-\alpha}}\,((1+\lambda)(1+\theta))^{\frac{\alpha(1-\phi)}{1-\alpha}}, \tag{C.9}$$

The growth rate of aggregate energy use must be equal to $(1+g)/(1+\gamma_p)$. Using (C.7), it follows that the price of capital services (that is, the price of capital adjusted by quality) falls at the rate:

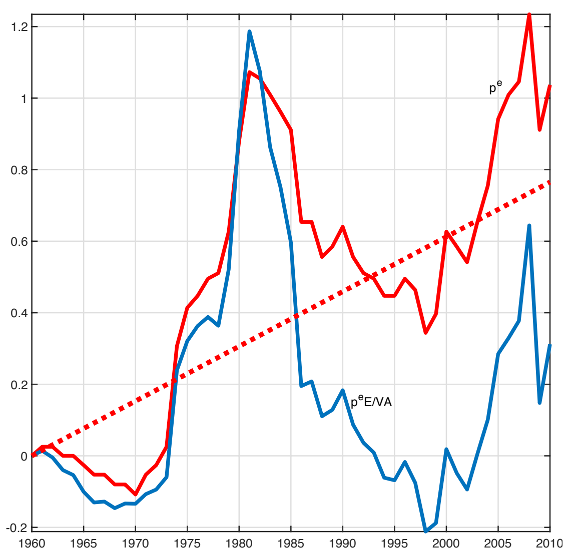
$$\frac{q_t^{\mathcal{K}}}{q_{t+1}^{\mathcal{K}}} = [(1+\lambda)(1+\theta)]^{1-\phi}\,(1+\gamma_p)^{-\phi}. \tag{C.10}$$



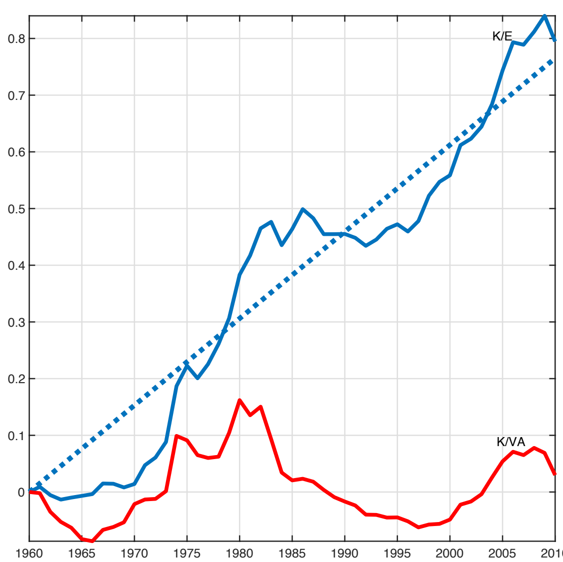
(a) Value Added, VA, and energy use, E, in logs. Trend shown in dashed line.



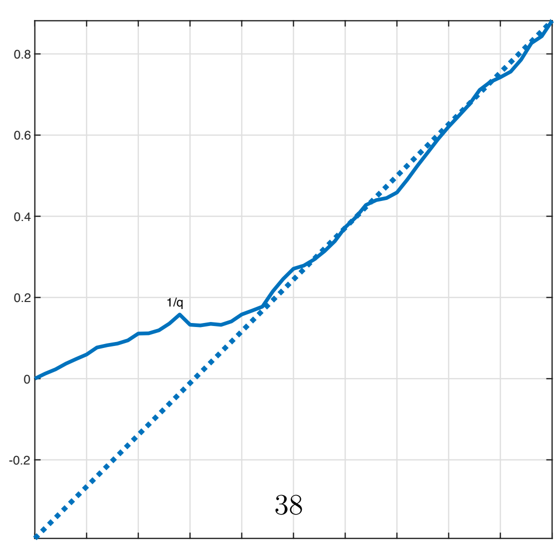
(b) Value added per unit of energy consumption, VA/E, in logs. Trend shown in dashed line.



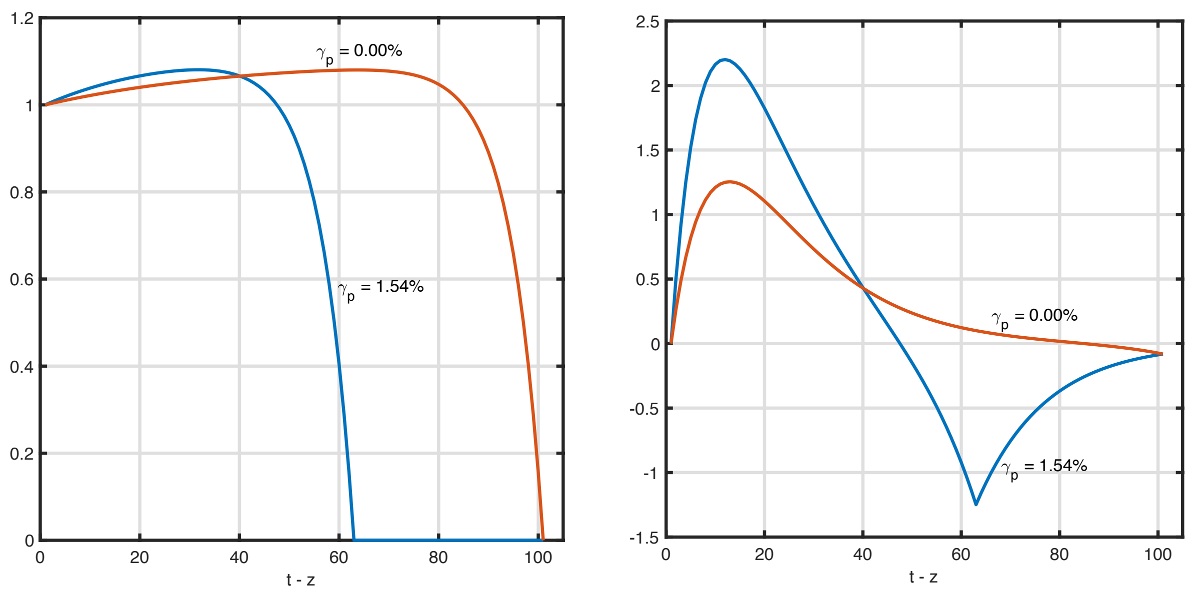
(c) The logged energy price, p^e and the share of energy, $p^e\,E/VA$.



(d) Capital-energy ratio, K/E, in logs, and capital to value added ratio, K/VA. Trend of VA/E shown in dashed line.



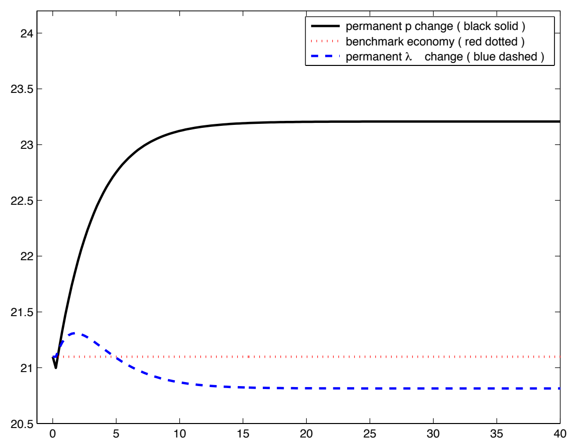
(e) The logged relative price of investment goods, $1/q$.



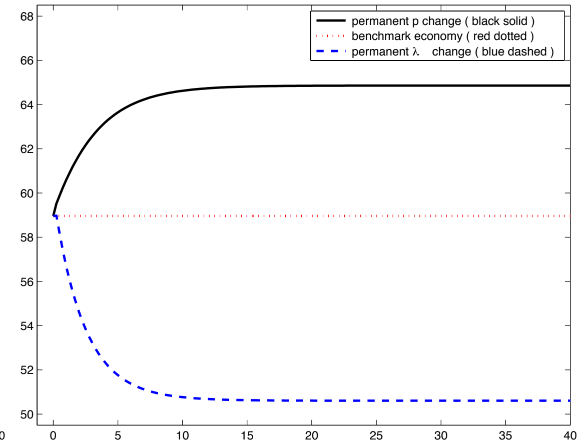
(a) $p_t(z, e_z)/q_t(z, e_z)$ by age.

(b) $\left(\frac{p_t(z,e_z)}{q_t(z,e_z)} - 1\right)(1-\varpi)^{t-z}$.

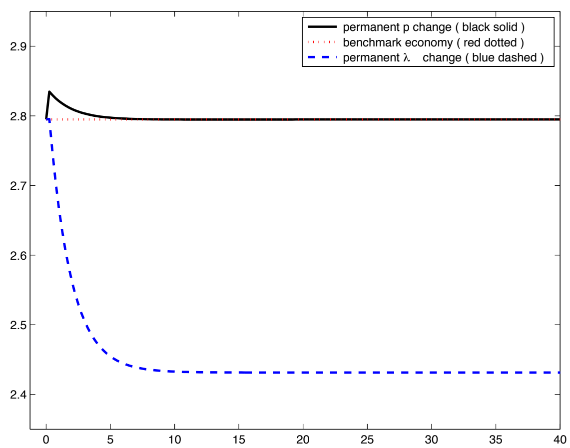
Figure 2: Market price, $p_t(z, e_z)$, versus price in units of gross output, $q_t(z, e_z)$ in the balanced growth path.



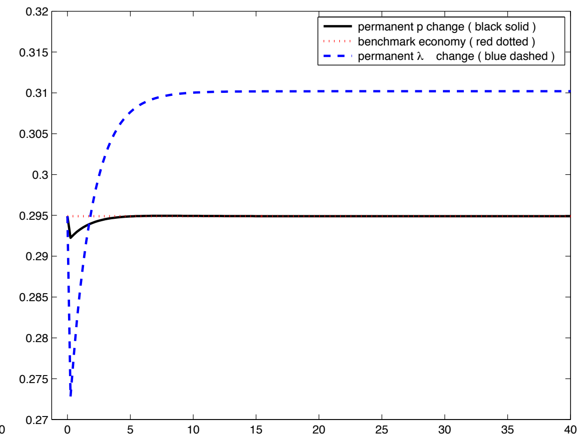
(a) VA/E



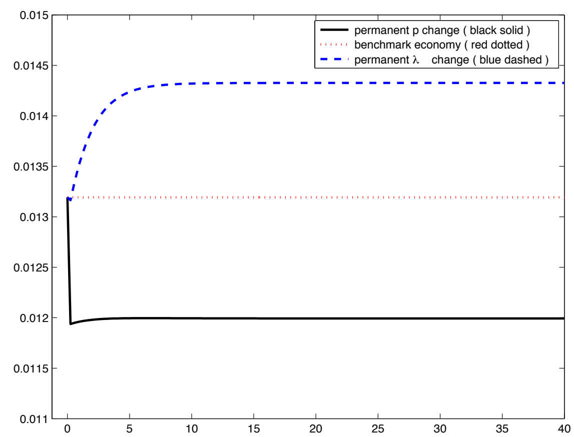
(b) K/E



(c) K/VA



(d) X/VA



(e) Detrended energy intensity of investment $\widehat{e}$

Figure 3: Responses to either a permanent 10% shock in the price of energy, p^e, or a permanent 50% shock in ISTC growth.

Param.	Observation		Putty-Putty	Putty-Clay	Proxy P-C
σ	-		1.0000	1.0000	1.000
α	$w\,L/VA$	$= 0.5845$	0.4419	0.4419	0.4419
p^e	-		1.0000	1.0000	1.0000
ϖ	I/VA	$= 0.2949$	0.0693	0.0693	0.0693
Scenario 1: No trend in the energy price, $\gamma_p = 0.0000$					
β	K/VA	$= 2.7948$	0.9591	0.9580	0.9579
ϕ	$p^e\,E/VA$	$= 0.0474$	0.1024	0.0949	0.0944
γ_a	g	$= 0.0127$	-0.0042	-0.0042	-0.0042
$(1+\lambda)(1+\theta)$	$q_t^\kappa/q_{t+1}^\kappa$	$= 1.0258$	1.0288	1.0286	1.0286
Scenario 2: Positive trend in the energy price, $\gamma_p = 0.0154$					
β	K/VA	$= 2.7948$	0.9591	0.9573	0.9569
ϕ	$p^e\,E/VA$	$= 0.0474$	0.1024	0.0902	0.0876
γ_a	g	$= 0.0127$	-0.0042	-0.0042	-0.0042
$(1+\lambda)(1+\theta)$	$q_t^\kappa/q_{t+1}^\kappa$	$= 1.0258$	1.0306	1.0311	1.0299

Notes: Averages for period 1960-2010. $u(c) = \ln c$, VA = measured GDP + services of consumer durables + services of public capital − VA of energy producing sectors.

Table 1: Calibration

	VA/E	p^eE/VA	K/E	K/VA	X/VA
50% Δ in ISTC					
Putty-Putty (S1 & S2)	0.0000	0.0000	-12.2310	-12.2310	4.6801
Putty-Clay (S1)	-1.1879	1.2022	-13.6702	-12.6323	5.0258
Putty-Clay(S2)	0.6681	-0.6637	-12.6650	-13.2446	5.2328
10% Δ in p^e					
All economies	10.0000	0.0000	10.0000	0.0000	0.0000
10% Δ in γ_p					
Putty-Putty (S2)	0.0000	0.0000	0.0875	0.0875	-0.0307
Putty-Clay (S2)	-0.6589	0.6633	-0.4721	0.1880	-0.0344

Notes: Percent changes with respect to the calibrated balanced growth path.

Table 2: Long run effects in aggregate energy use and energy efficiency.

	T	$\widehat{e}$	$\delta_{\mathbf{k}}$
Scenario 1			
Benchmark	100.0000	0.0133	0.0928
50% Δ in ISTC	68.0000	0.0144	0.1040
10% Δ in p^e	100.0000	0.0120	0.0928
Scenario 2			
Benchmark	67.0000	0.0110	0.0927
50% Δ in ISTC	51.0000	0.0121	0.1046
10% Δ in p^e	67.0000	0.0100	0.0927
10% Δ in γ_p	65.0000	0.0109	0.0926

Notes: The value of ψ, the weight of the oldest vintage used in production in capital services is always below $1e^{-5}$.

Table 3: Long run effects in service life and efficiency in the Putty-Clay Vintage economy.

	$\widehat{\mathcal{E}}$	$g_{\mathcal{E}}(\%)$	$g(\%)$
Scenario 1			
Benchmark	1.0000	1.2746	1.2746
50% Δ in ISTC	0.9088	2.2811	2.2811
10% Δ in p^e	0.9946	1.2746	1.2746
Scenario 2			
Benchmark	1.0000	-0.2636	1.2746
50% Δ in ISTC	0.9132	0.7805	2.3348
10% Δ in p^e	0.9952	-0.2636	1.2746
10% Δ in γ_p	0.1321	-0.4257	1.2636

Notes: The values of the detrended energy use, $\mathcal{E}$, are shown relative to its value in the benchmark economy.

Table 4: Energy use in the Putty-Clay Vintage Economy.

	VA/E	p^eE/VA	K/E	K/VA	X/VA	$\widehat{e}$
50% Δ in ISTC(S1)	-1.3159	1.3334	-13.7714	-12.6216	5.0386	8.3036
50% Δ in ISTC (S2)	0.0526	-0.0526	-13.1183	-13.1640	5.2914	9.7647
10% Δ in p^e (S1 & S2)	10.0000	0.0000	10.0000	0.0000	0.0000	-9.0909
10% Δ in γ_p (S2)	-0.9039	0.9121	-0.7104	0.1952	-0.0207	-1.5147

Notes: Percent changes with respect to the calibrated balanced growth path.

Table 5: Long run effects in energy use in the Proxy Putty-Clay Vintage economy.

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