Vintage capital, energy intensity and the medium-term business cycle*

Antonia Díaz[a] and Luis A. Puch[b] †

[a]Universidad Carlos III de Madrid
[b]Universidad Complutense and ICAE

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Abstract

In this paper we propose a theory to investigate the importance of investment-specific technical change (ISTC) in shaping the short run response of macroeconomic aggregates to energy price shocks. In absence of ISTC, aggregate energy efficiency in an economy (i.e, the amount of energy use per unit of capital) responds to changes in energy prices. Here we show that ISTC may be a substitute of energy efficiency: embodied technical change rises the productivity of capital and increases the effective energy efficiency (i.e., the amount of energy use required per unit of quality-adjusted capital). Our model economy offers an explanation to the softened response of aggregate output to the 2003-08 oil price shock: By increasing investment in the years of high ISTC growth, the economy was increasing the average efficiency of the economy (the capital-energy ratio), shielding the economy against the impact of the 2003-08 price shock.

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†Corresponding Author: Antonia Díaz, Department of Economics, Universidad Carlos III de Madrid, 28093 Madrid, Spain; E-mail: andiaz@eco.uc3m.es

1 Introduction

The oil-price shocks of the 1970s and the 1980s and the subsequent recession experienced in most OECD countries prompted many macroeconomists to investigate the effect of energy price shocks on the aggregate economy.[1] Yet, macroeconomists do not agree about the main mechanism. For instance, there is a branch of the literature that relies on imperfect competition and price rigidities for energy price shocks to have significant effects on aggregate output (see Rotemberg and Woodford 1996, or Blanchard and Galí 2007). Another branch assumes that the technology to produce goods, not the market structure, is the relevant mechanism through which energy affects the aggregate economy. Even in this latter branch there is no agreement, as some authors assume imperfect substitution at the aggregate level (see Pindyck and Rotemberg 1983, or **?**), whereas others model complementarity at the micro level (Atkeson and Kehoe 1999, Finn 2000, or Díaz et al. 2004). Moreover, as argued by Kilian (2008), or Blanchard and Galí (2007), among others, the relationship between energy prices and the economy seems to have changed in the last two decades, since the response of GDP to the 2003-08 oil-price shock is much softer than in the 1970s and 1980s. This is corroborated by microeconomic evidence that suggests that the short run response of energy use to energy prices may have changed and that this change is due to the fact that capital has become more energy efficient over time; see Edelstein and Kilian (2007), Metcalf (2008), or Steinbuks and Neuhoff (2010).

There are some examples that are illuminating on this respect. For instance, Newell et al. (1999) and Popp (2002) find that energy prices have a strong positive effect on energy-efficient innovations.[2] Moreover, Knittel (2011) estimates that if car quality would have not improved from 1980 to 2006, energy efficiency of both passenger cars and light trucks could have increased by nearly 60 percent from 1980 to 2006, instead of the observed meager 15 percent. These examples suggest three things: First, that the level of energy efficiency responds to changes in energy prices and, second, that the existence of embodied technological change affects to the magnitude of that response. A more general implication is that investment specific technical change may affect the cyclical behavior of the economy. Following this line of thought, in this paper we propose a theory to investigate quantitatively the importance of investment specific technological change in shaping the short run

[1] See Hamilton (2008) or Kilian (2008) for a survey of the significant literature.

[2] Newell et al. (1999) estimate that during the period 1958-1993 about half of the total change in energy efficiency of the models of room air conditioners, central air conditioners, and gas water heaters was prompted by the high price shocks experienced during the 1970s and early 1980s.

response of energy efficiency and macroeconomic aggregates to energy price shocks.

In order to answer this question we have built a model economy in which production of the unique final good takes place in plants. A plant is created by installing a unit of capital. The final good is produced combining the services yielded by the unit of capital and labor. As in Atkeson and Kehoe (1999), for capital to yield services it has to be combined with energy. Once the energy-capital ratio is fixed, it cannot be changed. This ratio determines the energy efficiency of the unit of capital installed. Higher efficiency comes at the cost of less capital services. The only way of increasing the aggregate energy efficiency of the economy is by creating new plants where the unit of capital is operated more efficiently, and vice versa. We assume that there is investment specific technological change: first, the cost in units of output of producing new capital units falls over time and, secondly, new capital yields more services than vintage capital, as in Solow (1960), regardless of the energy-efficiency at which those new units are operated.

The reason why we have chosen to model complementarity between capital and energy at the plant level instead of imperfect substitution at the aggregate level is because, in this way, our model economy can deliver simultaneously a large short run response and a small long run response of aggregate output to energy price shocks. This is so because the aggregate energy-efficiency of the economy cannot be changed in the short run, but it responds to prices in the long run. In the language of Atkeson and Kehoe and other early studies (see, for instance, Pindyck and Rotemberg 1983) the model delivers simultaneously a low short run and a high long run price elasticity of energy use, as the early evidence suggested.[3] Assuming that capital has a vintage structure implies that the return of operating capital more efficiently depends not only on energy prices but also on the investment specific technological change (ISTC hereafter). This implies that the short run response of aggregate output and energy use to energy price shocks will depend on the ISTC growth rate. In other words, ISTC is a form of energy saving technical change. An additional advantage of modeling the economy at the microeconomic level is that, in this way, we can obtain prices of capital depending on vintage and energy-efficiency which will allow us to represent the aggregate economy in terms of aggregate capital services and energy used. We find that both the relative price of quality-adjusted and non quality-adjusted aggregate capital fall over time, consistently with the estimates of Gordon (1990) or Cummins and Violante (2002), among others, but it is also affected by the level of energy efficiency chosen, as argued by Gordon (1996). We also show that Atkeson

[3] See Berndt and Wood (1975), or Griffin and Gregory (1976), Pindyck (1979).

and Kehoe (1999) and Díaz et al. (2004) can be written as particular cases of our theory. As a matter of fact, our theory collapses to Atkeson and Kehoe (1999) in absence of investment specific technological change. By rewriting Díaz et al. (2004) in our framework we show that assuming the existence of adjustment costs in aggregate investment is a short cut for the existence of capital irreversibility at the micro level.

We calibrate our model economy to match aggregate statistics of the U.S. economy for the period 1960-2008. We simulate a calibrated version of our model economy in which there are energy price shocks as well as ISTC shocks. That is, there are three shocks: innovations in energy prices, the cost of producing capital goods and the level of technical progress embodied in new capital units. We disentangle the two ISTC shocks by exploiting the fact that the BEA adjust for quality the price of some (but not all) investment goods. We prefer to abstract from neutral technological progress to isolate the effects of ISTC shocks in energy use. We estimate the three types of shocks so that the relative price of energy and the relative prices of investment—both the non quality-adjusted as well as the quality-adjusted price—in the model have the statistical properties of their counterparts in the data. It is interesting to note that the innovations in ISTC shocks are larger and highly correlated right after the oil price shocks during the early 1980s and during the late 1990s, right before the 2003-08 oil price shock. In order to assess the importance of ISTC we compare our simulated economy with another in which there are only energy price shocks and another economy that is a version of Atkeson and Kehoe (1999) with energy price shocks and neutral progress shocks.

Our simulations show the mechanism through which energy price shocks are transmitted to the rest of the economy. Upon impact, the capital-energy ratio is already given, so that an increase in the price increases in the same amount the energy expenditure share and lowers value added accordingly. The only way the economy has to respond to the price shock is through investment; that is, allocating new machines to more energy efficient technologies. Thus, the short run response of energy use is very low and rises in the long run as the average capital-energy ratio is increased. Conversely, an unexpected energy price drop is tantamount to a wealth effect: investment rises and new machines will be allocated to less energy-efficient technologies, which lower the capital-energy ratio. This is also the mechanism investigated in Atkeson and Kehoe (1999), where there is no ISTC. The existence of ISTC increases the responsiveness of investment to energy price shocks and changes the return of energy efficient technologies. The reason is the following: When technology is investment specific, agents have to invest in order to receive its rewards. Given that energy and capital are complements, higher investment implies that the economy becomes more energy-dependent and, therefore, more exposed to energy price shocks. If energy prices are persistent, a high price today implies a very likely high price tomorrow, making production very costly in the future. Thus, agents respond by investing less (and less than in an economy with neutral technological progress) and allocating the new machines to more energy efficient technologies. The opposite occurs when energy prices are low. Additionally, a combination of low energy prices and an acceleration of the ISTC growth rate lowers the return of energy efficient technologies (since ISTC is a form of energy saving technical change), allowing the economy to fully rip the rewards of ISTC in the form of higher investment. This is exactly what happened during the 1990s, according to Metcalf (2008), or Steinbuks and Neuhoff (2010): an investment boom. Our model economy offers an explanation to the softened response of aggregate output to the 2003-08 oil price shock: By increasing investment in the years of high ISTC growth the economy was increasing the average efficiency of the economy (the capital-energy ratio), shielding the economy against the impact of the 2003-08 price shock.

The organization of the paper is as follows. Section 2 describes in detail the evidence about energy use, energy prices and expenditures and it discusses the microeconomic evidence about changes in the short run price elasticity of energy use and its likely relationship with improvements in the energy efficiency of capital. Section 3 describes our model economy and defines a quasi-social planner problem whose solution is the efficient allocation of our economy. In Section 4 we discuss the role of investment specific technological change and argue that Atkeson and Kehoe (1999) and Díaz et al. (2004) are particular cases of our theory. In Section **??** we describe how we calibrate our economy and our procedure to identify investment specific technological shocks. Section 6 presents our main results and some robustness exercises and Section 7 concludes.

2 The aggregate data

In this section, we discuss the time-series features of energy expenditure, energy prices, energy use, and the aggregate capital-energy ratio. We argue that energy use, as percentage of aggregate Value Added, and the capital-energy ratio are fairly inelastic, and the share of energy expenditure in value added follows very closely the evolution of the relative price of energy. We also argue that there is a change in the response of energy use and the capital-energy ratio to fluctuations of the energy price starting from the 1990s.

We need to start by providing suitable definitions of aggregate value added, capital stock and investment, as in Atkeson and Kehoe (1999) and our previous work, Díaz et al. (2004). As these authors, we take the view that our theory of energy use cannot account for the behavior of the energy-producing sectors. As a result, in our measure of aggregate Value Added, capital and investment, we exclude value added, investment, and capital stock of the energy sector of the U.S. economy.[4] Appropriate measures of aggregate value added, capital and investment are constructed following the procedure outlined by Cooley and Prescott (1995). In this paper, since we are interested in the response of value added, investment and energy use to energy prices, we do not distinguish between private and public assets. In particular, we impute to measured GDP the value of the services of the stock of consumer durable goods (where cars are included) and the government stock of capital. We define the capital stock and aggregate investment accordingly. In Appendix A we describe in detail the construction of of value added, investment and capital stock.

Secondly, we use current annual data on energy prices, energy use and energy expenditure for the U.S. economy for the period 1960-2010. For the same reason argued above, we exclude the energy use of the energy-producing sectors of the U.S. economy. Energy expenditure is the amount of British Termal Units (BTUs, hereafter) obtained from the final use of coal, petroleum, natural gas and electricity, valued at their corresponding real prices of each energy source for each year. Real prices are obtained by dividing current prices by the implicit price deflator of non durable consumption goods and services in NIPA. To decompose this measure of energy expenditure into price and energy use (i.e., quantity) we define the latter as the amount of used BTUs valued at a base-year real price. Our aggregate energy price is an implicit price deflator: it is the ratio of energy expenditure to energy use. Notice that our aggregate energy price is a relative price; i.e., measured in units of non durable consumption goods and services. In Appendix A we describe in detail the sources and methods used.

In Figure 1, we plot the logarithm of the ratio of energy expenditures to Value Added, pE/VA, the relative price of energy, p, and the energy use to Value Added ratio, E/VA for the period 1960-2010.[5] We will refer to the first measure as the energy share and the second one is a measure

[4] The energy producing sectors represent about 5 percent of measured GDP with very low variance, whereas their capital stock represent between 12 percent of private fixed assets in 1981 and 7 percent in 1999. Their share in private fixed investment has larger variance, going from 20 percent in 1982 to 5 percent in 1999.

[5] All aggregates have also been deflated using the implicit price deflator of non durable consumption goods and

of energy intensity. The share of energy has a very high volatility, in spite of begin small on average, 4.75 percent of our measure of output for the whole period 1960-2010, peaking 8.89 percent in 1981 and reaching a minimum of 3.18 percent in 1998. Let us turn to inspect our measure of energy price. The first thing that we notice is that the price tracks very closely the evolution of the energy share. The second thing is the size of the shocks experienced in the years 1974, 1979-81, and 2003-08 that dwarf other price shocks. It may be thought that the evolution of this price index hides significant differences in prices for the different types of energy considered. This is not the case; as pointed out by Kilian (2008), the pattern of fluctuations in the share of energy primarily reflects changes in the price of crude oil rather than shifts in energy use. This can be seen in Figure 4. Here we have plotted the nominal price divided by the deflator of non durable consumption goods and services for the different components of our aggregate measure of energy use. Of course, all prices differ in their absolute level and variance, but all of them follow the price of crude oil. Moreover, as shown in Figure 5, more than 50 percent of all energy used is petroleum-based. That is, the important source of fluctuations in our measure of the energy price is the price of oil and its importance in our price index has not changed significantly during the period considered. Since we have excluded in our measure of energy use the amount of energy used by the energy-producing sectors we take the view that the fluctuations in our measure of energy price are driven by exogenous shocks.[6]

Let us turn to inspect the evolution of energy use as a fraction of Value Added, E/VA. As we can see, after 1970s until late 1980s, energy use falls over time. This is a compounded effect of energy demand falling (as it happened in the late 1970s as we can see in Figure 2 and Value Added growing more rapidly than energy demand in the early 1980s. After 1985 there is a slight increase in the ratio E/VA, as a response to the fall in energy prices, but after 1990, the ratio starts falling, in spite of the substantial reduction in the energy price, and keeps doing so during the 2000s, when we witness the last upsurge in the energy price. This continuous fall in the energy use to Value Added ratio comes along with a roughly constant capital to Value Added ratio (when capital is measured in units of output). That is, energy prices do not seem to affect capital deepening much, but they do affect energy efficiency, as measured by the capital to energy ratio, as we can see in Figure 3. During the 1970s and early 1980s, this ratio rises while energy prices climb up but it decreases during the late 1980s, suggesting that this ratio respond belatedly to energy price changes. After the early 1990s, however, this measure of energy efficiency increases, seeming unresponsive to the

services.

[6]See Kilian and Murphy (2010) for an identification of structural shocks to the real price of oil.

energy price drop.

One question that arises is whether the fall in energy use and the corresponding increase in the capital-energy ratio are a result of sectoral changes. On this respect, Metcalf (2008) finds that more than two-thirds of the decline in energy intensity in the U.S. economy comes from improvements in the capital-energy ratio vis-a-vis changes in sectoral composition since 1970.[7] Another question that rises is how this macroeconomic measure of energy efficiency relates to its microeconomic counterparts. For instance, Steinbuks and Neuhoff (2010) show that between 1990 and 2005 the energy efficiency of physical capital has increased in all manufacturing sectors in a sample of 19 OECD countries. However, Knittel (2011) estimates that, in absence of improvements in car quality, fuel economy (i.e., energy efficiency measured as miles per gallon) of both passenger cars and light trucks could have increased by nearly 60 percent from 1980 to 2006, instead of the observed meager 15 percent. In other words, investment specific technological change is a sort of energy saving technical change. The period of increasing energy efficiency of physical capital found by Steinbuks and Neuhoff (2010) coincides with the period of acceleration of quality growth in equipment and software documented by Gordon (1990) and Cummins and Violante (2002). According to the latter authors, the percent annual growth in the quality of equipment and software in the postwar period accelerated from 4 percent to more than 6 percent during the 1990s. These authors also document that the acceleration in quality growth occurred in every industry. This acceleration can be seen in Figure 6, where we have plotted two measures of the relative price of investment goods in units of non durable consumption goods and services. In Appendix A we describe in detail how the price measures are constructed.

3 The benchmark model economy

The evidence presented in the previous section suggests that ISTC is key to understand the changes in the response of energy use and efficiency to energy prices. Here we present our benchmark model economy where investment specific technological change plays a central role.

[7] Metcalf's measure of energy intensity is the ratio of the amount of BTUs yielded by various energy sources, to measured GDP, which is different from our measure, which takes into account the differences in prices across different energy sources. Nevertheless, the behavior of both series is fairly similar.

3.1 Preferences

There is a continuum of households that seek to maximize expected discounted lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(c_t\right) + \xi \log\left(\ell_t\right)\right), \ \beta \in (0,1) \ \ \alpha > 0, \tag{3.1}$$

where c_t is consumption and ℓ_t is leisure t. Each household is endowed with $\hbar$ units of time and, therefore, works $\hbar - \ell_t$ hours every period.

3.2 Technology and the physical environment

Production of the unique final good is carried out at a continuum of autonomous plants. A plant is built by installing one unit of capital. Plants are indexed by the vintage of the unit of capital installed, denoted by z, and the energy efficiency with which that unit of capital is operated, denoted by v. Output is produced combining labor and the services of the unit of capital installed according to the technology

$$y_t\left(z,v\right) = A_t\, \kappa_t\left(z,v\right)^{\alpha} h_t\left(z,v\right)^{1-\alpha}, \ z \leq t, \ v \in \mathbf{R}_{++}, \tag{3.2}$$

with $\alpha \in (0,1)$. A_t is the disembodied technological change factor, which may change stochastically over time according to a process that we will specify in Section 6; whereas $\kappa_t\left(z,v\right)$ and $h_t\left(z,v\right)$ are, respectively, the amount of services provided by the unit of capital installed and the amount of labor services employed in the plant.

As we have already mentioned, capital is heterogeneous in two dimensions: Its vintage, z, and its energy efficiency, v. The vintage is given by the date at which the unit of capital was produced. Thus, $z \leq t$. The efficiency type v takes values in $\mathbf{R}_{++}$. In order to yield services the unit of capital needs to be combined with energy. The type v determines the amount of energy, $e_t(z,v)$, needed to produce the amount $\kappa_t(z,v)$ of capital services:

$$\kappa_t\left(z,v\right) = \Lambda_z\, \Gamma_z\, v^{1-\mu} \min\left\{e_t\left(z,v\right), \Gamma_z^{-1} v^{\mu}\right\}, \ z \leq t, \ v \in \mathbf{R}_{++}, \tag{3.3}$$

where $\mu > 1$. Λ_z and Γ_z refer to embodied investment specific technical change. Energy efficiency type v and embodied investment specific technological change are meant to refer to different factors.

We may think of type v as engine power of a car. When the car is used at optimal speed the energy used is $\Gamma_z^{-1}\, v^{\mu}$. In this case, the services yielded by the unit of capital are $\kappa_t(z, v) = \Lambda_z\, v$. Since $\mu > 1$, energy use at optimal speed is more than proportional to engine power. When the amount of energy used is less than the amount required to drive at optimal speed, the amount of services is lower, $\kappa_t(z, v) = \Lambda_z\, \Gamma_z\, v^{1-\mu}\, e_t(z, v)$, although increasing with the amount of energy. Notice that, in such a case, for a given amount of energy, services may be lower the larger is the type v. This assumption implies that a SUV car may yield less services when driven in downtown, at suboptimal speed, than a smaller car equipped with less engine power. Let us turn to discuss the effects of embodied technical change.

Λ_z refers to technological improvements that increase capital services which are not directed to save energy use, whereas Γ_z refers to innovations that save energy directly. For instance, any improvement in the ergonomics of car seats, so that driving is less tiring, would be an increase in Λ_z, whereas improvements in the aerodynamics of the car would rise its energy efficiency, and should be considered as increases in Γ_z.[8] Thus, embodied technological progress brings both higher productivity and lower energy requirement given the engine power, v. Either factor, Λ_z, and Γ_z, may vary stochastically over time in a manner specified in Section 5. For simplicity, we will call a pair (z, v) a *technology class*, and $\kappa_t(z, v)$ will be the amount services yielded by a unit of capital of class (z, v) at period t. Likewise, $k_t(z, v)$ is the amount of capital of class (z, v) at time t.

Additionally, there is a technology that allows agents to transform final good of period t into Θ_t units of capital of vintage $t+1$. We take the view that investment specific technological change not only brings higher quality capital goods but also the production of those goods becomes increasingly efficient with the passage of time. Specifically, we will denote as $x_t(v)$ the amount of final good invested in capital of vintage $t+1$ that will be operated with efficiency type $\in \mathbf{R}_{++}$ at period $t+1$. The factor Θ_t may vary stochastically over time.

The number of plants of class (z, v) is equal to the amount of capital of that class, $k_t(z, v)$. Notice that while the vintage is given exogenously by the time at which the unit of capital was produced, the measure of classes installed and operated at any period t respond to economic conditions. Capital is irreversible; that is, capital of vintage z_1 cannot be converted in capital of vintage z_2. Likewise, capital is irreversible across efficiency types. Capital of vintage z of efficiency type v_1 cannot be transformed into type v_2. This amounts to saying that plants of class (z, v) cannot be converted

[8] See Knittel (2011) for particular examples of both types of embodied technical innovations.

into plants of a different class. The plant, though, can be left idle by not allocating either energy or labor. Finally, once production has taken place, the plant faces a positive probability of death, $\varpi \in [0,1]$, which is i.i.d. across plants. This death implies the destruction of the unit of capital. This death probability plays the role of physical depreciation of capital. Therefore,

$$k_t(z,v) = (1-\varpi)^{t-z}\, k_z(z,v),\ z \leq t,\ v \in \mathbf{R}_{++}. \tag{3.4}$$

3.3 Energy use and energy prices

We will assume that energy is entirely bought in an international market at an exogenously given price ϱ_t. Therefore, from the point of view of the economic agents, the energy price follows a stochastic process. We assume that there is no international borrowing and lending. In absence of an international credit market we can think of the price of energy as given by nature. This implies that, under market completeness, the Second Welfare Theorem applies and, therefore, we can restrict our attention to efficient allocations.

3.4 Aggregate value added

The amount of aggregate labor used in the production of the final good satisfies

$$0 \leq \ell_t \leq \hbar - \sum_{z=-\infty}^{t} \int_0^{\infty} k_t(z,v)\, h_t(z,v) d\,v,\ 0 \leq h_t(z,v) \leq \hbar,\ z \leq t,\ v \in \mathbf{R}_{++}, \tag{3.5}$$

Aggregate value added is aggregate production of the final good net of energy expenditures,

$$c_t + \int_0^{\infty} x_t\,(v)\ d\,v \leq \sum_{z=-\infty}^{t} \int_0^{\infty} k_t(z,v)\,[y_t(z,v) - \varrho_t\, e_t(z,v)]\, d\,v, \tag{3.6}$$

where c_t denotes consumption, and $\int_0^{\infty} x_t\,(v)\ d\,v$ is aggregate investment. Thus,

$$0 \leq k_{t+1}(t+1,v) \leq \Theta_t\, x_t\,(v)\,. \tag{3.7}$$

3.5 Properties of the efficient allocation

The efficient allocation is found by maximizing (3.1) subject to (3.3)–(3.7). The planning problem is fully specified in Appendix B. In what follows we are going to characterize the efficient allocation and provide conditions that guarantee that the economy can be aggregated.

Lemma 1. *Marginal productivity of labor is the same across all plants used:*

$$(1-\alpha)\, A_t\, \kappa_t(z,v)^{\alpha} h_t(z,v)^{-\alpha} = \frac{u_\ell\,(c_t,\ell_t)}{u_c\,(c_t,\ell_t)}. \tag{3.8}$$

This is a direct consequence of labor being freely mobile across plants. In the decentralized version of our economy (shown in Appendix C) the price of one unit of labor services is $w_t \equiv \frac{u_\ell(c_t,\ell_t)}{u_c(c_t,\ell_t)}$. Thus, Lemma 1 implies that the labor share in gross output is the same in all plants. Now we turn to investigate which classes of capital are allocated energy.

Proposition 1. *If at time t a plant of class (z,v), $z \le t$, $v > 0$, is operated, $e_t(z,v) > 0$, then it is operated at the requirement level, $e_t(z,v) = \Gamma_z^{-1}\, v^{\mu}$ and $\kappa_t(z,v) = \Lambda_z\, v$.*

Proof. Consider the Lagrangian

$$\max_{e_t(z,v)} A_t \left(\Lambda_z\, \Gamma_z\, v^{1-\mu} e_t(z,v)\right)^{\alpha} h_t(z,v)^{1-\alpha} - \varrho_t\, e_t(z,v) + \Psi_t^{e_1}(z,v)\left(\Gamma_z^{-1} v^{\mu} - e_t(z,v)\right) + \Psi_t^{e_0}(z,v)\, e_t(z,v). \tag{3.9}$$

For any plant for which $h_t(z,v) > 0$, if $0 < e_t(z,v) < \Gamma_z^{-1}\, v^{\mu}$, then $\Psi_t^{e_0}(z,v) = 0$, and it must be the case that

$$\alpha\, A_t \left(\Lambda_z\, \Gamma_z\, v^{1-\mu}\right)^{\alpha} e_t(z,v)^{\alpha-1} h_t(z,v)^{1-\alpha} - \varrho_t = \Psi_t^{e_1}(z,v). \tag{3.10}$$

Using (3.3) and (3.8), expression (3.10) becomes

$$\alpha\, A_t^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}} \Lambda_z\, \Gamma_z\, v^{1-\mu} - \varrho_t = \Psi_t^{e_1}(z,v), \tag{3.11}$$

where $w_t \equiv \frac{u_\ell(c_t,\ell_t)}{u_c(c_t,\ell_t)}$. Notice that expression (3.11) does not depend on $e_t(z,v)$. Thus if $\Psi_t^{e_1}(z,v)$ is positive, then $e_t(z,v) = \Gamma_z^{-1}\, v^{\mu}$, and $\kappa_t(z,v) = \Lambda_z\, v$. □

This Proposition states that it is never efficient to operate a plant using less energy than the requirement level. Proposition 1, expressions (3.2), and (3.8) imply that production at the plant level only depends on its class:

Corollary 1. *The difference in the amount of output produced in a plant only depends on the class of the plant; thus,*

$$\frac{h_{t+i}\,(z, v_z)}{h_{t+i}\,(t, v_t)} = \frac{y_{t+i}\,(z, v_z)}{y_{t+i}\,(t, v_t)} = \frac{\Lambda_z}{\Lambda_t}\,\frac{v_z}{v_t},\ \textit{for all } i \geq 0. \tag{3.12}$$

This Corollary shows how labor is allocated across plants. Labor is gradually withdrawn from older and less efficient plants to newer and more efficient plants. Therefore, output produced by older and less efficient classes is reduced gradually. Labor allocated to plants (and energy used) becomes zero whenever production is too low compared to the energy expenditure needed to produce that output. The cut-off point is implied by the non-negativity of the first order condition shown in equation (3.11) in Proposition 1.

Corollary 2. *For a plant of class (z, v) to be utilized in equilibrium it must be the case that*

$$\alpha\, A_t^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}} \Lambda_z\, v - \varrho_t\, \Gamma_z^{-1}\, v^{\mu} \geq 0, \tag{3.13}$$

where $w_t \equiv \frac{u_\ell(c_t,\ell_t)}{u_c(c_t,\ell_t)}$. For each vintage z there exists a a type $\underline{v}_{z\,t}$ for which (3.13) *holds with strict equality. The type $\underline{v}_{z\,t}$ decreases with ϱ_t and increases with z.*

In the decentralized version of this economy, expression (3.13) is the profit accrued in a plant of class (z, v). Thus, expression (3.13) says that for a plant to be utilized, the rents of capital net of energy expenditure have to be non-negative. At the marginal type $\underline{v}_{z\,t}$ gross output is completely exhausted by compensating labor and energy. Thus, factor shares at the plant level are not constant in this economy. In particular, for a given vintage z, the capital share (in Value Added at the plant level) is zero at the marginal type $\underline{v}_{z\,t}$ and decreases with v. The fact that $\underline{v}_{z\,t}$ increases with z implies that newest vintages can be operated with lower energy efficiency compared with older ones since their services per unit of energy used are higher. In other words, the level of investment specific technological progress affects the energy efficiency of the economy. Higher Λ_z implies more services per unit of energy used; thus, implicitly, even non-energy related embodied technical progress is a form of energy saving. Let us turn now to analyze the investment decision and the characteristics of the new plants.

Proposition 2. *If all installed capital is utilized, $e_t(z,v) > 0$, for all t, z, and v, then, all units of new capital are operated with the same level of energy efficiency, $v_{t+1} > 0$.*

Proof. The first order condition with respect to investment allocated to type v, $x_t(v)$, is

$$\Theta_t^{-1}\varphi_t - \Psi_t^k(v) = E_t \sum_{i=1}^{\infty}(1-\varpi)^{i-1}\varphi_{t+i}\left[\alpha\, y_{t+i}(t+1,v) - \varrho_{t+i}\, e_{t+i}(t+1,v)\right], \tag{3.14}$$

where φ_t is marginal utility of consumption at time t. Investment in type v is positive only if $\Psi_t^k(v) = 0$. This multiplier is non negative so zero is its minimum value. Thus, we have to show that $\Psi_t^k(v)$ has a unique minimum. Equivalently, present value of capital income, shown in the right hand side of (3.14), has a unique maximum. Using (3.8) and the result of Proposition 1 and plugging them in the expression of output at the plant level, (3.2), we can write (3.14) as

$$\Theta_t^{-1}\varphi_t - \Psi_t^k(v) = E_t \sum_{i=1}^{\infty}(1-\varpi)^{i-1}\varphi_{t+i}\left[\alpha\, A_{t+i}^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{w_{t+i}}\right)^{\frac{1-\alpha}{\alpha}}\Lambda_{t+1}\, v - \varrho_{t+i}\,\Gamma_{t+1}^{-1}\, v^{\mu}\right], \tag{3.15}$$

where $w_{t+i} \equiv \frac{u_\ell(c_{t+i},\ell_{t+i})}{u_c(c_{t+i},\ell_{t+i})}$. The right hand side of (3.15) has a unique maximum in v, that we call v_{t+1}, and it satisfies

$$\frac{\alpha}{\mu}\, E_t \sum_{i=1}^{\infty}(1-\varpi)^{i-1}\varphi_{t+i}\, y_{t+i}\left(t+1, v_{t+1}\right) = E_t \sum_{i=1}^{\infty}(1-\varpi)^{i-1}\varphi_{t+i}\,\varrho_{t+i}\,\Gamma_{t+1}^{-1}\, v_{t+1}^{\mu}, \tag{3.16}$$

that is, the present value of all future energy expenditures in a plant that uses a unit of capital of vintage $t+1$ is the fraction α/μ of the present value of future gross output produced by that plant. Hence the price of one unit of new capital, Θ_t^{-1}, must be equal to the fraction $\alpha(\mu-1)/\mu$ of the present value of future gross output. □

This Proposition shows that if all plants are always utilized, it is welfare enhancing to place all new investment in only one efficiency type, the one that maximizes the present value of future output produced in the plant net of energy expenditures and imputed labor income. Notice that manipulating (3.16) we obtain that the efficiency v_{t+1} satisfies

$$v_{t+1}^{\mu-1} = \Lambda_{t+1}\,\Gamma_{t+1}\,\frac{E_t \sum_{i=1}^{\infty}(1-\varpi)^{i-1}\varphi_{t+i}\,\alpha\,A_{t+i}^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{w_{t+i}}\right)^{\frac{1-\alpha}{\alpha}}}{\mu\,E_t \sum_{i=1}^{\infty}(1-\varpi)^{i-1}\varphi_{t+i}\,\varrho_{t+i}}. \tag{3.17}$$

Thus, at any time t investment will be allocated to a unique technology class $(t+1, v_{t+1})$ and in period $t+1$ all capital of vintage $t+1$ will be operated with the same energy efficiency.

3.6 Aggregation

We would like to represent aggregate Value Added as a function of aggregate capital. In order to aggregate capital we need to define appropriate relative prices for each class. It is easy to do so for the latest vintage produced, since its shadow price must be equal to the inverse of the productivity in the production of capital goods, Θ_t^{-1}, but we do not have an equivalent measure for the previous vintages. Therefore, we define a price for any class of capital in the following way:

Definition 1. *Let the cost of one unit of capital of class (z, v), $z \leq t+1$, $v_z \in \mathbf{R}_{++}$, in units of gross output at time t be defined as*

$$q_t\,(z, v_z) \equiv \Theta_t^{-1}\frac{\Lambda_z}{\Lambda_{t+1}}\,\frac{v_z}{v_{t+1}},\ z \leq t+1,\ v \in \mathbf{R}_{++}. \tag{3.18}$$

Notice that capital goods of the same vintage may have different prices, depending on their energy efficiency. This is consistent with Gordon (1990, 1996), who argued that not all changes in the relative price of capital goods are due to investment specific technical change but also to changes in energy efficiency (see Gordon 1996, p. 262).

We need to emphasize that the price defined above is not the market price of a unit of capital of class (z, v) at time t, since it does not measure the present value of the expected return to one unit of capital net of energy expenditures. The price shown in (3.18) is the cost of capital of class (z, v_z) in units of gross output at period t. Appendix C shows the decentralized version of this economy and that the market price of one unit of capital of class (z, v) at time $t \geq z$ is

$$p_t(z, v_z) = \frac{\Lambda_z\, v_z}{\Lambda_{t+1}\, v_{t+1}}\, \Theta_t^{-1} + \left(\frac{\Lambda_z\, v_z}{\Lambda_{t+1}\, v_{t+1}} - \frac{\Gamma_z^{-1}\, v_z^{\mu}}{\Gamma_{t+1}^{-1}\, v_{t+1}^{\mu}} \right) \frac{\Theta_t^{-1}}{\mu - 1}. \tag{3.19}$$

This price takes into account the associated cost of energy; that is, $p_t(z, v_z)$ is the price of capital in units of Value Added. Actually, if the ratio of capital services to energy expenditure is higher than that built into the new investment goods, then $p_t(z, v_z) > q_t(z, v_z)$; otherwise the cost valuation is higher than the market price of the plant, $p_t(z, v_z) < q_t(z, v_z)$. Unless all capital is operated with the same energy efficiency the market price of capital and its cost in units of gross output are different.

It could be argued that we should find aggregate capital using prices $p_t(z, v_z)$, prices in units of Value Added. We would do so if we could represent aggregate Value Added solely as a function of labor and capital, which we cannot due to the complementarity of capital and energy at the plant level. This complementarity implies that factor shares in Value Added in each plant are different depending on the capital vintage and its energy efficiency, as we have already shown. As a consequence, we cannot write aggregate Value Added as a function of aggregate primary inputs. We can, however, find aggregate gross output since the share of labor in gross output is constant regardless of the capital vintage and energy efficiency. See Sato (1976) for a more detailed discussion on the matter of aggregation.

3.6.1 Aggregate capital and its relative price

Let us define as k_t the aggregate volume of capital, in per capita terms, in units of the latest class. Thus,

$$k_t = \sum_{z=-\infty}^{t} \frac{q_t(z, v_z)}{q_t(t, v_t)}\, k_t(z, v_z). \tag{3.20}$$

Taking into account that investment takes place in only one class of capital, we can express capital as

$$k_t = \sum_{z=-\infty}^{t} \frac{\Lambda_z}{\Lambda_t} \frac{v_z}{v_t} (1 - \varpi)^{t-z}\, \Theta_{z-1}\, x_{z-1}. \tag{3.21}$$

The product $\frac{\Lambda_z}{\Lambda_t}\frac{v_z}{v_t}(1-\varpi)^{t-z}$ can be interpreted as the remaining value of the stock of class (z, v_z) (per unit of capital) once physical depreciation and obsolescence are taken into account. In other words, the average depreciation rate of capital class (z, v_z) at time t is

$$\delta_t(z, v_z) = 1 - (1-\varpi)\left(\frac{\Lambda_z}{\Lambda_t}\frac{v_z}{v_t}\right)^{\frac{1}{t-z}}. \tag{3.22}$$

The average relative price (cost) of capital in units of consumption good is, by definition of k_t, equal to the relative price of vintage t and type v_t.

$$q_t = \sum_{z=-\infty}^{t} q_t(z, v_z)\frac{k_t(z, v_z)}{k_t} = \frac{\Lambda_t}{\Lambda_{t+1}}\frac{v_t}{v_{t+1}}\Theta_t^{-1}. \tag{3.23}$$

We can also find the aggregate amount of capital services or, in other words, the amount of quality-adjusted capital:

$$\kappa_t = \Lambda_t\, v_t \sum_{z=-\infty}^{t} q_t(z, v_z)\, k_t(z, v_z) = \Lambda_t\, v_t\, k_t. \tag{3.24}$$

We can also define the cost of quality-adjusted capital, which is equal to the cost of capital services,

Definition 2. *Let the cost of one unit of capital services of class (z, v_z), $z \leq t+1$, $v_z \in \mathbf{R}_{++}$, in units of gross output at time t be defined as*

$$q_t^{\kappa} \equiv \frac{q_t(z, v)}{\lambda_z\, v_z},\ z \leq t+1,\ v \in \mathbf{R}_{++}. \tag{3.25}$$

Notice that this price is equal to $\Theta_t^{-1}\Lambda_{t+1}^{-1}v_{t+1}^{-1}$ for all classes of capital. Consistenly with Gordon (1990) and Cummins and Violante (2002), this price falls when investment specific technical change rises, as measured by $\Theta_t\,\Lambda_{t+1}$, but it rises when energy efficiency rises; that is, when v_{t+1} falls. Thus, when the energy price increase, agents invest in improving energy efficiency of new capital, which makes it more costly in terms of output, as it was suggested by Gordon (1996).

This method to define the aggregate net stock of capital is consistent with the procedure used by the Bureau of Economic Analysis to measure the real value of the net stock of capital. The only difference is that BEA defines the stock of capital in units of output. In our notation:

$$\tilde{k}_t = \sum_{z=-\infty}^{t} (1-\varpi)^{t-z} \frac{\Lambda_z}{\Lambda_t} \frac{v_z}{v_t} \frac{\Theta_{z-1}}{\Theta_{t-1}} x_{z-1} = \sum_{z=-\infty}^{t} (1-\varpi)^{t-z} \frac{q^{\kappa}_{t-1}}{q^{\kappa}_{z-1}} x_{z-1}. \tag{3.26}$$

The BEA depreciates investment realized at time $z-1$ by applying the corresponding change in the price of quality-adjusted capital from time z to time t.[9] Notice that $\tilde{k}_t$ is equal to the stock of quality-adjusted capital valued at prices at the beginning of the period, $\tilde{k}_t = q^{\kappa}_{t-1}\,\kappa_t$, which is equal to $\Theta^{-1}_{t-1}\,k_t$.

3.6.2 Output, hours worked and energy use

Production of all plants of class (z, v) is the expected output of a plant of vintage z which, using (3.12) can be written as

$$y_t = \sum_{z=-\infty}^{t} \frac{\Lambda_z}{\Lambda_t} \frac{v_z}{v_t} k_t\left(z, v_z\right) y_t\left(t, v_t\right). \tag{3.27}$$

Likewise, using (3.8), and (3.18) we can write aggregate labor as

$$h_t = \sum_{z=-\infty}^{t} \frac{\Lambda_z}{\Lambda_t} \frac{v_z}{v_t} k_t\left(z, v_z\right) h_t\left(t, v_t\right) = k_t\, h_t\left(t, v_t\right). \tag{3.28}$$

Aggregate gross output is

$$y_t = A_t\, \Lambda_t^{\alpha} \left(v_t\, k_t\right)^{\alpha} h_t^{1-\alpha}. \tag{3.29}$$

Likewise, the aggregate use of energy is

$$e_t = \sum_{z=-\infty}^{t} \Gamma_z^{-1}\, v_z^{\mu}\, k_t\left(z, v_z\right). \tag{3.30}$$

[9] See Bureau of Economic Analysis (2003) for a detailed description of the perpetual inventory method used to construct the aggregate stock of capital measure.

3.6.3 The aggregated economy

The law of motion of capital is

$$k_{t+1} = \Theta_t\, x_t + (1-\varpi)\frac{\Lambda_t}{\Lambda_{t+1}}\,\frac{v_t}{v_{t+1}}\,k_t. \tag{3.31}$$

If all plants are utilized in equilibrium, the law of motion of energy use is

$$e_{t+1} = \Gamma_{t+1}^{-1}\, v_{t+1}^{\mu}\, \Theta_t\, x_t + (1-\varpi)\, e_t. \tag{3.32}$$

The quasi-social planner's problem as

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(c_t\right) + \xi \log\left(\hbar - h_t\right)\right) \\ \text{s. t.} \quad & c_t + x_t \le A_t \left(\Lambda_t\, v_t\, k_t\right)^{\alpha} h_t^{1-\alpha} - \varrho_t\, e_t, \\ & e_{t+1} \ge \Gamma_{t+1}^{-1}\, v_{t+1}^{\mu}\, \Theta_t\, x_t + (1-\varpi)\, e_t, \\ & 0 \le k_{t+1} \le \Theta_t\, x_t + (1-\varpi)\tfrac{\Lambda_t}{\Lambda_{t+1}}\,\tfrac{v_t}{v_{t+1}}\,k_t, \\ & k_0 \text{ given},\ v_t \ge 0,\ x_t \ge 0. \end{aligned} \tag{3.33}$$

It will be useful to write the quasi-social planner's problem in terms of quality-adjusted capital using (3.24):

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(c_t\right) + \xi \log\left(\hbar - h_t\right)\right) \\ \text{s. t.} \quad & c_t + x_t \le A_t\, \kappa_t^{\alpha}\, h_t^{1-\alpha} - \varrho_t\, e_t, \\ & e_{t+1} \ge \Gamma_{t+1}^{-1}\, v_{t+1}^{\mu}\, \Theta_t\, x_t + (1-\varpi)\, e_t, \\ & 0 \le \kappa_{t+1} \le v_{t+1}\, \Lambda_{t+1}\, \Theta_t\, x_t + (1-\varpi)\, \kappa_t, \\ & \kappa_0 \text{ given},\ x_t \ge 0. \end{aligned} \tag{3.34}$$

Hence, by solving this planner's problem we can find the aggregates of our decentralized economy. Notice that the evolution of the aggregate stock of capital is shown in (3.31). Finally, we need to add the condition that ensures that this aggregation works: all installed capital is used in equilibrium. Using (3.13) with strict inequality, we can write the following assumption:

Assumption 1. *The energy price is never too large,*

$$\alpha\, A_t\, \kappa_t^{\alpha-1}\, h_t^{1-\alpha} > \varrho_t\, \Gamma_z^{-1}\, \Lambda_z^{-1}\, \upsilon_z^{\mu-1}. \tag{3.35}$$

That is, the share of energy in value added cannot exceed the capital share.

4 The role of investment specific technological change

In this section we want to highlight the role of embodied technological progress. To this purpose, we want to compare our theory with that of Atkeson and Kehoe (1999) and our previous work Díaz et al. (2004).

4.1 The cost of saving energy in a putty-clay model economy

Let us turn now to the economy proposed by Atkeson and Kehoe (1999). They abstract from investment specific technological progress but retain the assumption about efficiency types and capital irreversibility. We show in Appendix E the mapping of their economy to our economy in more detail. Their economy behaves as ours in absence of investment specific technological change:

$$y_t = A_t\, \kappa_t^{\alpha}\, h_t^{1-\alpha}, \tag{4.1}$$

$$va_t = y_t - \varrho_t\, e_t, \tag{4.2}$$

$$\kappa_{t+1} = \upsilon_{t+1}\, x_t + (1-\varpi)\, \kappa_t, \tag{4.3}$$

$$e_{t+1} = \upsilon_{t+1}^{\mu}\, x_t + (1-\varpi)\, e_t. \tag{4.4}$$

We have denoted aggregate Value Added as va_t. In this economy capital and energy are complements in the short run. Thus, right after an unexpected positive energy price shock at a given period t, the economy cannot reduce its energy use (and, therefore, its energy bill) so that the immediate effect is a fall in aggregate value added. If the energy price has persistence, the expected price will rise, too. This reduces the future return to current investment. The only response that this economy has to reduce its energy use in future periods is by investing less (lower x_t) or by making new capital more efficient; i.e, by reducing v_{t+1} and increasing the capital-energy ratio. In either case the impact of an energy price rise is propagated into the future as less capital services and, therefore, less aggregate value added. The quality-adjusted capital to energy ratio will track the evolution of the energy price, with a delay which will be longer the lower the depreciation rate ϖ. Moreover, the relative price of quality-adjusted capital, which is just equal to v_{t+1}^{-1}, will increase, reflecting the rise in the energy price.

Let us turn to consider the effect of changes in neutral technological progress. In this case, a decrease in A_t reduces gross output. Hence, it renders all machines less productive today. If technological progress has persistence the economy will move to invest in more efficient machines, which will produce an increase in the quality-adjusted capital to energy ratio. As a matter of fact, if both the energy price and the level of neutral technological move in the same way, the detrended level of Value Added should remain the same. Let us turn now to examine our economy with investment specific technological change.

4.2 Investment specific technological change and energy efficiency

Now we can go back to our benchmark economy. The expression of aggregate gross output and aggregate value added is the same as in the Atkeson and Kehoe's economy. The evolution of capital services and energy use is given by the expressions:

$$\kappa_{t+1} = \quad v_{t+1}\,\Lambda_{t+1}\,\Theta_t\,x_t + (1-\varpi)\,\kappa_t, \tag{4.5}$$

$$e_{t+1} = \quad \Gamma_{t+1}^{-1}\,v_{t+1}^{\mu}\,\Theta_t\,x_t + (1-\varpi)\,e_t. \tag{4.6}$$

Ceteris paribus, the qualitative effect of an unexpected shock in the energy price or neutral technological progress is the same than the Atkeson and Kehoe's economy. The existence of ISTC makes the reaction of investment quantitatively different, though. In order to understand this interaction it will be best to review first the effect of ISTC shocks. As opposed to neutral progress, ISTC is only materialized if investment takes place. Given that, it has two effects of opposite sign: on the one hand, due to the complementarity of capital and energy, higher ISTC means rising future energy use, which leaves the economy more exposed to energy price increases. In our model economy this is captured by the effect of Θ_t, the productivity of the technology to produce investment goods. On the other hand, higher ISTC implies more capital services and higher efficiency, captured by Λ_{t+1} and Γ_{t+1}, respectively, which lowers the burden of such energy use. The first effect implies that after a positive ISTC shock energy efficiency should rise, whereas the second implies that energy efficiency should decrease. In other words, disembodied investment specific technical change and energy efficiency are complements. Embodied investment specific technical change and energy efficiency, however, are substitutes. One of the questions that we address here is how we can disentangle both effects. We will come back to this issue in sections 5 and 6.

The previous discussion about the effect of ISTC shocks sheds light about the behavior of the economy after an energy price shock. The response of investment and energy efficiency will depend on the nature of ISTC. If embodied technical change is low relative to disembodied technical change, an energy price rise will prompt investing in energy efficiency, more so that in an economy in which all technical progress is neutral. If, on the other hand, the price rise occurs in a period in which there is an acceleration in embodied technical change, the economy will not invest so much in energy efficiency. We will turn to discuss all these interactions in more detail in section 6, where we illustrate quantitatively the ability of our model economy to account for the observed behavior of energy use, investment and other macroeconomic aggregates.

4.3 Capital adjustment costs as a short-cut for capital irreversibility

Let us now turn to the specification by Díaz et al. (2004). This framework is a bit different, though. In this framework there are two types of capital, working capital and energy saving capital. The energy efficiency of the economy, υ, is given by the amount energy saving capital installed in the plant, which can be accumulated independently from working capital, k. Thus, capital services of one unit of working capital are

$$\kappa_t\left(u_t\right) = u_t \min\left\{e_t\left(u_t\right), \frac{\zeta}{u_t}\right\}. \tag{4.7}$$

Plants' managers hire both types of capital every period before the energy price is known. The key assumption is that accumulating energy saving capital is subject to adjustment costs which implies that energy use will respond sluggishly to energy prices. Appendix F describes this economy and its associated quasi-social planner's problem is

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(c_t\right) + \varphi \log\left(\ell_t\right)\right) \\
\text{s. t.} \quad & c_t + \iota_t \leq A_t \, \zeta^{\alpha} \, k_t^{\alpha} \, h_t^{1-\alpha} - \varrho_t e_t, \\
& c_t \geq 0, \;\ell_t \leq \hbar - h_t, \\
& e_{t+1} \geq \tfrac{\zeta\left(\iota_t - \psi(u_{t+1}k_{t+1}, u_t k_t)\right)}{u_{t+1}(1+u_{t+1})} + (1-\varpi)\tfrac{u_t(1+u_t)}{u_{t+1}(1+u_{t+1})}\, e_t, \\
& 0 \leq k_{t+1} \leq \tfrac{1}{1+u_{t+1}}\, \iota_t - \tfrac{\psi(u_{t+1}k_{t+1}, u_t k_t)}{1+u_{t+1}} + (1-\varpi)\tfrac{1+u_t}{1+u_{t+1}}\, k_t, \\
& k_0, \; u_0, \text{ and energy prices given, } t \geq 0,
\end{aligned} \tag{4.8}$$

where the total amount of energy saving capital is $\mathbf{u}_t = u_t \, k_t$. Services of working capital are just $\kappa_t = \zeta \, k_t$. The total amount of investment, net of reallocation costs is $x_t = \frac{(\iota_t - \psi(u_{t+1}k_{t+1}, u_t k_t))}{u_{t+1}(1+u_{t+1})}$, which has to allocated to working capital and energy saving capital. The amount of energy saving capital u_t plays the role of energy efficiency in our setup, $v_t = u_t^{-1}$. Notice that energy efficiency at the plant level is reversible but the existence of adjustment costs, $\frac{\psi(u_{t+1}k_{t+1}, u_t k_t)}{1+u_{t+1}}$, has an aggregate effect similar to irreversibility—it slows down the response of aggregate energy efficiency to changes in energy prices.

5 Calibration and estimation

In this section we describe our procedure to calibrate our economy and the estimation of the shocks. As noted otherwise, we are using annual data for the period 1960-2008.

5.1 The energy price shock

In Appendix A.1 we describe in detail the construction of the energy price as an implicit price deflator of the aggregate we have labeled "energy". We have fitted the following ARMA process

$$\log \varrho_{t+1} = (1-\rho) \log \overline{\varrho} + \rho \log \varrho_t + \phi \epsilon_t + \epsilon_{t+1}, \tag{5.1}$$

and find that $\rho \simeq 0.94$, $\phi \simeq 0.45$ for 1960-08.

5.2 The specification of the investment specific shocks processes

We turn to specify a structure for Θ_t, Λ_t and Γ_t. For the time being, we assume that Γ_t is constant. This is consistent with assuming that the energy price has no trend, as we have done in expression (5.1). Thus, we have left Θ_t, which affects the technology that transforms final good into investment good, and Λ_t, which affects the amount of capital services yielded by one unit of capital. We follow Justiniano et al. (2011) and assume the following processes:

$$\Theta_t = \Theta_{t-1} \, e^{(\theta + z_t^\theta)}, \tag{5.2}$$

$$\Lambda_{t+1} = (1+\lambda)^t e^{z_t^\lambda}. \tag{5.3}$$

That is, we assume that the log of Θ_t follows a random walk with drift θ, whereas the embodied investment specific shock has a deterministic trend. In other words, shocks to the technology to produce investment goods are permanent, whereas shocks to embodied technological progress are transitory. In order to estimate both processes we need to use our theory and confront it to the data available.[10]

The Bureau of Economic Analysis reports Producer Price Indices for investment goods. Some of them are adjusted by quality, as it is the case of equipment and software whereas others are not. According to our theory, the relative price of investment goods, not adjusted by quality, is $q_t\,(t+1, v_{t+1}) = \Theta_t^{-1}$, as shown in (3.15). We are going to estimate the process for (5.2) using the ratio of the chain weighted NIPA deflators for durable consumption expenditures and private investment in structures over non-durable consumption, which we are going to denote as Q_t. Thus, we take the view that $q_t^{-1}\,(t+1, v_{t+1}) = Q_t$ and estimate the process

$$Q_t = Q_{t-1} e^{(\theta + z_t^\theta)}. \tag{5.4}$$

We proceed in this way because the prices of durable expenditures and investment in structures are not adjusted by quality. Next, we need an estimate of the embodied process shown in (5.3). The relative price of the investment good adjusted by quality in our model economy is $q_t^\kappa\,(t+1, v_{t+1}) =$

[10] Justiniano et al. (2011) consider a new-Keynesian model with investment specific technical progress in which capital accumulation is affected by two disturbances. As these authors, we consider an investment specific shock that affects productivity in the investment good producing sector. Different from them we consider a vintage structure with embodied technical progress rather than specifying a capital good producing sector. In so doing, Justiniano et al. (2011) building upon Carlstrom and Fuerst (1997), consider a shock to the marginal efficiency of investment whose observable is a corporate sector stock index.

$q_t\,(t+1, v_{t+1})\,\Lambda_{t+1}^{-1} v_{t+1}^{-1}$. We use the estimates of Gordon (1990) and Cummins and Violante (2002) (hereafter the GCV deflator) in the following way: We call Q_t^{κ} the inverse of the quality-adjusted relative price of equipment and software and we construct the process

$$Q_t^{\kappa} = Q_t(1+\lambda)^t e^{z_{t-1}^{\tilde{\lambda}}}. \tag{5.5}$$

Notice that this procedure does not identify the innovation z_t^{λ} shown in (5.3), but the composite $z_t^{\lambda} + \log(v_{t+1})$. Thus, we are introducing a bias in our estimation since it includes the response of an endogenous variable to the shocks. Ex ante we have no manner to assess whether the bias is large or not, but our theory implies that the composite $z_t^{\lambda} + \log(v_{t+1})$ has higher variance that z_t^{λ}, since v_{t+1} should be negatively correlated with the embodied technical shock z_t^{λ}. That is, our theory implies that proceeding this way we are overestimating the magnitude of the embodied technical change shock; as a matter of fact, we are providing an upper bound for the magnitude of the embodied technical change. We will discuss this issue in more detail in Section 6.

5.3 The stationary version of our model economy

In our economy, output and capital grow at different rates. Moreover, since one of the ISTC shocks is permanent, our economy features a stochastic trend.

Proposition 3. *Suppose that Assumption 1 holds, so that in equilibrium all installed capital is utilized,* $e_t(z, v) > 0$*, for all* t*. Suppose that the energy price grows at the constant rate* γ^p *and that* Θ_t *and* Λ_t *are given by* (5.2) *and* (5.3)*, respectively. The efficiency type* v_{t+1} *has a stochastic trend given by*

$$\Psi_t = \left[\frac{\Gamma_{t+1}}{p_t}\,\Theta_t^{-1}\right]^{\frac{1}{\mu}}, \text{ for all } t. \tag{5.6}$$

Gross output, y_t*, investment and consumption,* x_t*,* c_t*, and* $p_t\,e_t$*, have a trend* Z_t *which satisfies*

$$Z_t = A_{t-1}^{\frac{1}{1-\alpha}}\left[\Lambda_t\,\Theta_{t-1}\right]^{\frac{\alpha}{1-\alpha}}\left[\frac{\Gamma_t}{p_{t-1}}\,\Theta_{t-1}^{-1}\right]^{\frac{\alpha}{\mu(1-\alpha)}}. \tag{5.7}$$

The trend of capital satisfies

$$T_{t-1} = Z_t\,\Theta_{t-1}. \tag{5.8}$$

The trend of capital services

$$T^{\kappa}_{t-1} = \Lambda_t \, \Psi_{t-1} \, T_{t-1}. \tag{5.9}$$

Proof. Taking into account (3.17) and that gross output and energy expenditure should grow at the same rate the result follows. □

5.4 Aggregate targets

The calibration of the model closely follows the methods discussed in Atkeson and Kehoe (1999) and Díaz et al. (2004). In this version of our benchmark economy we abstract from labor choice, $\xi = 0$, and concentrate on the behavior of investment. Parameter values are calibrated so that selected statistics of the steady state of our economy match their counterparts in the data. Table 1 shows the values of the calibrated parameters. In the data, the share of labor income over Value Added is about 60 percent. This implies a value for $\alpha = 0.4406$. Investment is 28.41 percent of gross output, which implies a depreciation rate $\varpi = 0.0523$. We have two other technological parameters: the stationary value for energy intensity, $\widetilde{v}_{t+1} \equiv v^{\mu}_{t+1}/\Theta_t$, which governs the capital-energy ratio at the steady state, and μ, which governs the dynamic response of energy use to shocks. The reason why we need to set the ratio v^{μ}_{t+1}/Θ_t is because of the complementarity of capital and energy at the plant level. Our theory accounts for the dynamics of such complementarity, but we need to set its steady state level. Hence, the ratio v^{μ}_{t+1}/Θ_t is set so that the aggregate share of energy in our economy, in absence of shocks, is constant and equal to the mean of the considered period. Finally, as we show in Appendix D the share of energy in gross output for the latest vintage is equal to α/μ, which should be lower than the share for any older vintage. As a matter of fact we show in Appendix D that

$$\frac{\varrho_t \, e_t\,(z, v_z)}{y_t\,(z, v_z)} = \frac{\alpha}{\mu} \, \frac{\Lambda_t \, \Gamma_t \, v_t^{1-\mu}}{\Lambda_z \, \Gamma_z \, v_z^{1-\mu}}, \text{ for all } z \leq t. \tag{5.10}$$

Thus we can use our theory to find an appropriate value for μ consistent with the aggregate share of energy being constant and equal to its counterpart in the data. This value is 16.2409.

5.5 Empirical strategy

Figure 7 shows the relative prices $q_t(.)$ and $q_t^{\kappa}(.)$ that correspond to the ratio of the chain weighted deflators of consumption and investment as defined above. That is, respectively, the NIPA deflator for private investment in structures and durable consumption expenditures (hereafter, St&D) and the GCV deflator for investment in equipment and software (hereafter E&S), over non-durable consumption and services. The former (NIPA St&D) is used as a proxy for disembodied investment-specific technical progress, whereas the latter (GCV E&S) proxies embodied investment-specific technical progress. Clearly, the GCV deflator exhibits a faster rate of decline, whereas the decline in the NIPA deflator mostly comes from the contribution of the deflator for durable consumption expenditures.

A stationary representation of the investment specific technical change processes can be described in a state space form according to

$$\xi_t = \Gamma + F\xi_{t-1} + \varepsilon_z, \quad \text{state equation,} \tag{5.11}$$

$$y_t = H\xi_t(+Az_t + e_t), \quad \text{observation equation.} \tag{5.12}$$

where $\xi_t = [\Delta \log Q_t^I \quad \Delta \log \tilde{Q}_t^I]'$, and $\varepsilon_z = (1 - \rho_x L)z_t^x$, with $x = \theta, \lambda$. The term $(+Az_t + e_t)$ accounts for measurement error if necessary. Such a representation is estimated with a Kalman filter, and under a likelihood based approach. Table 2 reports the corresponding parameter estimates obtained by using annual data as corresponds to the time frequency of the energy data in this paper, and by using the corresponding quarterly data from 1947:04 to 2009:01 as a robustness check. As indicated above, our empirical strategy implies a particular stationary inducing transformation of the aggregate representation of the economy.

Figure 8 shows the correlation of the innovations z_t^{θ} and z_t^{λ} to the Q_t^I and $\tilde{Q}_t^I$ process. To compute this correlation a window of 5 years (20 quarters) has been used. The vertical red lines aim to correspond to 1974, 1978 and 1992 events related to major oil shocks. The picture suggests that quality adjustments in durable goods and equipment, say, are correlated above average during periods of changing conditions in energy use: 1974-1980 and 1992-2002. We interpret this finding as an evidence in favor of the role of increased obsolesce measured as an acceleration in the rate of embodied investment-specific technical change in the response of energy use to energy price changes. By log-linear approximation and simulation of the aggregate vintage economy we asses the quantitative importance of such a channel, and therefore we empirically implement our theory of energy use.

6 Quantitative experiments

In this section we show the main experiments conducted. The way we proceed is the following: we feed into the model the observed path for the energy price and investment shocks and simulate our economy. The results are shown in logs and demeaned, as the figures showing the data.

6.1 The benchmark model economy

Figure 9 shows the behavior of the energy share and energy intensity in our model. As we can see the model performs remarkably well. The evolution of the capital-energy ratio is shown in figure 10. It is interesting to see that the capital-energy ratio responds to the energy price until the 1990s, period in which it starts increasing regardless of the energy price. In order to understand the effect of ISTC we show a version of our model economy in which we shut the channel of Investment specific technological change. Thus, there is only one shock and the economy can adjust to it by changing the efficiency of capital. The behavior of this economy is shown in figures 14 and 15. As we can see, energy use fluctuates significantly more than in the data. Moreover, after the mid 1980s, energy use increases responding to the observed fall in energy price up to the early 2000s. The capital-energy ratio moves slowly and responds, with some lag, to the behavior of the energy price. The lesson that we learn here is that investment specific technological change is needed to understand both the behavior of energy use and the increase in the capital-energy ratio starting in the mid-1990s.

To understand better the interaction between energy price shocks and ISTC we can turn to inspect figure 11. Here we have plotted the evolution of $1/v_t$, the index for energy efficiency in our model economy and its version without ISTC shocks. The first thing that we notice is that efficiency follows the energy price in absence of ISTC (the red dashed line). Efficiency in the world with ISTC, however, behaves differently. At the peak of the 1970s shocks, agents invest in higher efficiency in a world in which there is ISTC than in the alternative world without it. The reason is the following: In a world in which technological progress is investment specific, agents need to invest to materialize it. This implies increasing energy demand due to the complementarity between capital and energy. If the energy price is high and is very persistent, agents prefer to invest in energy efficiency. On the other hand, if embodied ISTC is high and expected prices are low, agents prefer to invest in very productive machines that use a lot of energy. This is what happens during the 1990s in our economy with ISTC. Recall that embodied ISTC shocks are transitory. Thus, there is an investment boom and the capital-energy ratio rises. We should point out the relationship between the efficiency index v_t and the evolution of the relative quality-adjusted price of investment. Recall expression (2), $q_t^{\kappa} = \Theta_t^{-1}\,\Lambda_{t+1}^{-1}\,v_{t+1}^{-1}$. Notice that higher efficiency (lower v_t) implies a higher relative price of investment. Thus, for a given growth rate of investment specific technological change (the change in Θ_t and Λ_t), measured ISTC growth will be lower in periods of high energy prices. The opposite occurs in periods of low energy prices. This is consistent with the estimates reported by Cummins and Violante (2002), who estimate a fall in the growth rate of ISTC during the 1970s and an upsurge in the 1990s.

Figures 12 and 13 show the evolution of the investment rate and the capital to value added ratio, respectively. Recall that we do not have neutral technological progress in our model economy. We cannot replicate the drop in the investment rate in the early 1993, suggesting that that recession was not related to investment specific technological shocks. Nevertheless, we think that ours is a very reasonable theory to understand the behavior of investment and capital.

A figure about relative prices

Key figures

1. Use and expenditures 2. Capital-energy ratio 3. Table of prices

6.2 The role of ISTC shocks

6.2.1 Identification

Figure of v benchmark versus Atkenson and Kehoe

6.2.2 Contribution

To make our point clearer, we strip our economy of investment specific technological shocks and leave the energy price shock. This is Atkeson and Kehoe (1999) with our calibration. Notice that energy use tracks, with a lag, the evolution of the energy price. Likewise happens to the capital-energy ratio.

Figure of capital-energy, benchmark versus Atkeson-Kehoe

6.2.3 Permanent versus transitory ISTC shocks

Figure of capital-energy, benchmark versus counterfactuals

7 Concluding remarks

[TO BE COMPLETED]

Appendix

A The data

In this Appendix we document the construction of the data series we use in the empirical part of the paper. We obtain data from two sources: Annual Energy Review (2000) and National Income and Product Accounts. The data we use can be accessed in the addresses: http://www.eia.gov/ and http://www.bea.gov. From now on we will refer to each source as AER, and NIPA, respectively. Our data set is available upon request.

A.1 Energy price, use, and expenditures series

Our energy data covers the primary energy consumption of end-users. We consider four forms of energy: coal, petroleum, natural gas and electricity. AER (Table 2.1a) gives data on total energy consumption by end users measured in British termal units (BTUs) disaggregated into the four forms of energy considered. We denote these data on energy use for each type of energy by Q_{it}, where the index i denotes the form of energy.

This measure E_{it} is already net of energy use of the electricity sector. We subtract from final energy use of the industrial sector that of four energy sectors: oil and gas extraction, electricity and gas services, petroleum and coal production, and pipeline transportation. In BEA we have information on the net stock of Fixed Assets by industry and we assumed that the amount of BTUs consumed by those sectors, as a proportion of BTUs consumed by the industrial sector, is the same that the amount of capital in those sectors as a proportion of assets in the industrial sector.

We construct a constant-price measure of energy use. We choose the base year to be 2005 and define energy use to be $E_t = \sum_i Q_{it} P_{i0}$, where P_{i0} is the price in dollars per BTUs of energy type in 2005 from AER, divided by the implicit price deflator of non durable consumption goods and services in NIPA (which is constructed as a weighted average of tht two implicit price deflators). For coal, natural gas and petroleum we use the production price series (AER, Table 3.1). For electricity, we use the retail price of electricity sold by electric utilities (see AER, Table 8.10). In Table 8.10 the price for electricity is in cents per kilowatt-hour. We use AER Table A.6 to convert the price to cents per BTUs. All prices are in real terms; i.e., divided by the implicit price deflator of non durable consumption goods and services. We construct the energy price deflator as

$$P_t = \frac{\sum_i Q_{it} P_{it}}{\sum_i Q_{it} P_{i0}}. \tag{A.1}$$

Finally, nominal expenditure is $P_t \cdot E_t = \sum_i Q_{it} P_{it}$.

A.2 Output, consumption, investment, and the capital stock

We follow the method described by Cooley and Prescott (1995) to construct broad measures of output, consumption, investment, and the capital stock. Specifically, our measure of capital includes private stock of capital, the stock of inventories, the stock of consumer durable goods and the government stock. Consequently, the measured value of GDP is augmented with the imputed flow of services from the stock of durable goods and the government stock. For output, investment, and capital we subtract from each of these series the corresponding series for the energy producing sectors: oil and gas extraction, electricity and gas services, petroleum and coal production, and pipeline transportation. We have information on the three variables for the last two sectors but about the first two sectors we only have information about the net stock of capital, and we use it to impute estimates of output and investment. Gross output is the sum of value added and the final expenditure on energy. Real variables are obtained by dividing the nominal variables by the implicit price deflator of non durable consumption goods and services.

B The quasi-social planner's problem

The efficient allocation for this economy can be found as the solution to the following planning problem:

$$\begin{aligned}
\max_{\substack{c_t, x_t(t+1,v),\\ \ell_t, k_{t+1}(t+1,v)}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(c_t\right) + \varphi \log\left(\ell_t\right)\right) \\
\text{s. t.} \quad & c_t + \int_0^{\infty} x_t(v)\, d\, v \leq \sum_{z=-\infty}^{t} \int_0^{\infty} (1-\varpi)^{t-z} k_z(z,v) \left[y_t(z,v) - \varrho_t\, e_t(z,v)\right] d\, v, \\
& 0 \leq y_t(z,v) \leq A_t\, \kappa_t\left(z,v\right)^{\alpha} h_t\left(z,v\right)^{1-\alpha}, \\
& 0 \leq \kappa_t(z,v) \leq \Lambda_z \Gamma_z v^{1-\mu} e_t\left(z,v\right), \\
& 0 \leq e_t(z,v) \leq \Gamma_z^{-1}\, v^{\mu}, \\
& 0 \leq \ell_t \leq \hbar - \sum_{z=-\infty}^{t} \int_0^{\infty} (1-\varpi)^{t-z} k_z(z,v)\, h_t(z,v) d\, v, \\
& 0 \leq h_t(z,v) \leq \hbar, \\
& 0 \leq k_{t+1}(t+1,v) \leq \Theta_t\, x_t(v),\ t \geq 0, \\
& k_z(z,v) \text{ given}, \quad z \leq t,\ v \geq 0.
\end{aligned} \tag{B.1}$$

C Decentralization of the efficient allocation

Here we show how to decentralize the economy described in Section 3.

C.1 Market arrangements

We assume that households are the owners of the plants and, therefore, of the capital installed. There is a market for plants that opens at the end of the period, once profits have been realized. Notice, though, that capital is not traded since it is already installed in a plant and it cannot be reallocated. Since there is a one to one correspondence between plants and units of capital, the price of a plant is also equal to the price of the unit of capital installed, $p_t(z,v)$, where $p_t(z,v)$ is the price of one unit of capital of vintage z and type v at the end of period t in units of consumption good at time t. We further assume that all households start out with the same amount of shares of the plants installed. Additionally, we assume that households trade a one risk free bond which is in zero net supply.

The timing is the following: At the end of period $t-1$ any prospective plant must install one unit of capital before the energy price is known. After this decision has been made, at the beginning of period t the uncertainty is resolved: agents learn the productivity of the investment technology Θ_t. The energy price is realized. Then, they decide the amount of energy used, $e_t(z,v)$, and the number of workers hired, $h_t(z,v)$. Households consume and save. A fraction ϖ of plants die.

C.1.1 The household's problem

Plants of any vintage and type can be traded at the individual level. New investment, however, comes in new vintage—it is a technological restriction, as in the one sector model TFP grows exogenously, we cannot help to be more productive. Agents can, though, choose the type of the new capital units to be installed. The household's problem can be written in the following way:

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left(\log(c_t) + \xi \log(\ell_t)\right) \\
\text{s. t.} \quad & c_t + \int_0^{\infty} x_t(v)\, dv + \sum_{z=-\infty}^{t+1} \int_0^{\infty} p_t(z,v)\, m_{t+1}(z,v)\, dv + b_{t+1} \le w_t(\hbar - \ell_t) + \left(1 + r_t^b\right) b_t + \\
& \int_0^{\infty} p_t(t+1,v)\, k_{t+1}(t+1,v)\, dv + \sum_{z=-\infty}^{t} \int_0^{\infty} \left[(1-\varpi)\, p_t(z,v) + \pi_t(z,v)\right] m_t(z,v)\, dv, \qquad \text{(C.1)} \\
& k_{t+1}(t+1,v) \le \Theta_t\, x_t(v), \ v > 0, \\
& x_t \ge 0, \ m_{t+1}(z,v) \ge 0, \ k_{t+1}(t+1,v) \ge 0, \ \text{for all } z \le t+1, \ v > 0, \\
& b_{t+1} \ge \underline{b}, \\
& m_0(z,v), \ b_0, \ \text{ and energy prices given.}
\end{aligned}$$

The constraint $k_{t+1}(t+1,v) \le \Theta_t\, x_t(v)$ implies that the amount of new capital that agents can sell in the market cannot be higher that the amount of good needed to create them.

C.1.2 The plant's problem

$$\begin{aligned}
\max_{\substack{y_t(z,v)\ge 0,\, h_t(z,v)\ge 0,\\ e_t(z,s)\ge 0}} \quad & \pi_t(z,v) = y_t(z,v) - w_t h_t(z,v) - \varrho_t\, e_t(z,v) \\
\text{s. t.} \quad & y_t(z,v) \le A_t\, \kappa_t(z,v)^{\alpha}\, h_t(z,v)^{1-\alpha}, \\
& \kappa_t(z,v) \le \Lambda_z\, \Gamma_z\, v^{1-\mu} \min\left\{e_t(z,v), \Gamma_z^{-1}\, v^{\mu}\right\}.
\end{aligned} \qquad \text{(C.2)}$$

C.1.3 Definition of equilibrium

An equilibrium for this economy, given the sequence of energy prices, $\{\varrho_t\}_{t=0}^{\infty}$, is a sequence of prices $\left\{\{p_t(z,v)\}_{z=-\infty}^{t+1}, w_t, r_t^b\right\}_{t=0}^{\infty}$, an allocation $\left\{c_t, \ell_t, \{m_t(z,v)\}_{z=-\infty}^{t+1}, x_t(v), k_{t+1}(t+1,v), b_{t+1}\right\}$ for each consumer, and an allocation for each plant of variety (z,v), $\{y_t(z,v), h_t(z,v), \kappa_t(z,v), e_t(z,v)\}_{z=-\infty}^{t}$, $v \in \mathbf{R}_{++}$, such that:

1. $\left\{c_t, \ell_t, \{m_t(z,.)\}_{z=-\infty}^{t+1}, x_t(v), b_{t+1}\right\}$ solves the household's problem shown in (C.1) given the sequence of prices,

2. $\{y_t(z,v), h_t(z,v), e_t(z,v)\}_{z=-\infty}^{t}$, $v \in \mathbf{R}_{++}$, solves the plant's problem given the sequence of prices,

3. the relative price of the latest vintage is $p_t(t+1,v) = \Theta_t^{-1}$, for any v,

4. markets clear,

   (a) the bond is in zero net supply, $b_{t+1} = 0$,

   (b) the amount of plants of class (z,v) must be equal to the amount of capital of that class, $m_t(z,v) = k_t(z,v)$, for all $z \leq t$, $v > 0$,

   (c) the labor market clears, $\hbar - \ell_t = \sum_{z=-\infty}^{t} \int_0^\infty m_t(z,v)\, h_t(z,v)\, d\, v$,

   (d) the final good market satisfies $c_t + x_t = \sum_{z=-\infty}^{t} \int_0^\infty m_t(z,v)\, y_t(z,v)\, d\, v$,

5. and the law of motion of capital of class (z,v) is $k_t(z,v) = (1-\omega)^{t-z}\Theta_{z-1}x_{z-1}(v)$, for all $t \geq z$, $v \in \mathbf{R}_+$, for all t.

C.1.4 Some properties of equilibrium

We have shown in Section 3.5 some properties of the efficient allocation. In this economy, given the energy prices, the Welfare Theorems hold; thus, in equilibrium Propositions 1 to 2, as well as Corollary 2, hold. Thus, we keep Assumption 1. Clearly, as shown in Proposition 2, the price of a new plant, $p_t\,(t+1, v_{t+1})$, is equal to the cost of producing the unit of capital installed in that plant, which was defined as $q_t\,(t+1, v_{t+1}) = \Theta_t^{-1}$. Moreover, we know, using Proposition 2 that agents only invest in one type of capital, that for which the present value of all future profits is maximized. The price of an old plant is equal to the present value of profits that the plant will accrue in the future:

$$p_t\,(z, v_z) = E_t \sum_{i=1}^{\infty} (1-\varpi)^{i-1} \frac{\varphi_{t+i}}{\varphi_t} \left[\alpha\, y_{t+i}\,(z, v_z) - \varrho_{t+i}\, \Gamma_z^{-1}\, v_z^{\mu}\right], \tag{C.3}$$

Using (3.12) we can write (C.3) as

$$p_t\,(z, v_z) = \frac{\Lambda_z\, v_z}{\Lambda_{t+1}\, v_{t+1}}\, p_t\,(t+1, v_{t+1}) + \left(\frac{\Lambda_z\, v_z}{\Lambda_{t+1}\, v_{t+1}} - \frac{\Gamma_z^{-1}\, v_z^{\mu}}{\Gamma_{t+1}^{-1}\, v_{t+1}^{\mu}}\right) E_t \sum_{i=1}^{\infty} (1-\varpi)^{i-1} \frac{\varphi_{t+i}}{\varphi_t}\, \varrho_{t+i}\, \Gamma_{t+1}^{-1}\, v_{t+1}^{\mu}. \tag{C.4}$$

Since for the latest class of capital the present value of all energy expenditures is equal to the fraction α/μ of gross output, using (3.16) we find that

$$p_t\,(z, v_z) = \frac{\Lambda_z\, v_z}{\Lambda_{t+1}\, v_{t+1}}\, \Theta_t^{-1} + \left(\frac{\Lambda_z\, v_z}{\Lambda_{t+1}\, v_{t+1}} - \frac{\Gamma_z^{-1}\, v_z^{\mu}}{\Gamma_{t+1}^{-1}\, v_{t+1}^{\mu}}\right) \frac{\Theta_t^{-1}}{\mu - 1}. \tag{C.5}$$

This price is different than the cost of of unit of capital installed, as defined in Definition 1, since it takes into account the associated cost of energy. Actually if the ratio of capital services to energy expenditure is higher than in a new plant, then $p_t(z, v_z) > q_t(z, v_z)$; otherwise the cost valuation is higher than the market price of the plant, $p_t(z, v_z) < q_t(z, v_z)$.

D Calibration issues

Let us write the planner's problem in the following way:

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left(\log(c_t) + \xi \log(\hbar - h_t)\right) \\
\text{s. t.} \quad & c_t + x_t \leq A_t \left(\Theta_{t-1}\, \Lambda_t\, v_t\, \tilde{\kappa}_t\right)^{\alpha} h_t^{1-\alpha} - \varrho_t\, e_t, \\
& e_{t+1} \geq \Gamma_{t+1}^{-1}\, v_{t+1}^{\mu}\, \Theta_t\, x_t + (1-\varpi)\, e_t, \\
& 0 \leq \tilde{\kappa}_{t+1} \leq x_t + (1-\varpi)\, \tfrac{v_t\, \Lambda_t\, \Theta_{t-1}}{v_{t+1}\, \Lambda_{t+1}\, \Theta_t}\, \tilde{\kappa}_t, \\
& \tilde{\kappa}_0 \text{ given, } x_t \geq 0.
\end{aligned} \tag{D.1}$$

$\tilde{\kappa}_t$ is the amount of capital services in units of gross output (i.e., multiplied by $q_{t-1}\kappa$). Now we can think about calibration. We calibrate α so that labor share in Value Added matches its corresponding counterpart in the data. Let us now turn to the law of motion of capital. Since capital is expressed in units of gross output, the ratio must be constant at the steady state,

$$\frac{y_{t+1}}{y_t}\,\frac{\tilde{\kappa}_{t+1}}{y_{t+1}} = \frac{x_t}{y_t} + (1-\varpi)\,\frac{v_t\,\Lambda_t\,\Theta_{t-1}}{v_{t+1}\,\Lambda_{t+1}\,\Theta_t}\,\frac{\tilde{\kappa}_t}{y_t}. \tag{D.2}$$

Matching the capital-output ratio and the investment ratio yields

$$1-\tilde{\varpi} = (1-\varpi)\,\frac{v_t\,\Lambda_t\,\Theta_{t-1}}{v_{t+1}\,\Lambda_{t+1}\,\Theta_t}. \tag{D.3}$$

Since $q_t^{\kappa} = \Theta_t^{-1}\,\Lambda_{t+1}^{-1}\,v_{t+1}^{-1}$, we can obtain

$$1-\varpi = (1-\tilde{\varpi})\,\frac{q_{t-1}^{\kappa}}{q_t^{\kappa}}. \tag{D.4}$$

At the steady state, the share of energy, $p_t\, e_t/y_t$, must be constant, as c_t/y_t and the investment ratio. Now we can rewrite the law of motion of energy as

$$\frac{\varrho_t}{\varrho_{t+1}}\,\frac{y_{t+1}}{y_t}\,\frac{p_{t+1}\,e_{t+1}}{y_{t+1}} = \left(\frac{\Gamma_{t+1}}{\varrho_t}\right)^{-1} v_{t+1}^{\mu}\,\Theta_t\,\frac{x_t}{y_t} + (1-\varpi)\,\frac{p_t\,e_t}{y_t}. \tag{D.5}$$

Thus, matching the energy share determines the stationary value of $\tilde{v}_t^{\mu}$,

$$\tilde{v}_t^{\mu} = \left(\frac{\Gamma_{t+1}}{\varrho_t}\right)^{-1} v_{t+1}^{\mu}\,\Theta_t. \tag{D.6}$$

We have left obtaining μ. Unless we assume that there is no neutral progress, we cannot use the gross output growth rate to calibrate μ. For the same reason, we cannot use q_t^{κ}. We know that for any new vintage installed, energy expenditures are the share α/μ of gross output. If, at the steady state, the energy price is constant, by using (3.16) it is easy to show that

$$\frac{\varrho_t\, e_t\,(t, v_t)}{y_t\,(t, v_t)} = \frac{\alpha}{\mu}, \text{ for all } t. \tag{D.7}$$

Likewise,

$$\frac{\varrho_t\, e_t\,(z, v_z)}{y_t\,(z, v_z)} = \frac{\alpha}{\mu}\,\frac{\Lambda_t\,\Gamma_t\, v_t^{1-\mu}}{\Lambda_z\,\Gamma_z\, v_z^{1-\mu}}, \text{ for all } z \leq t. \tag{D.8}$$

The energy share on aggregate gross output is

$$\frac{\varrho_t\, e_t}{y_t} = \sum_{z=-\infty}^{t} \frac{\varrho_t\, e_t\,(z, v_z)}{y_t\,(z, v_z)}\,\frac{y_t\,(z, v_z)\,(1-\varpi)^{t-z}\,\Theta_{z-1}\, x_{z-1}}{y_t}. \tag{D.9}$$

Now, using our aggregation strategy and the fact the economy is at a steady state, we find that

$$\frac{\varrho_t\, e_t}{y_t} = \frac{\alpha}{\mu}\,\frac{\sum_{z=-\infty}^{t}\left(\frac{1-\varpi}{1+g_y}\right)^{t-z}}{\sum_{z=-\infty}^{t}\left(\frac{1-\varpi}{1+g_\kappa(\mu)}\right)^{t-z}}. \tag{D.10}$$

Calling α^e to the share of energy in gross output in the data,

$$\alpha^e = \frac{\alpha}{\mu}\,\frac{\sum_{z=-\infty}^{t}\left(\frac{1-\varpi}{1+g_y}\right)^{t-z}}{\sum_{z=-\infty}^{t}\left(\frac{1-\varpi}{1+g_\kappa(\mu)}\right)^{t-z}}. \tag{D.11}$$

This is a non-linear equation in μ, since it affects the growth rate of capital services, $g_\kappa(\mu)$. It must be that α^e is greater than α/μ, since older vintages operate with higher energy requirements that the new one. This is a key difference with Atkeson and Kehoe (1999).

E The cost of saving energy in a putty-clay model economy

Atkeson and Kehoe (1999) abstract from investment specific technological progress but retain the assumption about efficiency types and capital irreversibility. In particular, the amount of capital services, $\kappa_t\,(u)$, depends on the amount of energy used in the plant, $e_t\,(u) \geq 0$, according to the technology

$$\kappa_t\,(u) = f(u) \min\left\{e_t\,(u)\,, \frac{1}{u}\right\}, \tag{E.1}$$

where $f(u)$ is a strictly increasing function of v, where $f'(u) \geq 0$, and $f''(u) < 0$. In this economy, the efficiency type u plays the same role that v^{-1} in our model economy. In this case, however, since all technological progress is disembodied, only gains in energy efficiency, (i.e., changes in v), help to save energy. In this framework, the production of one new unit of capital always takes one unit of output, which is equivalent to assuming in our framework that $\Theta_t = 1$, for all t.

The firms optimally choose $e_t(u) = u^{-1}$ and $\kappa_t(u) = f(u)/u$. By denoting $v = f(u)/u$, it is easy to show that $e_t(v)$ is an increasing and convex function of v. Thus, the social planner's problem in this case is exactly the one shown in expression (D.1). In this economy, the capital-energy ratio is solely governed by changes in capital type, v, which respond to changes in energy prices.

F A Putty-Putty model of energy with costly capital reallocation

This is a simplified version of Díaz et al. (2004) in our environment.

F.1 Preferences

There is a continuum of households that seek to maximize expected discounted lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(c_t\right) + \xi \log\left(\ell_t\right)\right), \ \beta \in (0,1), \ \xi > 0, \tag{F.1}$$

where c_t is consumption and ℓ_t is leisure t. Each household is endowed with $\hbar$ units of time and, therefore, works $\hbar - \ell_t$ hours every period.

F.2 Technology

Production of the unique final good is carried out at a continuum of autonomous plants which are indexed by the amount of energy-saving capital used, v. In each plant output is produced with labor, energy and the unit of working capital installed, according to the technology

$$y_t\left(v\right) = A_t \, \kappa_t\left(v\right)^{\alpha} h_t\left(v\right)^{1-\alpha}, \tag{F.2}$$

with $0 < \alpha < 1$, where A_t is the growth factor of the disembodied technological knowledge, $\kappa_t\left(v\right)$ is the amount of services provided by the unit of working capital installed, whereas $h_t\left(v\right)$ is the amount of labor services employed in the plant. The amount of services yielded by the unit of working capital, $\kappa_t\left(v\right)$, depends on the amount of energy used in the plant, $e_t\left(v\right)$, and the amount of energy-saving capital installed, v, according to the technology

$$\kappa_t\left(v\right) = v \, \min\left\{e_t\left(v\right), \frac{\zeta}{v}\right\}. \tag{F.3}$$

Each period households save and have the possibility of transforming final good into new units of working capital or new units of energy-saving capital. Investing in energy-saving capital, though, is subject to adjustment costs, which imply that working capital cannot be transformed on a one-to-one basis into energy-saving capital, and vice versa. Households rent out energy-saving capital to plants in period $t-1$ to be used in period t. Plants can be scrapped at no cost. Finally, at the end of the period, once production has taken place, the unit of working capital installed has a positive probability of death, $\varpi \in [0,1]$, which is i.i.d. across types and plants. This death probability plays the role of physical depreciation of working capital. To simplify the exposition of the model, energy-saving capital depreciates at the same rate $\varpi \in [0,1]$.

F.3 Planner's problem

Notice that since plants can be scrapped at no cost, and the amount of energy-saving capital can be changed every period, all plants are ex-ante identical at all periods. Moreover, the total number of plants is always equal to the amount of working capital, k_t. Thus, the problem of a household is

$$
\begin{aligned}
\max_{\substack{c_t, x_t(v), \\ \ell, k_{t+1}(v)}} \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(c_t\right) + \varphi \log\left(\ell_t\right) \right) \\
\text{s. t.} \quad & c_t + x_t \leq \int_0^{\infty} k_t(v) \left[y_t(v) - \varrho_t \, e_t(v) \right] d\, v, \\
& 0 \leq y_t(v) \leq A_t \, \kappa_t \left(v\right)^{\alpha} h_t\left(v\right)^{1-\alpha}, \\
& 0 \leq \kappa_t(v) \leq v \, e_t\left(v\right), \\
& 0 \leq e_t(v) \leq \tfrac{\zeta}{v}, \\
& 0 \leq \ell_t \leq 1 - \int_0^{\infty} k_t(v) \, h_t(v) d\, v, \\
& 0 \leq h_t(v) \leq 1, \\
& k_{t+1} + \mathbf{v}_{t+1} - (1-\varpi)\left(k_t + \mathbf{v}_t\right) + \psi\left(\mathbf{v}_{t+1}, \mathbf{v}_t\right) \leq x_t, \\
& k_{t+1}(v) \geq 0, \\
& \int_0^{\infty} k_{t+1}(v) \, d\, v \leq k_{t+1}, \\
& \mathbf{v}_{t+1} \geq \int_0^{\infty} v \, k_{t+1}(v) \, d\, v, \\
& k_0(v) \text{ given}, \;\; v \geq 0.
\end{aligned}
\tag{F.4}
$$

F.4 Properties of the efficient allocation

Proposition App. 1. *The ratio labor to working capital services is the same across all classes of working capital used:*

$$
(1-\alpha) \, A_t \, \kappa_t(v)^{\alpha} h_t(v)^{-\alpha} = \frac{u_{\ell}\left(c_t, \ell_t\right)}{u_c\left(c_t, \ell_t\right)}. \tag{F.5}
$$

Now we turn to investigate which classes are allocated energy.

Proposition App. 2. *If a time t the type $v > 0$, is allocated energy, $e_t(v) > 0$, then it must be the case that $e_t(v) = \zeta \, v^{-1}$ and $\kappa_t(v) = \zeta$.*

The previous Proposition implies a rule for the utilization of capital.

Corollary App. 1. *Only installed working capital of types $v \geq \underline{v}_t$ are utilized in equilibrium, where*

$\underline{v}_t$ *is defined as*

$$\alpha\, A_t^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}} v = \varrho_t, \tag{F.6}$$

and $w_t \equiv \frac{u_\ell(c_t,\ell_t)}{u_c(c_t,\ell_t)}$. *The type* $\underline{v}_t$ *increases with* ϱ_t.

Let us turn now to analyze the investment decision and the characteristics of the new plants.

Proposition App. 3. *Working capital is installed to one efficiency type* $v_{t+1} > 0$*,* $k_{t+1}(v_{t+1}) > 0$*.*

Proof. The first order condition with respect to $k_{t+1}(v)$ is

$$\Psi_t^k + v\,\Psi_t^{\mathbf{v}} = E_t \lambda_{t+1} \left[\alpha\, y_{t+1}(v) - \varrho_{t+1} e_{t+1}(v)\right] + \Psi_t^{k_0}(v), \tag{F.7}$$

where Ψ_t^k is the first order condition with respect the aggregate k_{t+1}, $\Psi_t^{\mathbf{v}}$ is the first order condition with respect to $\mathbf{v}_{t+1}$, $\Psi_t^{k_0}(v)$ is the multiplier associated to the non-negativity constraint on $k_{t+1}(v)$, and λ_{t+1} is marginal utility of consumption at time $t+1$. Using Proposition App. 2 we can write the previous expression as

$$\Psi_t^k + v\,\Psi_t^{\mathbf{v}} = E_t \lambda_{t+1} \left[\alpha\, A_{t+1}^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{w_{t+1}}\right)^{\frac{1-\alpha}{\alpha}} \zeta - \varrho_{t+1}\, \zeta\, v^{-1}\right] + \Psi_t^{k_0}(v), \tag{F.8}$$

If $k_{t+1}(v) > 0$, then $\Psi_t^{k_0}(v) = 0$. Continuity of both sides of (F.8) with respect to v imply that only one type is used in equilibrium. Since working capital can be reallocated across types, only a particular type v_{t+1} is used. □

F.5 Aggregation

In this economy, at any period t only one efficiency type is used at every period. Thus, all plants are alike and aggregate output is $y_t = y_t(v_t)\, k_t$; likewise happens to labor. Moreover, services of working capital are just $\kappa_t = \zeta\, k_t$. The total amount of capital is $k_t + \mathbf{v}_t = (1 + v_t)\, k_t$. The amount of energy used every period is $e_t = \zeta\, k_t / v_t$. Thus, we can write the planner's problem as

$$\begin{aligned}
\max \quad & E_0 \sum_{t=0}^{\infty} \beta^t \left(\log(c_t) + \xi \log(\ell_t)\right) \\
\text{s. t.} \quad & c_t + x_t \le A_t\, \zeta^\alpha\, k_t^\alpha\, h_t^{1-\alpha} - \varrho_t e_t, \\
& c_t \ge 0,\ \ell_t \le 1 - h_t, \\
& e_{t+1} \ge \tfrac{\zeta\,(x_t - \psi(v_{t+1} k_{t+1}, v_t k_t))}{v_{t+1}(1+v_{t+1})}\, x_t + (1-\varpi)\tfrac{v_t(1+v_t)}{v_{t+1}(1+v_{t+1})}\, e_t, \\
& 0 \le k_{t+1} \le \tfrac{1}{1+v_{t+1}}\, x_t - \tfrac{\psi(v_{t+1} k_{t+1}, v_t k_t)}{1+v_{t+1}} + (1-\varpi)\tfrac{1+v_t}{1+v_{t+1}}\, k_t, \\
& k0,\ v_0, \text{ and energy prices given, } t \ge 0.
\end{aligned} \tag{F.9}$$

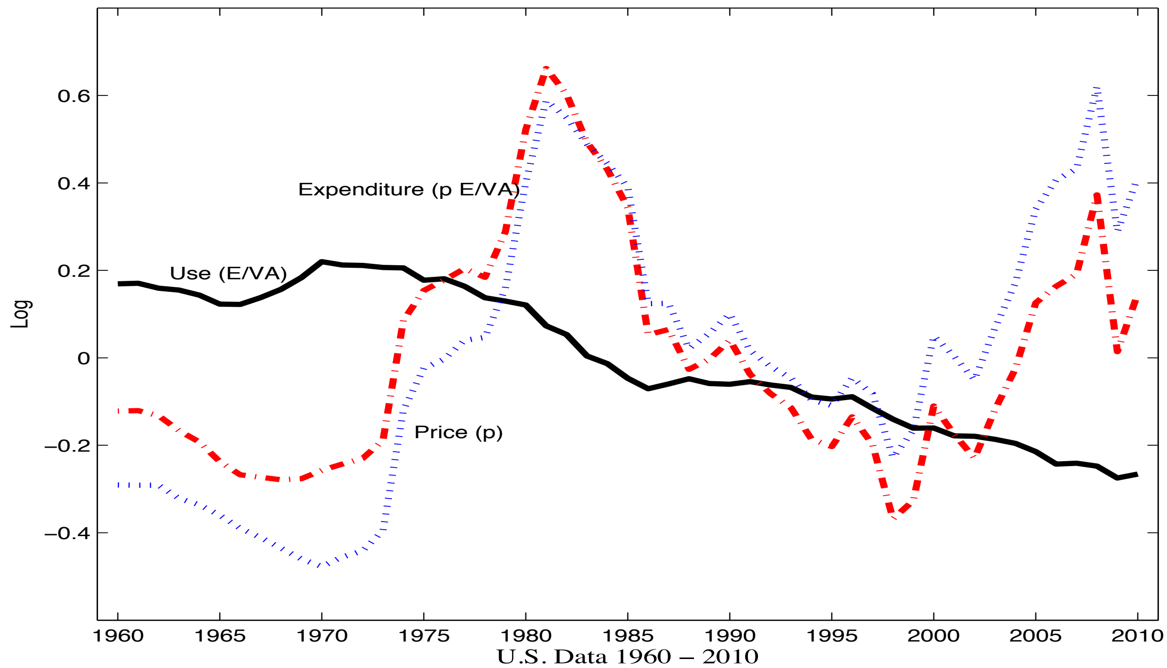

Figure 1: Energy use, energy expenditure and the price of energy (I).

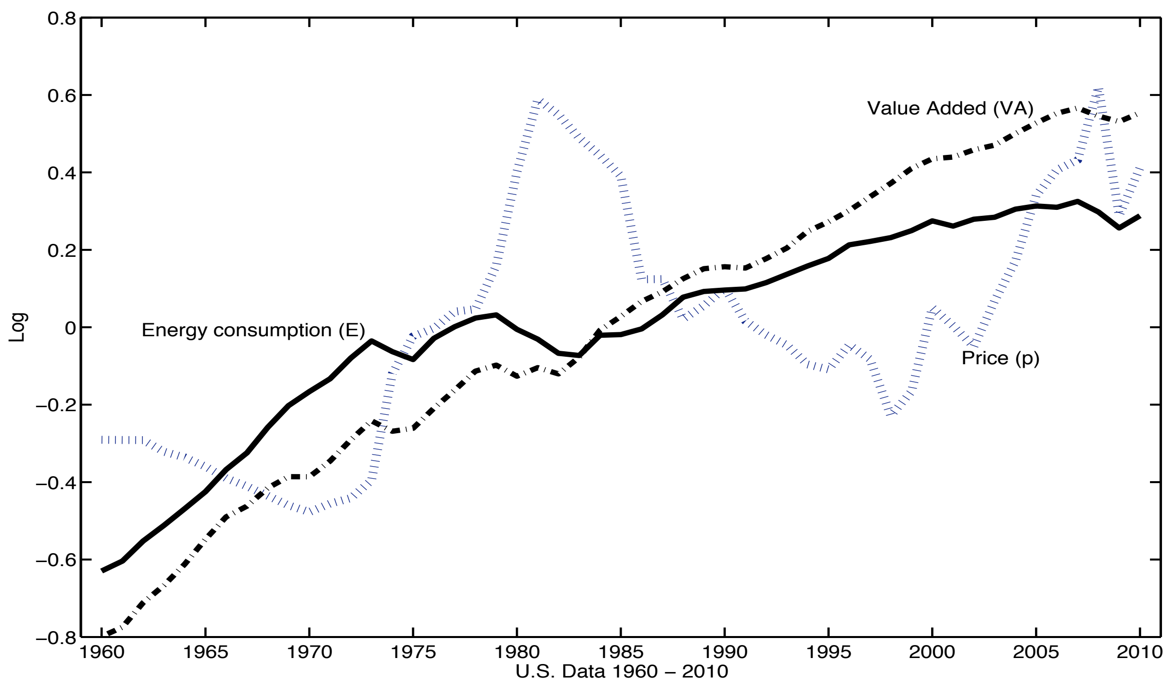

Figure 2: Energy use, energy expenditure and the price of energy (II).

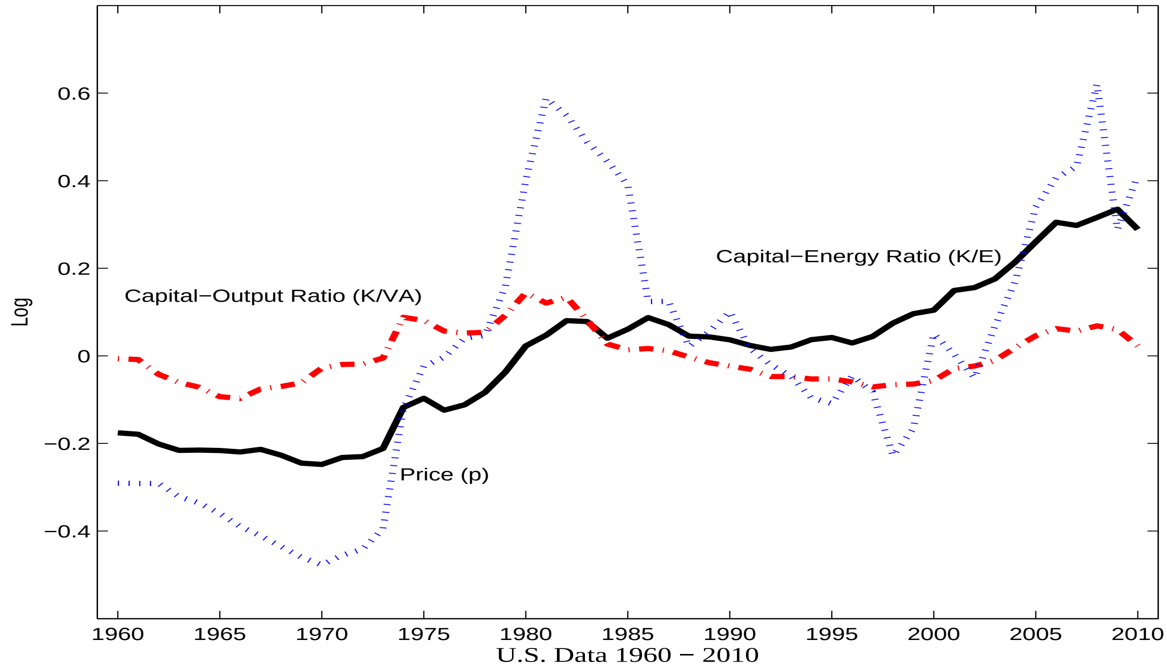

Figure 3: Capital-energy ratio and capital-output ratio.

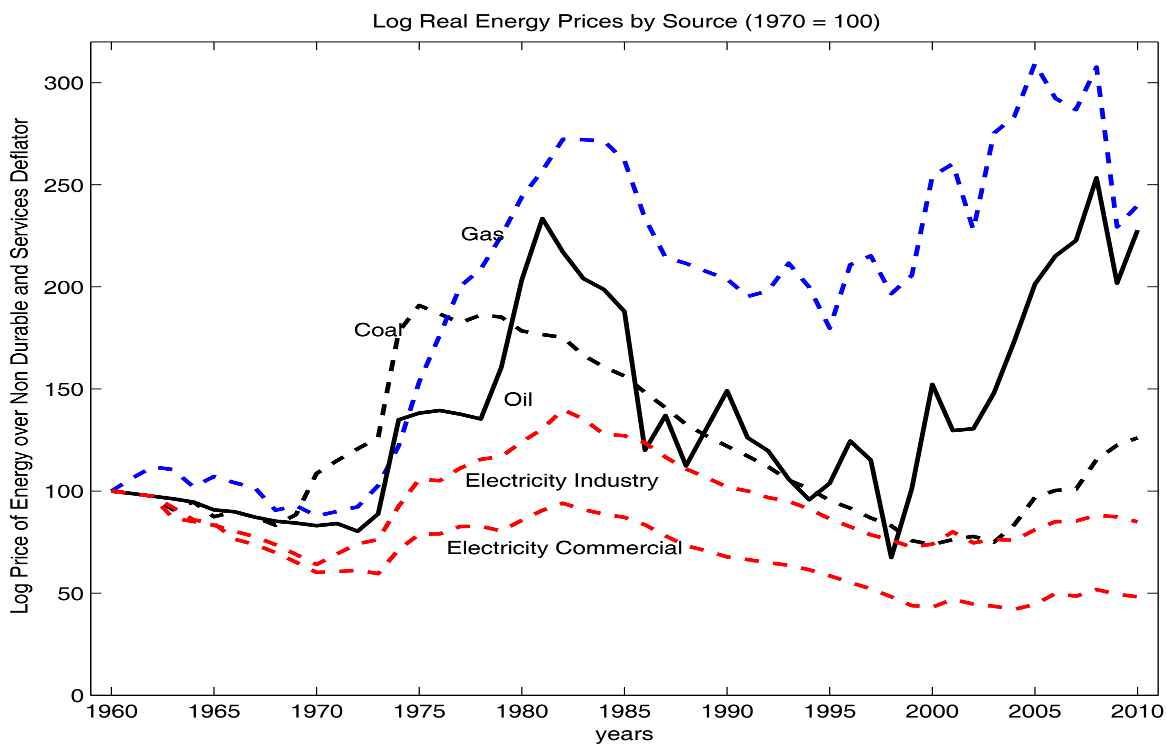

Figure 4: Real prices of energy sources.

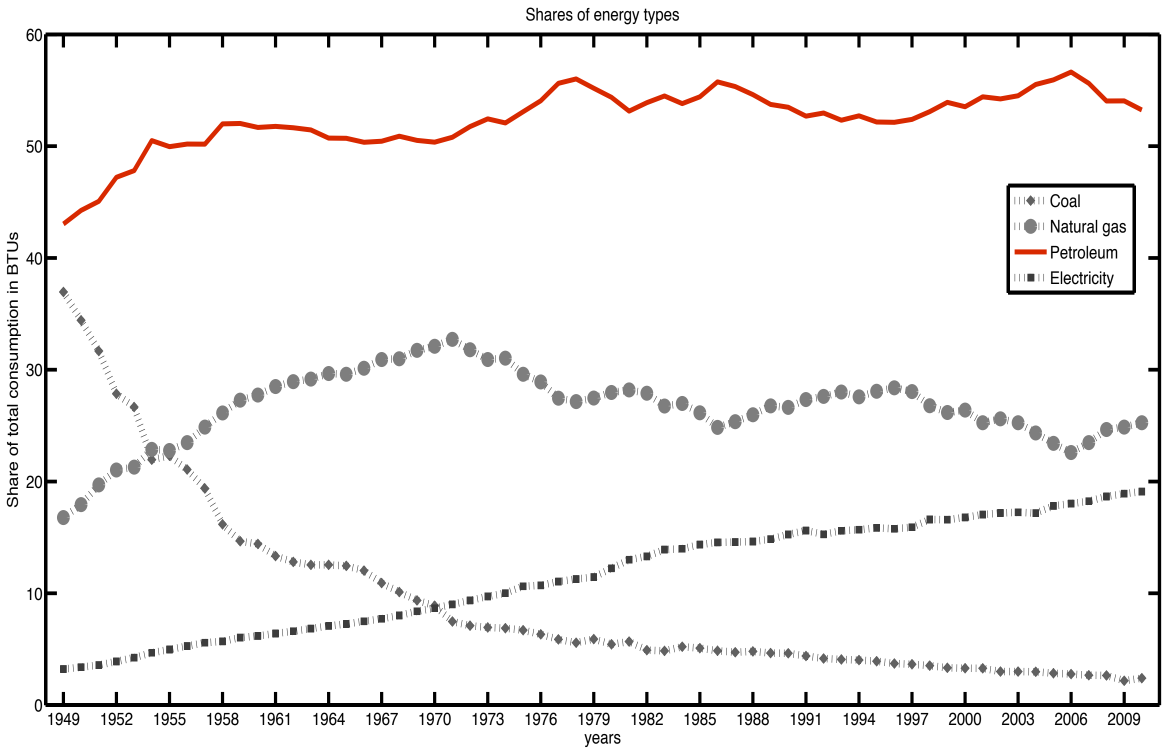

Figure 5: The amount consumed of the four types of energy as share of total consumption of BTUs.

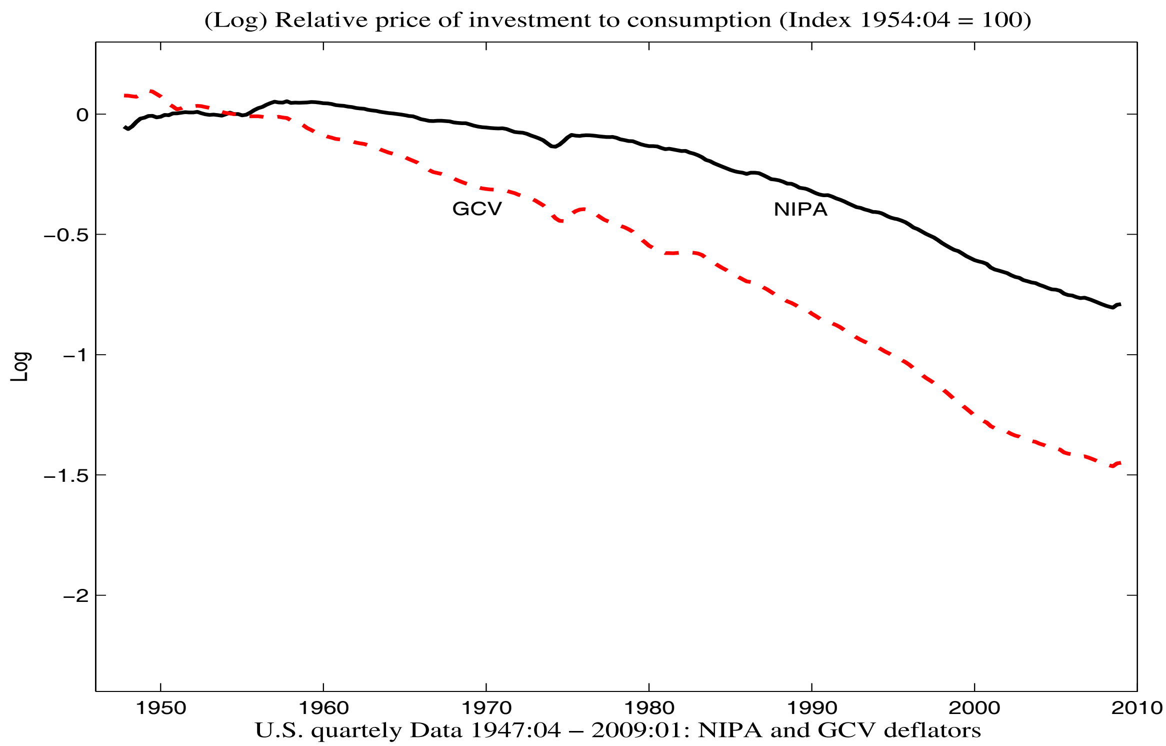

Figure 6: The relative price of investment.

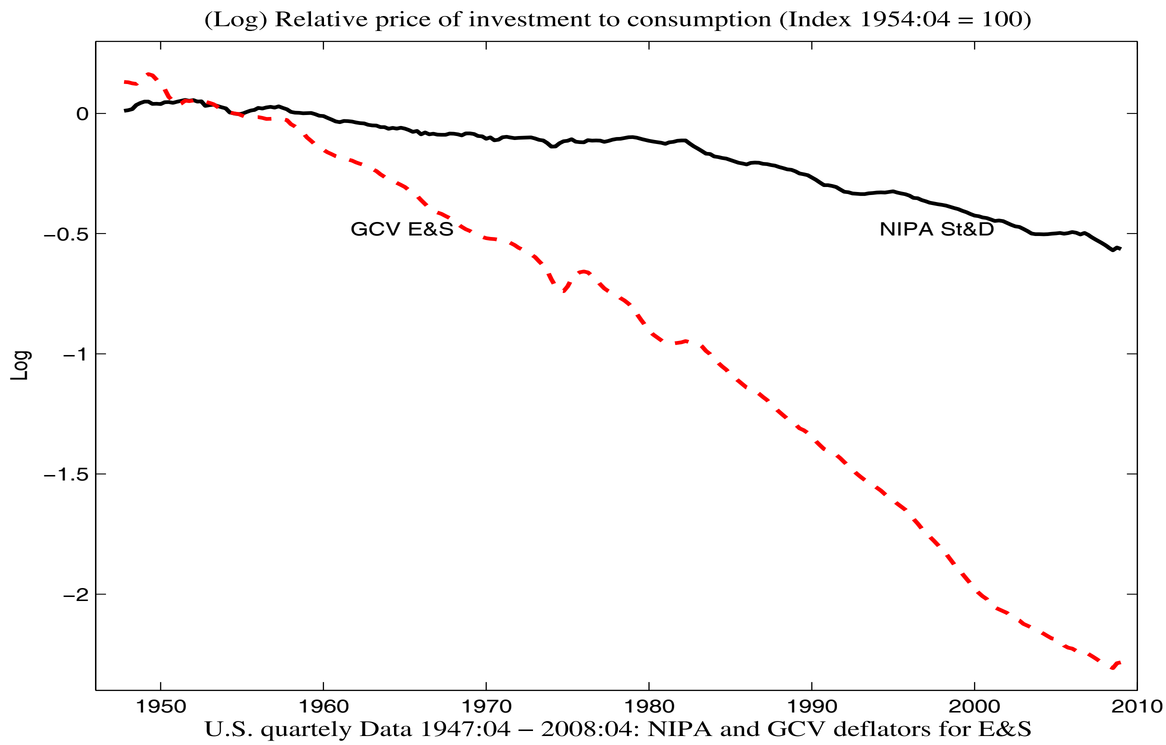

Figure 7: Alternative measures of the relative price of investment. NIPA St&D is based on the National Income and Product Accounts deflators for structures and durables. GCV E&S uses the Gordon-Cummins-Violante deflator for equipment and software.

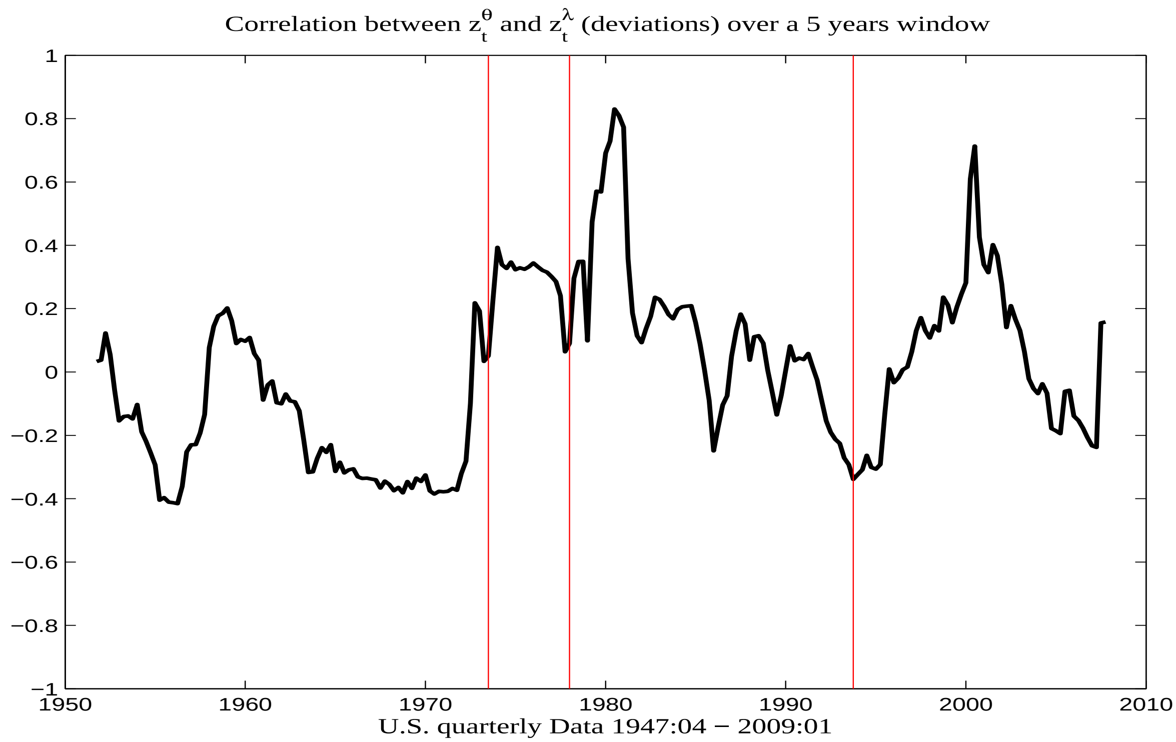

Figure 8: A measurement of the joint evolution of the innovations to investment specific disturbances.

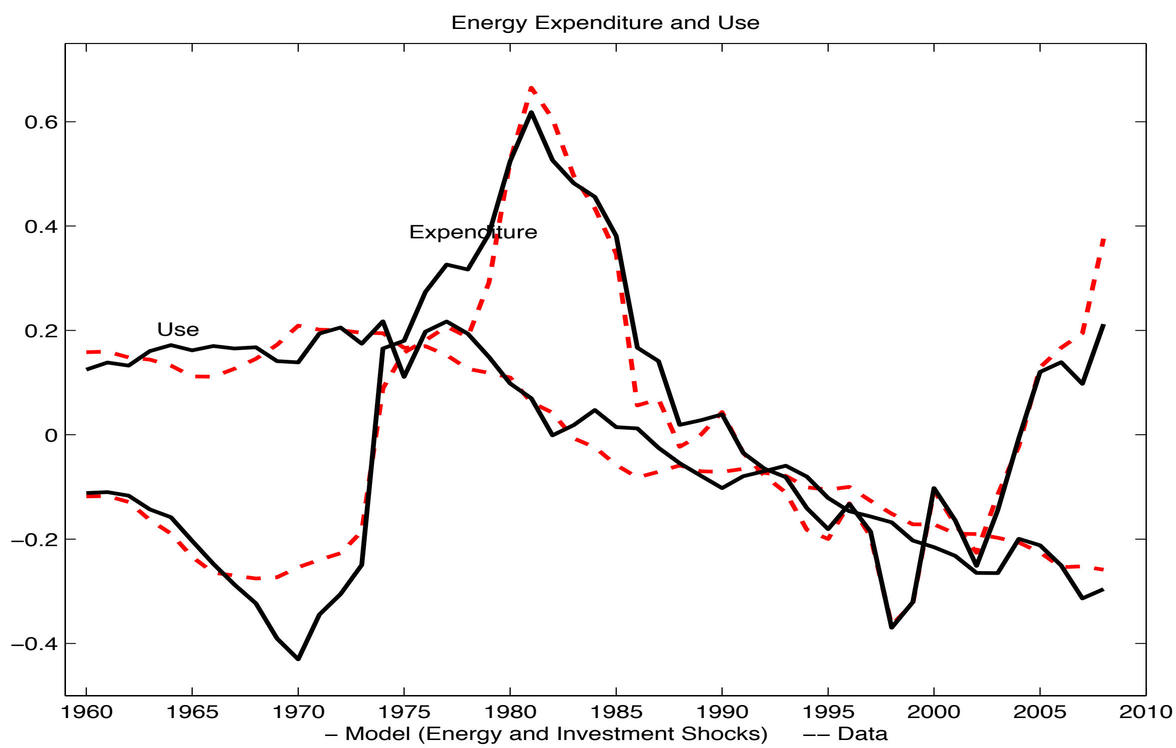

Figure 9: Energy expenditure and energy use in the vintage model and in the data

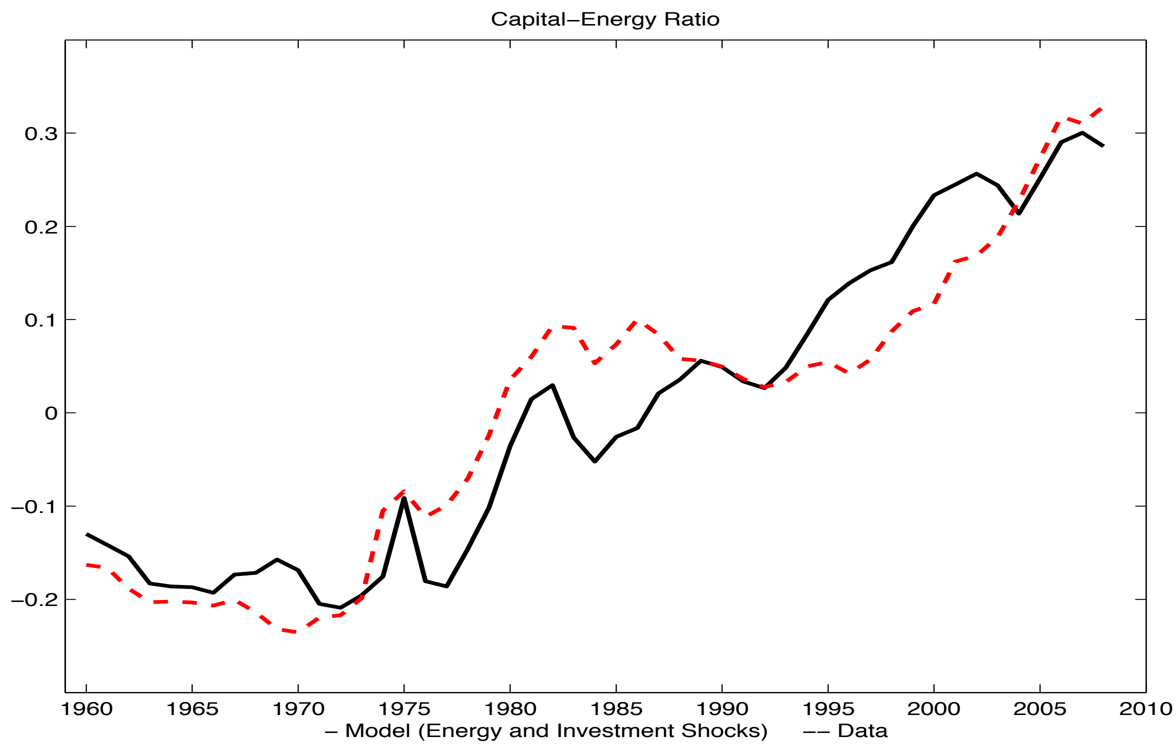

Figure 10: Evolution of the capital-energy ratio implied by the vintage model in comparison with data

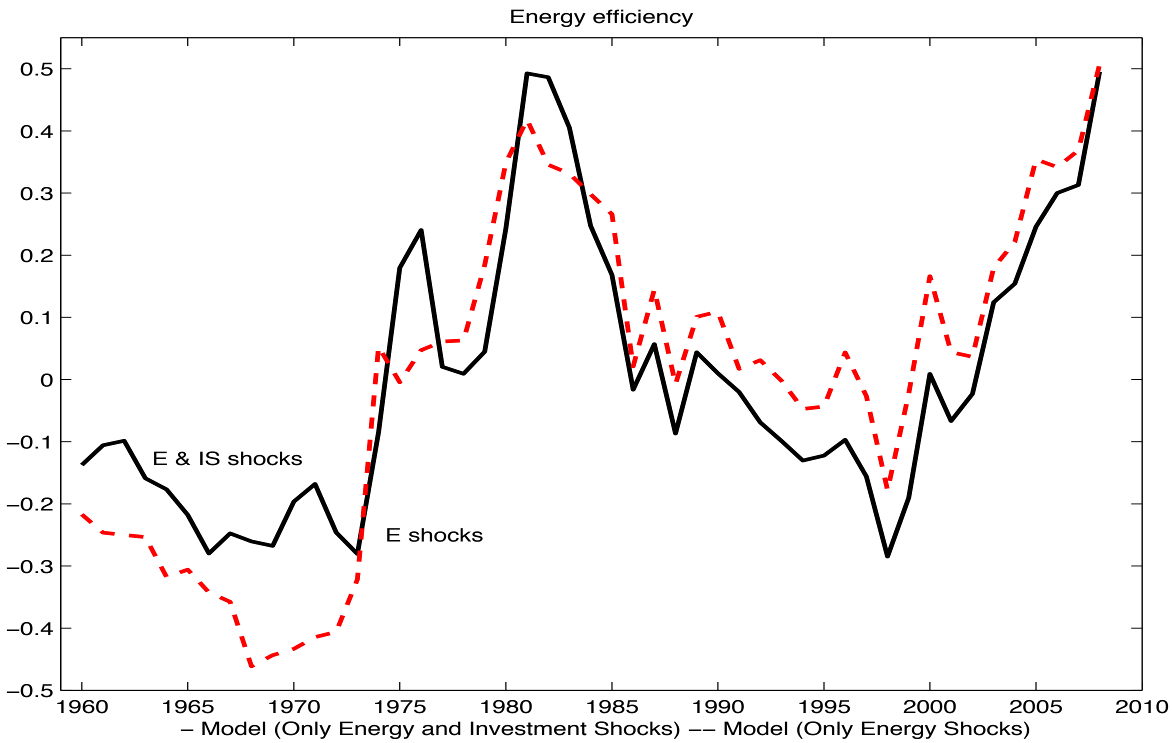

Figure 11: Efficiency in the newest capital installed

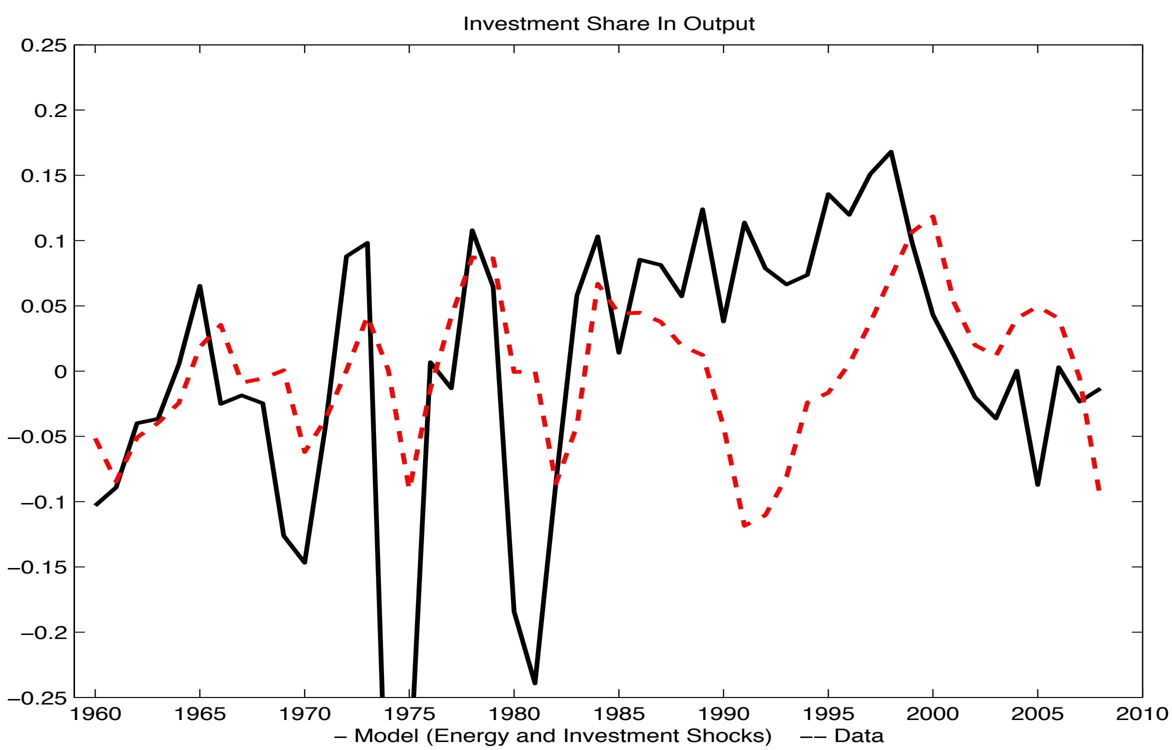

Figure 12: Investment to Value Added ratio

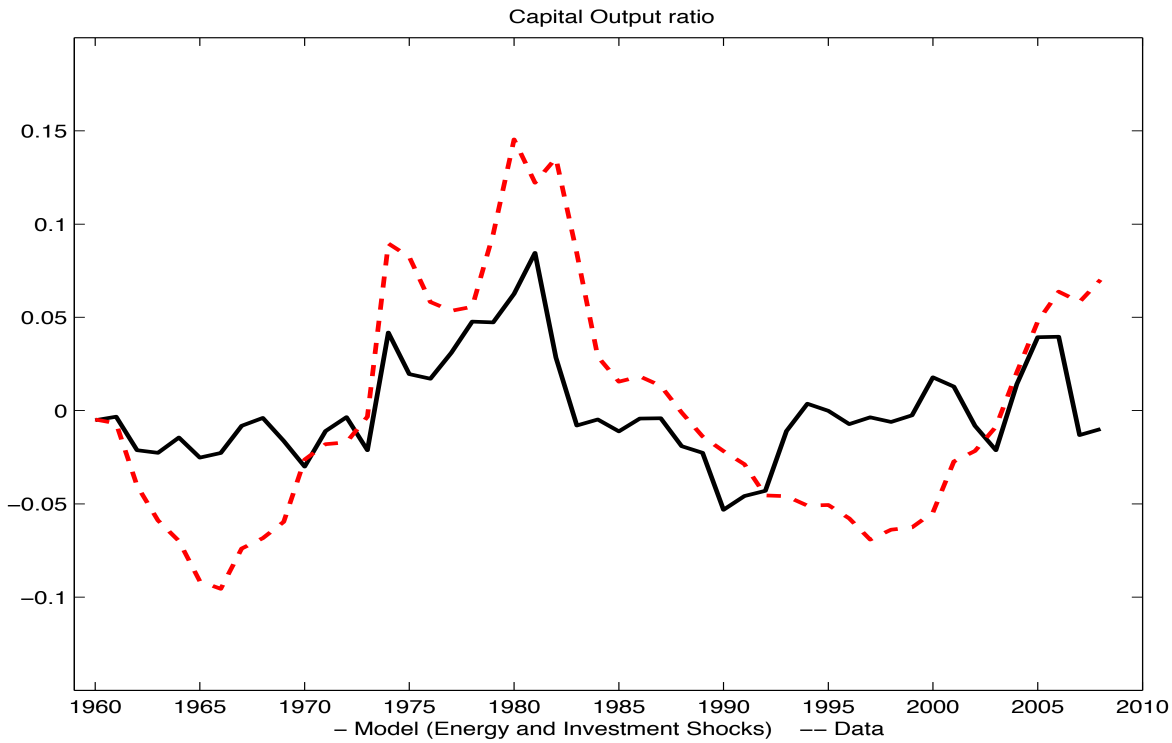

Figure 13: Capital to Value Added ratio

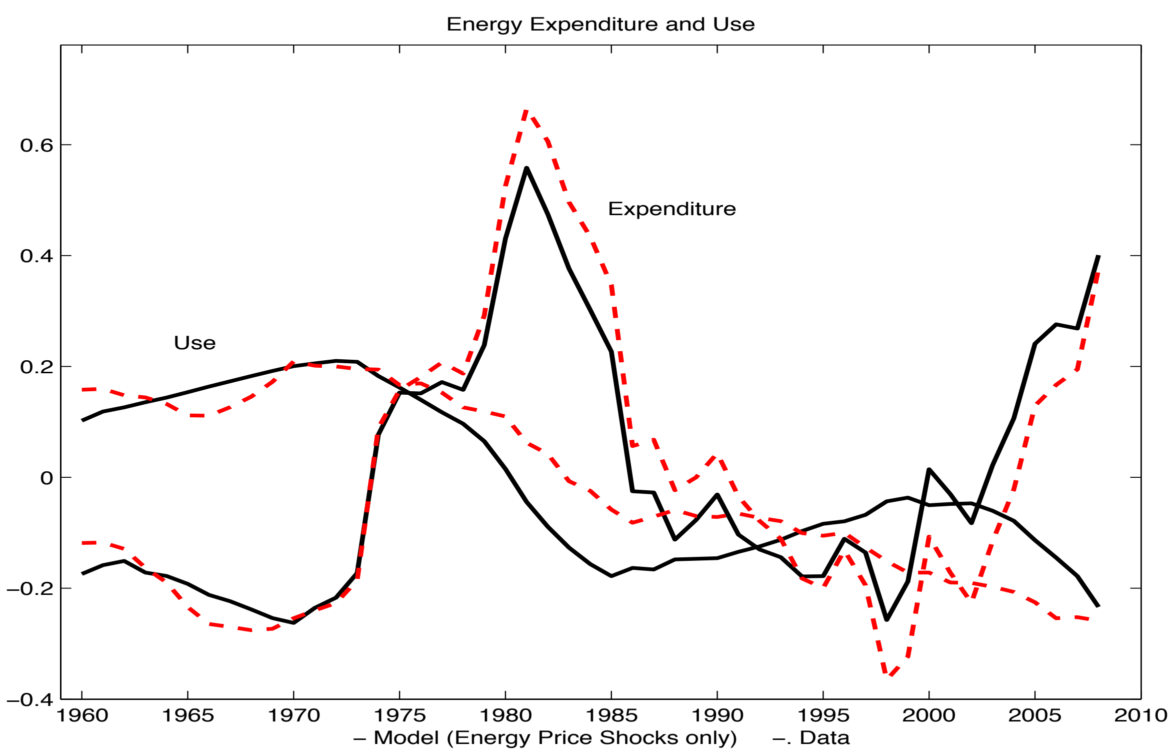

Figure 14: Energy expenditure and energy use in the vintage model and in the data

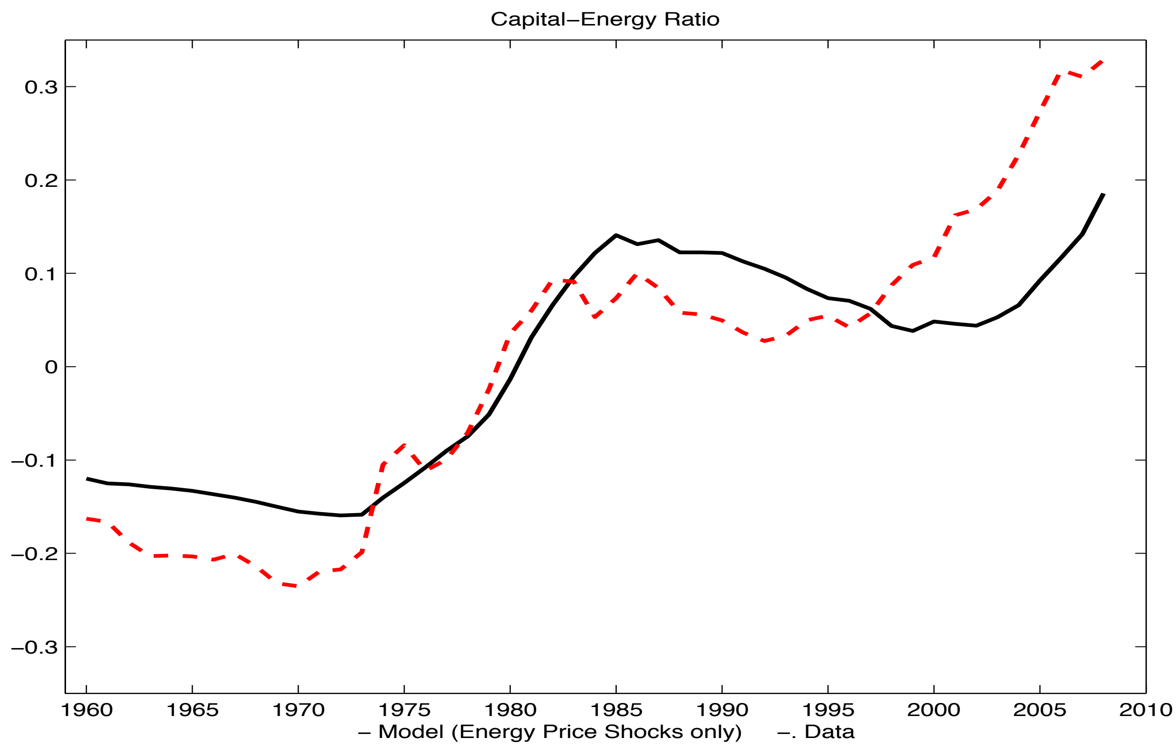

Figure 15: Evolution of the capital-energy ratio implied by the vintage model in comparison with data

Table 1: Aggregate targets

Param.	Observation		Value
	Preferences		
β	$K/(VA+p\,E)$	$= 2.66$	0.9545
	Technology		
α	$w\,L/(VA+p\,E)$	$= 55.94\%$	0.4406
ϖ	$I/(VA+p\,E)$	$= 28.41\%$	0.0523
$\tilde{v}^{\mu}$	$p\,E/(VA+p\,E)$	$= 4.51\%$	0.0160
μ	energy share in vintage t	$= \alpha/\mu$	16.2409

Notes: Average for period 1960-2008. $u(c) = \ln c$, VA = measured GDP + services of consumer durables + services of public capital − VA of energy producing sectors.

Table 2: State space representation of embodied and disembodied technical progress,1947-2009

	λ	θ	ρ_λ	ρ_θ	σ_λ	σ_θ
Quarterly Data	0.0073**	0.0023**	0.6622**	0.2856**	0.0063**	0.0052**
47:04 - 09:01	(0.0010)	(0.0001)	(0.0002)	(0.0001)	(0.0001)	(0.0001)
Quarterly Data	0.0078**	0.0028**	0.7223**	0.2633**	0.0054**	0.0049**
59:04 - 09:01	(0.0006)	(0.0001)	(0.0004)	(0.0002)	(0.0001)	(0.0001)
Annual Data	0.0303**	0.0092**	0.0368**	0.2336**	0.0282**	0.0139**
1947 - 2009	(0.0079)	(0.0005)	(0.0137)	(0.0020)	(0.0026)	(0.0004)
Annual Data	0.0323**	0.0112**	0.1444**	0.1463**	0.0272**	0.0125**
1959 - 2009	(0.0119)	(0.0007)	(0.0114)	(0.0021)	(0.0053)	(0.0006)
Observations	(1) 246	(2) 198	(3)	(4) 61	(5) 49	(6)
LL	(1) -1831	(2) -1522	(3)	(4) -306	(5) -253	(6)
Prob>F	0.000	0.000		0.000	0.000	

Notes: Standard errors in parentheses:* significant at 5%; ** significant at 1%. F-test for joint significance.

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