Non-Sincere Voting in Common Value Elections

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Abstract

We consider a common value election between two candidates where there is imperfect information about who is the best candidate. Before the election, apart from a common prior each voter receives a private signal of a certain idiosyncratic quality, where the quality measures how well the signal predicts the best candidate. Within this setting, we study when a voter has incentives to vote against his signal even if his signal provides useful information and abstention is allowed (non-sincere voting). A voter may be vote non-sincerely if his signal is of lower quality than that of the common prior. In this case the voter maximizes utility whenever pivotal by following such prior, thus disregarding the information provided by his private signal. We characterize the possible equilibria and find that non-sincere voting can be present in equilibrium and the election does not in general aggregate information efficiently. As the number of voters grows large, however, non-sincere voting vanishes and the best candidate wins the election with probability one.

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Keywords: Abstention, Elections, Private Information, Non-Sincere Voting.

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1 Introduction

Consider an election between two candidates where all voters have the same preferences: they want to choose the best candidate. However, voters are not perfectly informed about who is the best candidate. Instead, each voter receives information about the identity of best candidate from two sources, one public and one private. The public source of information is a common prior shared by all voters that specifies what is the probability that a certain candidate is the best one. The private source of information consists of an idiosyncratic signal of a certain quality. This private signal tells who is likely to be the best candidate, where how likely depends on the signal quality. Each voter knows the quality of his own signal but not the quality of the signals others receive nor these signals themselves.

If all voters shared their private signals and signal qualities, they would agree on which candidate to vote for as they all share the same preferences. In this case the winner of the election would be the best candidate with a probability increasing in the number of voters, the quality of the common prior and the quality of voters' private signals. However, in many real life settings, such sharing of information before the election may not be possible or feasible.

If there is no communication before the election, it may happen that some voters decide to abstain because they believe that their vote is going to harm the chances of the best candidate winning the election (this is known as *strategic abstention*, see for instance McMurray (2010, 2013) or Feddersen and Pesendorfer (1996)). This can be the case if the signal quality of these voters is low, so that they prefer leaving the decision of selecting a candidate to other, possibly better informed, voters. More importantly, it may happen that a voter ignores his signal and votes for the candidate that is supported by the common prior (*non-sincere voting*). Both of these behaviors may impede the efficient aggregation of information: with strategic abstention a signal is lost whereas with non-sincere voting a voter votes against his signal. In this paper we focus on non-sincere voting. Contrary to previous literature, non-sincere voting does not arise as a result of the different biases voters may have (see Feddersen and Pesendorfer (1999) or Rivas and Rodríguez-Álvarez (2013) among others); all voters have the same preferences and want the same thing out of the election: to choose the best candidate.

In our analysis we obtain two main results: first, we characterize the possible equilibria and find that a significant amount of non-sincere voting can be observed in equilibrium. Second, we find that as the number of voters grows large, non-informative voting disappears in the limit and the best candidate wins the election with probability one.

The reason behind the fact that non-sincere voting can be observed in equilibrium is the following. Consider a situation where the common prior puts a significant probability on a certain candidate to be the best one. Assume that among the voters who receive the signal that supports the other candidate, those with a low signal quality ignore their signal and vote for the candidate supported by the common prior, those with a moderate signal quality abstain, and those with a high signal quality vote following their signal. Voters who receive a signal that agrees with the common prior all vote according to their signal. Consider a voter who receives a low quality signal supporting the candidate that the common prior goes against. Whenever such voter can change the outcome of the election, i.e. he is pivotal, it must be that both candidates are at most one vote apart when counting the votes of all other voters. By voting for the candidate that is supported by the common prior, such voter knows that the common prior will decide the election. However, such common prior can be very informative as it puts a significant probability on a certain candidate. The voter has a signal against the common prior, but if such signal is of sufficiently low quality, the updated belief stills put a significant probability on the candidate supported by the common prior being the best one. A voter who receives a signal of moderate quality against the common prior may update his belief in a way that leaves him uncertain on which candidate to support. In this case such voter may be better off by abstaining and letting the other voters decide the election. A voter who receives a signal of sufficiently high quality against the common prior updates his belief in a way that leaves him convinced that the common prior is wrong and, hence, he will want to vote following his signal.

Our second main result deals with elections where there is a large number of voters. We show that in this case non-sincere voting vanishes in the limit. This is because in an equilibrium with a significant proportion of non-sincere voting, being pivotal implies that most voters received the signal that contradicts the common prior: we show that voters who vote non-sincerely are only those that receive a signal that goes against the prior, and that if some voters vote non-sincerely then no voter who receives a signal in favor of the prior abstains. Thus, if a voter is pivotal it must be that there are more voters who received a signal against the common prior that in favor. Thus, because there is a large number of voters, law of large numbers applies and the best candidate is the one not favored by the common prior with probability one. Thus, all voters have incentives to vote for such candidate, which implies that the original situation was not an equilibrium. We also show that although the information may not be used efficiently in equilibrium, with a large number of voters the best candidate wins the election with probability one in equilibrium.

The rest of the paper is organized as follows. Next we present a literature review. In section 2 we introduce the model while in section 3 we present our main result and some examples. In section 4 we study the limit case then the number of voters grows large. Finally, section 5 concludes. All mathematical proofs are presented in the appendix.

1.1 Literature

This paper contributes to the literature on the Condorcet's (1785) jury theorem. In particular, our work is very closely related to McMurray (2013). The main different between McMurray (2013) and the present paper is that we allow for the common prior to be biased: i.e. not all candidates are equally likely to be the best one a priori. This gives rise to a phenomenon not present in McMurray (2013): non-sincere voting. In McMurray (2013) the common prior puts probability a half on each candidate being the better one. Hence, any signal is at least as good as the prior in predicting the best candidate. This means that no voter has incentives to vote against his signal and their decision then reduces to whether to abstain or not. In our paper the fact that a signal may be less informative than the common prior means that some voters will choose to vote against their private signal. Moreover, even those who never vote against their signal behave differently depending on the signal they receive; we show that whether to abstain or not depends not only on the quality of the signal, but also on the value of the signal itself. In particular, those who receive a signal that agrees with the common prior are less likely to abstain than those who receive a signal against. When we consider elections with a large number of voters, we prove that in equilibrium the fraction of voters who vote non-sincerely, and the difference in behavior between those who receive a signal that supports the common prior and those that receive a signal against, vanishes to zero. Thus, the fact that candidates are not equally likely to be best a priori has no effect if and only if the electorate is large.

Feddersen and Pesendorfer (1996) present a model where voters are of three types: partisans, fully informed and uninformed. Partisans support a certain candidate irrespective of the information available while fully informed and uninformed voters prefer the best candidate. Fully informed voters know for certain who is the best candidate while uninformed voters have no information about the best candidate other than the common prior. They show that a positive fraction of uninformed voters abstain even when they strictly prefer one candidate over the other (swingers voter's curse). As in McMurray (2013) we find that being uninformed is not a requirement for the swingers voter's curse. Indeed, the fact that voters poses information of different qualities leads to a self selection in abstention; those with lower quality signals abstain, even if such signal is more informative than the prior and they strictly prefer one candidate over the other.

In Feddersen and Pesendorfer (1997) voters receive information from different sources, where each source may provide information of different qualities. However, they do not allow for abstention, which is a crucial addition to our model and the driving force behind most of our results. Feddersen and Pesendorfer (1999) allow voters to abstain. However, all voters receive information of the same quality. The results in Feddersen and Pesendorfer (1999) are similar to ours except for the fact that in their article the reason behind what each voter chooses given his signal is how biased towards either candidate he is. In our paper, no voter is biased and the driving force behind what each voter chooses given his signal is the quality of the signal. In this respect, there is a sense in which the bias of a voter can be related to the quality of his private information.

Ben-Yashar and Milchtaich (2007) present a paper where voters have homogeneous preferences and private signals are of different qualities. However, they do not consider the possibility of abstention; their focus is on computing the best monotone voting rule. Krishna and Morgan (2012) investigate the welfare effects of introducing voluntary voting when all voters have the same signal quality. Oliveros (2013) presents a model where voters can buy information of different qualities and studies the effects of different ideologies on information acquisition.

Another related paper is that of Austen-Smith and Banks (1996), who show that non-sincere voting can arise if abstention is not allowed and all voters have the same signal quality. From the technical point of view, our paper is different to some of the previous literature (McMurray (2013), Feddersen and Pesendorfer (1996, 1997, 1999), etc.) in that we do not need to consider an uncertain number of voters, i.e. Poisson games (see Myerson (1998)), to prove our results.

Finally, although it may seem more reasonable to assume that voters have certain ex-ante biases for or against each of the candidates (as it is done in some of the previous literature, see for instance Feddersen and Pesendorfer (1996, 1997, 1999)), the fact that we consider a common value election strengthens our finding that non-sincere voting is possible. If a voter votes against his private signal even when his preferences are aligned with other voters, he should have even more incentives to vote non-sincerely if he is biased in favor of the candidate supported by the common prior.

2 The Model

Consider a setting where $N+1 \geq 2$ voters have to decide between candidate A or candidate Q by simultaneously casting a vote for either candidate or abstaining. The candidate that the receives the most votes wins the election. In case of a tie each candidate wins with equal probability.

Each voter derives one unit of utility if the candidate who wins coincides with the state of nature and zero units of utility otherwise. The state of nature is a random variable $s \in \{A, Q\}$ where without loss of generality we assume that the probability that the state is A is given by $p \geq \frac{1}{2}$. We restrict our attention to situations where $p \in \left[\frac{1}{2}, 1\right)$ as if $p = 1$ then all voters agree that A is the best candidate and thus will vote for him regardless on any other information they may have available. The value of p is common knowledge and we refer to it as the common prior.

Before the election, each voter i receives a signal $\sigma_i \in \{A, Q\}$ with quality $q_i \in \left[\frac{1}{2}, 1\right]$ where

$$P(\sigma_i = s|s) = q_i.$$

Both the signal received by each voter as well as the quality of such signal are private information. The distribution of signal qualities for each voter in the population is common knowledge, identical, independently distributed and given by the strictly increasing cumulative density function $F : \left[\frac{1}{2}, 1\right] \to [0, 1]$ and probability density function $f : \left[\frac{1}{2}, 1\right] \to \mathbb{R}^+$. We assume that f is integrable in $\left[\frac{1}{2}, 1\right]$. Define the average signal quality $\mu = \int_{\frac{1}{2}}^{1} qf(q)\mathrm{d}q$ and consider $\mu \in \left(\frac{1}{2}, 1\right)$ to avoid the trivial case where all voters receive a useless signal or when all voters receive a perfectly informative signal.

Thus, before the election each voter knows the common prior, his own signal and the quality of such signal, as well as the distribution of the quality of other voters' signals. However, he ignores the state of nature, the signals received by other voters, and the quality of such signals.

A strategy for each voter is a map $v : \{A, Q\} \times \left[\frac{1}{2}, 1\right] \to \{\emptyset, A, Q\}$ where $v(\sigma_i, q_i)$ is the action of voter i who receives signal σ_i of quality q_i, and $\emptyset$ stands for the action of abstaining. Note that we focus on symmetric strategies: voters that are the same (same signal and quality) behave the same. The fact that we only consider symmetric equilibria does not undermine our main findings: if non-sincere voting is possible with symmetric strategies then it is also an equilibrium when asymmetric strategies are considered. Moreover, as we argue later on in section 4, when there is a large number of voters considering symmetric strategies is without loss of generality.

3 Analysis

Let $\pi_t(v, s)$ be the probability that candidate A receives the same number of votes as candidate Q (i.e. there is tie) when N voters use strategy v and the state is s. Similarly, let $\pi_A(v, s)$ be the probability candidate A receives exactly one vote less than candidate Q when N voters use strategy v and the state is s. Finally, let $\pi_Q(v, s)$ be the probability candidate Q receives exactly one vote less than candidate A when N voters use strategy v and the state is s.

A voter's vote can change the outcome of the election if and only if candidates A and Q are at most one vote apart when counting the votes of the other N voters. Thus, the utility voter i derives from voting for A compared to voting for Q when the other N voters use strategy v is given by

$$\begin{aligned} u_i(A,Q,v) &= P\left(s=A|\sigma_i,q_i\right)\left[\pi_t(v,A)+\frac{1}{2}\pi_Q(v,A)+\frac{1}{2}\pi_A(v,A)\right] \\ &\quad -P\left(s=Q|\sigma_i,q_i\right)\left[\pi_t(v,Q)+\frac{1}{2}\pi_Q(v,Q)+\frac{1}{2}\pi_A(v,Q)\right], \end{aligned} \tag{1}$$

where

$$\begin{aligned} P\left(s=A|\sigma_i=A,q_i\right) &= \frac{pq_i}{pq_i+(1-p)(1-q_i)}, \\ P\left(s=Q|\sigma_i=A,q_i\right) &= \frac{(1-p)(1-q_i)}{pq_i+(1-p)(1-q_i)}, \\ P\left(s=A|\sigma_i=Q,q_i\right) &= \frac{p(1-q_i)}{p(1-q_i)+(1-p)q_i}, \\ P\left(s=Q|\sigma_i=Q,q_i\right) &= \frac{(1-p)q_i}{p(1-q_i)+(1-p)q_i}. \end{aligned}$$

Notice that the private signal of voter i is more informative than the prior, $P\left(s|\sigma_i=s,q_i\right)\geq\frac{1}{2}$ for all $s\in\{A,Q\}$, if and only if $q_i\geq p$.

The utility voter i derives from voting for A or Q compared to abstaining when the other N voters use strategy v is given respectively by

$$\begin{aligned} u_i(A,\emptyset,v) &= P\left(s=A|\sigma_i,q_i\right)\left[\frac{1}{2}\pi_t(v,A)+\frac{1}{2}\pi_A(v,A)\right] \\ &\quad -P\left(s=Q|\sigma_i,q_i\right)\left[\frac{1}{2}\pi_t(v,Q)+\frac{1}{2}\pi_A(v,Q)\right], \end{aligned} \tag{2}$$

$$\begin{aligned} u_i(Q,\emptyset,v) &= P\left(s=Q|\sigma_i,q_i\right)\left[\frac{1}{2}\pi_t(v,Q)+\frac{1}{2}\pi_Q(v,Q)\right] \\ &\quad -P\left(s=A|\sigma_i,q_i\right)\left[\frac{1}{2}\pi_t(v,A)+\frac{1}{2}\pi_Q(v,A)\right]. \end{aligned} \tag{3}$$

To simplify the exposition, we assume that if a voter is indifferent between the two candidates he prefers the one that coincides with its signal. Similarly, if a voter is indifferent between voting for a certain candidate or abstaining, he prefers to vote if and only if such candidate coincides with his signal. As we it will be clear later on, the fact that f is integrable means that the probability that a voter is indifferent between two options (voting to one candidate or the other, or voting to either candidate or abstaining) is zero. As such, the way indifference ties are broken has no effect in our results and it allows us to ignore mixed strategies.

We have the following characterization:

Theorem 1. *There exists an equilibrium. The equilibrium is either of two types:*

- *Type* 1*, characterized by two cutpoints* $\frac{1}{2} \leq q_Q^- \leq q_Q^+ \leq 1$ *with* $q_Q^- \leq p$ *such that*

$$v(\sigma_i, q_i) \quad = \quad \begin{cases} A & \text{if either } \sigma_i = A \text{ or } \sigma_i = Q \text{ and } q_i < q_Q^-, \\ Q & \text{if } \sigma_i = Q \text{ and } q_i \geq q_Q^+, \\ \emptyset & \quad \text{otherwise.} \end{cases}$$

- *Type* 2*, characterized by two cutpoints* $\frac{1}{2} \leq q_A^+ \leq q_Q^+ \leq 1$ *such that*

$$v(\sigma_i, q_i) \quad = \quad \begin{cases} A & \text{if } \sigma_i = A \text{ and } q_i \geq q_A^+, \\ Q & \text{if } \sigma_i = Q \text{ and } q_i \geq q_Q^+, \\ \emptyset & \quad \text{otherwise.} \end{cases}$$

In equilibrium of Type 1 all voters who receive signal A vote and they do so for candidate A. These are the voters who receive a signal that agrees with the common prior. On the other hand, voters who receive a signal against the common prior, i.e. signal Q, behave as follows: those with a low quality signal ignore their signal and vote according to the common prior (non-sincere voting), those with a moderately informative signal abstain, and those with a sufficiently informative signal vote according to their signal.

In equilibrium of Type 2 there is no non-sincere voting, voters either vote according to their signal or abstain. Note that $q_A^+ \leq q_Q^+$ implies that those voters who receive a signal that agrees with the common prior are less likely to abstain than those who receive a signal against. This is the case because $p \geq \frac{1}{2}$ and, thus, if a voter receives signal A the common prior makes him trust is signal more whereas if voter receives signal Q he is less convinced about candidate Q than his signal quality suggests as the common prior goes against Q.

The reason why there is not an equilibrium where voters who receive signal A vote for Q is that we are assuming $p \geq \frac{1}{2}$ and, thus, a voter whose signal agrees with the common prior believes that A is the best candidate so he either abstains or votes for A. If $p \leq \frac{1}{2}$ then an equilibrium of Type 1 where the label A is swapped with Q could be possible. Figures 1 and 2 present a graphical representation of both types of equilibria.

Figure 1: Equilibrium of Type 1

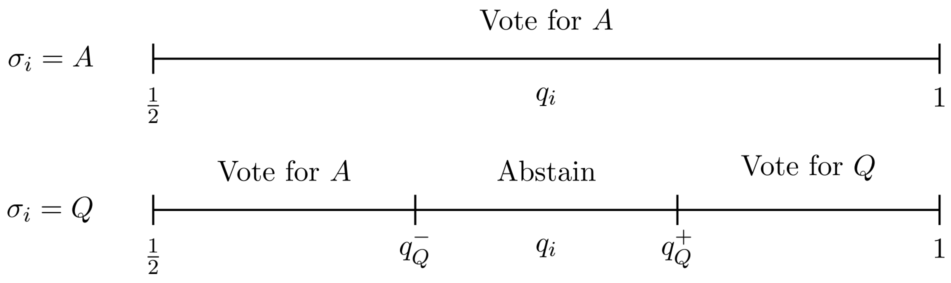
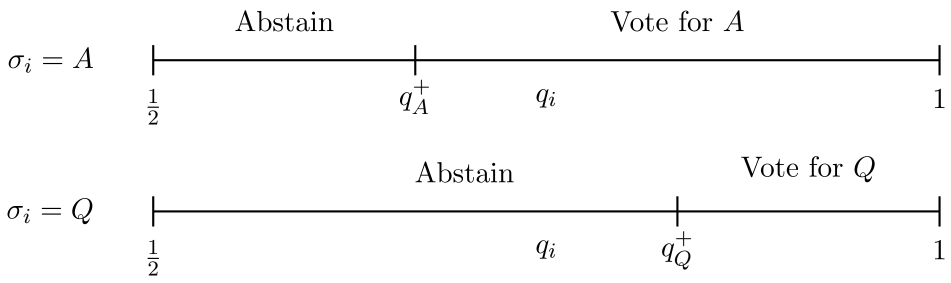


Figure 2: Equilibrium of Type 2



The expected fraction of non-sincere voting in equilibrium of Type 1 is given by $\int_{\frac{1}{2}}^{q_Q^-} f(q)$ which, as we shall see with examples, can be a strictly positive number. The fraction of voters who abstain is given by $\int_{q_Q^-}^{q_Q^+} f(q)$ in equilibrium of Type 1 and $\int_{\frac{1}{2}}^{q_Q^+} f(q) + \int_{\frac{1}{2}}^{q_A^+} f(q)$ in equilibrium of Type 2. This is the so-called strategic abstention (and swinger's vote curse), found for instance in McMurray (2013) and Feddersen and Pesendorfer (1996) (see McMurray (2010) for an empirical reference).

The reason behind non-sincere voting is the following. Consider a very simple example where there are only two voters. In this case a voter is always pivotal and thus learns very little from the fact that he is pivotal (he still does learn some information, as there are three different possibilities for a voter to be pivotal). In this case if a voter receives a low quality signal against the common prior, given that he learns very little from being pivotal, he may still prefer to vote for what the common prior suggests if the common prior is informative enough. This reasoning extends to more than just two voters: Assume that a voter receives a low quality signal supporting the candidate that goes against the common prior. If such voter is pivotal, he knows that there are mixed signals in the population, which suggests that the common prior may be wrong. However, his information may still support the same candidate as the common prior given that his updated belief stills put a significant probability on such candidate because the voter's signal is of low quality. Thus, the voter may have incentives to disobey his signal and vote non-sincerely.

The reason why strategic abstention is possible is that if a voter receives a signal of moderate quality and the common prior is not very informative (or he receives a signal of high quality against an informative the common prior, but not of sufficiently high quality), then if the voter is pivotal he may prefer to abstain and leave the decision to those who are better informed. This is because if the voter is pivotal there is a significant chance that the best candidate is ahead by one vote. Hence, by voting the voter runs the risk of contradicting the opinion of most other voters who do not abstain and who have a better signal quality than himself. In this situation the voter is better off by abstaining, even if he prefers one candidate over the other, and leaving the decision of electing a candidate to the other more informative voters.

Note that from the information revelation point, voting non-sincerely is worse than abstaining. When a voter abstains he reveals that his signal is not very informative. However, with non-sincere voting if a voter votes for A it is not clear whether such voter received signal A or Q. That is, non sincere-voting harms the chances of the best candidate winning the election more than abstention.

Theorem 1 states that in an equilibrium of Type 1, $q_Q^- \leq p$. Numerical examples shows that this inequality can be strict. If instead of a group of voters a single voter (dictator) chose the winning candidate, straightforward calculations show that this voter will choose to follow his signal if and only if his signal points at candidate A or if it points at candidate Q and the signal quality is at least p. In the language of the model, if $N+1 = 1$ then the unique equilibrium is Type 1 with $q_Q^- = q_Q^+ = p$. Thus, the fact that the group of voters includes more than just one voter means that voters are less likely to vote against their signal. That is, voters have more incentives to share their signal even if such signal is of a quality lower than the prior. Later in the paper we show that the fraction of voters who vote against their signal converges to zero as the number of voters increases.

As discussed in the introduction, McMurray (2013) considers a setting very similar to ours where the main difference is that he assumes $p = \frac{1}{2}$. The consequence of this is that in his setting the only possible equilibria is of Type 2 with $q_A^+ = q_Q^+$. The fact that $p > \frac{1}{2}$ is what produces both the possibility of an equilibrium of Type 1 with $q_Q^- > \frac{1}{2}$ and an equilibrium of Type 2 with $q_A^+ < q_Q^+$. The comparison of our results to McMurray (2013) is explored in more detail later on when we consider elections with a large number of voters.

It is worth pointing out the similarities between our result and that in Feddersen and Pesendorfer (1999). Particularly striking is the resemblance between figures 1 and 2 and figure 1 in Feddersen and Pesendorfer (1999). However, both results originate from very different sources. In our paper, voters' behavior depends on the signal they receive, but also on the quality of such signal. In Feddersen and Pesendorfer (1999), voters' behavior depends on the signal they receive and on their bias towards each of the candidates. Thus, the fact that unbiased voters receive signals of different qualities mimics the behavior observed when biased voters receive information of equal quality. A difference between the two situations is that a voter who is biased takes such bias as given while an unbiased voter is aware of the fact that his signal may or may not be very accurate.

3.1 Examples

Next we present some examples that illustrate the existence of non-sincere voting. In all the numerical examples shown in this section we assume that the signal qualities have a uniform distribution in $\left[\frac{1}{2}, 1\right]$. That is, $F(q) = 2q - 1$ and $f(q) = 2$. In tables 1 and 2 we calculate some of the possible equilibria when there are 5 and 6 voters respectively.

Table 1: Equilibria, $N+1=5$

$p=0.5$		$p=0.8$		$p=0.99$	
$q_A^+=0.5$	$q_A^+=0.68$	$q_A^+=0.60$	$q_Q^-=0.59$	$q_Q^-=0.57$	$q_Q^-=0.78$
$q_Q^+=0.5$	$q_Q^+=0.68$	$q_Q^+=0.75$	$q_Q^+=0.59$	$q_Q^+=0.88$	$q_Q^+=0.78$

Table 2: Equilibria, $N+1=6$

$p=0.5$	$p=0.8$	$p=0.99$
$q_A^+=0.68$	$q_A^+=0.62$	$q_Q^-=0.53$
$q_Q^+=0.68$	$q_Q^+=0.74$	$q_Q^+=0.85$

When $N+1=5$ there is multiplicity of equilibria, and for all common priors p considered an equilibrium without abstention exists (in equilibrium of Type 1 if $q_Q^- = q_Q^+$ then there is no abstention). On the other hand, if $N+1=6$ there is always abstention. We can prove the following result:

Proposition 1. *An equilibrium without abstention exists if and only if $N+1$ is odd.*

The reason for the result above is that if $N+1$ is even then we have that N is odd, which implies that if a voter is pivotal in a situation where no voter abstains, it must be that either of the two candidates received one vote more than the other one (i.e. candidates are not tied). In this case a voter with a low signal quality may be better off abstaining given that the best candidate is the one that is more likely to be ahead by one vote. On the other hand, if $N+1$ is odd then we have that N is even, which implies that if a voter is pivotal in a situation where no voter abstains, it must be that both candidates are tied. In this case the information the voter has (common prior plus private signal) becomes valuable, no matter how low his signal quality is. This is because conditional on being pivotal if both candidates are tied they are both equally likely to be the best one when the common prior and private signal are not taken into account. Thus, a voter has incentives to vote and break the tie.

Previous work on voting models assumed a random number of agents (Poisson games, see for instance Myerson (1998)). Such modeling assumption is mainly motivated by how the properties of the Poisson distribution significantly simplifies the analysis. Proposition 1 is not true in Poisson games because voters don't know with certainty how many voters there are and, hence, when there is no abstention they cannot tell apart between the situation where there is a tie and the situation where either candidate is ahead by one vote. A situation where the number of voters is unknown seems more plausible when the electorate is large where a situation where the number of voters is known is more realistic for small electorates. Even though we have assume that the population size is know, our predictions for a large electorate can be easily compared to those in previous literature that uses Poisson games. This will be clearer later on when we consider large elections. Moreover, the existence of non-sincere voting has no relation with the fact that the number of voters is known.

Numerical methods suggest that the equilibrium is unique if and only if $N+1$ is even. However, we have been unable to prove this. The problem of uniqueness of equilibrium in voting models such as this far from trivial (McMurray (2013)) and is often ignored (Feddersen and Pesendorfer (1996, 1997, 1999)). Nevertheless, uniqueness of equilibrium is not necessary for our results. Our characterization in Theorem 1 together with the examples above already illustrate the point of this paper: the existence of non-sincere voting in common value elections.

With respect to whether information is used efficiently in equilibrium, the examples in Tables 1 and 2 already tell us that this is not necessarily the case. For instance, if $N+1=5$ and $p=0.8$ then the probability with which the best candidate wins is 0.92 in Equilibrium of Type 1 and 0.93 in the equilibrium of Type 2. Numerical results show that this difference tends to increase if there are fewer voters. Moreover, as we shall show in the next section, for elections with a large number of voters the best candidate wins the election with probability one.

4 Large Elections

In this section we focus on elections where the number of voters tends to infinite. We refer to such elections as large elections. Our first result is that in large elections the fraction of voters who votes non-sincerely converges to zero and, moreover, the difference in behavior between those who receive different signals of the same quality also converges to zero.

Theorem 2. *The equilibrium in a large election is either Type* 1 *with* $\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q \to 0$ *or Type* 2 *with* $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q \to 0$.

The first part of the theorem states that in equilibrium of Type 1 we have $\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q \to 0$. Since in such equilibrium $q_Q^- \leq q_Q^+$, we have then that $\int_{\frac{1}{2}}^{q_Q^-} f(q)\mathrm{d}q \to 0$. Therefore, the proportion of voters that vote non-sincerely converges to zero. Note that it may happen that the number of voters who vote non-sincerely is bounded away from zero, that is, it could be that $\lim_{N\to\infty}(N+1)\int_{\frac{1}{2}}^{q_Q^-} f(q)\mathrm{d}q > \varepsilon$ for some $\varepsilon > 0$. However, the number of non-sincere voters in the population is insignificant compared with the number of voters who vote according to their signal.

Another implication of first part of the theorem is that the difference in behavior between those who receive signal A or Q converges to zero. A voter who receives signal A always votes for A while the fraction of voters who do not vote for Q when they receive signal Q is $\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q$, which converges to zero.

The second part of the theorem states that in equilibrium of Type 2 we have $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q \to 0$. Again this implies that the difference in behavior between those who receive signal A or Q converges to zero. Therefore, Theorem 2 implies that when the number of voters tends to infinity the fraction of voters who vote non-sincerely ($\int_{\frac{1}{2}}^{q_Q^-} f(q)\mathrm{d}q$) and the fraction of voters whose behavior depend on the specific signal received ($\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q$) vanishes in the limit. That is, as the number of voters increases the effect of an asymmetric common prior ($p > \frac{1}{2}$) vanishes in the limit and the results in McMurray (2013) apply. (i.e. the equilibrium is characterized by a cut-point q that determines who abstains and who votes for his signal independently on the particular signal received).

The reason behind the result in Theorem 2 is the following. Assume that $\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q$ does not converge to zero. In this case if a voter is pivotal then it must be that in proportion more voters received signal Q than A: as $\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q$ does not converge to zero, not all voters who receive signal Q vote for Q yet all voters who receive signal A vote for A. If more voters receive signal Q than A then since the average signal quality μ is greater than $\frac{1}{2}$ by law of large numbers the state of nature is Q with probability one. This implies that all voters should vote for Q, contradicting the fact that all voters who receive signal A prefer to vote for A.

A similar argument shows that as the number of voters grows large in equilibrium of Type 2 we must have $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q \to 0$. If $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q$ does not converge to zero then if a voter is pivotal it must be that a greater proportion of voters receive signal Q than signal A. This is because a higher fraction of those voters who receive signal Q compared to those who receive signal A abstain. Law of large number then means that the state is Q with probability one, which implies that all voters should vote for Q. This represents a contradiction to the characterization of equilibrium of Type 2.

In this paper we consider only symmetric strategies. The fact that an equilibrium with non-sincere voting exists in symmetric strategies implies that it also exists when non-symmetric strategies are considered. Moreover, the fact that we consider only symmetric strategies is without loss of generality in case there is a large number of voters. This is because in large elections the probability that the vote of an specific voter determines the outcome is zero. Thus, all voters face the same distribution of strategies played by other voters. Given that voters are never (with probability zero) indifferent between the available options (voting for either candidate and abstaining), all voters chose the same strategy conditional on the signal received and its quality. That is, voters follow symmetric strategies. For more on this see McMurray (2013).

Our final result states that in large elections the best candidate wins with probability one. This result is in line with the Condorcet Jury Theorem and the findings in previous literature (see for instance Feddersen and Pesendorfer (1996, 1997, 1999) and McMurray (2013)).

Proposition 2. *The equilibrium in a large election is such that the best candidate wins with probability one.*

Given the result in Theorem 2, whether a voter chooses to vote or to abstain depends on the quality of his signal, not on the value of the signal itself. Thus, for a given state of nature and given level of abstention, the best candidate is expected to receive a share μ of the votes while the other candidate is expected to receive a share $1-\mu$ of the votes. Since $\mu > \frac{1}{2}$ law of large numbers implies that the best candidate wins with probability one.

5 Conclusions

We present a common value election setting where voters have private information of different qualities. We showed that even when voters all have the same preferences, they may still have incentives to vote non-sincerely, i.e. against their private information, even if such private information is useful. Moreover, we found the behavior of voters when they receive information of different qualities resembles the behavior of voters who receive information of the same quality but that have difference preferences (they are biased towards either candidate). This suggest that the observed different biases towards candidates could be the result of different rational beliefs about who is the best candidate.

This paper extends the literature about voting in elections by finding a type of behavior that to our knowledge had not been observed before, non-sincere voting in common value elections. Future work could look at empirical or experimental evidence of this phenomenon in small and large elections as well as exploring the existence of such behavior in other settings.

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Appendix

The following lemma is used in the proof of Theorem 1.

Lemma 1. *The best response of any voter i against any strategy v played by the other N voters is given v', which is characterized by four cutpoints q_A^-, q_A^+, q_Q^- and q_Q^+ in $\left[\frac{1}{2}, 1\right]$ such that*

$$v'(\theta_i, q_i) = \begin{cases} A & \text{if either } \sigma_i = A \text{ and } q_i \geq q_A^+ \text{ or } \sigma_i = Q \text{ and } q_i < q_Q^-, \\ Q & \text{if either } \sigma_i = A \text{ and } q_i < q_A^- \text{ or } \sigma_i = Q \text{ and } q_i \geq q_Q^+, \\ \emptyset & \text{otherwise.} \end{cases}$$

Proof. Take any arbitrary voter i and assume all voters except i use strategy v. Consider equations (1), (2) and (3) and assume that $\sigma_i = A$. We have that both $Eu(A, v) - Eu(Q, v)$ and $Eu(A, v) - Eu(\emptyset, v)$ are increasing in q_i. Therefore, there exists a $x \in [0, 1]$ such that both equations are positive and voter i votes for A whenever $q_i \geq x$. Since $q_i \in \left[\frac{1}{2}, 1\right]$ if we define $q_A^+ = \max\left\{\frac{1}{2}, x\right\}$ we have that voter i votes for A whenever $q_i \geq q_A^+$.

Moreover, both $Eu(Q, v) - Eu(A, v)$ and $Eu(Q, v) - Eu(\emptyset, v)$ are decreasing in q_i. Therefore, there exists a y with $0 \leq y \leq x$ such that both equations are positive and voter i votes for Q whenever $q_i < y$. If we define $q_A^- = \max\left\{\frac{1}{2}, y\right\}$ we have that voter i votes for A whenever $q_i < q_A^-$.

The final possibility is that both $Eu(A, v) - Eu(\emptyset, v)$ and $Eu(Q, v) - Eu(\emptyset, v)$ are negative, which can happen if and only if $q_i \in [y, x)$ or, in other words, $q_i \in [q_A^-, q_A^+)$. In this case, voter i prefers to abstain.

A similar reasoning when $\sigma_i = Q$ leads to the conclusion in the lemma. □

Proof of Theorem 1. **An Equilibrium Exists**

First we demonstrate existence. Given the result in Lemma 1, we know that for any strategy v employed by the other N voters every voter employs a strategy that is characterized by four cutpoints q_A^-, q_A^+, q_Q^- and q_Q^+. Define the function $\phi : \left[\frac{1}{2}, 1\right]^4 \to \left[\frac{1}{2}, 1\right]^4$ where $\phi(q_A^-, q_A^+, q_Q^-, q_Q^+)$ is the best response of any voter to a situation where all other N voters employ an strategy characterized the four cutpoints $q_A^-, q_A^+, q_Q^-, q_Q^+$. We have to prove that ϕ has a fixed point. By the fixed point theorem, since the set $\left[\frac{1}{2}, 1\right]^4$ is convex and compact in the Euclidean space we are left to show that ϕ is continuous.

When N voters are using strategy v characterized by the four cutpoints q_A^-, q_A^+, q_Q^- and q_Q^+ we have that

$$
\begin{aligned}
\pi_t(v,A) \quad = \quad & \sum_{s_A(A)=0}^{\left\lfloor \frac{N}{2} \right\rfloor} \sum_{s_Q(A)=0}^{\left\lceil \frac{N}{2} \right\rceil - s_A(A)} \sum_{s_A(\emptyset)=0}^{N-2(s_A(A)+s_Q(A))} \sum_{s_A(Q)=0}^{s_A(A)+s_Q(A)} \\
& \frac{N!}{s_A(A)!s_Q(A)!s_A(\emptyset)!s_A(Q)!(s_A(A)+s_Q(A)-s_A(Q))!(N-2(s_A(A)+s_Q(A))-s_A(\emptyset))!} \\
& \times \left[\int_{q_A^+}^{1} qf(q)\mathrm{d}q\right]^{s_A(A)} \left[\int_{\frac{1}{2}}^{q_Q^-} (1-q)f(q)\mathrm{d}q\right]^{s_Q(A)} \\
& \times \left[\int_{q_A^-}^{q_A^+} qf(q)\mathrm{d}q\right]^{s_A(\emptyset)} \left[\int_{\frac{1}{2}}^{q_A^-} qf(q)\mathrm{d}q\right]^{s_A(Q)} \\
& \times \left[\int_{q_Q^+}^{1} (1-q)f(q)\mathrm{d}q\right]^{s_A(A)+s_Q(A)-s_A(Q)} \left[\int_{q_Q^-}^{q_Q^+} (1-q)f(q)\mathrm{d}q\right]^{N-2(s_A(A)+s_Q(A))-s_A(\emptyset)} \qquad (4)
\end{aligned}
$$

Since $F(q) = \int_{\frac{1}{2}}^{q} f(q)\mathrm{d}q$ we have that F is continuous and, because it is a cumulative density function, it is bounded in $[0,1]$. Therefore, F is integrable and moreover continuous with respect to the integration limits. Thus, $\int qf(q)\mathrm{d}(q) = qF(q) - \int F(q)\mathrm{d}q$ is continuous with respect to the integration limits. As a result, $\pi_t(v,A)$ is continuous with respect to the cutpoints q_A^-, q_A^+, q_Q^- and q_Q^+.

It can be shown in a similar fashion that $\pi_t(v,s), \pi_Q(v,s)$ and $\pi_A(v,s)$ are continuous with respect to the cutpoints q_A^-, q_A^+, q_Q^- and q_Q^+ for all $s \in \{A, Q\}$. Hence, we have that $Eu(A,v) - Eu(Q,v), Eu(A,v) - Eu(\emptyset, v)$ and $Eu(A,v) - Eu(\emptyset, v)$ are continuous with respect to the cutpoints q_A^-, q_A^+, q_Q^- and q_Q^+. Thus, ϕ is continuous as we wanted to show.

Equilibrium is of Two Types

Given the result in Lemma 1, any equilibrium is characterized by the four threshold values q_A^-, q_A^+, q_Q^- and q_Q^+. Assume that $q_Q^- > \frac{1}{2}$, then we have that $Eu(A,v) - Eu(Q,v) > 0$ and $Eu(A,v) - Eu(\emptyset,v) > 0$ for all i with $\sigma_i = Q$ and $q_i \in \left[\frac{1}{2}, q_Q^-\right)$, which implies that $Eu(A,v) - Eu(Q,v) > 0$ and $Eu(A,v) - Eu(\emptyset,v) > 0$ for all i with $\sigma_i = A$ and $q_i \in \left[\frac{1}{2}, q_Q^-\right)$. This means that $q_A^-, q_A^+ = \frac{1}{2}$, which leads to equilibrium of Type 1 in the proposition.

Assume now that $q_Q^- = \frac{1}{2}$ and $q_Q^+ > \frac{1}{2}$. In this case we have that $Eu(Q,v) - Eu(\emptyset,v) < 0$ for all i with $\sigma_i = Q$ and $q_i \in \left[\frac{1}{2}, q_Q^+\right)$, which implies that $Eu(Q,v) - Eu(\emptyset,v) < 0$ for all i with $\sigma_i = A$ and $q_i \in \left[\frac{1}{2}, q_Q^+\right)$. This means that $q_A^- = \frac{1}{2}$, which leads to equilibrium of type 2 in the proposition.

Finally, assume that $q_Q^- = q_Q^+ = \frac{1}{2}$. We proceed by showing that $\pi_t(v,A) + \pi_Q(v,A) \geq \pi_t(v,Q) + \pi_Q(v,Q)$. If this were true, and since $q_Q^+ = \frac{1}{2}$ implies that $Eu(Q,v) - Eu(\emptyset, v) \geq 0$ for all i with $\sigma_i = Q$ and $q_i \in \left[\frac{1}{2}, 1\right]$, equation (3) together with the fact that $p \geq \frac{1}{2}$ implies that $\pi_t(v,Q) + \pi_Q(v,Q) \geq \pi_t(v,A) + \pi_Q(v,A)$, which would represent a contradiction (unless $q_Q^- = q_Q^+ = q_A^- = q_A^+ = p = \frac{1}{2}$, which is an equilibrium of either Type in the proposition).

First we show that $\pi_t(v,A) - \pi_t(v,Q) \geq 0$ for all $\frac{1}{2} \leq q_A^- \leq q_A^+ \leq 1$. Note that

$$\begin{aligned}
\pi_t(v,A) &= \sum_{j=0}^{\lfloor \frac{N}{2} \rfloor} \sum_{r=0}^{j} \frac{N!}{j!(j-r)!r!(N-2j)!} \\
&\left[\int_{q_A^+}^{1} qf(q)\mathrm{d}q\right]^j \left[\int_{\frac{1}{2}}^{q_A^-} qf(q)\mathrm{d}q\right]^{j-r} \left[\int_{\frac{1}{2}}^{1} (1-q)f(q)\mathrm{d}q\right]^r \left[\int_{q_A^-}^{q_A^+} qf(q)\mathrm{d}q\right]^{N-2j}, \\
\pi_t(v,Q) &= \sum_{j=0}^{\lfloor \frac{N}{2} \rfloor} \sum_{r=0}^{j} \frac{N!}{j!(j-r)!r!(N-2j)!} \\
&\left[\int_{q_A^+}^{1} (1-q)f(q)\mathrm{d}q\right]^j \left[\int_{\frac{1}{2}}^{q_A^-} (1-q)f(q)\mathrm{d}q\right]^{j-r} \left[\int_{\frac{1}{2}}^{1} qf(q)\mathrm{d}q\right]^r \left[\int_{q_A^-}^{q_A^+} (1-q)f(q)\mathrm{d}q\right]^{N-2j}.
\end{aligned}$$

Given that $q_i \geq \frac{1}{2}$ for all voter i we have that

$$\begin{aligned}
\pi_t(v,A) - \pi_t(v,Q) \geq & \sum_{j=0}^{\lfloor \frac{N}{2} \rfloor} \sum_{r=0}^{j} \frac{N!}{j!(j-r)!r!(N-2j)!} \\
& \left(\left[\int_{q_A^+}^{1} qf(q)\mathrm{d}q\right]^j \left[\int_{\frac{1}{2}}^{1} (1-q)f(q)\mathrm{d}q\right]^r \right. \\
& \left. - \left[\int_{q_A^+}^{1} (1-q)f(q)\mathrm{d}q\right]^j \left[\int_{\frac{1}{2}}^{1} qf(q)\mathrm{d}q\right]^r \right).
\end{aligned}$$

Thus, if $q_A^+ = \frac{1}{2}$ or $q_A^+ = 1$ then $\pi_t(v,A) - \pi_t(v,Q) \geq 0$. Consider now the cases where $q_A^+ \in \left(\frac{1}{2}, 1\right)$. Using once more that $q_i \geq \frac{1}{2}$ for all i, a necessary condition for $\pi_t(v,A) - \pi_t(v,Q) \geq 0$ is that

$$\left[\int_{q_A^+}^{1} qf(q)\mathrm{d}q\right]^r \left[\int_{\frac{1}{2}}^{1} (1-q)f(q)\mathrm{d}q\right]^r \geq \left[\int_{q_A^+}^{1} (1-q)f(q)\mathrm{d}q\right]^r \left[\int_{\frac{1}{2}}^{1} qf(q)\mathrm{d}q\right]^r.$$

This can be written as

$$\begin{aligned}
&\left[\int_{q_A^+}^{1} qf(q)\mathrm{d}q\right] \left[\int_{q_A^+}^{1} (1-q)f(q)\mathrm{d}q + \int_{\frac{1}{2}}^{q_A^+} (1-q)f(q)\mathrm{d}q\right] \geq \\
&\qquad \left[\int_{q_A^+}^{1} (1-q)f(q)\mathrm{d}q\right] \left[\int_{q_A^+}^{1} qf(q)\mathrm{d}q + \int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q\right].
\end{aligned}$$

In other words,

$$
\begin{aligned}
\frac{\int_{\frac{1}{2}}^{q_A^+}(1-q)f(q)\mathrm{d}q}{\int_{q_A^+}^{1}(1-q)f(q)\mathrm{d}q} &\geq \frac{\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q}{\int_{q_A^+}^{1} qf(q)\mathrm{d}q}, \\
\frac{\int_{\frac{1}{2}}^{q_A^+}(1-q)f(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q} &\geq \frac{\int_{q_A^+}^{1}(1-q)f(q)\mathrm{d}q}{\int_{q_A^+}^{1} qf(q)\mathrm{d}q}, \\
\frac{\int_{\frac{1}{2}}^{q_A^+} f(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q} &\geq \frac{\int_{q_A^+}^{1} f(q)\mathrm{d}q}{\int_{q_A^+}^{1} qf(q)\mathrm{d}q}, \\
\frac{\int_{q_A^+}^{1} qf(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q} &\geq \frac{\int_{q_A^+}^{1} f(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_A^+} f(q)\mathrm{d}q}.
\end{aligned}
$$

Since it is true that

$$
\begin{aligned}
\frac{\int_{q_A^+}^{1} qf(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q} &\geq \frac{\int_{q_A^+}^{1} q_A^+ f(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_A^+} q_A^+ f(q)\mathrm{d}q} \\
&= \frac{\int_{q_A^+}^{1} f(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_A^+} f(q)\mathrm{d}q},
\end{aligned}
$$

we have that $\pi_t(v, A) - \pi_t(v, Q) \geq 0$.

Proceeding in a similar fashion, it can be shown that $\pi_Q(v, A) - \pi_Q(v, Q) \geq 0$. Thus, we have that $\pi_t(v, A) - \pi_Q(v, A) \geq \pi_i(v, Q) - \pi_Q(v, Q)$ as required.

In Equilibrium of Type 1 $q_Q^- \leq p$

We can use the algebra from the previous part of the proof to show that in equilibrium of Type 1 $\pi_t(v, A) - \pi_t(v, Q) \leq 0$ and $\pi_A(v, Q) - \pi_A(v, A) \geq 0$ for all $\frac{1}{2} \leq q_Q^- \leq q_Q^+ \leq 1$. Hence, equation (2) together with the definition of q_Q^- implies $p(1 - q_Q^-) \geq (1 - p)q_Q^-$, which in turn implies $q_Q^- \leq p$.

In Equilibrium of Type 2 $q_Q^+ \geq q_A^+$

Next we prove that in any equilibrium of type 2 it must be that $q_Q^+ \geq q_A^+$. Assume the opposite, $q_A^+ > q_Q^+$. Note that in any Type 2 equilibrium we have that

$$\begin{aligned}
\pi_t(v,A) &= \sum_{j=0}^{\lfloor \frac{N}{2} \rfloor} \left[\int_{q_A^+}^{1} qf(q)\mathrm{d}q\right]^j \left[\int_{q_Q^+}^{1} (1-q)f(q)\mathrm{d}q\right]^j \\
&\quad \times \sum_{r=0}^{N-2j} \frac{N!}{j!j!r!(N-2j-r)!} \\
&\quad \times \left[\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q\right]^r \left[\int_{\frac{1}{2}}^{q_Q^+} (1-q)f(q)\mathrm{d}q\right]^{N-2j-r}.
\end{aligned}$$

Thus, it is true that

$$\begin{aligned}
\pi_t(v,A) &= \sum_{j=0}^{\lfloor \frac{N}{2} \rfloor} \left[\int_{q_A^+}^{1} qf(q)\mathrm{d}q\right]^j \left[\int_{q_Q^+}^{1} (1-q)f(q)\mathrm{d}q\right]^j \\
&\quad \times \sum_{k=0}^{\lfloor \frac{N-2j}{2} \rfloor} \frac{N!}{j!j!k!(N-2j-k)!} \\
&\quad \times \left[\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q\right]^k \left[\int_{\frac{1}{2}}^{q_Q^+} (1-q)f(q)\mathrm{d}q\right]^k \\
&\quad \times \left(\left[\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q\right]^{N-2j-2k} + \left[\int_{\frac{1}{2}}^{q_Q^+} (1-q)f(q)\mathrm{d}q\right]^{N-2j-2k}\right), \qquad (5) \\
\pi_t(v,Q) &= \sum_{j=0}^{\lfloor \frac{N}{2} \rfloor} \left[\int_{q_A^+}^{1} (1-q)f(q)\mathrm{d}q\right]^j \left[\int_{q_Q^+}^{1} qf(q)\mathrm{d}q\right]^j \\
&\quad \times \sum_{k=0}^{\lfloor \frac{N-2j}{2} \rfloor} \frac{N!}{j!j!k!(N-2j-k)!} \\
&\quad \times \left[\int_{\frac{1}{2}}^{q_A^+} (1-q)f(q)\mathrm{d}q\right]^k \left[\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q\right]^k \\
&\quad \times \left(\left[\int_{\frac{1}{2}}^{q_A^+} (1-q)f(q)\mathrm{d}q\right]^{N-2j-2k} + \left[\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q\right]^{N-2j-2k}\right). \qquad (6)
\end{aligned}$$

We now show that $\pi_t(v,A) - \pi_t(v,Q) \geq 0$ in three steps. First, we have that $q_A^+ > q_Q^+$ implies

$$\left[\int_{q_A^+}^{1} qf(q)\mathrm{d}q\right]^j \left[\int_{q_Q^+}^{1} (1-q)f(q)\mathrm{d}q\right]^j \geq \left[\int_{q_A^+}^{1} (1-q)f(q)\mathrm{d}q\right]^j \left[\int_{q_Q^+}^{1} qf(q)\mathrm{d}q\right]^j \qquad (7)$$

for all $j \in \{0, 1, \ldots\}$ if and only if

$$\int_{q_A^+}^1 qf(q)\mathrm{d}q \left[\int_{q_A^+}^1 (1-q)f(q)\mathrm{d}q + \int_{q_Q^+}^{q_A^+} (1-q)f(q)\mathrm{d}q\right] \geq \int_{q_A^+}^1 (1-q)f(q)\mathrm{d}q \left[\int_{q_A^+}^1 qf(q)\mathrm{d}q + \int_{q_Q^+}^{q_A^+} qf(q)\mathrm{d}q\right].$$

which can be rewritten as

$$\begin{aligned}
\int_{q_A^+}^1 qf(q)\mathrm{d}q \int_{q_Q^+}^{q_A^+} (1-q)f(q)\mathrm{d}q &\geq \int_{q_A^+}^1 (1-q)f(q)\mathrm{d}q \int_{q_Q^+}^{q_A^+} qf(q)\mathrm{d}q, \\
\frac{\int_{q_Q^+}^{q_A^+} (1-q)f(q)\mathrm{d}q}{\int_{q_Q^+}^{q_A^+} qf(q)\mathrm{d}q} &\geq \frac{\int_{q_A^+}^1 (1-q)f(q)\mathrm{d}q}{\int_{q_A^+}^1 qf(q)\mathrm{d}q}, \\
\frac{\int_{q_Q^+}^{q_A^+} f(q)\mathrm{d}q}{\int_{q_Q^+}^{q_A^+} qf(q)\mathrm{d}q} &\geq \frac{\int_{q_A^+}^1 f(q)\mathrm{d}q}{\int_{q_A^+}^1 qf(q)\mathrm{d}q}, \\
\frac{\int_{q_A^+}^1 qf(q)\mathrm{d}q}{\int_{q_Q^+}^{q_A^+} qf(q)\mathrm{d}q} &\geq \frac{\int_{q_A^+}^1 f(q)\mathrm{d}q}{\int_{q_Q^+}^{q_A^+} f(q)\mathrm{d}q},
\end{aligned}$$

which given that

$$\begin{aligned}
\frac{\int_{q_A^+}^1 qf(q)\mathrm{d}q}{\int_{q_Q^+}^{q_A^+} qf(q)\mathrm{d}q} &\geq \frac{\int_{q_A^+}^1 q_A^+ f(q)\mathrm{d}q}{\int_{q_Q^+}^{q_A^+} q_A^+ f(q)\mathrm{d}q}, \\
&= \frac{\int_{q_A^+}^1 f(q)\mathrm{d}q}{\int_{q_Q^+}^{q_A^+} f(q)\mathrm{d}q},
\end{aligned}$$

proves that equation (7) holds true when $q_A^+ > q_Q^+$.

Second, we have that $q_A^+ > q_Q^+$ implies

$$\left[\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q\right]^k \left[\int_{\frac{1}{2}}^{q_Q^+} (1-q)f(q)\mathrm{d}q\right]^k \geq \left[\int_{\frac{1}{2}}^{q_A^+} (1-q)f(q)\mathrm{d}q\right]^k \left[\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q\right]^k \quad (8)$$

for all $k \in \{0, 1, \ldots\}$ if and only if

$$\begin{aligned}
\frac{\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_A^+} f(q)\mathrm{d}q} &\geq \frac{\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q}, \\
\frac{\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q} &\geq \frac{\int_{\frac{1}{2}}^{q_A^+} f(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q}, \\
\frac{\int_{q_Q^+}^{q_A^+} qf(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q} &\geq \frac{\int_{q_Q^+}^{q_A^+} f(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q},
\end{aligned}$$

which given that

$$\begin{aligned}
\frac{\int_{q_Q^+}^{q_A^+} qf(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q} &\geq \frac{\int_{q_Q^+}^{q_A^+} q_Q^+ f(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_Q^+} q_Q^+ f(q)\mathrm{d}q}, \\
&= \frac{\int_{q_Q^+}^{q_A^+} f(q)\mathrm{d}q}{\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q},
\end{aligned}$$

proves that equation (8) holds true when $q_A^+ > q_Q^+$.

Third, we have that $q_A^+ > q_Q^+$ implies

$$\begin{aligned}
&\left[\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q\right]^m + \left[\int_{\frac{1}{2}}^{q_Q^+} (1-q)f(q)\mathrm{d}q\right]^m \geq \\
&\quad \left[\int_{\frac{1}{2}}^{q_A^+} (1-q)f(q)\mathrm{d}q\right]^m + \left[\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q\right]^m
\end{aligned} \tag{9}$$

for all $m \in \{0, 1, \ldots\}$ if and only if

$$\begin{aligned}
&\left[\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q + \int_{q_Q^+}^{q_A^+} qf(q)\mathrm{d}q\right]^m - \left[\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q\right]^m \geq \\
&\left[\int_{\frac{1}{2}}^{q_Q^+} (1-q)f(q)\mathrm{d}q + \int_{q_Q^+}^{q_A^+} (1-q)f(q)\mathrm{d}q\right]^m - \left[\int_{\frac{1}{2}}^{q_Q^+} (1-q)f(q)\mathrm{d}q\right]^m
\end{aligned}$$

which is always true for $m = 0$ and true for $m \in \{1, 2, \ldots\}$ if and only if

$$\begin{aligned}
&\sum_{l=1}^{m} \binom{m}{l} \left[\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q\right]^{m-l} \left[\int_{q_Q^+}^{q_A^+} qf(q)\mathrm{d}q\right]^l \geq \\
&\sum_{l=1}^{m} \binom{m}{l} \left[\int_{\frac{1}{2}}^{q_Q^+} (1-q)f(q)\mathrm{d}q\right]^{m-l} \left[\int_{q_Q^+}^{q_A^+} (1-q)f(q)\mathrm{d}q\right]^l.
\end{aligned}$$

Since the expression above is true we have that $q_A^+ > q_Q^+$ implies equation (9) as required.

Therefore, we have shown that $q_A^+ > q_Q^+$ implies equations (7), (8) and (9) are true. Hence, from equations (5) and (6) we have that $q_A^+ > q_Q^+$ implies $\pi_t(v, A) - \pi_t(v, Q) \geq 0$.

Equations (2) and (3) together with the fact that $q_A^+ > q_Q^+$ and $p \geq \frac{1}{2}$ imply that

$$\begin{aligned} q_A^+ \left(\pi_t(v, A) + \pi_A(v, A)\right) &\leq \left(1 - q_A^+\right) \left(\pi_t(v, Q) + \pi_A(v, Q)\right), \\ q_A^+ \left(\pi_t(v, Q) + \pi_Q(v, Q)\right) &> \left(1 - q_A^+\right) \left(\pi_t(v, A) + \pi_Q(v, A)\right). \end{aligned}$$

Given that, as we have just shown, $q_A^+ > q_Q^+$ implies $\pi_t(v, A) - \pi_t(v, Q) \geq 0$, the two expressions above imply

$$\left(1 - q_A^+\right) \left(\pi_A(v, Q) - \pi_Q(v, A)\right) > q_A^+ \left(\pi_A(v, A) - \pi_Q(v, Q)\right). \tag{10}$$

Note now that

$$\begin{aligned} \pi_Q(v, A) &= \sum_{j=1}^{\lceil \frac{N}{2} \rceil} \left[\int_{q_A^+}^{1} q f(q) \mathrm{d}q\right]^j \left[\int_{q_Q^+}^{1} (1-q) f(q) \mathrm{d}q\right]^{j-1} \\ &\times \sum_{k=0}^{\lfloor \frac{N-2j+1}{2} \rfloor} \frac{N!}{j! j! k! (N-2j+1-k)!} \\ &\times \left[\int_{\frac{1}{2}}^{q_A^+} q f(q) \mathrm{d}q\right]^k \left[\int_{\frac{1}{2}}^{q_Q^+} (1-q) f(q) \mathrm{d}q\right]^k \\ &\times \left(\left[\int_{\frac{1}{2}}^{q_A^+} q f(q) \mathrm{d}q\right]^{N-2j+1-2k} + \left[\int_{\frac{1}{2}}^{q_Q^+} (1-q) f(q) \mathrm{d}q\right]^{N-2j+1-2k}\right), \\ \pi_A(v, Q) &= \sum_{j=1}^{\lceil \frac{N}{2} \rceil} \left[\int_{q_A^+}^{1} (1-q) f(q) \mathrm{d}q\right]^{j-1} \left[\int_{q_Q^+}^{1} q f(q) \mathrm{d}q\right]^j \\ &\times \sum_{k=0}^{\lfloor \frac{N-2j+1}{2} \rfloor} \frac{N!}{j! j! k! (N-2j+1-k)!} \\ &\times \left[\int_{\frac{1}{2}}^{q_A^+} (1-q) f(q) \mathrm{d}q\right]^k \left[\int_{\frac{1}{2}}^{q_Q^+} q f(q) \mathrm{d}q\right]^k \\ &\times \left(\left[\int_{\frac{1}{2}}^{q_A^+} (1-q) f(q) \mathrm{d}q\right]^{N-2j+1-2k} + \left[\int_{\frac{1}{2}}^{q_Q^+} q f(q) \mathrm{d}q\right]^{N-2j+1-2k}\right), \end{aligned}$$

and similarly for $\pi_A(v,A)$ and $\pi_Q(v,Q)$. Define

$$\begin{aligned}
K_A &= \sum_{k=0}^{\lfloor\frac{N-2j+1}{2}\rfloor} \frac{N!}{j!j!k!(N-2j-k+1)!} \\
&\times \left[\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q\right]^k \left[\int_{\frac{1}{2}}^{q_Q^+} (1-q)f(q)\mathrm{d}q\right]^k \\
&\times \left(\left[\int_{\frac{1}{2}}^{q_A^+} qf(q)\mathrm{d}q\right]^{N-2j+1-2k} + \left[\int_{\frac{1}{2}}^{q_Q^+} (1-q)f(q)\mathrm{d}q\right]^{N-2j+1-2k}\right), \\
K_Q &= \sum_{k=0}^{\lfloor\frac{N-2j+1}{2}\rfloor} \frac{N!}{j!j!k!(N-2j-k+1)!} \\
&\times \left[\int_{\frac{1}{2}}^{q_A^+} (1-q)f(q)\mathrm{d}q\right]^k \left[\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q\right]^k \\
&\times \left(\left[\int_{\frac{1}{2}}^{q_A^+} (1-q)f(q)\mathrm{d}q\right]^{N-2j+1-2k} + \left[\int_{\frac{1}{2}}^{q_Q^+} qf(q)\mathrm{d}q\right]^{N-2j+1-2k}\right).
\end{aligned}$$

Then, equations (8) and (9) imply $K_A \geq K_Q$. Moreover, as $q_A^+ > q_Q^+$ implies equation (7), we have that

$$\begin{aligned}
(1-q_A^+)\left(\pi_A(v,Q)-\pi_Q(v,A)\right) &\leq (1-q_A^+)\left(\int_{q_Q^+}^1 qf(q)\mathrm{d}q - \int_{q_A^+}^1 qf(q)\mathrm{d}q\right)K_Q, \\
q_A^+\left(\pi_A(v,A)-\pi_Q(v,Q)\right) &\geq q_A^+\left(\int_{q_Q^+}^1 (1-q)f(q)\mathrm{d}q - \int_{q_A^+}^1 (1-q)f(q)\mathrm{d}q\right)K_A.
\end{aligned}$$

This means that equation (10) holds only if

$$\begin{aligned}
(1-q_A^+)\left(\int_{q_Q^+}^1 qf(q)\mathrm{d}q - \int_{q_A^+}^1 qf(q)\mathrm{d}q\right)K_Q &\geq q_A^+\left(\int_{q_Q^+}^1 (1-q)f(q)\mathrm{d}q - \int_{q_A^+}^1 (1-q)f(q)\mathrm{d}q\right)K_A, \\
(1-q_A^+)\left(\int_{q_Q^+}^{q_A^+} qf(q)\mathrm{d}q\right) &\geq q_A^+\left(\int_{q_Q^+}^{q_A^+} (1-q)f(q)\mathrm{d}q\right), \\
\left(\int_{q_Q^+}^{q_A^+} qf(q)\mathrm{d}q\right) &\geq \left(\int_{q_Q^+}^{q_A^+} q_A^+f(q)\mathrm{d}q\right),
\end{aligned}$$

holds. However, given that $q_A^+ > q_Q^+$ the expression above is false. This leads to a contradiction, which means that the claim $q_A^+ > q_Q^+$ is false as required. □

Proof of Proposition 1. Define σ_{xy} as the probability with which a random voter votes for $x \in \{A, Q, \emptyset\}$ in state $s \in \{A, Q\}$. Assume first that $N+1$ is even. In this case we have that

N is odd and, therefore, $\pi_t(v, A) = \pi_t(v, Q) = 0$. Moreover, if there is no abstention we can write

$$\begin{aligned}
\pi_A(v, A) &= \binom{N}{\lceil \frac{N}{2} \rceil} \sigma_{AA}^{\lceil \frac{N}{2} \rceil - 1} \sigma_{QA}^{\lceil \frac{N}{2} \rceil}, \\
\pi_A(v, Q) &= \binom{N}{\lceil \frac{N}{2} \rceil} \sigma_{AQ}^{\lceil \frac{N}{2} \rceil - 1} \sigma_{QQ}^{\lceil \frac{N}{2} \rceil}, \\
\pi_Q(v, A) &= \binom{N}{\lceil \frac{N}{2} \rceil} \sigma_{AA}^{\lceil \frac{N}{2} \rceil} \sigma_{QA}^{\lceil \frac{N}{2} \rceil - 1}, \\
\pi_Q(v, Q) &= \binom{N}{\lceil \frac{N}{2} \rceil} \sigma_{AQ}^{\lceil \frac{N}{2} \rceil} \sigma_{QQ}^{\lceil \frac{N}{2} \rceil - 1}.
\end{aligned}$$

Consider an equilibrium of Type 1 with no abstention, i.e. $q_Q^- = q_Q^+ = \bar{q}$. The fact that $\pi_t(v, A) = \pi_t(v, Q) = 0$ together with equations (2) and (3) imply that

$$\begin{aligned}
p(1 - \bar{q})\pi_A(v, A) &= (1 - p)\bar{q}\pi_A(v, Q), \\
p(1 - \bar{q})\pi_Q(v, A) &= (1 - p)\bar{q}\pi_Q(v, Q),
\end{aligned}$$

which implies

$$\begin{aligned}
\frac{\sigma_{QA}}{\sigma_{AA}} &= \frac{\sigma_{QQ}}{\sigma_{AQ}}, \\
\frac{1 - \sigma_{QQ}}{\sigma_{QQ}} &= \frac{1 - \sigma_{QA}}{\sigma_{QA}}, \\
\sigma_{QQ} &= \sigma_{QA}, \\
\int_{\bar{q}}^1 q f(q) \mathrm{d}q &= \int_{\bar{q}}^1 (1 - q) f(q) \mathrm{d}q.
\end{aligned}$$

The expression above is true if and only if either $\bar{q} = 1$ or $f(q) = 0$ for all $q \in (\bar{q}, 1)$. Both case imply that effectively all voters vote for A, a situation that cannot be an equilibrium as if all voters vote for A a voter is never pivotal and, hence, he is indifferent between whether to vote for either candidate or to abstain. By assumption (how indifference ties are broken) in this case voters who receive signal Q vote for Q which represents a contradiction to the fact that all voters vote for A.

Consider now an equilibrium of Type 2 with no abstention, i.e. $q_A^+ = q_Q^+ = \frac{1}{2}$. Proceeding as above we have that $\sigma_{QQ} = \sigma_{QA}$, which in turn implies that $\int_{\frac{1}{2}}^1 q f(q) \mathrm{d}q = \int_{\frac{1}{2}}^1 (1 - q) f(q) \mathrm{d}q$. This is true if and only if $f(q) = 0$ for all $q \in (0, 1)$, which contradicts the fact that f is integrable and $\int_{\frac{1}{2}}^1 f(q) \mathrm{d}q = 1$.

Therefore, we have just shown that there can be no equilibrium without abstention when $N + 1$ is even. Consider now the case where $N + 1$ is odd. In this case the fact that there is no abstention implies that $\pi_A(v,A) = \pi_A(v,Q) = \pi_Q(v,A) = \pi_Q(v,Q) = 0$ and

$$\begin{aligned}\pi_t(v,A) &= \binom{N}{\frac{N}{2}} \sigma_{AA}^{\frac{N}{2}} \sigma_{QA}^{\frac{N}{2}},\\ \pi_t(v,Q) &= \binom{N}{\frac{N}{2}} \sigma_{AQ}^{\frac{N}{2}} \sigma_{QQ}^{\frac{N}{2}}.\end{aligned}$$

Consider a situation with no abstention such that $q_Q^- = q_Q^+ = \bar{q} \in \left[\frac{1}{2}, 1\right)$. Define the function $\eta(\bar{q}) = p(1-\bar{q})\pi_t(v,A) - (1-p)\bar{q}\pi_t(v,Q)$, we have that

$$\begin{aligned}\eta(\bar{q}) &= p(1-\bar{q})\binom{N}{\frac{N}{2}} \sigma_{AA}^{\frac{N}{2}} \sigma_{QA}^{\frac{N}{2}} - (1-p)\binom{N}{\frac{N}{2}} \sigma_{AQ}^{\frac{N}{2}} \sigma_{QQ}^{\frac{N}{2}},\\ &= \binom{N}{\frac{N}{2}} \left((p(1-\bar{q}))^{\frac{2}{N}} \sigma_{AA}\sigma_{QA} - ((1-p)\bar{q})^{\frac{2}{N}} \sigma_{AQ}\sigma_{QQ} \right).\end{aligned}$$

Note that η is continuous. Moreover, it is true that if $\bar{q} = \frac{1}{2}$ then $\sigma_{AA}\sigma_{QA} = \sigma_{AQ}\sigma_{QQ}$ and, hence, $\eta\left(\frac{1}{2}\right) = \binom{N}{\frac{N}{2}}(2p-1)\sigma_{AA}\sigma_{QA} \geq 0$. On top of that,

$$\begin{aligned}\lim_{\varepsilon\to 0} \eta(1-\varepsilon) &= \binom{N}{\frac{N}{2}} \lim_{\varepsilon\to 0} (p\varepsilon)^{\frac{2}{N}} \sigma_{AA} \int_{1-\varepsilon}^{1} (1-q)f(q)\mathrm{d}q - ((1-p)(1-\varepsilon))^{\frac{2}{N}} \sigma_{AQ} \int_{1-\varepsilon}^{1} qf(q)\mathrm{d}q\\ &= \binom{N}{\frac{N}{2}} \lim_{\varepsilon\to 0} (p\varepsilon)^{\frac{2}{N}} \sigma_{AA}\varepsilon(F(1) - F(1-\varepsilon))\\ &\quad - ((1-p)(1-\varepsilon))^{\frac{2}{N}} \sigma_{AQ}(1-\varepsilon)(F(1) - F(1-\varepsilon))\\ &\leq 0.\end{aligned}$$

Therefore, because η is continuous, there exists a $\bar{q} \in \left(\frac{1}{2}, 1\right)$ such that $\eta(\bar{q}) = 0$ which implies that $p(1-\bar{q})\pi_t(v,A) - (1-p)\bar{q}\pi_t(v,Q)$. This, together with equations (2) and (3) and the fact that $\pi_A(v,A) = \pi_A(v,Q) = \pi_Q(v,A) = \pi_Q(v,Q) = 0$ implies that the cutpoints $q_Q^- = q_Q^+ = \bar{q}$ are an equilibrium. □

The following Lemma from Feddersen and Pessendorfer (1996) is used in the proof of the Theorem 2.

Lemma 2 (Lemma 0 in Feddersen and Pessendorfer (1996)). *Let $(a_N, b_N, c_N)_{N=1}^{\infty}$ a sequence that satisfies $(a_N, b_N, c_N) \in [0,1]^3$ and $a_N < b_N - \delta$ and $\delta < c_N$ for all N and some $\delta > 0$. Then, for $i = 0, 1$ as $N \to \infty$*

$$\frac{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!j!(N-2j-i)!} c_N^{N-2j-i} a_N^j}{\sum_{j=0}^{\frac{N}{2}-i} \frac{N!}{(j+i)!j!(N-2j-i)!} c_N^{N-2j-i} b_N^j} \to 0.$$

Proof of Theorem 2. **Equilibrium of Type 1**

First we show that as $N \to \infty$ the equilibrium of Type 1 is such that $\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q \to 0$. Assume for now that there exists a $\rho > 0$ such that $\int_{q_Q^+}^{1} f(q)\mathrm{d}q > \rho$ for all N and consider an equilibrium of Type 1 and assume that there exists a $\varepsilon > 0$ such that either $\int_{\frac{1}{2}}^{q_Q^-} f(q)\mathrm{d}q \geq \varepsilon$ or $\int_{q_Q^-}^{q_Q^+} f(q)\mathrm{d}q \geq \varepsilon$ for all N. We have that there exists a $\delta_1 > 0$ such that $\sigma_{QQ}\sigma_{AQ} - \delta_1 > \sigma_{AA}\sigma_{QA}$ if and only if

$$\begin{aligned}
&\int_{q_Q^+}^{1} qf(q)\mathrm{d}q \left(\int_{\frac{1}{2}}^{1} (1-q)f(q)\mathrm{d}q + \int_{\frac{1}{2}}^{q_Q^-} qf(q)\mathrm{d}q\right) - \delta_1 > \\
&\int_{q_Q^+}^{1} (1-q)f(q)\mathrm{d}q \left(\int_{\frac{1}{2}}^{1} qf(q)\mathrm{d}q + \int_{\frac{1}{2}}^{q_Q^-} (1-q)f(q)\mathrm{d}q\right), \\
&\int_{q_Q^+}^{1} qf(q)\mathrm{d}q \left(\int_{q_Q^-}^{1} (1-q)f(q)\mathrm{d}q + \int_{\frac{1}{2}}^{q_Q^-} f(q)\mathrm{d}q\right) - \delta_1 > \\
&\qquad \int_{q_Q^+}^{1} (1-q)f(q)\mathrm{d}q \left(\int_{q_Q^-}^{1} qf(q)\mathrm{d}q + \int_{\frac{1}{2}}^{q_Q^-} f(q)\mathrm{d}q\right), \\
&\int_{q_Q^+}^{1} qf(q)\mathrm{d}q \left(\int_{q_Q^-}^{q_Q^+} (1-q)f(q)\mathrm{d}q + \int_{\frac{1}{2}}^{q_Q^-} f(q)\mathrm{d}q\right) - \delta_1 > \\
&\qquad \int_{q_Q^+}^{1} (1-q)f(q)\mathrm{d}q \left(\int_{q_Q^-}^{q_Q^+} qf(q)\mathrm{d}q + \int_{\frac{1}{2}}^{q_Q^-} f(q)\mathrm{d}q\right).
\end{aligned}$$

A necessary condition for this is

$$\begin{aligned}
\int_{q_Q^+}^{1} (q - q_Q^+)f(q)\mathrm{d}q \int_{q_Q^-}^{q_Q^+} f(q)\mathrm{d}q + \int_{q_Q^+}^{1} (2q-1)f(q)\mathrm{d}q \int_{\frac{1}{2}}^{q_Q^-} f(q)\mathrm{d}q - \delta_1 &> 0, \\
\int_{q_Q^+}^{1} (q - q_Q^+)f(q)\mathrm{d}q \int_{q_Q^-}^{q_Q^+} f(q)\mathrm{d}q + (2q_Q^- - 1) \int_{q_Q^+}^{1} f(q)\mathrm{d}q \int_{\frac{1}{2}}^{q_Q^-} f(q)\mathrm{d}q - \delta_1 &> 0.
\end{aligned}$$

By assumption $\int_{q_Q^+}^{1} f(q)\mathrm{d}q > \rho$ for some $\rho > 0$. Therefore, if $\int_{\frac{1}{2}}^{q_Q^-} f(q)\mathrm{d}q \geq \varepsilon$ then $q_Q^- \geq F^{-1}(\varepsilon)$ and the expression above is true for any $\delta_1 \in (0, (2F^{-1}(\varepsilon) - 1)\rho\varepsilon)$.[1]

Assume $\int_{\frac{1}{2}}^{q_Q^-} f(q)\mathrm{d}q < \varepsilon$, which implies that $\int_{q_Q^-}^{q_Q^+} f(q)\mathrm{d}q \geq \varepsilon$. Note that $\int_{q_Q^+}^{1} f(q)\mathrm{d}q > \rho$ for some $\rho > 0$ implies that for all $\bar{\rho} \in (0, \rho)$ there exists a $\beta > 0$ such that $\int_{q_Q^+ + \beta}^{1} f(q)\mathrm{d}q > \bar{\rho}$, fix such ρ and consider its corresponding β. Thus, a necessary condition for $\sigma_{QQ}\sigma_{AQ} - \delta_1 >$

[1] F^{-1} exists because f is integrable and, hence, F is continuous.

$\sigma_{AA}\sigma_{QA}$ is

$$\begin{aligned}\int_{q_Q^+}^{1}(q-q_Q^+)f(q)\mathrm{d}q\int_{q_Q^-}^{q_Q^+}f(q)\mathrm{d}q-\delta_1 &> 0,\\ \int_{q_Q^++\beta}^{1}\beta f(q)\mathrm{d}q\int_{q_Q^-}^{q_Q^+}f(q)\mathrm{d}q-\delta_1 &> 0,\\ \beta\bar{\rho}\int_{q_Q^-}^{q_Q^+}f(q)\mathrm{d}q-\delta_1 &> 0,\\ \beta\bar{\rho}\varepsilon-\delta_1 &> 0.\end{aligned}$$

Hence, for any $\delta_1 \in (0, \min\{(2F^{-1}(\varepsilon)-1)\rho\varepsilon, \beta\bar{\rho}\varepsilon\})$ we have that $\sigma_{QQ}\sigma_{AQ}-\delta_1 > \sigma_{AA}\sigma_{QA}$. If $\int_{q_Q^-}^{q_Q^+}f(q)\mathrm{d}q < \varepsilon$ for all $\varepsilon > 0$ then $\sigma_{\emptyset s}\to 0$. Since $\sigma_{QQ}\sigma_{AQ}-\delta_1 > \sigma_{AA}\sigma_{QA}$ implies

$$\lim_{N\to\infty}\frac{(\sigma_{AA}\sigma_{QA})^{\frac{N}{2}-i}}{(\sigma_{QQ}\sigma_{AQ})^{\frac{N}{2}-i}} \quad\to\quad 0$$

for $i=0,1$, we have $\frac{\pi_t(v,A)}{\pi_t(v,Q)}\to 0$ and, if $\int_{q_Q^-}^{q_Q^+}f(q)\mathrm{d}q \neq 0$, also that $\frac{\pi_A(v,A)}{\pi_A(v,Q)}\to 0$ and $\frac{\pi_Q(v,A)}{\pi_Q(v,Q)}\to 0$. If $\int_{q_Q^-}^{q_Q^+}f(q)\mathrm{d}q = 0$ then $\pi_A(v,A)=\pi_A(v,Q)=\pi_Q(v,A)=\pi_Q(v,Q)=0$.

On the other hand, if $\int_{q_Q^-}^{q_Q^+}f(q)\mathrm{d}q \geq \varepsilon$ then there exists a $\delta_2 > 0$ such that $\sigma_{\emptyset s} > \delta_2$. Define $\delta = \min\{\delta_1, \delta_2\}$. By Lemma 2 we have that as N grows large $\frac{\pi_t(v,A)}{\pi_t(v,Q)}\to 0$, $\frac{\pi_A(v,A)}{\pi_A(v,Q)}\to 0$ and $\frac{\pi_Q(v,A)}{\pi_Q(v,Q)}\to 0$.

Therefore, equations (2) and (3) then imply $q_Q^-\to\frac{1}{2}$ and $q_Q^+\to\frac{1}{2}$ which in turn implies $\int_{\frac{1}{2}}^{q_Q^-}f(q)\mathrm{d}q\to 0$ and $\int_{q_Q^-}^{q_Q^+}f(q)\mathrm{d}q\to 0$, which contradicts the fact that either $\int_{\frac{1}{2}}^{q_Q^-}f(q)\mathrm{d}q\geq\varepsilon$ or $\int_{q_Q^-}^{q_Q^+}f(q)\mathrm{d}q\geq\varepsilon$ for a fixed ε.

Assume now that for all $\rho > 0$ there exists an $\bar{N}$ such that for all $N \geq \bar{N}$, we have $\int_{q_Q^+}^{1}f(q)\mathrm{d}q \leq \rho$. Fix a $\rho \in \left(0,\frac{1}{2}\right)$ and the corresponding $\bar{N}$. This means that at most a fraction ρ of voters vote for Q for any $N \geq \bar{N}$. In equilibrium of Type 1, $q_A^+ = \frac{1}{2}$ and all voters who receive signal A vote for A. Hence, if a voter is pivotal it must be that at most a fraction ρ of voters plus one received signal A. Since ρ can be chosen as small as desired and N as large as desired, we have that if a voter is pivotal then the fraction of voters who received signal A is negligible compared to the fraction of voters who received signal Q and, hence, the probability that the state of nature is Q converges to one when a voter is pivotal by law of large numbers. By equation (3) this implies $q_Q^+\to\frac{1}{2}$ which contradicts the fact that $\int_{q_Q^+}^{1}f(q)\mathrm{d}q\leq\rho$.

Equilibrium of Type 2

We prove next that in an equilibrium of Type 2 we must have $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q \to 0$. Assume for now that there exists a $\rho > 0$ such that $\int_{q_Q^+}^{1} f(q)\mathrm{d}q > \rho$ for all N. Consider an equilibrium of Type 2 and suppose there exists a $\varepsilon > 0$ such that $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q \geq \varepsilon$ for all N. We have that a necessary condition for there to be a $\delta_1 > 0$ such that $\sigma_{QQ}\sigma_{AQ} - \delta_1 > \sigma_{AA}\sigma_{QA}$ is

$$
\begin{aligned}
\int_{q_A^+}^{1} qf(q)\mathrm{d}q \int_{q_Q^+}^{1} (1-q)f(q)\mathrm{d}q - \delta_1 &> \int_{q_A^+}^{1} (1-q)f(q)\mathrm{d}q \int_{q_Q^+}^{1} qf(q)\mathrm{d}q, \\
\int_{q_A^+}^{1} qf(q)\mathrm{d}q \int_{q_Q^+}^{1} f(q)\mathrm{d}q - \delta_1 &> \int_{q_A^+}^{1} f(q)\mathrm{d}q \int_{q_Q^+}^{1} qf(q)\mathrm{d}q, \\
\int_{q_Q^+}^{1} qf(q)\mathrm{d}q \int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q - \delta_1 &> \int_{q_Q^+}^{1} f(q)\mathrm{d}q \int_{q_A^+}^{q_Q^+} qf(q)\mathrm{d}q, \\
\int_{q_Q^+}^{1} (q - q_Q^+)f(q)\mathrm{d}q \int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q &> \delta_1.
\end{aligned}
$$

Since $\int_{q_Q^+}^{1} f(q)\mathrm{d}q > \rho$ for some $\rho > 0$ then for all $\bar{\rho} \in (0, \rho)$ there exists a $\beta > 0$ such that $\int_{q_Q^+ + \beta}^{1} f(q)\mathrm{d}q > \bar{\rho}$, fix such ρ and consider its corresponding β. Moreover, by assumption $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q \geq \varepsilon$. Thus, if we choose any $\delta_1 \in (0, \beta\hat{\rho}\varepsilon)$ then $\sigma_{QQ}\sigma_{AQ} - \delta_1 > \sigma_{AA}\sigma_{QA}$.

Given that $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q \geq \varepsilon$ it is true that $\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q \geq \varepsilon$ and, hence, there exists a $\delta_2 > 0$ such that $\sigma_{\emptyset s} > \delta_2$. Define $\delta = \min\{\delta_1, \delta_2\}$. Then by Lemma 2 we have then that as N grows large $\frac{\pi_t(v,A)}{\pi_t(v,Q)} \to 0$, $\frac{\pi_A(v,A)}{\pi_A(v,Q)} \to 0$ and $\frac{\pi_Q(v,A)}{\pi_Q(v,Q)} \to 0$. Equations (2) and (3) then imply $q_Q^+ \to \frac{1}{2}$ which in turn implies $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q \to 0$, this contradicts the fact that $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q > \varepsilon$ for a fixed ε.

Assume now that for all $\rho > 0$ there exists an $\bar{N}$ such that for all $N \geq \bar{N}$, we have $\int_{q_Q^+}^{1} f(q)\mathrm{d}q \leq \rho$. Fix a $\rho \in \left(0, \frac{1}{2}\right)$ and the corresponding $\bar{N}$. This means that at most a fraction ρ of voters vote for Q for any $N \geq \bar{N}$. In equilibrium of Type 2, $q_Q^- = \frac{1}{2}$ and we have two possibilities. If for all $\rho > 0$ there exists an N such that for all $N \geq \bar{N}$ we have $\int_{q_A^+}^{1} f(q)\mathrm{d}q \leq \rho$, then $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q \leq \rho$ which is what the result in the Theorem states. If, on the other hand, there exists a $\varepsilon > 0$ such that $\int_{q_A^+}^{1} f(q)\mathrm{d}q \geq \varepsilon$ for all N, then at least a fraction ε of voters who receive signal A vote for A. If a voter is pivotal, it must be because at most a fraction ρ of voters plus one receive signal A. However, since ρ can be chosen as small as desired and $N + 1$ as large as desired, the fraction of voters who receive signal A must be arbitrarily small as otherwise a fraction ε of them vote for A against the fraction ρ that vote for Q and the voter is not pivotal. Therefore, the probability that the state of nature is Q converges to one when a voter is pivotal by law of large numbers. By equation (3) this implies $q_Q^+ \to \frac{1}{2}$ which contradicts the fact that $\int_{q_Q^+}^{1} f(q)\mathrm{d}q \leq \rho$. $\square$

Proof of Proposition 2. Using the proof of Theorem 2 we have that either $\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q \to 0$ or $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q \to 0$. Assume first that $\int_{\frac{1}{2}}^{q_Q^+} f(q)\mathrm{d}q \to 0$. In this case almost all voters vote for the candidate that coincides with their signal (for all $\delta > 0$ there exists a N for which the proportion of voters who do not is smaller than δ). Therefore, by law of large numbers the proportion of voters who vote for the candidate that coincides with the state of nature is μ while the proportion of voters who vote for the other candidate is $1-\mu$. Since $\mu > \frac{1}{2}$ implies that there exists a $\varepsilon > 0$ such that $\mu - \varepsilon > \frac{1}{2}$, we have that most voters vote for the candidate that coincides with the state of nature which gives the desired result.

Assume now $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q \to 0$. In this case all voters who do not abstain vote for the candidate that coincides with their signal and, furthermore, the decision on whether to vote or not is independent on the signal received (for all $\delta > 0$ there exists a N for which the number of voters choose whether to abstain or not depending on their signal is smaller than δ). Therefore, by law of large numbers the proportion of voters who vote for the candidate that coincides with the state of nature is $(N+1)\int_{q_Q^+}^{1} qf(q)\mathrm{d}q$ while the proportion of voters who vote for the other candidate is $(N+1)\int_{q_Q^+}^{1}(1-q)f(q)\mathrm{d}q$. If there exists a $\rho > 0$ such that $\int_{q_Q^+}^{1} f(q)\mathrm{d}q > \rho$ for all $N+1$ then for all $\bar{\rho} \in (0,\rho)$ there exists a $\beta > 0$ such that $\int_{q_Q^+ + \beta}^{1} f(q)\mathrm{d}q > \bar{\rho}$ we have

$$\begin{aligned}
\int_{q_Q^+}^{1} qf(q)\mathrm{d}q - \int_{q_Q^+}^{1}(1-q)f(q)\mathrm{d}q &= \int_{q_Q^+}^{1}(2q-1)f(q)\mathrm{d}q \\
&\geq \int_{q_Q^+ + \beta}^{1}(2q-1)f(q)\mathrm{d}q \\
&\geq 2\beta \int_{q_Q^+ + \beta}^{1} f(q)\mathrm{d}q \\
&> 2\beta\bar{\rho}.
\end{aligned}$$

Thus, most voters vote for the candidate that coincides with the state of nature as we wanted to show.

Consider now the case where for all $\rho > 0$ there exists a $\bar{N}$ such that $\int_{q_Q^+}^{1} f(q)\mathrm{d}q \leq \rho$ for all $n > \bar{N}$. By monotonicity of F and the fact that $\int_{q_A^+}^{q_Q^+} f(q)\mathrm{d}q \to 0$ we have $q_A^+ \to q_Q^+ \to 1$.

Moreover,

$$\begin{aligned}
\lim_{q_Q^+ \to 1} \frac{\sigma_{AQ}}{\sigma_{QQ}} &= \lim_{q_Q^+ \to 1} \frac{\int_{q_Q^+}^{1} (1-q) f(q) \mathrm{d}q}{\int_{q_Q^+}^{1} q f(q) \mathrm{d}q}, \\
&= \lim_{q_Q^+ \to 1} \frac{(1-q_Q^+) f(q_Q^+)}{q_Q^+ f(q_Q^+)}, \\
&= \lim_{q_Q^+ \to 1} \frac{1}{q_Q^+} - 1, \\
&= 0,
\end{aligned}$$

and similarly $\lim_{q_A^+ \to 1} \frac{\sigma_{QA}}{\sigma_{AA}} = 0$, where we have used L'Hôpital's rule for computing the limit above. That is, the probability that a random voter votes for the candidate that does not match the state of nature is insignificant compared to the probability that a random voter votes for the candidate that does, which implies $P(V = S) \to 1$. □