On the measurement of multidimensional poverty in multiple domain contexts

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Abstract: We develop the measurement of multidimensional poverty for the case in which the different dimensions taken into account are partitioned in several domains–an issue with crucial implications for the identification and aggregation of the poor which has been neglected in the literature. First, we introduce a general method to identify the poor that makes room for the non-trivial interactions that might exist between dimensions depending on the domains they belong to. Consistent with the former, we then present a new aggregation method that allows for the possibility of having domain-specific elasticities of substitution among pairs of dimensions. Our empirical findings using 48 Demographic and Health Surveys across the developing world suggest that when considering the alternative identification and aggregation methodologies proposed here, the set of households that are identified as poor and the corresponding multidimensional poverty levels can differ to a considerable extent with respect to currently existing approaches.

Keywords: Multidimensional poverty measurement, Multiple domains, Identification, Aggregation, Counting approach, Complementarity, Substitutability.

JEL Classification: I3; I32; D63; O1

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1. Introduction

Who is poor and who is not poor? How poor are the poor? These are the fundamental 'identification' and 'aggregation' questions suggested by Amartya Sen that must be addressed before any poverty eradication program can be implemented (Sen 1976). While the answer to these questions has been quite satisfactorily addressed when poverty is measured in the space of income distributions (after the seminal contribution by Sen in 1976 the literature on income poverty measurement is huge and is based on a very solid footing – see, for instance, Chakravarty 2009 for a recent survey on the topic), matters become much more complicated when the poverty status and its levels are determined using several dimensions at the same time. When well-being is conceptualized using both monetary and non-monetary attributes and the corresponding poverty measures are multidimensional (see Bourguignon and Chakravarty 2003, Alkire and Foster 2011 and Alkire et al 2015 for a motivation of the approach), it is customary to partition the variables composing such measures in mutually exclusive *domains* (e.g: the domains of 'Health', 'Education' or 'Standard of Living', with several variables within each domain). Such partition aims at imposing certain coherence and structure to the variables one is dealing with by clustering them in conceptually related areas. Our main concern here is that the separation of variables across domains in currently existing multidimensional poverty measures is merely nominal and, contrary to what one might a priori expect, has no implications whatsoever when determining: (i) who is poor and who is not, and (ii) the corresponding poverty levels. Yet, we contend that these two exercises are strongly influenced by the domains partition, which implicitly imposes a hierarchical structure between variables – with the variables belonging to the same (resp. alternative) domain being more similar (resp. dissimilar) among themselves – that has been sistematically ignored in current approaches to multidimensional poverty measurement. In this paper we introduce a flexible framework where the separation of variables in multiple domains plays a central role both in the 'identification' and 'aggregation' steps.

Assuming one is able to define dimension-specific poverty thresholds to determine whether individuals are deprived or not in the corresponding dimensions (that is: when one works in the deprivation space[2]), there are currently three well-known approaches for the identification of the poor in a multi-attribute framework. According to the 'union approach', an individual is said to be multidimensionally poor if there is at least one dimension in which the person is deprived. At the other extreme, the 'intersection approach' states that an individual is 'poor' if s/he is deprived in all dimensions simultaneously. Respectively, these approaches are likely to over-estimate and under-estimate the set of individuals that should be considered as 'poor', particularly when the number of dimensions considered is large. While the union approach might include individuals that are only deprived in one relatively unimportant dimension among many, the intersection approach might fail to identify those individuals that are experiencing extensive but not universal deprivation. A natural alternative suggested by Alkire and Foster (2011) (which is inspired by the work of Atkinson 2003) is to use an intermediate cutoff level that lies somewhere between the two extremes. According to the so-called 'intermediate approach', an individual is poor if the number of dimensions in which s/he is deprived is above a given poverty threshold – denoted as k – that is exogenously chosen by the analyst (note that both the union and intersection approaches are particular cases of the intermediate approach). Since this counting methodology uses

[2] Whenever the different dimensions are commensurable, some scholars suggest working in the attainment space (that is: aggregate individuals' attainments into a unidimensional welfare indicator and identify them as 'poor' whenever their aggregate well-being level falls below a given poverty threshold). This is the route advocated by Ravallion (2011) and implicitly used by Duclos et al (2006). As argued by Alkire and Foster (2011) and many others, a key conceptual drawback of viewing multidimensional poverty through a unidimensional lens is the loss of information on the dimension-specific shortfalls. In addition, the problem of identification of the poor becomes trivial in the unidimensional setting, so it will not be considered in this paper.

deprivation thresholds *within* dimensions and an overall poverty threshold k *across* dimensions, it has been denoted as the 'dual cutoff' identification method – also referred to in the literature as the 'counting approach', the 'AF identification method', or the 'AF method'.

There is much to praise in the dual cutoff method and there are several factors that have contributed to its widespread acceptance and implementation – indeed, it is the state-of-the-art methodology currently employed by researchers, policy-makers and institutions around the world to identify the poor in multidimensional settings.[3] These factors include (i) its flexibility to accommodate many reasonable alternatives lying between the – admittedly extreme – 'union' and 'intersection' perspectives, (ii) the possibility of incorporating the ordinal data that commonly arise in multidimensional settings and (iii) its plasticity in adapting to alternative contexts where different variables are available.

However, since the use of the dual cutoff method is becoming so predominant it is also important to highlight some of its limitations. The counting approach that underlies the AF identification method is a procedure that, roughly speaking, adds up the number of deprivations across dimensions to decide whether the individuals experiencing them should be considered poor or not. While this is reflective of the current state of the literature, such aggregation exercises are a crude way of proceeding that sidestep many of the subtle and more qualitative considerations that have to be incorporated when deciding what combinations of deprivations should be included in the identification of the poor. Inter alia, currently existing identification methods fail to take into consideration the hierarchical structure of multidi-

[3] To illustrate: the AF method is currently being implemented by the governments of Bhutan, Brazil, Chile, China, Colombia, El Salvador, Malaysia, Mexico or the Philippines to complement their income poverty measures, with many other countries to follow soon, and the United Nations Development Program (UNDP) has since 2010 annually published the worldwide distribution of the Multidimensional Poverty Index – which is based on the AF method. The book 'Multidimensional Poverty Measurement and Analysis', published in 2015 by Oxford University Press, describes in detail the AF method and its applications and will further contribute to settle and reinforce the global diffusion of the approach.

mensional poverty indices, where different variables are nested within mutually exclusive domains. Such partitions imply some degree of similarity between variables within the same domain and some degree of dissimilarity with respect to those belonging to different domains, an issue that has a bearing on the ways to identify the poor in multidimensional settings. The flexible framework introduced in this paper – containing the counting approach as a particular case – takes into account the domains structure and allows modelling previously unexplored ways of identifying the multidimensionally poor. In addition, our approach gives ample room to model non-trivial compensation patterns taking place between deprived and non-deprived attributes either belonging to the same or to the different domains in which composite indices are typically partitioned (see sections 2 and 3 for the formal definitions). As a consequence, our approach allows introducing different levels of complementarity / substitutability between different groups of variables when measuring poverty levels – an improvement with respect to the measurement strategies introduced so far in multidimensional poverty. An obvious – yet crucial – implication of our results is that the set of poor individuals targeted by the dual cutoff method and the other criteria proposed in this paper do not necessarily coincide, an issue that over- or under-represents certain sectors of the population as potential beneficiaries of poverty eradication programs worldwide and which generates very different estimates in the aggregate levels of multidimensional poverty.

Another attractive feature of the AF method is the alleged possibility of knowing the contribution of each dimension to overall poverty levels once the identification step is over (see Alkire and Foster 2011: 481-482). According to this model, it is possible to conclude that deprivations in variable V_i have contributed to overall multidimensional poverty levels by, e.g., $v_i\%$ – thereby giving an apparently clear and appealing message to researchers or policy-makers aiming to identify the single most important dimension that contributes to poverty so as to eradicate it in the most effective way. We argue that this dimension-decomposability approach might give a misleading picture of the ways in which multidimensional poverty is articulated because it disregards the *joint* patterns of deprivation that individuals must experience in order to classify them as poor. We suggest complementing the potentially misleading dimension-decomposability property by another decomposability property – referred to as 'profile decomposability' – that is naturally derived from the identification method suggested in this paper. Profile decomposability is superior to its dimension-wise counterpart in informing about the structure of multidimensional poverty and in conveying clearer and more focused messages to those working toward its eradication. The rest of the article is organized as follows. The next section introduces notation and formally describes the problem. Section 3 discusses new methods to identify the multidimensionally poor in a multiple domain context and Section 4 discusses the implications that such methods have for poverty measurement. Section 5 presents two empirical applications illustrating our results and Section 6 provides some concluding remarks. The proofs are relegated to the appendix.

2. Notation and Definitions

We introduce some notation that are used in the rest of the paper. Let N be the set of individuals[4] and D the set of dimensions (also referred to as 'attributes') under consideration, with $n := |N| \geq 1, d := |D| \geq 2$. For any natural number $G \leq \lfloor |D|/2 \rfloor$, let $\Pi_{D,G}$ denote the set of partitions of D into G exhaustive and mutually exclusive groups $D_1, \ldots, D_G$ (i.e.: $D_i \cap D_j = \emptyset \forall i \neq j$ and $D = \bigcup_{g=1}^{g=G} D_g$) where each group has at least two members (i.e.: $d_g := |D_g| \geq 2 \forall g$). A generic element of $\Pi_{D,G}$ is denoted as $(D_1, \ldots, D_G)$. Throughout this paper, let $X := \{0, 1\}$. For any natural number $m \in \mathbb{N}$, let X^m denote the set of

[4] The word 'individuals' refers to the basic unit of analysis – even if such unit involves households or other aggregates.

$m-$dimensional vectors whose elements can either be 0 or 1, that is $X^m := \{0,1\}^m$. Given any vector $\mathbf{x} = (x_1, \ldots, x_m) \in X^m$, the number $\sum x_i$ will be denoted as the *size* of $\mathbf{x}$. Let $\tau_m : X^m \rightarrow 2^{\{1,\ldots,m\}}$ be the function that for any $\mathbf{x} = (x_1, \ldots, x_m) \in X^m$ assigns the set of dimensions within $\{1, \ldots, m\}$ for which $x_i = 1$. The inverse function of τ_m (denoted as τ_m^{-1}) converts any subset $S \subseteq \{1, \ldots, m\}$ into the $m-$dimensional vector in X^m whose $i-$th element is equal to 1 if $i \in S$ and 0 otherwise. To illustrate: if $d = 5$, one can define the partition of $D = \{1, \ldots, 5\}$ into $D_1 = \{1, 2\}$ and $D_2 = \{3, 4, 5\}$. Then $(D_1, D_2) \in \Pi_{D,2}$, $\tau_5^{-1}(D_1) = (11000)$ and $\tau_5^{-1}(D_2) = (00111)$. $\mathbb{R}^q, \mathbb{R}^q_+, \mathbb{R}^q_{++}$ are the $q-$dimensional Euclidean space and its nonnegative and strictly positive counterpart respectively. Let $\mathbf{a} = (a_1, \ldots, a_d)$ be a $d-$dimensional vector of positive numbers summing up to 1, whose j^{th} coordinate a_j is interpreted as the normalized weight associated with dimension j. The set of all possible $d-$dimensional weigthing schemes summing up to 1 is called the *$d-$dimensional simplex*, and will be denoted by Δ_d (i.e.: $\Delta_d = \left\{(a_1, \ldots, a_d) \in \mathbb{R}^d_+ | \sum_i a_i = 1\right\}$). $[0, 1]$ is the closed interval of real numbers between 0 and 1.

The achievement of individual i in attribute j will be denoted by y_{ij}. The results in this paper are independent of the measurement scale of our attributes: They can either be ordinal or cardinal. Therefore, the range of values of y_{ij}, denoted as I_j, can either be the set of non-negative real numbers $\mathbb{R}_+$ (an almost universal assumption in both unidimensional and multidimensional cardinal poverty measurement) or a discrete subset of it. The vector $\mathbf{y}_i = (y_{i1}, \ldots, y_{id}) \in I_1 \times \cdots \times I_d$ contains individual i's achievements across dimensions and is called the *achievement vector*. In this context, an *achievement matrix* M is a $n \times d$ matrix containing the achievement vectors of n individuals in the different rows. The set of all $n \times d$ achievement matrices is denoted as $\mathcal{M}_{n\times d}$. More generally, we define $\mathcal{M} := \bigcup\limits_{n\in\mathbb{N}} \bigcup\limits_{d\in\mathbb{N}} \mathcal{M}_{n\times d}$. For each attribute j we consider a poverty threshold z_j representing a minimum attainment in that attribute that is needed for subsistence – which in this paper we consider as exogenously given. The vector of dimension-specific poverty thresholds is denoted by $\mathbf{z} = (z_1, \ldots, z_d) \in I_1 \times \cdots \times I_d$. Whenever $y_{ij} \leq z_j$, we say that individual i is *deprived* in attribute j.

In this context, a *multidimensional poverty index* f is a non-trivial function that converts an element M from the space of achievement matrices $\mathcal{M}$ and a vector of dimension-specific deprivation thresholds $\mathbf{z}$ (with as many elements as the number of columns in M) into a real number $f(M; \mathbf{z})$ indicating the extent of poverty in the corresponding distribution. According to Sen (1976), when defining a specific multidimensional poverty index f one should first identify who is poor and who is not and then aggregate the information about the extent of deprivation of the poor. In the remainder of this section we will deal with the issue of identification, leaving until section 4 the issue of aggregation.

Following Bourguignon and Chakravarty (2003), an *identification function* $\rho : (I_1 \times \cdots \times I_d) \times (I_1 \times \cdots \times I_d) \to \{0, 1\}$ is a non-trivial mapping from individual i's achievement vector $\mathbf{y}_i$ and the poverty thresholds vector $\mathbf{z}$ to an indicator variable in such a way that $\rho(\mathbf{y}_i, \mathbf{z}) = 1$ if person i is multidimensionally poor and $\rho(\mathbf{y}_i, \mathbf{z}) = 0$ if person i is not multidimensionally poor. For analytical clarity, it will be convenient to write the identification function ρ as the composite $\rho = \rho^b \circ \rho^w$, with

$$\rho^w : (I_1 \times \cdots \times I_d) \times (I_1 \times \cdots \times I_d) \to X^d \tag{1}$$

and

$$\rho^b : X^d \to \{0, 1\}. \tag{2}$$

The function ρ^w converts the achievement vector $\mathbf{y}_i$ and the vector of poverty thresholds $\mathbf{z}$ into a $d-$dimensional vector of 0s and 1s indicating whether individual i is deprived or not in the different dimensions taken into account (where 1 denotes deprivation and 0 non-deprivation). Such object is called *individual's i deprivation profile*, and is denoted as $\mathbf{x}_i = (x_{i1}, \ldots, x_{id})$, with $x_{ij} \in \{0,1\}$. The set X^d contains all possible combinations of deprivations/non-deprivations across d dimensions, and we refer to it as the *set of deprivation profiles.* Its generic members are denoted as $\mathbf{x} = (x_1, \ldots, x_d)$, with $x_j \in \{0,1\}$ indicating the deprivation status in dimension j. Therefore, the profile $(0, ..., 0)$ corresponds to someone who is not deprived in any dimension and $(1, ..., 1)$ to someone who is deprived in all dimensions. Clearly, $\left|X^d\right| = 2^d$. By construction, ρ^w only considers the deprivation status of individuals *within* dimensions according to the criterion introduced in the previous paragraph. On the other hand, the function ρ^b identifies who is multidimensionally poor and who is not on the basis of individuals' list of deprivations *between* dimensions. Therefore ρ^w and ρ^b are referred to as *within*- and *between*-dimension identification functions, respectively. In this paper, we consider ρ^w as exogenously given,[5] and we focus on the different ways in which ρ^b can be defined. Given the set of deprivation profiles X^d and any between-dimension identification function $\rho^b : X^d \to \{0,1\}$, we derive the partition $X^d = P_d \sqcup R_d$, where

$$P_d := \left\{\mathbf{x} \in X^d | \rho^b(\mathbf{x}) = 1\right\} = \left(\rho^b\right)^{-1}(1) \tag{3}$$

and

$$R_d := \left\{\mathbf{x} \in X^d | \rho^b(\mathbf{x}) = 0\right\} = \left(\rho^b\right)^{-1}(0) = X^d \backslash P_d. \tag{4}$$

Whenever an individual experiences a combination of deprivations like those included in P_d (resp. R_d), that individual is identified as poor (resp. non-poor) according to ρ^b. For this reason, we refer to P_d (resp. R_d) as a *set of poor profiles* (resp. *non-poor profiles*). Since there is a one-to-one correspondence between "sets of poor profiles" and "sets of

[5] Implicitly, this assumes that we are working in the space of deprivations (i.e.: taking into account the dimension-specific gaps between attainments and the corresponding poverty threshold – see footnote #4). The alternative approach advocated by Ravallion (2011) of working in the space of attainments is not followed in this paper because the collapse of multivariate distributions into unidimensional ones trivially simplifies the problem of identification of the poor.

between-dimensions identification functions" (see equation (3)), we use both sets of objects interchangeably when no confusion arises. For any $\mathbf{x} \in X^d$, let $N_{\mathbf{x}} \subseteq N$ denote the set of individuals experiencing deprivations as described in $\mathbf{x}$. Clearly, $\bigcup_{\mathbf{x} \in X^d} N_{\mathbf{x}} = N$. The number of elements in $N_{\mathbf{x}}$ is denoted as $n_{\mathbf{x}}$. For any set of poor profiles $P_d \subset X^d$ let $Q(P_d) := \{i \in N | \rho^w(\mathbf{y}_i, \mathbf{z}) \in P_d\} = \bigcup_{\mathbf{x} \in P_d} N_{\mathbf{x}}$ be the set of individuals considered poor according to P_d. The number of 'P_d-poor' individuals is defined as $q := |Q(P_d)| = \sum_{\mathbf{x} \in P_d} n_{\mathbf{x}}$.

The elements of X^d can be partially ordered by $\preceq$, the partial order[6] generated by vector dominance in $X^d \times X^d$. That is: For any $\mathbf{x}, \mathbf{y} \in X^d$, $\mathbf{x} \preceq \mathbf{y}$ if and only if $x_i \leq y_i$ for all $i \in \{1, \ldots, d\}$. When this happens, we say that $\mathbf{y}$ vector-dominates $\mathbf{x}$. Observe that when a given deprivation profile $\mathbf{x}$ is vector-dominated by another deprivation profile $\mathbf{y}$ (i.e.: when $\mathbf{x} \preceq \mathbf{y}$), we might reasonably say that the state of affairs represented by the former is better than the one represented by the latter. Let Z be any subset of X^d. On the one hand, the *up-set* of Z (denoted as $Z^{\uparrow}$) is defined as $Z^{\uparrow} := \{\mathbf{x} \in X^d | \exists \mathbf{z} \in Z \text{ s.t. } \mathbf{z} \preceq \mathbf{x}\}$ (i.e.: it is the set of deprivation profiles vector-dominating at least one member of Z). On the other hand, the *set of undominating elements of* Z (denoted as $U(Z)$) is defined as $U(Z) := \{\mathbf{x} \in Z | \nexists \mathbf{y} \in Z \backslash \{\mathbf{x}\} \text{ s.t. } \mathbf{y} \preceq \mathbf{x}\}$ (i.e.: it is the set of elements in Z that do not vector-dominate any other element in Z). By construction, if $\mathbf{x} \in U(P_d)$ and $\mathbf{y} \in X^d$ is such that $\mathbf{y} \preceq \mathbf{x}$, then $\mathbf{y} \in R_d$. In words: for a given set of poor profiles P_d, the members of $U(P_d)$ are the elements representing the least deprived situation among the poor.

For any natural number $m \in \mathbb{N}$, let $\delta : X^m \times X^m \to \{1, \ldots, m\}$ be a function such that for any $\mathbf{x}, \mathbf{y} \in X^m$, $\delta(\mathbf{x}, \mathbf{y}) \subseteq \{1, \ldots, m\}$ is the set of dimensions where $\mathbf{x}$ is deprived but $\mathbf{y}$

[6] A *partial order* over a set S is a binary relation $\preceq$ which, for any $a, b, c \in S$, satisfies the following conditions: (i) $a \preceq a$ (Reflexivity); (ii) If $a \preceq b$ and $b \preceq a$ then $a = b$ (Antisymmetry); (iii) If $a \preceq b$ and $b \preceq c$ then $a \preceq c$ (Transitivity).

is not (i.e: when $i \in \delta(\mathbf{x}, \mathbf{y})$ then $x_i = 1$ and $y_i = 0$). The following technical Lemma will be used when presenting the main results of the paper.

Lemma 1: For any natural number $m \in \mathbb{N}$, let Z be any subset of X^m with $|U(Z)| \geq 2$ and let $\mathbf{x}, \mathbf{y} \in U(Z)$ denote two of its elements, with $\mathbf{x} \neq \mathbf{y}$. One has that $\delta(\mathbf{x}, \mathbf{y}) \neq \emptyset$ and $\delta(\mathbf{y}, \mathbf{x}) \neq \emptyset$.

Proof: See the appendix.

To clarify ideas, it is useful to graph the Hasse diagram corresponding to the set X^d (whose elements are the nodes of the diagram) and the partial order $\preceq$ (represented by the edges between nodes). The different deprivation profiles (i.e.: the nodes) are ordered in rows depending on the number of deprivations they contain: The first row includes the profile with no deprivations, the second one the profiles with at most one deprivation, and so on. In these diagrams, it is useful to distinguish whether the different nodes belong to P_d or R_d. In Figure 1 we show two examples of Hasse diagrams for the case $d = 4$ that will be useful to illustrate other sections of the paper. In the first one (Fig. 1a), the set of poor profiles is $P_4^1 = \{$1100, 1010, 1001, 0110, 0101, 0011, 1110, 1101, 1011, 0111, 1111$\}$ and in the second one (Fig. 1b) $P_4^2 = \{$1100, 1011, 1110, 1101, 1111$\}$. Observe that $U(P_4^1) = \{$1100, 1010, 1001, 0110, 0101, 0011$\}$, $U(P_4^2) = \{$1100, 1011$\}$.

[[[Figure 1a,b]]]

2.1 The dual cutoff identification method

The identification function suggested by Alkire and Foster (2011), based on the counting approach and denoted as $\rho_{\mathcal{C},\mathbf{a},k}$, can be written as the composite $\rho_{\mathcal{C},\mathbf{a},k} = \rho^b_{\mathcal{C},\mathbf{a},k} \circ \rho^w$, where

$\rho^b_{\mathcal{C},\mathbf{a},k}$ is in turn defined as the composite $\rho^b_{\mathcal{C},\mathbf{a},k} = \iota_k \circ c_\mathbf{a}$, with

$$c_\mathbf{a} : X^d \to [0,1] \tag{5}$$

and

$$\iota_k : [0,1] \to \{0,1\}\,. \tag{6}$$

For any $\mathbf{x} \in X^d$, the function $c_\mathbf{a}$ is defined as $c_\mathbf{a}(\mathbf{x}) = \sum_{j=1}^{j=d} a_j x_j$, that is: $c_\mathbf{a}$ simply counts the weighted proportion of deprivations experienced by someone with deprivation profile $\mathbf{x}$. Following the terminology of Alkire and Foster (2011), $c_\mathbf{a}(\mathbf{x})$ is referred to as *deprivation score*. Lastly, for any $s \in [0,1]$ and for any $k \in (0,1]$, ι_k is defined as

$$\iota_k(s) = \left\{ \begin{array}{c} 1 \text{ if } s \geq k \\ \\ 0 \text{ if } s < k \end{array} \right\}. \tag{7}$$

The $\iota_k \circ c_\mathbf{a}$ function takes a value of 1 whenever the weighted proportion of deprivations attains a certain threshold k (which is exogenously given) and a value of 0 otherwise. Summing up, the dual cutoff identification method $\rho_{\mathcal{C},\mathbf{a},k}$ is defined as a composite of three functions

$$(I_1 \times \cdots \times I_d) \times (I_1 \times \cdots \times I_d) \xrightarrow{\rho^w} X^d \xrightarrow{c_\mathbf{a}} [0,1] \xrightarrow{\iota_k} \{0,1\} \tag{8}$$

that identifies individual i as being 'poor' whenever the deprivation score associated with the deprivation profile $\rho^w(\mathbf{y}_i, \mathbf{z})$ is not lower than k (the poverty threshold across dimensions) and as 'non-poor' otherwise. Parameter k indicates the proportion of weighted deprivations a person needs to experience in order to be considered multidimensionally poor. Therefore, the sets of '$\mathcal{C}$-poor' and 'non-$\mathcal{C}$-poor' profiles can be written as

$$P_{d,\mathcal{C}(\mathbf{a},k)} \ : \ = \left\{ \mathbf{x} \in X^d | \sum_{j=1}^{j=d} a_j x_j \geq k \right\} \tag{9}$$

$$R_{d,\mathcal{C}(\mathbf{a},k)} \ : \ = \left\{ \mathbf{x} \in X^d | \sum_{j=1}^{j=d} a_j x_j < k \right\}. \tag{10}$$

The higher the value of k, the more difficult it is that an individual ends up being classified as poor. When $k \leq \min_j a_j$, $\rho_{\mathcal{C},\mathbf{a},k}$ corresponds to the union identification approach, and when $k = 1$, $\rho_{\mathcal{C},\mathbf{a},k}$ is equivalent to the intersection approach. The Hasse diagrams shown in Figures 1a and 1b illustrate examples of sets of poor profiles $P_{d,\mathcal{C}(\mathbf{a},k)}$ for certain combinations of $d, \mathbf{a}$ and k. In Figure 1a, we have chosen $d = 4, a_1 = a_2 = a_3 = a_4 = 1/4$ and $k = 1/2$ and in Figure 1b, $d = 4, a_1 = 1/2, a_2 = 1/4, a_3 = 1/8, a_4 = 1/8$ and $k = 3/4$. If one chooses equal weights, whenever a deprivation profile belongs to $P_{d,\mathcal{C}(\mathbf{a},k)}$, all other deprivation profiles in the same row are included in $P_{d,\mathcal{C}(\mathbf{a},k)}$ as well (see Fig. 1a). Alternatively, when the weights are allowed to be different it is possible that not all members of the same row are included in $P_{d,\mathcal{C}(\mathbf{a},k)}$ (see Fig. 1b).

3. Identifying the poor: Beyond the counting approach

Roughly, the AF method is a procedure stipulating that if the number of deprivations experienced by an individual exceeds a certain threshold, that individual should be considered poor – irrespective of the specific combination of deprivations contributing to the count. The main aim of this section is to go beyond this counting approach suggesting more general and less stringent identification procedures that are better equipped to capture the subtelties and intrincacies involved in such a delicate matter. Initially, we assume that all variables belong to a single domain but later (from section 3.1 onwards) we consider the more general case where variables are partitioned across several domains.

Among all potential partitions of X^d into the disjoint sets P_d and R_d (i.e., when identifying what deprivation profiles should fall into the 'poor' or 'non-poor' categories), not all possibilities are meaningful. Whenever a certain $\mathbf{x}$ belongs to P_d, one would expect that those deprivation profiles $\mathbf{y}$ containing at least the same set of deprivations as those in $\mathbf{x}$

should also be included in P_d. That is: if an individual i is labeled as poor, another individual j experiencing deprivations at least in the same dimensions as those where i experiences deprivations, and possibly in others, should also be labeled as poor. Formally, it seems reasonable to impose that the set of poor profiles P_d should respect the partial order $\preceq$ generated by vector dominance, that is:

Definition: A set of poor profiles P_d satisfies the *Consistency Condition* (CC) if and only if for any $\mathbf{x} \in P_d$ and any $\mathbf{y} \in \mathbf{x}^{\uparrow}$, then $\mathbf{y} \in P_d$.

In terms of the corresponding between-dimension identification functions[7] (i.e.: in terms of ρ^b), the Consistency Condition stipulates that for any $\mathbf{x}, \mathbf{y} \in X^d$ with $\mathbf{x} \preceq \mathbf{y}$, one must have $\rho^b(\mathbf{x}) \leq \rho^b(\mathbf{y})$. Because of its logical solidity, we posit that the class of between dimension identification functions satisfying CC should be the universe of reference from which identification functions should be drawn[8] . Indeed, the incorporation of identification functions not satisfying CC seems extremely hard to justify on logical and ethical grounds. We denote by $\mathcal{P}_d$ the set of all sets of poor profiles P_d satisfying CC. Given their relevance for this paper, we now characterize the elements of $\mathcal{P}_d$.

Proposition 1. One has that $P_d \in \mathcal{P}_d \Leftrightarrow (U(P_d))^{\uparrow} = P_d$.

Proof: See the appendix.

According to Proposition 1, the set of poor profiles satisfying CC are the sets that coincide with the up-set of their undominating elements. This implies that the sets of poor

[7] Given the one-to-one correspondence between 'poor profiles' and 'between-dimension identification functions', we will interchangeably use the expressions 'P_d satisfies CC' and 'ρ^b satisfies CC'.

[8] The Consistency Condition is reminiscent of the so-called 'poverty consistency' property introduced by Lasso de la Vega (2010) in the context of the counting approach (see section 2.1). According to that author, an identification function is 'poverty consistent' if, when identifying a person with a deprivation score equal to s as poor, it also considers as poor anybody whose deprivation score count is at least as high as s. Clearly, the Consistency Condition proposed here is more general than the 'poverty consistency' property.

profiles P_d satisfying CC are univocally characterized and represented by the corresponding subsets of undominating elements $U(P_d)$. When choosing a sensible set of poor profiles P_d, the subsets $U(P_d)$ are particularly important because their elements determine the least deprived conditions that individuals should experience in order to be considered as poor. Indeed, the sets $U(P_d)$ can be thought as a generalization of the concept of a poverty line to the multidimensional context (i.e.: they determine the boundary separating the poor from the non-poor). For this reason, the sets $U(P_d)$ obtained from the different $P_d \in \mathcal{P}_d$ are referred to as the sets of *boundary profiles*, and are denoted as $\mathcal{Z}$. As a consequence of Proposition 1, we say that $\mathcal{P}_d$ is the same as $\{Z^{\uparrow}\}_{Z\in\mathcal{Z}}$, that is: Any poor profile $P_d \in \mathcal{P}_d$ corresponds to the up-set of some Z belonging to $\mathcal{Z}$ and vice-versa. Since $\mathcal{Z}$ contains the undominating elements of the sets of poor profiles satisfying CC, it can be written as

$$\mathcal{Z} := \left\{Z \subset X^d | \forall \mathbf{x} \in Z, \nexists \mathbf{y} \in Z \backslash \{\mathbf{x}\} \ s.t.\ \mathbf{y} \preceq \mathbf{x}\right\}. \tag{11}$$

That is: $\mathcal{Z}$ contains all subsets of X^d such that any two of its members never vector-dominate one another (in particular, it contains all singletons of X^d).

What can be said about the dual cutoff method in this broader identification context? The sets of poor profiles $P_{d,\mathcal{C}(\mathbf{a},k)}$ generated by the dual cutoff method satisfy the Consistency Condition for any $\mathbf{a} \in \Delta_d$ and any $k \in (0,1]$ (i.e., $P_{d,\mathcal{C}(\mathbf{a},k)} \in \mathcal{P}_d$ because, whenever $\mathbf{x} \preceq \mathbf{y}$, one clearly has that $\rho^b_{\mathcal{C},\mathbf{a},k}(\mathbf{x}) \leq \rho^b_{\mathcal{C},\mathbf{a},k}(\mathbf{y})$). However, the following result proves that $\mathcal{P}_d$ contains other elements that cannot be generated via the dual cutoff method.

Theorem 1: For any $d \geq 2$ let $\mathcal{C}_d$ be the set of all sets of poor profiles generated by the counting approach, that is: $\mathcal{C}_d := \left\{P_{d,\mathcal{C}(\mathbf{a},k)}\right\}_{\mathbf{a}\in\Delta_d, k\in(0,1]}$. Then, if $d \in \{2,3\}$, $\mathcal{C}_d = \mathcal{P}_d$. For any $d \geq 4, \mathcal{C}_d \subsetneq \mathcal{P}_d$.

Proof: See the appendix.

Theorem 1 stipulates that the set of sets of poor profiles generated by the dual cutoff identification method is strictly included within the set of sets of poor profiles satisfying CC whenever the number of dimensions taken into account is greater than 3. This implies that the counting approach underlying the AF method leaves aside certain sets of poor profiles P_d belonging to $\mathcal{P}_d$ that might represent a sensible way of deciding who is poor and who is not. In the following section we describe a wide class of sets of poor profiles belonging to $\mathcal{P}_d$ that the AF method fails to identify.

3.1 Identification of the poor in a multiple domain context

Consider the following hypothetical example taken from Alkire and Foster (2011: 483), where the multidimensional poverty levels of individuals are assessed with the following variables: V_1 = 'Income', V_2 = 'Years of Schooling', V_3 = 'Self-assessed Health' and V_4 = 'Health insurance' (that is: $d = 4$). Assume that, for each variable, there is a threshold below which individuals should be considered deprived (in the case of V_4, an individual is deprived if s/he has no health insurance). In this framework, one might say that V_1 and V_2 capture alternative aspects of a broader domain one might call "Capacity to make a living" (denoted as D_1) while V_3 and V_4 capture different aspects within the domain of "Health " (denoted as D_2). When deciding who is poor and who is not, one might reasonably argue that if someone is only deprived in V_1, then she should not be identified as 'poor' because her high level of education might somehow compensate and potentially offer some alternatives for the current lack of income in the capacity to make a living. If someone is only deprived in V_2, then he might not be identified as 'poor' because his lack of education can be compensated by his high level of income. Here, one might say that in order to be identified as 'poor', an individual should experience deprivations at least in V_1 *and* V_2 simultaneously – something which would severly hinder that individual's capacity to make a decent living. Analogously, one could argue that an individual is poor whenever she experiences deprivations in at least V_3 *and* V_4 simultaneously (an alarming circumstance for that individual's health), but not poor if she only experiences deprivation in one of the two variables separately (good self-assessed health might somehow compensate for the lack of health insurance and vice-versa). Lastly, one could also argue that when an individual is only deprived in one variable within D_1 *and* in one variable within D_2, then that individual should *not* be identified as 'poor' because the variable within each domain where that individual attains a good achievement somehow compensates for the deprivation experienced in the other variable. For instance: an individual deprived in V_2 and V_3 only might not be classified as poor because her high income and health insurance might compensate in a way for her low levels of education and low self-assessed health respectively. Formally, all the previous arguments are summarized stating that the set of poor profiles $P_4^* = \{1100, 0011, 1110, 1101, 1011, 0111, 1111\}$ can be a reasonable choice when deciding who is poor and who is not for the case $d = 4$ (to illustrate, the Hasse diagram corresponding to P_4^* is shown in Figure 2a; observe that $U(P_4^*) = \{1100, 0011\}$). Interestingly, it is straightforward to check that while P_4^* satisfies the Consistency Condition (i.e.: $P_4^* \in \mathcal{P}_4$), P_4^* does not belong to $\mathcal{C}_4$ for any $\mathbf{a} \in \Delta_4$ and any $k \in (0, 1]$ (see Theorem 2 for a proof of this statement). In other words: No matter what weighting scheme $\mathbf{a}$ or what deprivation score threshold k we choose, the AF identification method never generates a set of poor profiles like P_4^*.

The set P_4^* identifies as poor those individuals that are either 'completely' deprived in D_1 or in D_2 (i.e: experiencing deprivations in the corresponding two constituent variables), but it does not identify as poor those individuals that are 'partially' deprived in both domains simultaneously (i.e: experiencing deprivation in only one variable within each domain). Alternatively, one might be interested in targeting those individuals experiencing some deprivations both in D_1 and D_2, even if they are not completely deprived in any of the two. Since these individuals experience some level of deprivation in all life domains considered in this example, it also seems reasonable to define a poverty profile to identify them. Using the notation introduced in this paper, such profile would be written as $P_4^{**} = \{1010$, 1001, 0110, 0101, 1110, 1101, 1011, 0111, $1111\}$ (see illustration in Figure 2b; observe that $U(P_4^{**}) = \{1010$, 1001, 0110, $0101\}$). Again, even if P_4^{**} satisfies the Consistency Condition, it turns out that there is no weighting scheme **a** and any deprivation score threshold k such that P_4^{**} belongs to $\mathcal{C}_4$ (see the proof of Theorem 2).

[[[Figure 2a + 2b]]]

As the two preceding examples suggest, the counting approach seems ill-suited to identify the poor in those settings where the different variables we are taking into account are nested within mutually exclusive domains and where there might be non-trivial compensation patterns between deprived and non-deprived variables depending on whether they belong to the same domain or not. Indeed, we show below that the counting approach fails to identify the poor in virtually any setting where the variables are nested within multiple domains and the identification functions explicitly take into consideration such partition. In order to push forward the intuitions laid out in the previous two examples it becomes necessary to formalize what it means 'being deprived in a given domain' and how this relates to the identification of the multidimensionally poor. For that purpose we will now introduce some further notation and definitions.

From now onwards, we assume that the set of variables D of size d is partitioned into G domains $(D_1, \ldots, D_G)$ with at least two variables within each domain ($d_g := |D_g| \geq 2 \forall g$), i.e:

$(D_1, \ldots, D_G) \in \Pi_{D,G}$. In addition, we assume that there are at least two domains, i.e: $G \geq 2$ (so far we have been implicitly working as if there was one domain only). Since $(D_1, \ldots, D_G)$ is a partition of D, a deprivation profile $\mathbf{x} = (x_1, \ldots, x_d) \in X^d$ can be rewritten without loss of generality as $(\mathbf{x}_1, \ldots, \mathbf{x}_g, \ldots, \mathbf{x}_G) := (x_{11}, \ldots, x_{1d_1}, \ldots, x_{g1}, \ldots, x_{gd_g}, \ldots, x_{G1}, \ldots, x_{Gd_G})$, where $\mathbf{x}_g = (x_{g1}, \ldots, x_{gd_g}) \in X^{d_g}$ and $x_{gv} \in \{0, 1\}$ indicates the deprivation status in variable v within domain g. Formally, we denote by $\iota : X^d \to X^{d_1} \times \ldots \times X^{d_G}$ the one-to-one function that transforms any deprivation profile $\mathbf{x}$ into $(\mathbf{x}_1, \ldots, \mathbf{x}_G)$. Clearly, $d = \sum_g d_g$.

Now, how does one identify those individuals deprived in domain D_g and, more generally, those who are multidimensionally poor? Loosely speaking, an individual is deprived in domain D_g when the deprivations she experiences are 'large enough' to prevent enjoying a decent living *in that sphere of life*. Deciding what bundles of deprivations are sufficient to prevent enjoying such a 'decent living' is again a complex and elusive matter that is formally performed by the following function

$$\rho_g^w : X^{d_g} \to \{0, 1\}. \tag{12}$$

ρ_g^w converts the vector $\mathbf{x}_g = (x_{g1}, \ldots, x_{gd_g})$ into a 0 or a 1 indicating whether someone experiencing such deprivations pattern in domain D_g should be considered to be deprived in that domain or not (with a value of 1 meaning 'deprivation in that domain'). Extending the domain deprivation definition to all domains $D_1, \ldots, D_G$ we obtain

$$\rho_G^\omega = \left(\rho_1^w, \ldots, \rho_g^w, \ldots, \rho_G^w\right) : X^{d_1} \times \ldots \times X^{d_G} \to X^G. \tag{13}$$

The function ρ_G^ω converts the vector $(\mathbf{x}_1, \ldots, \mathbf{x}_g, \ldots, \mathbf{x}_G)$ into a $G-$dimensional vector of 0s and 1s indicating the deprivation status of individuals across domains. Formally, the vector $(\rho_1^w(\mathbf{x}_1), \ldots, \rho_G^w(\mathbf{x}_G)) \in X^G$ will be denoted as *domain deprivation profile*. Lastly, one must decide whether an individual experiencing a certain domain deprivation profile should be identified as 'multidimensionally poor' or not. For that purpose, we consider the function

$$\rho_G^b : X^G \to \{0,1\} \tag{14}$$

with a value of 1 indicating that the corresponding individual has been identified as multidimensionally poor. To sum up, in the multiple domain context the between dimension identification function ρ^b introduced in (2) can be rewritten as a composite of three functions $\rho^b = \rho_G^b \circ \rho_G^\omega \circ \iota$, as illustrated in the following diagram:

$$\begin{array}{ccc} X^d & \xrightarrow{\rho^b} & \{0,1\} \\ \downarrow \iota & & \uparrow \rho_G^b \\ X^{d_1} \times \ldots \times X^{d_G} & \xrightarrow{\rho_G^\omega} & X^G \end{array}$$

In words: to decide whether an individual is multidimensionally poor we take three steps: (i) Consider the partition of variables across domains (ι); (ii) Examine whether individuals are deprived or not within those domains (ρ_G^ω); and (ii) Examine whether individuals are deprived across domains (ρ_G^b). Because of the way in which they have been defined, ρ_G^ω and ρ_G^b are referred to as *within*- and *between-domain* identification functions respectively. From the previous definitions we can naturally derive the following sets

$$P_{d_g}^{w,g} := \left\{\mathbf{x}_g \in X^{d_g} | \rho_g^w(\mathbf{x}_g) = 1\right\} = \left(\rho_g^w\right)^{-1}(1) \tag{15}$$

$$R_{d_g}^{w,g} := \left\{\mathbf{x}_g \in X^{d_g} | \rho_g^w(\mathbf{x}_g) = 0\right\} = \left(\rho_g^w\right)^{-1}(0) = X^{d_g} \backslash P_{d_g}^{w,g} \tag{16}$$

$$P_G^b := \left\{\mathbf{v} \in X^G | \rho_G^b(\mathbf{v}) = 1\right\} = \left(\rho_G^b\right)^{-1}(1) \tag{17}$$

and

$$R_G^b := \left\{\mathbf{v} \in X^G | \rho_G^b(\mathbf{v}) = 0\right\} = \left(\rho_G^b\right)^{-1}(0) = X^G \backslash P_G^b. \tag{18}$$

These are the analogues of the sets of poor and non-poor profiles shown in (3) and (4) adapted to the context of within- and between-domain deprivation (as is clear, whenever there is only one domain, equations (15) and (16) reduce to equations (3) and (4) respectively). The set $P^{w,g}_{d_g}$ (resp. $R^{w,g}_{d_g}$) indicates what combinations of deprivations should one experience to be considered as being deprived (resp. non-deprived) in domain D_g. On the other hand, P^b_G (resp. R^b_G) contain the combination of domain deprivations one should experience to be considered as being multidimensionally poor (resp. non-poor). Since the vector dominance order $\preceq$ can be defined both within and across domains (i.e: both in $X^{d_g} \times X^{d_g}$ and $X^G \times X^G$), the Consistency Condition (CC) will be imposed as well when defining the sets $P^{w,g}_{d_g}$ and P^b_G. That is: whenever $\mathbf{x}_g \in P^{w,g}_{d_g}$ (resp. $\mathbf{v} = (v_1, \ldots, v_G) \in P^b_G$) and $\mathbf{y}_g \in \mathbf{x}_g^{\uparrow}$ (resp. $\mathbf{t} = (t_1, \ldots, t_G) \in \mathbf{v}^{\uparrow}$), then $\mathbf{y}_g \in P^{w,g}_{d_g}$ (resp. $\mathbf{t} \in P^b_G$). Analogously, the notion of 'undominating elements' can be defined as well both in the within- and between-domain contexts (i.e.: $U(P^{w,g}_{d_g}) := \{\mathbf{x}_g \in P^{w,g}_{d_g} | \nexists \mathbf{y}_g \in P^{w,g}_{d_g} \setminus \{\mathbf{x}_g\} \text{ s.t. } \mathbf{y}_g \preceq \mathbf{x}_g\}$ and $U(P^b_G) := \{\mathbf{v} \in P^b_G | \nexists \mathbf{t} \in P^b_G \setminus \{\mathbf{v}\} \text{ s.t. } \mathbf{t} \preceq \mathbf{v}\}$). While the choice of $P^{w,g}_{d_g}$ and P^b_G determine who is poor and who is not poor within and between domains, the elements of $U(P^{w,g}_{d_g})$ and $U(P^b_G)$ determine the least deprived conditions that individuals should experience to be considered deprived within and between domains respectively.

Examples: As an illustration, we are going to rewrite the sets of poor profiles P^*_4, P^{**}_4 with which we started this section using the multiple domains terminology we have just introduced. In those examples there are two domains (D_1 and D_2) with two variables each (V_1, V_2 and V_3, V_4 respectively). For P^*_4 one has that $P^{w,1}_{d_1} = P^{w,2}_{d_2} = \{11\}$ (i.e: one has to be deprived in both variables to be deprived in the corresponding domain) while $P^b_G = \{10, 01, 11\}$ (i.e: it suffices to be deprived in one domain to be considered multidimensionally poor). On the other hand, for P^{**}_4 one has that $P^{w,1}_{d_1} = P^{w,2}_{d_2} = \{10, 01, 11\}$ (i.e: it suffices to be deprived in one variable only to be deprived in the corresponding domain) while $P^b_G = \{11\}$ (i.e: one has to be deprived in both domains to be considered multidimensionally poor).

The following results determine how does the counting approach fare in the multiple domain context proposed in this paper.

Proposition 2: Let $(D_1, \ldots, D_G) \in \Pi_{D,G}$. When $U(P^b_G)$ and the $U(P^{w,g}_{d_g})$ are singletons $\forall g \in \{1, \ldots, G\}$, there exists some $\mathbf{a} \in \Delta_d$ and some $k \in (0, 1]$ such that $P_{d,AF(\mathbf{a},k)}$ coincides with the set of poor profiles generated by the different $P^{w,g}_{d_g}$ and P^b_G.

Proof: See the appendix.

This result basically states that when the within- and between-domain deprivation functions (ρ^ω_G and ρ^b_G) are very 'simple', the counting approach is still valid as a method to identify the poor in a multiple domain context. The fact that $U(P^b_G)$ and the different $U(P^{w,1}_{d_1}), \ldots, U(P^{w,G}_{d_G})$ only contain one element implies that the set of dimensions D can be split in two groups: (i) those which are essential for assessing multidimensional poverty[9] and (ii) those which are subsidiary to the former (i.e: when an individual is only deprived in the dimensions of the second group, she is not considered to be multidimensionally poor). When this happens, the counting approach still works because it suffices to give a sufficiently large weight to the dimensions included in the first group (see the proof of Proposition 2). However, the following result shows that this is no longer the case when ρ^ω_G and ρ^b_G have a slightly richer structure and depart from the trivial case.

Theorem 2: Let $(D_1, \ldots, D_G) \in \Pi_{D,G}$. Consider the following sets of conditions

[9] To identify these dimensions it suffices to look at the non-zero elements of the vectors in $U(P^b_G)$ and the different $U(P^{w,1}_{d_1}), \ldots, U(P^{w,G}_{d_G})$.

(i) Assume that $\left|U(P_G^b)\right| \geq 2$. There exists a pair of elements $\mathbf{p}, \mathbf{q} \in U(P_G^b)$ ($\mathbf{p} \neq \mathbf{q}$) such that the following happens: if $g_1 \in \delta(\mathbf{p}, \mathbf{q})$ and $g_2 \in \delta(\mathbf{q}, \mathbf{p})$ (where $\delta(\mathbf{p}, \mathbf{q})$ and $\delta(\mathbf{q}, \mathbf{p})$ are the elements referred to in Lemma 1), then there exist $\mathbf{x}_{g_1} \in U(P_{d_{g_1}}^{w,g_1}), \mathbf{x}_{g_2} \in U(P_{d_{g_2}}^{w,g_2})$ with $\sum_{v=1}^{v=d_{g_1}} x_{g_1 v} > 1, \sum_{v=1}^{v=d_{g_2}} x_{g_2 v} > 1$.

(ii) Assume that in P_G^b there exists an element $\mathbf{x}_G$ with $|\tau_G(\mathbf{x}_G)| \geq 2$ such that for at least two domains $g_1, g_2 \in \tau_G(\mathbf{x}_G)$ one has that $\left|U(P_{d_{g_1}}^{w,g_1})\right| \geq 2$ and $\left|U(P_{d_{g_2}}^{w,g_2})\right| \geq 2$.

Whenever (i) or (ii) hold, there is no weighting scheme $\mathbf{a}$ and any deprivation score threshold k such that $P_{d,\mathcal{C}(\mathbf{a},k)}$ coincides with the set of poor profiles generated by the different $P_{d_g}^{w,g}$ and P_G^b.

Proof: See the appendix.

Theorem 2 shows that under very mild restrictions, the counting approach is essentially unable to identify the multidimensionally poor in the multiple domain context suggested in this paper. Condition (i) presents a scenario where there are at least two groups of domains in which individuals have to experience deprivation to be considered multidimensionally poor. In addition, it requires that in order to be considered deprived within some of these domains, individuals have to experience deprivations in more than one variable simultaneously. Essentially, this is a generalization of the set of poor profiles P_4^* adapted to the multiple domain context. On the other hand, condition (ii) presents a scenario where individuals have to experience deprivation in at least two domains simultaneously to be considered multidimensionally poor but where deprivation needs not to be universal within at least two of these domains (i.e: individuals are not required to be deprived in all variables to be considered deprived within those domains). This generalizes the set of poor profiles P_4^{**} to the multiple domain context.

Rather than piling up deprivations irrespective of the domains to which they belong, in this paper we suggest taking these domains into account when identifying the poor. Selecting the appropriate combination of within- and between-domain identification functions (i.e: ρ_G^{ω} and ρ_G^{b}) there is ample room to generate poor identification functions that can accomodate non-trivial patterns of compensation that might exist between deprived and non-deprived attributes belonging to the same or alternative domains – a possibility that is not feasible when one relies on the counting approach only.

3.2 The generalized counting approach

Despite the limitations of the counting approach highlighted in theorem 2, it can be naturally extended to the multi-domain framework in the following way. When constructing the within-domain identification functions, one might decide that an individual must at least experience m_g deprivations within domain D_g in order to be deprived in that domain (with $m_g \leq d_g$). Analogously, in order to be identified as multidimensionally poor, an individual must be deprived in at least M domains ($M \leq G$). Rather than having a single poverty threshold across all dimensions ($k \in (0, 1]$) as in the classical counting approach, here we need to specify one threshold within each domain (m_g) and an overall threshold across domains (M). This natural way of extending the counting approach to the multiple domain context will be referred to as 'generalized counting approach'[10] . The specification of the thresholds vector $(m_1, \ldots, m_G; M)$ will univocally indicate what combination of thresholds will be used within and across domains. The choice of the thresholds vector $(1, \ldots, 1; 1)$ is equivalent to the 'classical' (i.e: single domain context) union approach, while the thresholds vector

[10]When generalizing the counting approach, one could of course introduce different weights within the dimensions of a given domain and different weights between domains to generate the corresponding within- and between-domain deprivation scores. Since this would further complicate notations but give no additional insights we have kept the simpler weightless version.

$(d_1, \ldots, d_G; G)$ leads to the 'classical' intersection approach.

The generalized counting approach allows great flexibility when modelling non-trivial tradeoffs between deprived and non-deprived attributes either belonging to the same or to alternative domains. While it is by no means the only way of identifying the multidimensionally poor – the class of CC identification functions is broader than that – it has the advantage of being simple to understand and implement. Indeed, both P_4^* and P_4^{**} can be described as members of the generalized counting approach in a context where there are two domains and two variables within each domain. While P_4^* uses the intersection approach within domains and the union approach between domains (i.e: it is characterized by the thresholds vector $(2, 2; 1)$), P_4^{**} uses the union approach within domains and the intersection approach between domains (i.e: it is characterized by the thresholds vector $(1, 1; 2)$). The following corollary of Theorem 2 basically illustrates the limitations of the classical counting approach vis-à-vis its generalized counterpart.

Corollary 1: Assume we are using the generalized counting approach to identify the multidimensionally poor via the thresholds vector $(m_1, \ldots, m_G; M)$. Consider the following sets of conditions

(i) Let $M < G$ and let there be at least two domains $g_1, g_2 \in \{1, \ldots, G\}$ with $m_{g_1} \geq 2$ and $m_{g_2} \geq 2$.

(ii) Let $M = G$ and let there be at least two domains $g_1, g_2 \in \{1, \ldots, G\}$ with $m_{g_1} < d_{g_1}$ and $m_{g_2} < d_{g_2}$.

It turns out that the classical counting approach fails to generate the same set of poor profiles as the ones generated by the generalized counting approach described in (i) and (ii).

Proof: See the appendix.

Corollary 1 clearly shows the limitations of the classical counting approach in the multiple domains framework. Essentially, it is only when the union or intersection approaches are used both within and across domains (i.e: either $(1,\dots 1;1)$ or $(d_1,\dots,d_G;G)$) that the classical counting approach is still valid to identify the multidimensionally poor in the multiple domain context.

4. How poor are the poor?

So far we have been discussing how the partition of dimensions within domains affects the identification of the poor. We are now going to explore the implications of the multiple domains approach for the 'agregation step' – where the information about the poor is summarized into a single number. Basically we suggest aggregating information about the poor in a way that is naturally linked with the identification method presented in the previous section. For that purpose, we start with some basic definitions and measures that are quite standard in the poverty measurement literature and then proceed with the new measures proposed in this paper (see section 4.1).

In most of this section it will prove useful to express our measures in terms of deprivations rather than achievements. When an individual i is deprived in attribute j, there are several ways of capturing the extent of that deprivation, usually referred to as 'deprivation shortfall' or 'deprivation gap'. For the purposes of this paper, it will suffice[11] to consider the following definition of individual's i deprivation gap in dimension j:

$$\gamma_{ij}^{c} := Max\left\{\left(\frac{z_j - y_{ij}}{z_j}\right)^{c}, 0\right\}, \tag{19}$$

[11] Other definitions of the deprivation gaps are also feasible (see Table 1 in Permanyer 2014 for other examples). However, since alternative definitions do not alter the findings of the paper, we have chosen the one that is more commonly used in the literature for the sake of concreteness and simplicity.

where $c \geq 0$. Observe that γ_{ij}^c is well-defined for any $c \geq 0$ whenever y_{ij} is measured in a cardinal scale. When this happens, $\gamma_{ij}^c \in [0,1]$. In particular, when $c = 1$, γ_{ij}^1 is the so-called *normalized deprivation gap*, which measures in a $[0,1]$ scale the distance between a given achievement y_{ij} and the corresponding poverty line z_j. However, when y_{ij} is measured in an ordinal scale, γ_{ij}^c is only meaningful when $c = 0$. In that case, $\gamma_{ij}^0 = 1$ whenever $y_{ij} \leq z_j$, while $\gamma_{ij}^0 = 0$ otherwise. For an individual i, we define the corresponding *vector of deprivations gaps* as $\boldsymbol{\gamma}_i^c := (\gamma_{i1}^c, \ldots, \gamma_{id}^c)$. Observe that when $c = 0$, $\boldsymbol{\gamma}_i^0 := (\gamma_{i1}^0, \ldots, \gamma_{id}^0)$ corresponds to individual's i deprivation profile $\mathbf{x}_i = (x_{i1}, \ldots, x_{id}) \in X^d$, with $x_{ij} \in \{0, 1\}$.

When considering one domain only ($G = 1$), there are several methods of aggregating information to construct a multidimensional poverty index (see Permanyer (2014:4) for a review of different aggregation procedures in that context). We now present some of them adapted to the identification methods suggested in the previous section. Because of its popularity we start considering the multidimensional headcount ratio H and then proceed with the family of multidimensional poverty indices M_α suggested by Alkire and Foster (2011), which is currently being used in the construction of UNDP's MPI. Assume that individuals are identified as poor if they experience a combination of deprivations like those included in the set of poor profiles P_d (with $P_d \in \mathcal{P}_d$). Then

$$H(P_d) := \frac{1}{n} \sum_{i \in Q(P_d)} 1 = \frac{1}{n} \sum_{\mathbf{x} \in P_d} n_{\mathbf{x}} = \frac{q}{n}. \tag{20}$$

The index $H(P_d)$ is simply the share of individuals that are multidimensionally poor according to the set of poor profiles P_d. On the other hand, the family of poverty indices suggested by Alkire and Foster (2011) can be written as

$$M_\alpha(P_d) := \frac{1}{n} \sum_{i \in Q(P_d)} c_{\mathbf{a}}(\boldsymbol{\gamma}_i^\alpha). \tag{21}$$

Essentially, $M_\alpha(P_d)$ is an average of the deprivation gaps γ_{ij}^α across dimensions and across the set of individuals that are multidimensionally poor according to P_d. When $\alpha = 0$, $M_0(P_d)$ is called the 'adjusted headcount ratio' and it can be used with cardinal or ordinal variables[12] . However, when $\alpha > 0, M_\alpha(P_d)$ is only well defined for cardinal variables. In particular, $M_1(P_d)$ and $M_2(P_d)$ are named 'adjusted poverty gap' and 'adjusted FGT measure' respectively. Observe that whenever $P_d = P_{d,\mathcal{C}(\mathbf{a},k)}$ for some $\mathbf{a} = (a_1, \ldots, a_d) \in \Delta_d, k \in (0,1]$ (i.e: $P_d \in \mathcal{C}_d$), the measures $H(P_d)$ and $M_\alpha(P_d)$ coincide exactly with the original identification and aggregation measures proposed by Alkire and Foster (2011). Otherwise, when $P_d \in \mathcal{P}_d \backslash \mathcal{C}_d$, $H(P_d)$ and $M_\alpha(P_d)$ can be seen as slight generalizations of the former. It turns out that the $M_\alpha(P_d)$ measures can be seen as members of the following general class of functions

$$\Pi_\theta(P_d) = \frac{1}{n} \sum_{i \in Q(P_d)} \left(\sum_{j=1}^{d} a_j (\gamma_{ij}^c)^\theta \right)^{1/\theta}, \tag{22}$$

where $\theta > 0$. This measure estimates individuals' poverty levels averaging the corresponding deprivation gaps vector $\boldsymbol{\gamma}_i^c$ using a *weighted generalized mean of order* θ.[13] Clearly, when $\theta = 1$, $\Pi_1(P_d) = M_c(P_d)$. Interestingly, $\Pi_\theta(P_d)$ can also be seen as a member of the class of multidimensional poverty indices proposed by Bourguignon and Chakravarty (2003). While the original measure was defined under the assumption that the poor were identified via the union approach, the new measure shown in (22) has been adapted to incorporate the more general identification functions embodied in P_d. The choice of different values for θ allows modelling different elasticities of substitution between pairs of deprivations. However – as highlighted by Bourguignon and Chakravarty (2003:40) – such elasticity of substitution is

[12] Interestingly, $M_0(P_d)$ can also be written in the following alternative ways:

$$\frac{1}{n} \sum_{i \in Q(P_d)} c_{\mathbf{a}}(\rho^w(\mathbf{y}_i; z)) = \frac{1}{n} \sum_{i \in Q(P_d)} c_{\mathbf{a}}(\boldsymbol{\gamma}_i^0) = \frac{1}{n} \sum_{\mathbf{x} \in P_d} n_{\mathbf{x}} c_{\mathbf{a}}(\mathbf{x}).$$

[13] The class of weighted generalized means is well-known and has been widely used in welfare analysis. Higher values of θ give more importance to the upper tails of the distribution and vice versa. In the limit, as $\theta \to \infty$ (resp. $\theta \to -\infty$) the generalized mean converges towards the maximum (resp. minimum) of the distribution.

the same across *all* pairs of deprivations, a restriction that in most occasions might not be very realistic. In a recent contribution, Lasso de la Vega and Urrutia (2011) axiomatically characterize the family of poverty measures Π_θ under the assumption that the poor are identified using the union approach[14] .

4.1 Measuring poverty in a multiple domains context

From now on, we assume that the number of domains we are taking into account can be greater than one ($G \geq 1$), with each domain D_g containing d_g variables. In order to introduce our new poverty measures, we need to relabel individual's i deprivation gaps vector $\boldsymbol{\gamma}_i^c = (\gamma_{i1}^c, \ldots, \gamma_{id}^c)$ to identify the specific domains where the different deprivations belong to. Without loss of generality we can rewrite $\boldsymbol{\gamma}_i^c$ as $(\boldsymbol{\gamma}_{i1}^c, \ldots, \boldsymbol{\gamma}_{ig}^c, \ldots, \boldsymbol{\gamma}_{iG}^c)$, where $\boldsymbol{\gamma}_{ig}^c = (\gamma_{ig1}^c, \ldots, \gamma_{igd_g}^c)$ is the vector of deprivation gaps in domain D_g for each $g \in \{1, \ldots, G\}$. Hence γ_{igv}^c is individual's i deprivation gap in variable v belonging to domain g. Like all existing multidimensional poverty measures (see Permanyer 2014:4) we posit that overall poverty is the average of individuals' poverty levels[15] . In order to estimate individuals' poverty levels we suggest a two-step procedure. In the first step we aggregate individuals' deprivations within domains only. In the second one, we aggregate the previous quantities across domains. For obvious reasons, we call this the *domain-first two-stage aggregation* method. Formally, the method can be defined as follows.

[14]Such characterization states that a multidimensional poverty index has the form shown in equation (22) if and only it satisfies the axioms of Continuity, Monotonicity, Weak dimension separability, Homotheticity, Subgroup decomposability and Normalization. *Continuity* ensures that the poverty measure is continuous in its arguments. *Monotonicity* requires the poverty measure to increase with its arguments. *Weak dimension separability* stipulates that the effect of any attribute on the deprivation level can be determined independently from the values of the other attributes. *Homotheticity* imposes that a common proportional change in all the individual's deprivations in all the attributes will not affect the ordering of the social deprivations. *Subgroup decomposability* states that overall poverty is equal to the population weighted average of the subgroup poverty levels. *Normalization* imposes that the poverty index should be bounded between zero and one. The detailed definitions are given in Lasso de la Vega and Urrutia (2011).

[15]This is a consequence of the Subgroup Decomposability axiom, which states that overall poverty is equal to the population weighted average of the subgroup-specific poverty levels (see Foster and Shorrocks 1991).

Domain-first two-stage aggregation. There exist functions $\Phi : [0,1]^G \to [0,1]$ and $\varphi_g : [0,1]^{d_g} \to [0,1]$ for each $g \in \{1,\ldots,G\}$ such that, for all sets of poor profiles $P_d \in \mathcal{P}_d$, multidimensional poverty can be measured as

$$\frac{1}{n}\sum_{i\in Q(P_d)} \Phi\left(\varphi_1(\gamma^c_{i11},\ldots,\gamma^c_{i1d_1}),\ldots,\varphi_G(\gamma^c_{iG1},\ldots,\gamma^c_{iGd_G})\right). \tag{23}$$

Clarly, φ_g are the domain-specific aggregation functions (which are allowed to vary across domains) while Φ is the across-domains aggregation function. Whenever individual i is poor, her poverty level is measured as $\Phi\left(\varphi_1(\gamma^c_{i11},\ldots,\gamma^c_{i1d_1}),\ldots,\varphi_G(\gamma^c_{iG1},\ldots,\gamma^c_{iGd_G})\right)$. Proceeding in this way, we are making room for the possibility of having pairs of variables that are complements or substitutes depending on whether they belong to the same or alternative domains. When deciding what aggregators φ_g, Φ should be included in the previous equation, it is natural to choose the same functional form as in equation (22). In that case, the following multidimensional poverty measure obtains:

$$\Pi_{\boldsymbol{\theta}}(P_d) := \frac{1}{n}\sum_{i\in Q(P_d)} \left(\sum_{g=1}^{g=G} a_{g\cdot}\left[\sum_{v=1}^{d_g} w_{gv}(\gamma^c_{igv})^{\theta_g}\right]^{\theta/\theta_g}\right)^{1/\theta}, \tag{24}$$

where $\boldsymbol{\theta} = (\theta,\theta_1,\ldots,\theta_G) \in \mathbb{R}^{G+1}_{++}$, $a_{g\cdot} := \sum_{v=1}^{v=d_g} a_{gv}$, $w_{gv} := a_{gv}/a_{g\cdot}$. Clearly, when $G = 1$, equation (24) reduces to equation (22). The new poverty measure depends on parameter θ (governing the complementarity or substitutability across domains) and the different θ_g (governing the complementarity or substitutability between attributes within domain D_g). As is clear, whenever $\theta = \theta_1 = \ldots = \theta_G$ the '$G-$domain measure' $\Pi_{\boldsymbol{\theta}}(P_d)$ is equivalent to the '$1-$domain measure' $\Pi_\theta(P_d)$ shown in equation (22), so all pairs of deprivations have the same elasticity of substitution. However, when one departs from that trivial case the levels of complementarity / substitutability between deprivations in the poverty measure $\Pi_{\boldsymbol{\theta}}(P_d)$ vary across domains[16] . More specifically, we have the following result.

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Proposition 3: Consider the multidimensional poverty measure $\Pi_{\boldsymbol{\theta}}(P_d)$. (i) For any domain D_g ($g \in \{1, \ldots, G\}$), two attributes u, v belonging to that domain (i.e: $u, v \in D_g$) are complements whenever $\theta_g < \min\{1, \theta\}$. On the other hand, the same two attributes are substitutes whenever $\theta_g > \max\{1, \theta\}$. (ii) Assume now the two attributes u, v belong to different domains D_g, D_h ($g, h \in \{1, \ldots, G\}$). Then u, v are complements whenever $\theta < 1$ and substitutes when $\theta > 1$.

Proof of Proposition 3: See the Appendix.

Such flexibility allows having poverty measures with different pairs of attributes being complements or substitutes depending on the domain they belong to. In the empirical section we will illustrate how this possibility can make a difference in our assessments of multidimensional poverty levels.

4.2 Profile decomposability

An attractive characteristic of the Multidimensional Poverty Index suggested by Alkire and Foster (2011) is its purported ability to assess the contribution of each dimension to the values of the index. Once the identification step is over, the additive separability of the index allows decomposing its values according to the percent contribution of its basic constituents, a property referred to as 'dimensional decomposability'. Clearly, this property is motivated by the desire of facilitating the design of the most effective poverty eradication strategy.

Despite the apparent simplicity and intuitive appeal behind dimensional decomposability, we contend that the property is reflective of an identification procedure in which deprivations across dimensions are freely interchangeable as long as they add up to the corresponding

In this paper we use the standard ALEP definition of complementarity / substitutability. That is: when the cross partial derivative of the individual poverty function is positive (resp. negative), the attributes are considered complements (resp. substitutes).

deprivation score. Because of the way in which it is defined, dimensional decomposability disregards the complex patterns in which dimensions are interwoven to generate the partition of deprivation profiles (X^d) into poor and non-poor profiles (P_d and R_d). In other words, it does not take into account the possibility that deprivations in some dimensions might have to be experienced jointly with deprivations in other dimensions if someone is to be identified as being multidimensionally poor. After performing a dimensional decomposability exercise, policy makers have incentives to focus on reducing deprivations in the dimension that contributes the most to multidimensional poverty levels – e.g., V_i. However, the reduction of deprivations in V_i might require entirely different policies if those deprivations are jointly experienced with deprivations in V_j, or with deprivations in V_l. Therefore, we suggest complementing dimension decomposability by another decomposability property that is in line with the identification method suggested in this paper.

The set of deprivation profiles naturally generates a partition of the population under study, N, into $\left|X^d\right| = 2^d$ groups (each individual i is assigned via ρ^w to the corresponding element in X_d on the basis of her achievement vector $\mathbf{y}_i$). For any $\mathbf{x} \in X^d$ let $M_{\mathbf{x}}$ denote the achievement matrix corresponding to the set of individuals experiencing deprivations as in $\mathbf{x}$ (i.e., $N_{\mathbf{x}}$). The multidimensional poverty level corresponding to the members of $N_{\mathbf{x}}$ is written as $f(M_{\mathbf{x}}; \mathbf{z})$. According to the axiom of Subgroup decomposability (see footnote #15),

$$f(M; \mathbf{z}) = \sum_{\mathbf{x} \in X^d} \frac{n_{\mathbf{x}}}{n} f(M_{\mathbf{x}}; \mathbf{z}) \tag{25}$$

The percent contribution of the members of $N_{\mathbf{x}}$ to overall poverty levels is thus calculated as

$$C_{\mathbf{x}} = 100 \left(\frac{n_{\mathbf{x}}}{n} f(M_{\mathbf{x}}; \mathbf{z}) \right) / f(M; \mathbf{z}). \tag{26}$$

Clearly, $\sum_{\mathbf{x}\in X^d} C_{\mathbf{x}} = 100$. The exercise of breaking down overall poverty into the set of contributions $\{C_{\mathbf{x}}\}_{\mathbf{x}\in X^d}$ is referred to as profile decomposability. We contend that profile decomposability conveys a clearer message than its dimension-wise counterpart with respect to understanding the articulation of multidimensional poverty. Since the different population subgroups in $\{N_{\mathbf{x}}\}_{\mathbf{x}\in X^d}$ might require profile-specific anti-poverty strategies (i.e., anti-poverty strategies specifically crafted for them), profile decomposability can be particularly informative for the design of efficient poverty erradication programs.

5. Empirical illustrations

In this section we present two empirical examples to illustrate the differences between the new identification and aggregation methods suggested in this paper and the ones used in standard multidimensional poverty measures. The first example uses data from the United States and the second one focuses on 48 countries from the developing world.

5.1 United States

In order to illustrate the usefulness of their multidimensional poverty measures, Alkire and Foster (2011) presented an empirical example using the 2004 National Health Interview Survey from the US. In that exercise, the authors used the following four variables to assess multidimensional poverty levels among adults aged 19 and above: V_1 = 'Income measured in poverty line increments and grouped into 15 categories', V_2 = 'Years of Schooling', V_3 = 'Self-assessed Health' and V_4 = 'Health insurance'. The dimension-specific deprivation thresholds were defined as follows. A person is deprived in V_1 if she lives in a household falling below the standard income poverty line, in V_2 if he lacks a high school diploma, in V_3 if she reports 'fair' or 'poor' health and in V_4 if he lacks health insurance. The population is partitioned into four groups (Hispanic/Latino, (Non-Hispanic) White, (Non-Hispanic) African American

/ Black and Others) and the sample size is n =45884.

To identify poor individuals, Alkire and Foster basically use the dual cutoff method assuming equal weights across dimensions (i.e.: $a_1 = a_2 = a_3 = a_4 = 1/4$) and a deprivation threshold $k = 1/2$.[17] This way, whenever an individual is deprived in at least two dimensions (any), she will be considered poor. With the notation introduced in this paper, this generates the set of poor profiles $P_4^1 = \{1100, 1010, 1001, 0110, 0101, 0011, 1110, 1101, 1011, 0111, 1111\}$.[18] However, if one is willing to allow for the role of compensation within domains (see section 3.1), there are good reasons to argue that in order to be considered poor an individual has to experience deprivation at least in V_1 and V_2 or in V_3 and V_4 simultaneously, therefore generating the set of poor profiles $P_4^* = \{1100, 0011, 1110, 1101, 1011, 0111, 1111\}$. Since P_4^* can never be generated via the classical counting identification method, it is informative to compare the poverty levels derived from it with the poverty levels reported by Alkire and Foster (2011) when using P_4^1.[19]

We start reporting the shares of individuals that are coherently identified as poor or non-poor according to P_4^1 and P_4^* together with the shares of individuals that are misclassified according to the two criteria across the four racial groups (see Table 1). Since $P_4^* \subset P_4^1$, the set of individuals that are coherently identified as poor by the two methods corresponds to the set of individuals with deprivation profiles $\mathbf{x}$ belonging to P_4^* (their percentages are reported in column A). The individuals that are coherently identified as non-poor by the two methods must have a deprivation profile belonging to {0000, 1000, 0100, 0010, 0001}

[17] At the end of the exercise, they show alternative results when choosing $k \in \{1/4, 1/2, 3/4, 1\}$.

[18] To strictly follow the notation introduced in this paper, this set of poor profiles should be written as $P_{4,\mathcal{C}((1/4,1/4,1/4,1/4),1/2)}$. For the sake of simplicity, we simply write P_4^1.

[19] It would also be possible to perform the same illustrative exercise using the other set of poor profiles that can not be generated via the counting approach mentioned in section 3.1 (i.e: $P_4^{**} = \{1010,1001,0110,0101,1110,1101,1011,0111,1111\}$. However, since this does not yield particularly interesting insights, we have kept the illustration short using P_4^* only.

(their percentages are reported in column B). The individuals that are identified as poor by P_4^1 but as non-poor by P_4^* are the ones with deprivation profiles in $\{1010, 1001, 0110, 0101\}$ (the respective percentages are reported in column C). As shown in the last row of Table 1, the shares of individuals that are coherently identified as poor and non-poor are 7.1% and 83.9%, respectively. The share of individuals that are misclassified according to the two identification methods is 9%, and is particularly high among Hispanics (20%). In Table 1 we also show the values of two multidimensional poverty indices resulting from alternative identification methods. One of them is the multidimensional headcount ratio H (see equation (20)) and the other one is the adjusted headcount ratio M_0 proposed by Alkire and Foster (2011) (see equation (21)). In Columns D and E we show the values of H when using P_4^1 and P_4^* as identification methods respectively, while the analogous results corresponding to M_0 are shown in Columns F and G. The values of the headcount index vary substantially between P_4^1 and P_4^*: They more than halve the original levels (since $P_4^* \subset P_4^1$, the values of H are necessarily smaller). A similar pattern is observed when computing the values of M_0: When moving from P_4^1 to P_4^* the values of the adjusted heacount ratio more than halve. We observe no changes between multidimensional poverty *rankings* when moving from one identification method to the other. However, the set of people that could potentially be the target of anti-poverty programs varies substantially across methods.

[[[Table 1]]]

We turn now to the issue of decomposability. According to dimension decomposability, the values of M_0 can be broken down by the contribution of the four variables taken into account.[20] More specifically, we write $M_0 = \sum_j H_j/d$, where H_j is the share of the

[20]See Table 2 in Alkire and Foster 2011 for the specific results.

respective population that is both poor (according to the AF identification method using P_4^1) and deprived in variable j. However, among the people that are both 'AF-poor' and deprived in variable j, there is a subgroup of individuals that are not poor according to the identification method P_4^*. To compute the relative size of this subgroup for the case where $j = 1$ (i.e.: in the case of V_1 = 'Income') we simply need to compute the following quantity: $(N_{1010} + N_{1001})/(N_{1100} + N_{1010} + N_{1001} + N_{1110} + N_{1101} + N_{1011} + N_{1111})$. The respective denominator contains all individuals that are 'AF-poor' *and* deprived in terms of income while the numerator counts how many of them are considered to be non-poor according to P_4^*. That would be the share of people contributing to H_1 that are mistargeted according to P_4^*. In Table 2 we show the percentage of mis-targeted individuals for the four variables taken into account across the different racial groups. The presence of mis-targeted individuals is quite substantial, on many occasions with values above 50%. This suggests that the alternative methods discussed in this section identify groups of individuals differing to a great extent.

[[[Table 2]]]

We conclude this empirical illustration with the results of the profile decomposability exercise suggested in section 4.2. In Table 3 we show the multidimensional poverty levels (as measured with M_0; they are reported in the third column) corresponding to each group $N_{\mathbf{x}}$ for the different $\mathbf{x} \in P_4^*$ and the corresponding contribution to overall poverty ($C_{\mathbf{x}}$, shown in the fourth column). The shares of the different groups $N_{\mathbf{x}}$ are reported in the second column. The deprivation profile experienced by the largest share of individuals is 1110 (that is: those having health insurance but deprived in all other variables) and the one experienced by the smallest share of individuals is 1011 (i.e., those having a high school diploma but deprived in all other variables). As expected, the groups experiencing more deprivations tend to be poorer, so their contribution to overall poverty levels is higher. This is the reason why even if the set of individuals experiencing deprivation in income and education only are four times more numerous than those individuals deprived in all dimensions ($N_{1100}/N = 1.68\%$ and $N_{1111}/N = 0.43\%$), the contribution of the former to overall poverty levels is barely twice that of the latter ($C_{1100} = 19.44\%$ vs. $C_{1111} = 8.33\%$).

[[[Table 3]]]

5.2 Developing World

Since 2010, the UNDP presents the values of the Multidimensional Poverty Index (MPI) on a yearly basis to rank more than a hundred countries in terms of multidimensional poverty levels (see Alkire and Santos 2010). The UNDP's MPI mainly draws from three sources of data: the Demographic and Health Surveys (DHS), the Multiple Indicators Cluster Survey and the World Health Survey. In order to avoid the potential comparability problems arising from the use of alternative sources of data, in this paper we focus our attention on 48 out of the 50 DHS used in the construction of the 2014 MPI [21] (totaling n=761,909 households, which are the basic units of analysis). The MPI is a hierarchically structured index of multidimensional poverty, with ten variables partitioned in three domains: 'Health' (H), 'Education' (E) and 'Standard of Living' (S). In Table 4 we show the variables included in

[21] The DHS for Nicaragua 2012 and Tajikistan 2012 were not accessible to the authors of this paper. The remaining 48 countries included in the dataset and the year/s in which the DHS was taken are: Albania 2008/2009; Armenia 2010; Azerbaijan 2006; Bangladesh 2011; Benin 2006; Bolivia 2008; Burkina Faso 2010; Burundi 2010; Cambodia 2010; Cameroon 2011; Colombia 2010; Congo 2011/2012; Cote d'Ivoire 2011/2012; Dominican Republic 2007; Egypt 2008; Ethiopia 2011; Gabon 2012; Guinea 2005; Guyana 2009; Haiti 2012; Honduras 2011/2012; India 2005/2006; Indonesia 2012; Jordan 2009; Kenya 2008/2009; Lesotho 2009; Liberia 2007; Madagascar 2008/2009; Malawi 2010; Maldives 2009; Mali 2006; Moldova 2005; Mozambique 2011; Namibia 2006/2007; Nepal 2011; Niger 2012; Pakistan 2012/2013; Peru 2012; Philippines 2008; Rwanda 2010; Sao Tome and Principe 2008/2009; Senegal 2010/2011; Tanzania 2010; Timor-Leste 2009/2010; Uganda 2011; Ukraine 2007; Zambia 2007 and Zimbabwe 2010/2011.

each domain. The ‘Health’ and ‘Education’ domains are composed of two variables each: One referring to adults and the other to children in the corresponding household. The six variables in the ‘Standard of Living’ domain include several household characteristics. In Table 4, we also show the conditions that must be met in order to consider a household deprived in the corresponding variable. Lastly, the table also shows the weight that the AF method assigns to each variable.

[[[Table 4]]]

In this section we are going to compare how the new identification and aggregation methods suggested in this paper fare vis-à-vis currently existing measures. To do that, we will separate our analysis in two parts. In the first part, we will compare alternative identification methods keeping the aggregation method constant. In the second one, we will fix the identification method and compare alternative aggregation methods (see below). Before starting, we perform a validation check to assess the quality and soundness of the 48-country dataset created for this section of the paper. More specifically, we compare the official UNDP’s 2014 MPI value, restricted to the 48 countries whose MPI values where estimated using DHS, with the MPI values obtained using the Alkire and Foster (2011) M_0 index applied to this dataset. Unsurprisingly, both sets of measures give highly consistent results. As shown in Figure 3 both measures tend to rank countries in a strongly linear fashion: The correlation coefficient is as high as 0.94. The differential treatment of missing values and some slight differences in the definition of the Nutrition variable[22] explain the

[22]In the official MPI, a household is deprived in the nutrition variable if any adult or child for whom there is nutritional information is malnourished. In the MPI measure constructed in this paper, the nutritional information has only been collected for the adult household members.

differences observed between both measures. These results suggest that the dataset we use is of reasonable quality.

[[[Figure 3]]]

New identification methods

To decide whether a household should be identified as poor or not, the MPI uses the AF method with the weights shown in Table 4. The three domains are equally weighted at 1/3, with a deprivation threshold of $k = 1/3$. This way, a household experiencing deprivation in one of the Health variables and in one of the Education variables (each with a weight of 1/6) is identified as poor. Analogously, a household experiencing deprivation in any one of the Health or Education variables and in any three of the Standard of Living variables is identified as poor. However, if one has reasons to consider that the lack of deprivation in some variables within some domain could somehow compensate for the deprivations experienced in the other variables of that domain, then the AF identification method is not the most appropriate (for instance: one might argue that the deprivation experienced by parents might somehow be compensated by the lack of deprivation of the children or vice versa). If this were the case, one might prefer to define a houshold as being poor whenever the corresponding deprivation profile belongs to the set of poor profiles $P^*_{10} = \{1100000000, 0011000000, 0000111111\}^{\uparrow}$. Observe that in order to be considered multidimensionally poor according to P^*_{10}, a household must be deprived in all variables of at least one domain. Since one might argue that requiring a household to be deprived in all six 'Standard of Living' indicators to be considered as poor is too stringent a condition, one could also relax this assumption and define another set of poor profiles, Q^*_{10}, as follows. A '$Q^*_{10}-$poor household' must be deprived in two variables in at least one of the domains of 'Health' and 'Education' (i.e.: $U(Q^*_{10}) \supset \{1100000000,\ 0011000000\}$) *or* it must be deprived in at least four of the six variables comprising the 'Standard of Living' domain. Because of the way in which it is defined, Q^*_{10} can be seen as a mixed case between P^*_{10} and the counting approach. While both P^*_{10} and Q^*_{10} satisfy the Consistency Condition, neither of them can be generated via the counting identification method (i.e: $P^*_{10}, Q^*_{10} \in \mathcal{P}_{10} \backslash \mathcal{C}_{10}$, see Theorem 2). Since both illustrate reasonable criteria to identify poor households, in this section we compare how they perform vis-à-vis the dual cutoff method (keeping the same aggregation method for all of them).

In the first three columns of Table 5 we present the country values of Alkire and Foster's poverty index $M_0(P_d)$ shown in equation (21) under three different poor identification functions: (i) The 'classical' dual cutoff method that weights the three domains of the MPI equally at 1/3 and uses a deprivation threshold of $k = 1/3$; (ii) P^*_{10} and (iii) Q^*_{10}. We denote them as $M_0(P_{10,\mathcal{C}((1/3,1/3,1/3),1/3)}), M_0(P^*_{10})$ and $M_0(Q^*_{10})$, respectively. As can be seen, the values of the different M_0 go in the same direction for the three cases: Countries with low or high poverty levels coincide substantially. The correlation coefficient between the 48 values of $M_0(P_{10,\mathcal{C}((1/3,1/3,1/3),1/3)})$ and $M_0(P^*_{10})$ and the correlation coefficient between the 48 values of $M_0(P_{10,\mathcal{C}((1/3,1/3,1/3),1/3)})$ and $M_0(Q^*_{10})$ are very high: 0.95 and 0.98, respectively. Since these correlation coefficients implicitly depend on the weights used for each domain $\mathbf{w} = (w_1, w_2, w_3) \in \Delta_3$ and the deprivation threshold $k \in (0, 1]$, they are denoted as $r_{\mathcal{C},P^*_{10}}((w_1, w_2, w_3), k)$ and $r_{\mathcal{C},Q^*_{10}}((w_1, w_2, w_3), k)$, respectively. Therefore, we can write $r_{\mathcal{C},P^*_{10}}((1/3, 1/3, 1/3), 1/3) = 0.95$, and $r_{\mathcal{C},Q^*_{10}}((1/3, 1/3, 1/3), 1/3) = 0.98$.

Even if the three measures tend to rank countries in a highly consistent way, it turns out that the corresponding poor identification functions operate in a distinct manner. In

Table 5 we show for each country the percentage of households where the AF-method and P^*_{10} disagree (i.e., we quantify the share of households that are misclassified as 'poor' or 'non-poor' according to the $P_{10,\mathcal{C}((1/3,1/3,1/3),1/3)}$-method and P^*_{10}). Since these percentages implicitly depend on the weights that are used for each domain $\mathbf{w} = (w_1, w_2, w_3) \in \Delta_3$ and the deprivation threshold $k \in (0, 1]$, we denote them as $m_{\mathcal{C},P^*_{10},l}((w_1, w_2, w_3), k)$, where l indexes the 48 countries taken into account. It turns out that the degree of disagreement between both identification methods is substantial: Averaging across countries (i.e., computing $\overline{m}_{\mathcal{C},P^*_{10}}((1/3, 1/3, 1/3), 1/3) := \sum_{l=1}^{l=48} m_{\mathcal{C},P^*_{10},l}((1/3, 1/3, 1/3), 1/3)/48)$, we find that 24% of households are classified inconsistently between the two criteria. In some countries the percentage of disagreement is greater than 50%. Repeating the same exercise comparing the AF-method with Q^*_{10}, we obtain a cross-country average of $\overline{m}_{\mathcal{C},Q^*_{10}}((1/3, 1/3, 1/3), 1/3) = 13\%$ of misclassified households. The size of these percentages implies that the potential beneficiaries of poverty alleviation programs can differ dramatically when choosing one identification method or the other.

[[[Table 5]]]

In Table 5 we have compared the performance of P^*_{10} and Q^*_{10} with the 'official' AF-method that weights the three domains of the MPI equally at 1/3 and uses $k = 1/3$. In this context, one might wonder whether the results shown in Table 5 are highly dependent on the specific choice of these parameters or if they are robust to other specifications. Since the dual cutoff method does not *a priori* impose any restrictions on the choice of weights $\mathbf{w}$ or the deprivation threshold k, we complete our comparative analysis allowing these parameters to take all possible values within their respective domains. In other words, we compare the performance of P^*_{10} and Q^*_{10} with the dual cutoff method considering *all* possible weighting schemes for the three domains of the MPI[23] , and under *any* deprivation threshold $k \in (0,1]$. It turns out that, even if the correlation coefficients $r_{\mathcal{C},P^*_{10}}((w_1,w_2,w_3),k)$ and $r_{\mathcal{C},Q^*_{10}}((w_1,w_2,w_3),k)$ tend to be very high, they never reach the value of 1 (a consequence of the fact that neither P^*_{10} nor Q^*_{10} belong to $\mathcal{C}_{10}$). The average of $r_{\mathcal{C},P^*_{10}}((w_1,w_2,w_3),k)$ and $r_{\mathcal{C},Q^*_{10}}((w_1,w_2,w_3),k)$ across the entire domain $\Delta_3 \times (0,1]$ equal 0.91 and 0.89 respectively. From this analysis, we conclude that the identification methods P^*_{10} and Q^*_{10} tend to rank countries in the same direction as the dual cutoff method does.

The fact that $M_0(P_{10,\mathcal{C}((w_1,w_2,w_3),k)}), M_0(P^*_{10})$ and $M_0(Q^*_{10})$ tend to rank countries similarly does not necessarily imply that the three methods agree when deciding whether a given household should be considered 'poor' or 'non-poor'. Since neither P^*_{10} nor Q^*_{10} belong to $\mathcal{C}_{10}$, it turns out that there is always some degree of disagreement, so all values of $\overline{m}_{\mathcal{C},P^*_{10}}((w_1,w_2,w_3),k)$ and $\overline{m}_{\mathcal{C},Q^*_{10}}((w_1,w_2,w_3),k)$ are strictly positive. Indeed, the average value of $\overline{m}_{\mathcal{C},P^*_{10}}((w_1,w_2,w_3),k)$ and $\overline{m}_{\mathcal{C},Q^*_{10}}((w_1,w_2,w_3),k)$ across the entire domain $\Delta_3 \times (0,1]$ equal 27% and 32% respectively (i.e: around one third of the households are misclassified). From these analyses we can conclude that the level of disagreement between the identification functions considered here are generally quite substantial, a result with strong implications for the identification of the potential beneficiaries of poverty eradication programs.

New aggregation methods

We are now going to compare the performance of the new aggregation methods suggested in this paper vis-à-vis currently existing approaches in multidimensional poverty measurement. In this context, we are particularly interested in investigating the extent to which

[23]To simplify matters, we allow all possible weights across domains only (i.e.: not across all 10 indicators). Once a weight is assigned to each domain, we assume that all indicators within that domain are weighted equally.

the new aggregation methods allowing domain-specific elasticities of substitution differ with respect to the traditional methods imposing fixed elasticities of substitution across all pairs of variables. When performing these comparisons, we will use the same identification methods overall to ensure that the observed changes are solely attributable to the changes in aggregation methods. More specifically, we will compare the values of $\Pi_{\boldsymbol{\theta}}(P_d)$ vis-à-vis the values of $M_0(P_d)$ for different values of $\boldsymbol{\theta} = (\theta, \theta_1, \theta_2, \theta_3) \in \mathbb{R}^4_{++}$ while keeping fixed $P_d = P_{10,\mathcal{C}((1/3,1/3,1/3),1/3)}$ (which is the set of poor profiles used in the original definition of UNDP's MPI – see Table 4) for both cases.

As is clear from equations (21), (22) and (24), the measure $M_0(P_d)$ is a particular case of $\Pi_{\boldsymbol{\theta}}(P_d)$ that obtains when one chooses $\boldsymbol{\theta} = (1, 1, 1, 1)$. In this case, the different pairs of variables are neither complements nor substitutes (according to Alkire and Foster (2011:485), M_α is 'neutral' in that individuals' poverty functions have vanishing cross partial derivatives for the pairs of variables in which they are deprived). Yet, there might be good reasons to argue that some pairs of variables should be substitutes (e.g: those within the Standard of Living domain) and other pairs complements (e.g: those within the Health and Education domains). According to Proposition 3, one possible way of accomplishing this is by choosing the vector of parameters $\boldsymbol{\theta}^* = (2, 3, 3, 0.5)$ in $\Pi_{\boldsymbol{\theta}}(P_d)$ (in this case, since $\theta^* > 1$ we are also assuming complementarity between pairs of variables belonging to different domains). The values of this new index of multidimensional poverty for the 48 countries considered in this section are shown in Table 5. It turns out that the correlation coefficient between the 48 values of $M_0(P_d)$ and those of $\Pi_{\boldsymbol{\theta}^*}(P_d)$ is extremely high: 0.99. Since the correlation coefficient between the 48 values of $M_0(P_d)$ and those of $\Pi_{\boldsymbol{\theta}}(P_d)$ depend on $\boldsymbol{\theta}$, it will be written as $r(\boldsymbol{\theta})$ (hence $r(2, 3, 3, 0.5) = 0.99$).

Even if $M_0(P_d)$ and $\Pi_{\boldsymbol{\theta}^*}(P_d)$ tend to rank countries pretty much in the same way, the extent of multidimensional poverty arising from both measures can be substantially different. To illustrate this point, for any $\boldsymbol{\theta} \in \mathbb{R}^4_{++}$ we define the following indicator:

$$\delta(\boldsymbol{\theta}) = \frac{100}{48} \sum_{l=1}^{l=48} \left| 1 - \frac{\Pi_{\boldsymbol{\theta}}(P_d)_l}{M_0(P_d)_l} \right|, \tag{27}$$

where l indexes the 48 countries considered in this section. Clearly, $\delta(\boldsymbol{\theta})$ averages the relative difference (in absolute value) between the values of $M_0(P_d)$ and $\Pi_{\boldsymbol{\theta}}(P_d)$ across the 48 countries, so it gives an idea of the extent of dissimilarity that exists between both measures. As is clear, $\delta(\boldsymbol{\theta}) = 0$ whenever the two measures are exactly the same and it is strictly positive otherwise. For the case at hand, it turns out that $\delta(\boldsymbol{\theta}^*) = 36.5\%$. In words: when switching from the values of $M_0(P_d)$ to those of $\Pi_{\boldsymbol{\theta}^*}(P_d)$, the values of the former change, on average, a 36.5% from their original level (in the last column of Table 5 we show the values of $100|1 - (\Pi_{\boldsymbol{\theta}}(P_d)/M_0(P_d))|$ for each of the 48 countries considered here).

Since there does not seem to be a standard procedure for determining the extent of complementarity and substitutability across poverty dimensions, the choice of the parameters $(\theta, \theta_1, \theta_2, \theta_3)$ might be somehow arbitrary. While we have chosen $\boldsymbol{\theta}^*$ for illustrative purposes, there might be many other reasonable choices as well. For this reason, we will explore the behavior of $\delta(\boldsymbol{\theta})$ when $\boldsymbol{\theta}$ can freely move within Θ, a 'reasonably large' subset of $\mathbb{R}^4_{++}$. Given the unbounded nature of $\mathbb{R}^4_{++}$, we restict our attention to the bounded case where $\Theta = (0, 3]^4$, i.e: when θ and the θ_i can not be larger than 3.[24] In Figure 4 we plot the values of $\delta(\boldsymbol{\theta})$ when $\boldsymbol{\theta} \in (0, 3]^4$. As can be seen, the extent of multidimensional poverty can differ to a large extent depending on the values of $\boldsymbol{\theta}$. By continuity, since $\delta(\boldsymbol{\theta}) = 0$ when $\boldsymbol{\theta} = (1, 1, 1, 1)$, the values of $\delta(\boldsymbol{\theta})$ approach 0 as $\boldsymbol{\theta}$ approaches $(1, 1, 1, 1)$. At the other extreme, for many values of $\boldsymbol{\theta} \in \Theta$ that are farther appart from $(1, 1, 1, 1)$, $\delta(\boldsymbol{\theta})$ can take

[24]Other choices of Θ are certainly feasible, but the results they offer are not particularly insightful.

values well over 50% (e.g: see some of the regions in the plots of the first and last rows respectively). The average of $\delta(\boldsymbol{\theta})$ over the entire domain Θ is 28.7%, thus showing that the alternative aggregation procedures suggested here can generate substantially different levels of multidimensional poverty. Interestingly, the average of $r(\boldsymbol{\theta})$ across the entire domain $(0,3]^4$ is 0.99. Therefore, while $\Pi_{\boldsymbol{\theta}}(P_d)$ tends to rank countries in the same way as $M_0(P_d)$ does, the values of the former can differ to a considerable extent with respect to the latter.

[[[Figure 4]]]

Summing up, we have seen that when considering the alternative identification or aggregation methodologies suggested here, the set of housholds that are identified as poor and the corresponding multidimensional poverty levels can differ to a considerable extent with respect to currently existing approaches. Presumably, such differences would be even larger if we considered a multidimensional poverty measure incorporating our new identification *and* aggregation methods (that is: something like $\Pi_{\boldsymbol{\theta}}(Q_d)$, with $Q_d \in \mathcal{P}_d \backslash \mathcal{C}_d$ and $\boldsymbol{\theta} \neq (1,1,1,1)$), an issue we have not investigated in this paper.

6. Discussion and concluding remarks

The success of any poverty eradication program crucially depends on its ability to identify who is poor and who is not. In this paper, we have shown that *the* state-of-the-art methodology that is pervasively used to identify the poor in multidimensional contexts, the dual cutoff or AF method (Alkire and Foster 2011), is a basic method that precludes many of the subtle and complex considerations that should be incorporated in such consequential decisions. One of the main findings of this work is that the simplicity of the counting approach that underlies the dual cutoff method – an algorithm-like approach that counts the number of deprivations experienced by individuals to decide about their poverty status – comes at a high price because it precludes the possibility of generating 'poor-identification rules' that are sensitive to potential interactions between the sets of dimensions taken into account. Depending on the nature of the variables considered, it could be the case that one might want the lack of deprivation in some dimension X to compensate for the deprivation experienced in some dimension Y *but* not in Z. If one is willing to allow for the possibility of such compensation phenomena within or between certain domains, there is reason to make room for the more sophisticated identification and aggregation methods proposed in this paper. We contend that such patterns of dimension-specific interactions naturally arise when multidimensional indices are hierarchically structured in exhaustive and mutually exclusive domains, as is increasingly the case in all areas of the social sciences.

To overcome the limitations of the dual cutoff method, we have suggested a much broader and less stringent identification method based on the so-called Consistency Condition (which contains the former approach as a particular case). The conditions imposed under CC are flexible enough to allow capturing the intertwined relationships between groups of variables one might observe in diverse empirical applications. In addition, the measurement framework suggested in this paper allows introducing alternative levels of complementarity or substitutability between pairs of variables depending on the domain they belong to when assessing poverty levels–an improvement with respect to the current state of the literature, which assumes the same degree of complementarity or substitutability across dimensions.

An attractive characteristic of the dual cutoff method is its purported ability to explain the contribution of each dimension to the overall values of the poverty index (a property known as 'dimensional decomposability'). However, this property implicitly ignores the interaction patterns existing between dimensions (that is, the fact that deprivations in some dimensions must be experienced jointly with deprivations in other dimensions if someone is to be identified as being multidimensionally poor). Decision-makers guided by 'dimensional decomposability' have incentives to allocate resources to reduce deprivations in the dimension contributing the most to overall poverty levels (say, X), irrespective of the huge difference it may make to experience deprivations in X jointly with deprivations in Y rather than experiencing deprivations in X and Z. We suggest complementing 'dimensional decomposability' with 'profile decomposability', another property that is naturally derived from the CC identification methods suggested in this paper and which conveys a clearer message to understand the articulation of multidimensional poverty. More specifically, 'profile decomposability' explicitly accounts for patterns of joint deprivation, so it is particularly useful for the design of 'profile-specific anti-poverty strategies', i.e: anti-poverty strategies specifically crafted for a group experiencing a certain pattern of multiple deprivations.

The ideas introduced in this paper allow modelling in previously unexplored ways crucial aspects related to the identification and aggregation of the multidimensionally poor. Yet, when it comes to empirically implement them one must face difficult questions forcing analysts and policy makers to reflect upon the meaning of being multiply deprived in different contexts (e.g: ¿how to choose the sets of poor profiles P_d?, ¿How to determine the degree of complementarity / substitutability across and within domains?). These questions are highly context-specific, so each serious empirical study should attempt to find its own answers when implementing its multidimensional poverty measures. In the empirical section of the paper we investigate the performance of the new identification and aggregation methodologies in two separate illustrative examples. The first uses data from the US in 2004, and the second uses data from 48 Demographic and Health Surveys collected around 2010. In both cases we reach similar conclusions. It turns out that the dual cutoff method and the alternative CC methods that can not be generated via the AF methodology tend to consistently rank the populations compared in terms of poverty levels. In other words: the populations experiencing high or low poverty levels using both identification methods coincide substantially. Even if the relative position of the populations that are being compared does not change substantially, what *does* substantially change is the corresponding *level* of poverty observed under alternative identification methods. The percentage of households that are inconsistenly identified as 'poor' according to both criteria is considerably high (for the 48 developing countries example, it is around 30%). In addition, the extent of multidimensional poverty can be substantially different when considering the alternative aggregation methods suggested here. We reiterate that these differences can have enormous implications for the identification of the potential beneficiaries of poverty eradication programs and the assessment of the extent of their poverty levels.

The ideas introduced in this paper can be extended in several interesting directions, among which we highight the following two. (i) Enlarge the hierarchical structure of the multidimensional poverty indices we are dealing with to include more finely-grained partitions beyond the one discussed here (for instance: each domain might be partitioned in several sub-domains, and so on). Clearly, the identification and aggregation ideas introduced here can be straightforwardly applied to those more complicated structures as well. (ii) Apply the domain-first two-stage aggregation method in the space of achievements (rather than deprivations as in (23)) to generate multidimensional indices of well-being. This way, assuming the different well-being indicators are partitioned in $G \geq 1$ domains, the following class of 'multiple domain well-being indices' can be defined

$$W(Y) := \frac{1}{n} \sum_{i=1}^{n} \left(\sum_{g=1}^{g=G} a_{g\cdot} \left[\sum_{v=1}^{d_g} w_{gv} (y_{igv})^{\theta_g} \right]^{\theta/\theta_g} \right)^{1/\theta}, \tag{28}$$

where y_{igv} denotes individual i achievement level in the variable v that belongs to domain g. The application of this approach can be very fruitful in multidimensional welfare analysis as it allows introducing domain-specific elasticities of substitution and improve many of the limitations of currently existing 'single-domain' composite indices of well-being (see, for instance, Ravallion (2012) in his criticism against the Human Development Index).

To the extent that the success of micro level anti-poverty programs depends on targeting the right individuals and properly assessing their deprivation levels, and that current international cooperation, development, and aid programs are guided by the macro level results derived from the corresponding measures, the issues analyzed in this paper have practical and financial implications for the design of effective poverty eradication strategies. Having recently reached the Millennium Development Goals (MDGs) target year, many scholars and policy-makers are currently engaged in an intense debate about what kind of headline poverty indicator should be the most appropriate to guide poverty eradication strategies in the post-2015 global development agenda. Like its predecessor, the first of the so-called Sustainable Development Goals (the SDGs) aims to 'End Poverty in all its forms everywhere'. This is a good moment to take stock and reflect before uncritically extending use of the dual cutoff method. Other procedures, such as the ones suggested here, exist to identify recipients and assess their poverty levels under one of the greatest international endeavours of our time to eradicate poverty.

7. Appendix

Proof of Lemma 1: Consider any pair $\mathbf{x}, \mathbf{y} \in X^m$ of distinct elements in $U(Z)$. Assume $\delta(\mathbf{x}, \mathbf{y}) = \emptyset$. If this were the case, it would mean that $\mathbf{y}$ would vector dominate $\mathbf{x}$, which is a contradiction since both $\mathbf{x}, \mathbf{y} \in U(Z)$. Therefore, one must have that $\delta(\mathbf{x}, \mathbf{y}) \neq \emptyset$.

Analogously, one must also have that $\delta(\mathbf{y}, \mathbf{x}) \neq \emptyset$.

Q.E.D.

Proof of Proposition 1: We start with the 'if' part of the proof. Assume $P_d \subset X^d$ satisfies the CC condition. We have to prove that $(U(P_d))^{\uparrow} = P_d$.

1) We start proving $(U(P_d))^{\uparrow} \subset P_d$. Take $\mathbf{x} \in (U(P_d))^{\uparrow}$. Then, there exists some $\mathbf{z} \in U(P_d)$ such that $\mathbf{z} \preceq \mathbf{x}$ (if $\mathbf{x} \in U(P_d)$, then $\mathbf{z} = \mathbf{x}$). Since $U(P_d) \subset P_d$, $\mathbf{z} \in P_d$. In addition, since $\mathbf{x} \in \mathbf{z}^{\uparrow}$ and $P_d \in \mathcal{P}_d$, one can conclude that $\mathbf{x} \in P_d$.

2) We now prove $(U(P_d))^{\uparrow} \supset P_d$. Take $\mathbf{x} \in P_d$. If it turns out that $\mathbf{x} \in U(P_d)$ then we are done. If $\mathbf{x} \notin U(P_d)$ then there must exist some $\mathbf{y} \in P_d \backslash \{\mathbf{x}\}$ such that $\mathbf{y} \preceq \mathbf{x}$. Now, if $\mathbf{y} \in U(P_d) \subset P_d$ then $\mathbf{x} \in \mathbf{y}^{\uparrow}$. Since $P_d \in \mathcal{P}_d$, one can conclude that $\mathbf{x} \in (U(P_d))^{\uparrow}$. Otherwise, if $\mathbf{y} \notin U(P_d)$ then we can proceed iteratively until reaching an element belonging to $U(P_d)$. That is: since X^d is finite ($|X^d| = 2^d$) there must exist a finite sequence of vector dominations $\mathbf{z}_i \preceq \mathbf{z}_{i+1}$ from some element $\mathbf{z}_1 \in U(P_d)$ up to $\mathbf{x}$ (i.e.: $\mathbf{z}_1 \preceq \mathbf{z}_2 \ldots \preceq \mathbf{z}_n \preceq \mathbf{x}$), so that $\mathbf{x} \in \mathbf{z}_1{}^{\uparrow}$. Since $P_d \in \mathcal{P}_d$, one can conclude that $\mathbf{x} \in (U(P_d))^{\uparrow}$.

This proves the 'if' part of the proposition. The 'only if' part of the proof goes as follows. Assume P_d is a subset of X^d such that $(U(P_d))^{\uparrow} = P_d$. We have to prove that $P_d \in \mathcal{P}_d$. Take any $\mathbf{x} \in P_d$. Since $(U(P_d))^{\uparrow} = P_d$ we can say that $\mathbf{x} \in \mathbf{z}^{\uparrow}$ for some $\mathbf{z} \in U(P_d)$. Consider now any $\mathbf{y} \in \mathbf{x}^{\uparrow}$. By the transitivity of $\preceq$ one has that $\mathbf{y} \in \mathbf{z}^{\uparrow}$. Since $(U(P_d))^{\uparrow} = P_d$, we can conclude that $\mathbf{y} \in P_d$.

Q.E.D.

Proof of Theorem 1: We start showing that when $d \in \{2, 3\}$, $\mathcal{C}_d = \mathcal{P}_d$. By definition, every element in $\mathcal{C}_d$ automatically belongs to $\mathcal{P}_d$. Therefore, we only need to show that for every element P_d in $\mathcal{P}_d$ there exists some weighting scheme $\mathbf{a} \in \Delta_d$ and a deprivation threshold k such that $P_{d,\mathcal{C}(\mathbf{a},k)} = P_d$. In Tables A1 and A2 we show the list of all possible elements of $\mathcal{P}_d$ and an example of the corresponding weights and deprivation thresholds such that $P_{d,\mathcal{C}(\mathbf{a},k)} = P_d$ for the cases $d = 2$ and $d = 3$ respectively (the weights and deprivation thresholds are not unique, but it suffices to show at least one solution).

Table A1 for $d = 2$

$P_2 \in \mathcal{P}_2$	$\mathbf{a}$	k
{10,11}	$(3/4, 1/4)$	$3/4$
{01,11}	$(1/4, 3/4)$	$3/4$
{11}	$(1/2, 1/2)$	$3/4$

Table A2 for $d = 3$

$P_3 \in \mathcal{P}_3$	$\mathbf{a}$	k
{100,110,101,111}	$(2/3, 1/6, 1/6)$	2/3
{010,110,011,111}	$(1/6, 2/3, 1/6)$	2/3
{001,101,011,111	$(1/6, 1/6, 2/3)$	2/3
{100,010,110,101,011,111}	$(2/5, 2/5, 1/5)$	2/5
{100,001,110,101,011,111}	$(2/5, 1/5, 2/5)$	2/5
{010,001,110,101,011,111}	$(1/5, 2/5, 2/5)$	2/5
{100,010,001,110,101,011,111}	$(1/3, 1/3, 1/3)$	1/3
{100,011,110,101,111}	$(1/2, 1/4, 1/4)$	1/2
{010,101,110,011,111}	$(1/4, 1/2, 1/4)$	1/2
{001,110,101,011,111}	$(1/4, 1/4, 1/2)$	1/2
{110,111}	$(2/5, 2/5, 1/5)$	4/5
{101,111}	$(2/5, 1/5, 2/5)$	4/5
{011,111}	$(1/5, 2/5, 2/5)$	4/5
{110,101,111}	$(1/2, 1/4, 1/4)$	3/4
{110,011,111}	$(1/4, 1/2, 1/4)$	3/4
{101,011,111}	$(1/4, 1/4, 1/2)$	3/4
{111}	$(1/3, 1/3, 1/3)$	3/4

This proves that $\mathcal{C}_2 = \mathcal{P}_2$ and $\mathcal{C}_3 = \mathcal{P}_3$. Now, let $d \geq 4$. We will prove that $\mathcal{C}_d \subset \mathcal{P}_d$. Consider a set of poor profiles $P_d^* \in \mathcal{P}_d$ such that $U(P_d^*) = \{\tau_d^{-1}(D_1), \ldots, \tau_d^{-1}(D_G)\}$, with $(D_1, \ldots, D_G) \in \Pi_{D,G}$ for some $2 \leq G \leq \lfloor |D|/2 \rfloor$ and $|D_g| = d_g \geq 2 \forall g \in \{1, \ldots, G\}$ (recall that for any natural number $m \in \mathbb{N}$, $\tau_m : X^m \to 2^{\{1,\ldots,m\}}$ is defined as $\tau_m(\mathbf{x}) = \{i \in \{1, \ldots, m\} | x_i = 1\}$ for all $\mathbf{x} = (x_1, \ldots, x_m) \in X^m$). We will denote the elements of the weighting vector $\mathbf{a}$ as a_{gv}, where $g \in \{1, \ldots, G\}$ indexes the member of the partition D_g to which the weight belongs and $v \in \{1, \ldots, d_g\}$ indexes the members within domain D_g. Since $(D_1, \ldots, D_G)$ is a partition of the set of dimensions D, all the elements of $\mathbf{a}$ can be written as a_{gv} for some g and some v. Clearly, $\sum_{g=1}^{g=G} \sum_{v=1}^{v=d_g} a_{gv} = 1$. Without loss of generality, we will assume that *within* each domain D_g the weights are sorted in a non-ascending order, i.e.: $a_{gv} \geq a_{gv+1}$ for all $g \in \{1, \ldots, G\}$ and all $v \in \{1, \ldots, d_g - 1\}$.

We need to show that there does *not* exist any weighting scheme $\mathbf{a} \in \Delta_d$ and deprivation threshold $k \in (0, 1]$ such that $P_{d,\mathcal{C}(\mathbf{a},k)} = P_d^*$. For that purpose, we will assume the contrary to arrive at a contradiction. If we assume that there exists some $\mathbf{a} \in \Delta_d$ and $k \in (0, 1]$ such that $P_{d,\mathcal{C}(\mathbf{a},k)} = P_d^*$, then one must have

$$\sum_{v=1}^{v=d_g} a_{gv} \geq k \tag{A1}$$

for all $g \in \{1, \ldots, G\}$. By definition, if an individual is deprived in all but one dimensions within each domain, s/he should not be considered as being poor according to P_d^*. Therefore, one must have that

$$\sum_{g=1}^{g=G} \sum_{v=1}^{v=d_g-1} a_{gv} < k. \tag{A2}$$

According to (A1), one can say that

$$\sum_{v=1}^{v=d_g-1} a_{gv} \geq k - a_{gd_g} \tag{A3}$$

for all $g \in \{1, \ldots, G\}$. Plugging (A3) into (A2) G times (one per group) one has that

$$Gk - \sum_{g=1}^{g=G} a_{gd_g} < k, \tag{A4}$$

which can be rewritten as

$$(G-1)k < \sum_{g=1}^{g=G} a_{gd_g}. \tag{A5}$$

Since $\mathbf{a} \in \Delta_d$ and because of (A2), one has that

$$\sum_{g=1}^{g=G} \sum_{v=1}^{v=d_g-1} a_{gv} = 1 - \sum_{g=1}^{g=G} a_{gd_g} < k. \tag{A6}$$

Inserting (A6) into (A5) one has that

$$(G-1)\left(1 - \sum_{g=1}^{g=G} a_{gd_g}\right) < (G-1)k < \sum_{g=1}^{g=G} a_{gd_g}. \tag{A7}$$

Comparing the extreme ends in (A7) and manipulating algebraically one deduces that

$$\frac{G-1}{G} < \sum_{g=1}^{g=G} a_{gd_g} \tag{A8}$$

must hold. Since the weights are written in a non-ascending order, using (A8) we obtain

$$1 \leq 2\left(\frac{G-1}{G}\right) < \sum_{g=1}^{g=G} (a_{gd_g} + a_{gd_g-1}) \leq 1 \tag{A9}$$

which is a contradiction. Therefore, we must conclude that it is not possible to find a weighting scheme $\mathbf{a} \in \Delta_d$ and a deprivation threshold $k \in (0,1]$ such that $P_{d,\mathcal{C}(\mathbf{a},k)} = P_d^*$ (i.e: $P_d^* \in \mathcal{P}_d \backslash \mathcal{C}_d$).

Q.E.D.

Proof of Proposition 2: Recall that, given the partition of D in G domains $(D_1, \ldots, D_G) \in \Pi_{D,G}$, we can rewrite without loss of generality any deprivation profile $\mathbf{x} = (x_1, \ldots, x_d) \in X^d$ as $(\mathbf{x}_1, \ldots, \mathbf{x}_g, \ldots, \mathbf{x}_G) := (x_{11}, \ldots, x_{1d_1}, \ldots, x_{g1}, \ldots, x_{gd_g}, \ldots, x_{G1}, \ldots, x_{Gd_G})$. That is: any dimension $j \in D$ can be uniquely re-labeled using two indices: $g \in \{1, \ldots, G\}$ (to index the domain it belongs to) and $v \in \{1, \ldots, d_g\}$ (indexing the variables within domain D_g).

Since $U(P_G^b)$ and the $U(P_{d_g}^{w,g})$ are singletons $\forall g \in \{1, \ldots, G\}$, we will write $U(P_G^b) = \mathbf{u} = (u_1, \ldots, u_G)$ and $U(P_{d_g}^{w,g}) = \mathbf{x}_g = (x_{g1}, \ldots, x_{gd_g})$ for some $\mathbf{u} \in X^G, \mathbf{x}_g \in X^{d_g}$. When this happens, we can partition the set of dimensions D in two groups A and B. In A, we have the dimensions $j \in D$ indexed by $g \in \tau_G(\mathbf{u})$ and $v \in \tau_{d_g}(\mathbf{x}_g)$ while in B we have the remaining ones. The variables in A are the ones which are essential for assessing multidimensional poverty (i.e: the ones identified by the 1s in $\mathbf{u}$ and the $\mathbf{x}_g$) and those in B are subsidiary to the former. Let $n_1 := |A|, n_2 := |B|$, which are assumed to be fixed throughout the proof. Clearly, $n_1 + n_2 = d$.

We are now going to prove that there exists some $\mathbf{a} \in \Delta_d$ and some $k \in (0, 1]$ such that $P_{d,\mathcal{C}(\mathbf{a},k)}$ coincides with the set of poor profiles generated by the different $P_{d_g}^{w,g}$ and P_G^b. For simplicity, we attach the same weight (a_1) to all variables included in A, and the same weight a_2 to the remaining variables. According to the counting approach, it is enough to show that there exist some $a_1, a_2 \in [0, 1]$ and some $k \in (0, 1]$ such that:

$$n_1 a_1 \geq k \tag{A10}$$

$$(n_1 - 1)a_1 + n_2 a_2 < k \tag{A11}$$

$$n_1 a_1 + n_2 a_2 = 1 \tag{A12}$$

Condition (A10) states that when an individual is deprived in all dimensions included in A, then he is multidimensionally poor. Condition (A11) states that when an individual is not deprived in all dimensions included in A, then she is not multidimensionally poor (even if she is deprived in all variables included in B). Condition (A12) is simply a normalization restriction stating that the sum of the weights attached to the different variables must add up to 1. If these three conditions are satisfied, we will have expressed the sets of poor profiles generated by the different $P_{d_g}^{w,g}$ and P_G^b via the counting approach.

According to (A12), a_2 can be rewritten as $(1 - n_1 a_1)/n_2$. Plugging this expression in (A11) and after basic algebraic manipulation one obtains

$$1 - a_1 < k \tag{A13}$$

Putting together (A10) and (A13) one has that $1 - a_1 < k \leq n_1 a_1$, which implies

$$a_1 > 1/(1 + n_1) \tag{A14}$$

One implication of (A12) is that

$$a_1 \leq 1/n_1 \tag{A15}$$

Imposing (A14) and (A15), one has that $1/(1 + n_1) < a_1 \leq 1/n_1$. These two inequalities are clearly satisfied if one chooses the following value for a_1 :

$$a_1^* := (1/(1 + n_1) + 1/n_1)/2 = (2n_1 + 1)/(2n_1(n_1 + 1)) \tag{A16}$$

Imposing (A13) one has that $1 - a_1^* < k \leq 1$. Analogously, these two inequalities are satisfied choosing the following value for k:

$$k^* := ((1 - a_1^*) + 1)/2 = (4n_1^2 + 2n_1 - 1)/(4n_1(n_1 + 1)) \tag{A17}$$

Finally, imposing (A12) one has that

$$a_2^* := (1 - n_1 a_1^*)/n_2 \tag{A18}$$

Since the values a_1^*, a_2^*, k^* satisfy the conditions (A10), (A11) and (A12), we have been able to find a weighting scheme $\mathbf{a}$ and a deprivation score threshold k generating the same set of poor profiles as the one generated by the different $P_{d_g}^{w,g}$ and P_G^b, so we are done.

Q.E.D.

Proof of Theorem 2: To prove the two parts of the theorem ((i) and (ii)) we will follow the same strategy: we start assuming that there *is* a weighting scheme $\mathbf{a}$ and a deprivation score threshold k such that $P_{d,\mathcal{C}(\mathbf{a},k)}$ coincides with the set of poor profiles generated by the different $P_{d_g}^{w,g}$ and P_G^b to arrive at a contradiction. As before, given the partition of D in G

domains $(D_1, \ldots, D_G) \in \Pi_{D,G}$, we will denote the elements of the weighting vector $\mathbf{a}$ as a_{gv}, where $g \in \{1, \ldots, G\}$ indexes the member of the partition D_g to which the weight belongs and $v \in \{1, \ldots, d_g\}$ indexes the members within domain D_g. Again, we can assume without loss of generality that *within* each domain D_g the weights are sorted in a non-ascending order, i.e.: $a_{gv} \geq a_{gv+1}$ for all $g \in \{1, \ldots, G\}$ and all $v \in \{1, \ldots, d_g - 1\}$.

Let's start with the set of conditions stated in (i) and let $\mathbf{p} = (p_1, \ldots, p_G), \mathbf{q} = (q_1, \ldots, q_G) \in U(P_G^b)$ ($\mathbf{p} \neq \mathbf{q}$) be the two elements referred to therein. On the basis of these two vectors, we will now consider the following partition of the domains $\{1, \ldots, G\}$ into four groups: H_1, H_2, H_3 and H_4. In H_1, we have the domains $g \in \{1, \ldots, G\}$ in which $p_g = 1$ and $q_g = 0$. H_2 contains the domains in which $p_g = 0$ and $q_g = 1$. Lastly, H_3 and H_4 contain the domains in which $p_g = 1, q_g = 1$ and $p_g = 0, q_g = 0$ respectively. Since $\mathbf{p}, \mathbf{q} \in U(P_G^b)$, H_1 *and* H_2 must be non-empty (see Lemma 1).

For each domain D_g we consider an element $\mathbf{x}_g \in U(P_{d_g}^{w,g})$ with m_g ones and $d_g - m_g$ zeroes. Without loss of generality, we assume that the m_g ones are the first elements of $\mathbf{x}_g$, with the last $d_g - m_g$ positions being zeroes. By the conditions stated in (i), there must exist at least one domain $g_1 \in H_1$ with $m_{g_1} \geq 2$ and at least one domain $g_2 \in H_2$ with $m_{g_2} \geq 2$. If we assume that there exists some $\mathbf{a} \in \Delta_d$ and $k \in (0, 1]$ such that $P_{d,\mathcal{C}(\mathbf{a},k)}$ corresponds to the set of poor profiles generated by the different $P_{d_g}^{w,g}$ and P_G^b, then the following conditions must hold:

$$a_{g_1 1} + a_{g_1 2} + \sum_{v=3}^{v=m_{g_1}} a_{g_1 v} + \sum_{\substack{g \in H_1 \\ g \neq g_1}} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{v=3}^{v=m_{g_2}} a_{g_2 v} + \sum_{\substack{g \in H_2 \\ g \neq g_2}} \sum_{v=2}^{v=m_g} a_{gv} + \sum_{g \in H_3} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g \in H_4} \sum_{v=2}^{v=m_g} a_{gv} \geq k \tag{A19}$$

$$a_{g_2 1} + a_{g_2 2} + \sum_{v=3}^{v=m_{g_2}} a_{g_2 v} + \sum_{\substack{g \in H_2 \\ g \neq g_2}} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{v=3}^{v=m_{g_1}} a_{g_1 v} + \sum_{\substack{g \in H_1 \\ g \neq g_1}} \sum_{v=2}^{v=m_g} a_{gv} + \sum_{g \in H_3} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g \in H_4} \sum_{v=2}^{v=m_g} a_{gv} \geq k \tag{A20}$$

Equation (A19) states the condition that must be satisfied by the weights vector $\mathbf{a}$ and the deprivation score threshold k if one wants an individual that is deprived in the dimensions indexed by $\mathbf{p}$ and the different $\mathbf{x}_g$ to be considered as multidimensionally poor. Therefore, such individual must be deprived in all the domains included in H_1 and H_3 but non-deprived in those included in H_2 and H_4. Analogously, equation (A20) does the same for the dimensions indexed by $\mathbf{q}$ and the different $\mathbf{x}_g$. In that case, individuals are deprived in all the domains included in H_2 and H_3 but non-deprived in those included in H_1 and H_4.

So far, we have presented the conditions to identify those who are multidimensionally poor: let us now impose conditions to identify those who are not. Consider the vector $\mathbf{p}' = (p'_1, \ldots, p'_G)$ defined as follows: $p'_{g_1} = 0$ and $p'_g = p_g$ for all domains $g \neq g_1$. Since, by definition $p_{g_1} = 1$, and $\mathbf{p} \in U(P^b_G)$, it turns out that $\mathbf{p}' \in R^b_G$. Therefore, the following inequalities must hold:

$$a_{g_1 1} + \sum_{v=3}^{v=m_{g_1}} a_{g_1 v} + \sum_{\substack{g \in H_1 \\ g \neq g_1}} \sum_{v=1}^{v=m_g} a_{gv} + a_{g_2 1} + \sum_{v=3}^{v=m_{g_2}} a_{g_2 v} + \sum_{\substack{g \in H_2 \\ g \neq g_2}} \sum_{v=2}^{v=m_g} a_{gv} + \sum_{g \in H_3} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g \in H_4} \sum_{v=2}^{v=m_g} a_{gv} < k \tag{A21}$$

$$a_{g_1 1} + \sum_{v=3}^{v=m_{g_1}} a_{g_1 v} + \sum_{\substack{g \in H_1 \\ g \neq g_1}} \sum_{v=1}^{v=m_g} a_{gv} + a_{g_2 2} + \sum_{v=3}^{v=m_{g_2}} a_{g_2 v} + \sum_{\substack{g \in H_2 \\ g \neq g_2}} \sum_{v=2}^{v=m_g} a_{gv} + \sum_{g \in H_3} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g \in H_4} \sum_{v=2}^{v=m_g} a_{gv} < k \tag{A22}$$

$$a_{g_1 2} + \sum_{v=3}^{v=m_{g_1}} a_{g_1 v} + \sum_{\substack{g \in H_1 \\ g \neq g_1}} \sum_{v=1}^{v=m_g} a_{gv} + a_{g_2 1} + \sum_{v=3}^{v=m_{g_2}} a_{g_2 v} + \sum_{\substack{g \in H_2 \\ g \neq g_2}} \sum_{v=2}^{v=m_g} a_{gv} + \sum_{g \in H_3} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g \in H_4} \sum_{v=2}^{v=m_g} a_{gv} < k \tag{A23}$$

$$a_{g_1 2} + \sum_{v=3}^{v=m_{g_1}} a_{g_1 v} + \sum_{\substack{g \in H_1 \\ g \neq g_1}} \sum_{v=1}^{v=m_g} a_{gv} + a_{g_2 2} + \sum_{v=3}^{v=m_{g_2}} a_{g_2 v} + \sum_{\substack{g \in H_2 \\ g \neq g_2}} \sum_{v=2}^{v=m_g} a_{gv} + \sum_{g \in H_3} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g \in H_4} \sum_{v=2}^{v=m_g} a_{gv} < k \tag{A24}$$

Analogously, we can also construct the vector $\mathbf{q}' = (q'_1, \ldots, q'_G)$ defined as follows: $q'_{g_2} = 0$ and $q'_g = q_g$ for all domains $g \neq g_2$. Again, since $q_{g_1} = 1$, and $\mathbf{q} \in U(P^b_G)$, it turns out that

$\mathbf{q}' \in R^b_G$. Therefore, the following inequalities must hold:

$$a_{g_1 1} + \sum_{v=3}^{v=m_{g_1}} a_{g_1 v} + \sum_{\substack{g\in H_1\\ g\neq g_1}} \sum_{v=2}^{v=m_g} a_{gv} + a_{g_2 1} + \sum_{v=3}^{v=m_{g_2}} a_{g_2 v} + \sum_{\substack{g\in H_2\\ g\neq g_2}} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g\in H_3} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g\in H_4} \sum_{v=2}^{v=m_g} a_{gv} < k \tag{A25}$$

$$a_{g_1 1} + \sum_{v=3}^{v=m_{g_1}} a_{g_1 v} + \sum_{\substack{g\in H_1\\ g\neq g_1}} \sum_{v=2}^{v=m_g} a_{gv} + a_{g_2 2} + \sum_{v=3}^{v=m_{g_2}} a_{g_2 v} + \sum_{\substack{g\in H_2\\ g\neq g_2}} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g\in H_3} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g\in H_4} \sum_{v=2}^{v=m_g} a_{gv} < k \tag{A26}$$

$$a_{g_1 2} + \sum_{v=3}^{v=m_{g_1}} a_{g_1 v} + \sum_{\substack{g\in H_1\\ g\neq g_1}} \sum_{v=2}^{v=m_g} a_{gv} + a_{g_2 1} + \sum_{v=3}^{v=m_{g_2}} a_{g_2 v} + \sum_{\substack{g\in H_2\\ g\neq g_2}} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g\in H_3} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g\in H_4} \sum_{v=2}^{v=m_g} a_{gv} < k \tag{A27}$$

$$a_{g_1 2} + \sum_{v=3}^{v=m_{g_1}} a_{g_1 v} + \sum_{\substack{g\in H_1\\ g\neq g_1}} \sum_{v=2}^{v=m_g} a_{gv} + a_{g_2 2} + \sum_{v=3}^{v=m_{g_2}} a_{g_2 v} + \sum_{\substack{g\in H_2\\ g\neq g_2}} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g\in H_3} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g\in H_4} \sum_{v=2}^{v=m_g} a_{gv} < k \tag{A28}$$

If one defines

$$k' := k - \left(\sum_{v=3}^{v=m_{g_1}} a_{g_1 v} + \sum_{v=3}^{v=m_{g_2}} a_{g_2 v} + \sum_{\substack{g\in H_1\\ g\neq g_1}} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{\substack{g\in H_2\\ g\neq g_2}} \sum_{v=2}^{v=m_g} a_{gv} + \sum_{g\in H_3} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g\in H_4} \sum_{v=2}^{v=m_g} a_{gv} \right) \tag{A29}$$

$$k'' := k - \left(\sum_{v=3}^{v=m_{g_1}} a_{g_1 v} + \sum_{v=3}^{v=m_{g_2}} a_{g_2 v} + \sum_{\substack{g\in H_1\\ g\neq g_1}} \sum_{v=2}^{v=m_g} a_{gv} + \sum_{\substack{g\in H_2\\ g\neq g_2}} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g\in H_3} \sum_{v=1}^{v=m_g} a_{gv} + \sum_{g\in H_4} \sum_{v=2}^{v=m_g} a_{gv} \right) \tag{A30}$$

the inequalities (A19)-(A24) can be rewritten as

$$\left\{ \begin{array}{c} a_{g_1 1} + a_{g_1 2} \geq k' \\ a_{g_2 1} + a_{g_2 2} \geq k'' \\ a_{g_1 1} + a_{g_2 1} < k' \\ a_{g_1 1} + a_{g_2 2} < k' \\ a_{g_1 2} + a_{g_2 1} < k' \\ a_{g_1 2} + a_{g_2 2} < k' \end{array} \right\} \tag{A31}$$

Analogously, inequalities (A19), (A20), (A25)-(A28) can be rewritten as

$$\left\{\begin{array}{l} a_{g_1 1} + a_{g_1 2} \geq k' \\ a_{g_2 1} + a_{g_2 2} \geq k'' \\ a_{g_1 1} + a_{g_2 1} < k'' \\ a_{g_1 1} + a_{g_2 2} < k'' \\ a_{g_1 2} + a_{g_2 1} < k'' \\ a_{g_1 2} + a_{g_2 2} < k'' \end{array}\right\} \tag{A32}$$

When comparing k' and k'', one must either have (a) $k' \leq k''$ or (b) $k' \geq k''$. Assume that $k' \leq k''$. When this happens, the inequalities system (A31) can be rewritten as

$$\left\{\begin{array}{l} a_{g_1 1} + a_{g_1 2} \geq k' \\ a_{g_2 1} + a_{g_2 2} \geq k' \\ a_{g_1 1} + a_{g_2 1} < k' \\ a_{g_1 1} + a_{g_2 2} < k' \\ a_{g_1 2} + a_{g_2 1} < k' \\ a_{g_1 2} + a_{g_2 2} < k' \end{array}\right\} \tag{A33}$$

It is trivial to show that the inequalities system shown in (A33) does not have feasible solutions. In the first inequality of the system, either $a_{g_1 1}$ or $a_{g_1 2}$ must be greater or equal than $k'/2$. The same goes for $a_{g_2 1}, a_{g_2 2}$ in the second inequality of the system: at least one of them must be greater or equal than $k'/2$. Picking the largest elements between $a_{g_1 1}, a_{g_1 2}$ and $a_{g_2 1}, a_{g_2 2}$ and adding them up results in a number that is greater or equal than k', therefore contradicting at least one of the four last inequalities of the system. In case (b) ($k' \geq k''$), the same reasoning applies for the inequalities system shown in (A32). We have reached the contradiction we were looking for, so the classical counting approach can not generate the same set of poor profiles as the ones generated by the generalized counting approach as described in (i).

Let us now consider case (ii). Without loss of generality, let us assume that the two domains $g_1, g_2 \in \tau_G(\mathbf{x}_G)$ for which $\left|U(P^{w,g_1}_{d_{g_1}})\right| \geq 2$ and $\left|U(P^{w,g_2}_{d_{g_2}})\right| \geq 2$ are $g_1 = 1$ and $g_2 = 2$. Let $\mathbf{s} = (s_1, \ldots, s_{d_1}), \mathbf{t} = (t_1, \ldots, t_{d_1}) \in U(P^{w,1}_{d_1})$ ($\mathbf{s} \neq \mathbf{t}$) be two of the elements belonging to $U(P^{w,1}_{d_1})$. On the basis of these two vectors, we will now consider the following partition of $\{1, \ldots, d_1\}$ into four groups: H^1_1, H^1_2, H^1_3 and H^1_4. In H^1_1, we have the variables $v \in \{1, \ldots, d_1\}$ in which $s_v = 1$ and $t_v = 0$. H^1_2 contains the domains in which $s_v = 0$ and $t_v = 1$. Lastly, H^1_3 and H^1_4 contain the domains in which $s_v = 1, t_v = 1$ and $s_v = 0, t_v = 0$ respectively. Since $\mathbf{s}, \mathbf{t} \in U(P^{w,1}_{d_1})$, H^1_1 *and* H^1_2 must be non-empty (see Lemma 1). We can now repeat the same for $U(P^{w,2}_{d_2})$. Let $\mathbf{s}' = (s'_1, \ldots, s'_{d_1}), \mathbf{t}' = (t'_1, \ldots, t'_{d_1}) \in U(P^{w,2}_{d_2})$ ($\mathbf{s}' \neq \mathbf{t}'$) be two of its members. Again, this leads to the partition of $\{1, \ldots, d_2\}$ into four groups: H^2_1, H^2_2, H^2_3 and H^2_4. Now, in H^2_1, we have the variables $v \in \{1, \ldots, d_2\}$ in which $s'_v = 1$ and $t'_v = 0$. H^2_2 contains the domains in which $s'_v = 0$ and $t'_v = 1$. Lastly, H^2_3 and H^2_4 contain the domains in which $s'_v = 1, t'_v = 1$ and $s'_v = 0, t'_v = 0$ respectively. Since $\mathbf{s}', \mathbf{t}' \in U(P^{w,2}_{d_2})$, H^2_1 *and* H^2_2 must be non-empty.

For the other domains D_g besides D_1 and D_2 (i.e: $g \in \{3, \ldots, G\}$), we consider an element $\mathbf{y}_g = (y_{g1}, \ldots, y_{gd_g}) \in U(P^{w,g}_{d_g})$. If we assume that there exists some $\mathbf{a} \in \Delta_d$ and $k \in (0, 1]$ such that $P_{d,\mathcal{C}(\mathbf{a},k)}$ corresponds to the set of poor profiles generated by the conditions presented in (ii), then the following conditions must hold:

$$\sum_{v \in H^1_1} a_{1v} + \sum_{v \in H^1_3} a_{1v} + \sum_{v \in H^2_1} a_{2v} + \sum_{v \in H^2_3} a_{2v} + \sum_{g \in \tau_G(\mathbf{x}_G)} \sum_{v \in \tau_{d_g}(\mathbf{y}_g)} a_{gv} \geq k \tag{A34}$$

$$\sum_{v \in H^1_1} a_{1v} + \sum_{v \in H^1_3} a_{1v} + \sum_{v \in H^2_2} a_{2v} + \sum_{v \in H^2_3} a_{2v} + \sum_{g \in \tau_G(\mathbf{x}_G)} \sum_{v \in \tau_{d_g}(\mathbf{y}_g)} a_{gv} \geq k \tag{A35}$$

$$\sum_{v \in H^1_2} a_{1v} + \sum_{v \in H^1_3} a_{1v} + \sum_{v \in H^2_1} a_{2v} + \sum_{v \in H^2_3} a_{2v} + \sum_{g \in \tau_G(\mathbf{x}_G)} \sum_{v \in \tau_{d_g}(\mathbf{y}_g)} a_{gv} \geq k \tag{A36}$$

$$\sum_{v\in H_2^1} a_{1v} + \sum_{v\in H_3^1} a_{1v} + \sum_{v\in H_2^2} a_{2v} + \sum_{v\in H_3^2} a_{2v} + \sum_{g\in\tau_G(\mathbf{x}_G)} \sum_{v\in\tau_{d_g}(\mathbf{y}_g)} a_{gv} \geq k \tag{A37}$$

Inequality (A34) states the condition that must be satisfied by the weights vector $\mathbf{a}$ and the deprivation score threshold k if one wants an individual that is deprived in the variables included in H_1^1 and H_3^1 for domain D_1 and in the variables included in H_1^2 and H_3^2 for domain D_2 (plus the corresponding deprivations in the other domains) to be considered as multidimensionally poor. Equations (A35),(A36) and (A37) do the same for other combinations of deprivations within domains D_1 and D_2. Consider now the following $G-$dimensional binary vectors: $\mathbf{u} = (1\ 0\ 1 \ldots 1)$, $\mathbf{u}' = (0\ 1\ 1 \ldots 1)$. Since $\mathbf{u}, \mathbf{u}' \in R_G^b$, the following inequalities must hold:

$$\sum_{v\in H_1^1} a_{1v} + \sum_{v\in H_2^1} a_{1v} + \sum_{v\in H_3^1} a_{1v} + \sum_{v\in H_3^2} a_{2v} + \sum_{g\in\tau_G(\mathbf{x}_G)} \sum_{v\in\tau_{d_g}(\mathbf{y}_g)} a_{gv} < k \tag{A38}$$

$$\sum_{v\in H_3^1} a_{1v} + \sum_{v\in H_1^2} a_{2v} + \sum_{v\in H_2^2} a_{2v} + \sum_{v\in H_3^2} a_{2v} + \sum_{g\in\tau_G(\mathbf{x}_G)} \sum_{v\in\tau_{d_g}(\mathbf{y}_g)} a_{gv} < k \tag{A39}$$

Inequality (A38) imposes that when an individual is only deprived in the variables included in H_3^2 for domain D_2, the she should not be considered as multidimensionally poor. Analogously, inequality (A39) imposes that those individuals that are only deprived in the variables included in H_3^1 for domain D_1 should be neither considered as being multidimensionally poor. Defining

$$k' := k - \left(\sum_{v\in H_3^1} a_{1v} + \sum_{v\in H_3^2} a_{2v} + \sum_{g\in\tau_G(\mathbf{x}_G)} \sum_{v\in\tau_{d_g}(\mathbf{y}_g)} a_{gv} \right) \tag{A40}$$

the inequalities (A34)-(A39) ca be rewritten as

$$\left\{\begin{array}{l} A_{11} + A_{21} \geq k' \\ A_{11} + A_{22} \geq k' \\ A_{12} + A_{21} \geq k' \\ A_{12} + A_{22} \geq k' \\ A_{11} + A_{12} < k' \\ A_{21} + A_{22} < k' \end{array}\right\} \tag{A41}$$

where $A_{11} := \sum_{v \in H_1^1} a_{1v}$, $A_{12} := \sum_{v \in H_2^1} a_{1v}$, $A_{21} := \sum_{v \in H_1^2} a_{2v}$, $A_{22} := \sum_{v \in H_2^2} a_{2v}$. Again, it it trivial to prove that the inequalities system shown in (A41) does not have feasible solutions. In the second to last inequality of the system, either A_{11} or A_{12} must be smaller than $k'/2$. The same goes for A_{21}, A_{22} in the last inequality of the system: at least one of them must be smaller than $k'/2$. Picking the smallest elements between A_{11}, A_{12} and A_{21}, A_{22} and adding them up results in a number that is smaller than k', therefore contradicting at least one of the four first inequalities of the system. We have reached the contradiction we were looking for, so the classical counting approach can not generate the same set of poor profiles as the ones generated by the generalized counting approach as described in (ii). This concludes the proof of the theorem.

Q.E.D.

Proof of Corollary 1: This corollary is a quasi-immediate consequence of Theorem 2. We simply need to show that when conditions (i) and (ii) in Corollary 1 are satisfied, then the corresponding conditions (i) and (ii) in Theorem 2 are satisfied as well, so the result applies automatically.

Condition (i): In the generalized counting approach characterized by the thresholds vector $(m_1, \ldots, m_G; M)$, the elements of $U(P_G^b)$ are all possible $G-$dimensional binary vectors with M ones and $G-M$ zeroes. Therefore, whenever $M < G$ and $G \geq 2$, $U(P_G^b)$ must at least have two elements, so the first assumption is satisfied. Since there are at least two domains $g_1, g_2 \in \{1, \ldots, G\}$ including elements of size greater than one ($m_{g_1} \geq 2$ and $m_{g_2} \geq 2$), the second assumption is satisfied as well, so the set of conditions (i) established in Theorem 2 applies.

Condition (ii): When $M = G$, one clearly has that $P_G^b = \{(1 \ldots 1)\}$, so the first assumption in Theorem 2 part (ii) is satisfied. Since there are at least two domains $g_1, g_2 \in \{1, \ldots, G\}$ with $m_{g_1} < d_{g_1}, m_{g_2} < d_{g_2}$ and one has that $d_{g_1} \geq 2, d_{g_2} \geq 2$, then one must necessarily have that $\left|U(P_{d_{g_1}}^{w,g_1})\right| \geq 2, \left|U(P_{d_{g_2}}^{w,g_2})\right| \geq 2$. Therefore, the set of conditions (ii) established in Theorem 2 applies as well, so we are done.

Q.E.D.

Proof of Proposition 3: Let

$$\pi_{\boldsymbol{\theta}} := \left(\sum_{g=1}^{g=G} a_{g\cdot} \left[\sum_{v=1}^{d_g} w_{gv} (\gamma_{gv}^c)^{\theta_g} \right]^{\theta/\theta_g} \right)^{1/\theta} \tag{A42}$$

be the individual level poverty function corresponding to (24). Therefore, one has that

$$\frac{\partial \pi_{\boldsymbol{\theta}}}{\partial \gamma_{gv}^c} = \left(\sum_{g=1}^{g=G} a_{g\cdot} \left[\sum_{v=1}^{d_g} w_{gv} (\gamma_{gv}^c)^{\theta_g} \right]^{\theta/\theta_g} \right)^{\frac{1}{\theta}-1} \left[\sum_{v=1}^{d_g} w_{gv} (\gamma_{gv}^c)^{\theta_g} \right]^{\frac{\theta}{\theta_g}-1} w_{gv} (\gamma_{gv}^c)^{\theta_g - 1} \tag{A43}$$

After several algebraic manipulations it is easy to show that

$$\frac{\partial^2 \pi_{\boldsymbol{\theta}}}{\partial \gamma_{gv}^c \partial \gamma_{gu}^c} \equiv \left[(1-\theta) \left(\sum_{v=1}^{d_g} w_{gv} (\gamma_{gv}^c)^{\theta_g} \right)^{\theta/\theta_g} + (\theta - \theta_g) \left(\sum_{g=1}^{g=G} \left[\sum_{v=1}^{d_g} w_{gv} (\gamma_{gv}^c)^{\theta_g} \right]^{\theta/\theta_g} \right) \right] \tag{A44}$$

The last expression can be rearranged and written as follows:

$$\frac{\partial^2 \pi_{\boldsymbol{\theta}}}{\partial \gamma_{gv}^c \partial \gamma_{gu}^c} \equiv \left[(1-\theta_g) \left(\sum_{v=1}^{d_g} w_{gv} (\gamma_{gv}^c)^{\theta_g} \right)^{\theta/\theta_g} + (\theta - \theta_g) \left(\sum_{h=1, h\neq g}^{h=G} \left[\sum_{v=1}^{d_g} w_{hv} (\gamma_{hv}^c)^{\theta_h} \right]^{\theta/\theta_h} \right) \right] \tag{A45}$$

Therefore, one can basically say that

$$\frac{\partial^2 \pi_{\boldsymbol{\theta}}}{\partial \gamma_{gv}^c \partial \gamma_{gu}^c} \equiv A(1-\theta_g) + B(\theta - \theta_g) \tag{A46}$$

for some real constants $A, B > 0$. Hence, whenever $\theta_g < \min\{1, \theta\}$, $(\partial^2 \pi_{\boldsymbol{\theta}}) / \left(\partial \gamma_{gv}^c \partial \gamma_{gu}^c\right) > 0$, so the attributes u, v belonging to the same domain are complements. On the other hand, whenever $\theta_g > \max\{1, \theta\}$, $(\partial^2 \pi_{\boldsymbol{\theta}}) / \left(\partial \gamma_{gv}^c \partial \gamma_{gu}^c\right) < 0$, so the attributes u, v belonging to the same domain are substitutes. This proves part (i). For part (ii), we need to compute $(\partial^2 \pi_{\boldsymbol{\theta}}) / \left(\partial \gamma_{gv}^c \partial \gamma_{hu}^c\right)$. After algebraic manipulations it can be shown that

$$\frac{\partial^2 \pi_{\boldsymbol{\theta}}}{\partial \gamma_{gv}^c \partial \gamma_{hu}^c} \equiv (1-\theta) \left(\sum_{g=1}^{g=G} a_{g\cdot} \left[\sum_{v=1}^{d_g} w_{gv} (\gamma_{gv}^c)^{\theta_g} \right]^{\theta/\theta_g} \right)^{\frac{1}{\theta}-2} \left[\sum_{u=1}^{d_h} w_{hu} (\gamma_{hu}^c)^{\theta_g} \right]^{\frac{\theta}{\theta_h}} w_{hu} (\gamma_{hu}^c)^{\theta_h - 1} \tag{A47}$$

From the previous equation we can say that

$$\frac{\partial^2 \pi_{\boldsymbol{\theta}}}{\partial \gamma_{gv}^c \partial \gamma_{hu}^c} \equiv C(1-\theta) \tag{A48}$$

for some real constant $C > 0$. Therefore, whenever $\theta < 1$, $(\partial^2 \pi_{\boldsymbol{\theta}}) / \left(\partial \gamma_{gv}^c \partial \gamma_{hu}^c\right) > 0$, so the attributes u, v belonging to different domains are complements. Analogously, when $\theta > 1$, $(\partial^2 \pi_{\boldsymbol{\theta}}) / \left(\partial \gamma_{gv}^c \partial \gamma_{hu}^c\right) < 0$, so the attributes u, v belonging to different domains are substitutes. This proves part (ii).

Q.E.D.

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Figures

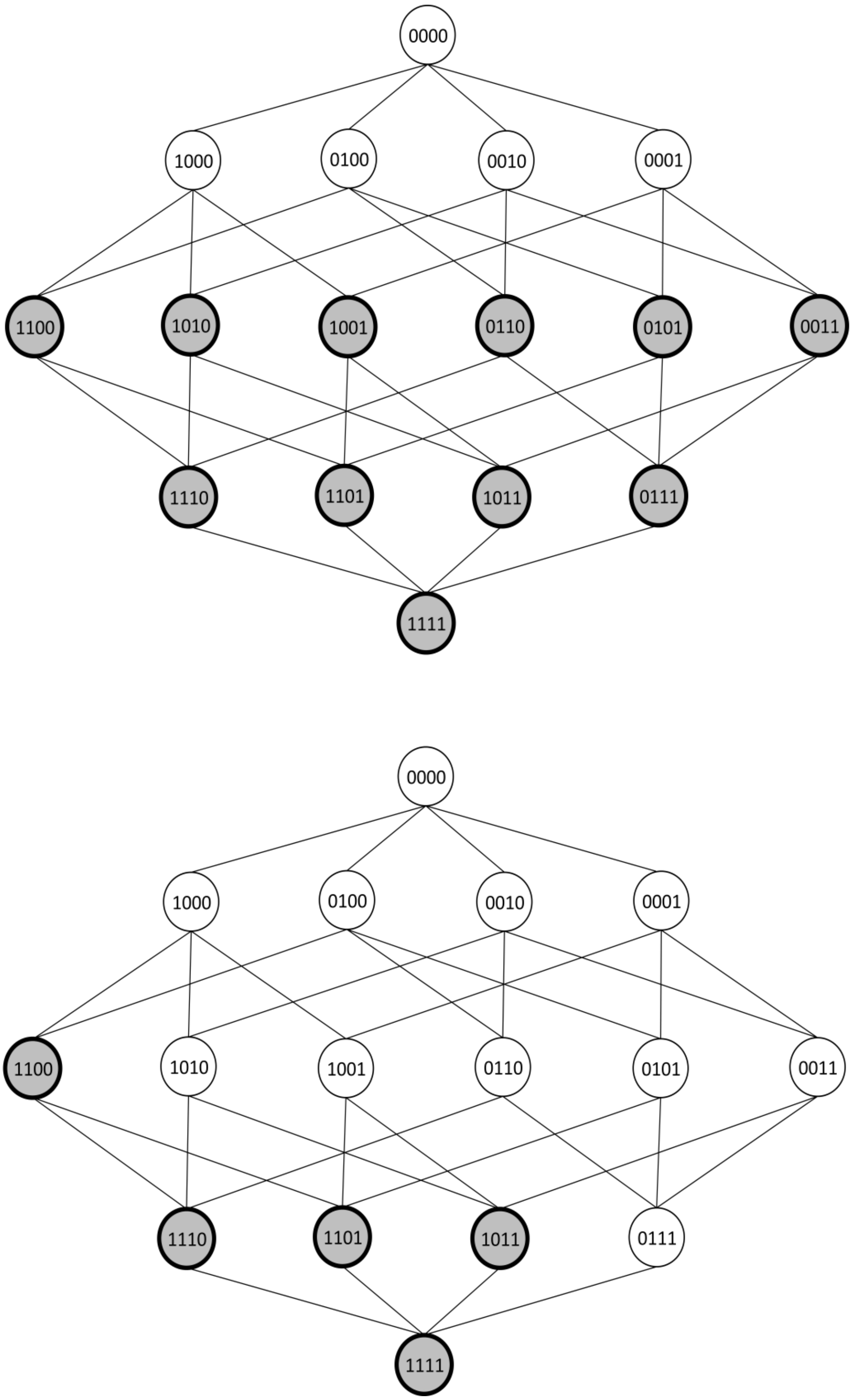

Figure 1a (top), 1b (bottom). Two examples of sets of poor profiles for the partially ordered set (X_4,≼). The shaded circles in the top and bottom panels are the members of P_4^1 and P_4^2 respectively.

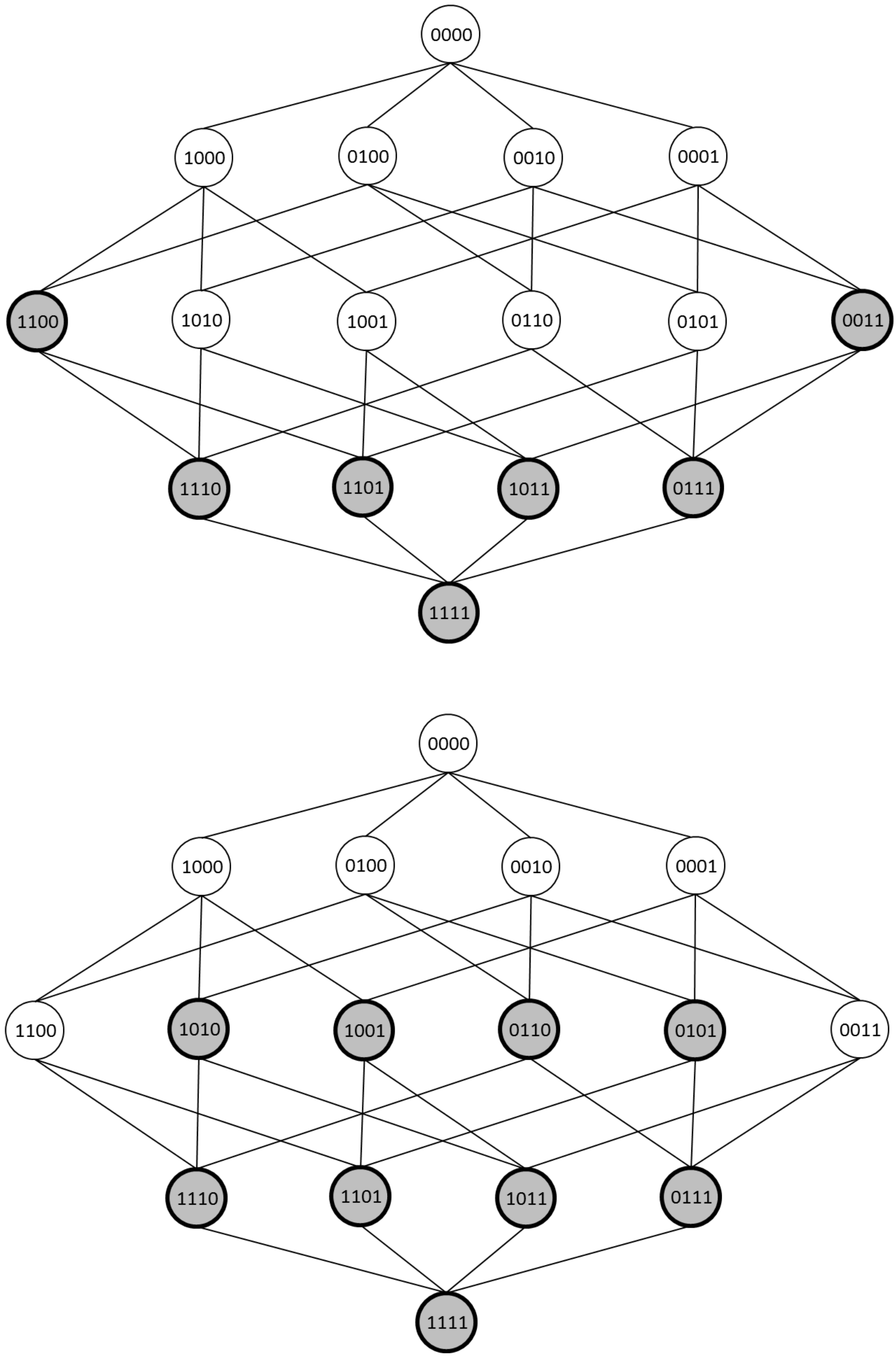

Figure 2a (top) ,2b (bottom). Two examples of sets of poor profiles for the partially ordered set (X_4,≼). The shaded circles in the top and bottom panels are the members of P_4^* and P_4^{**} respectively.

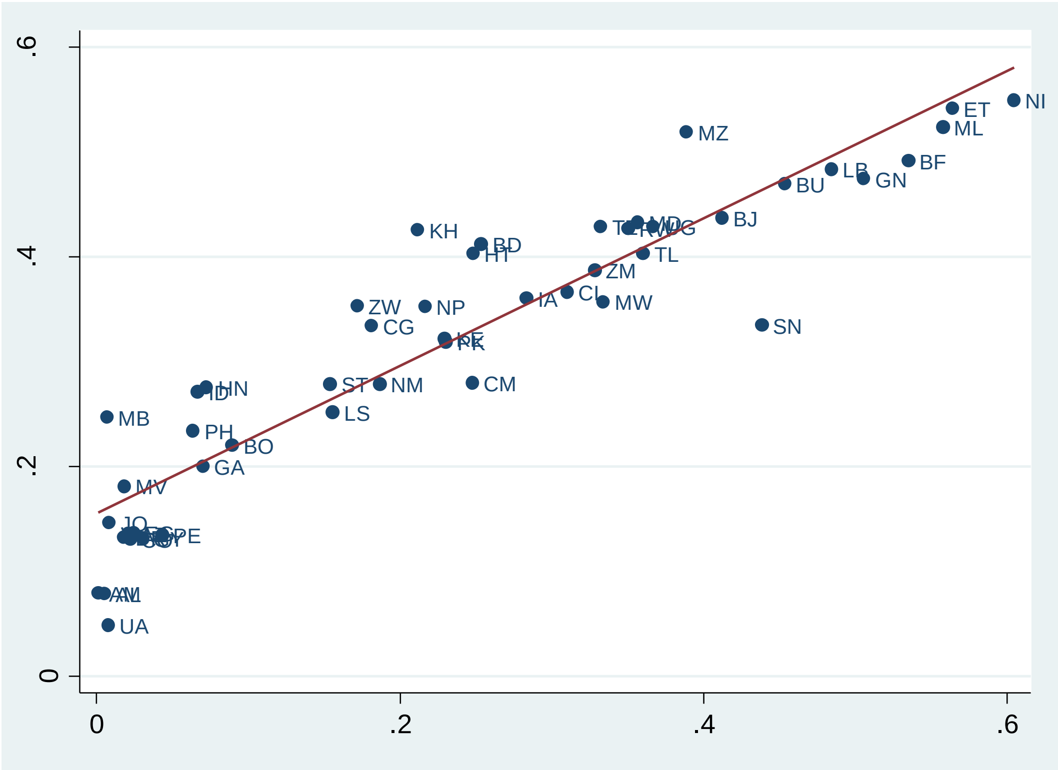

Figure 3. Comparison of UNDP's official values of the MPI (horizontal axis) with the multidimensional poverty values M_0 estimated in this paper (vertical axis). Country labels follow the ISO-3166 coding scheme. The solid line corresponds to the least squares best linear fit. Source: Author's calculations using UNDP and DHS data.

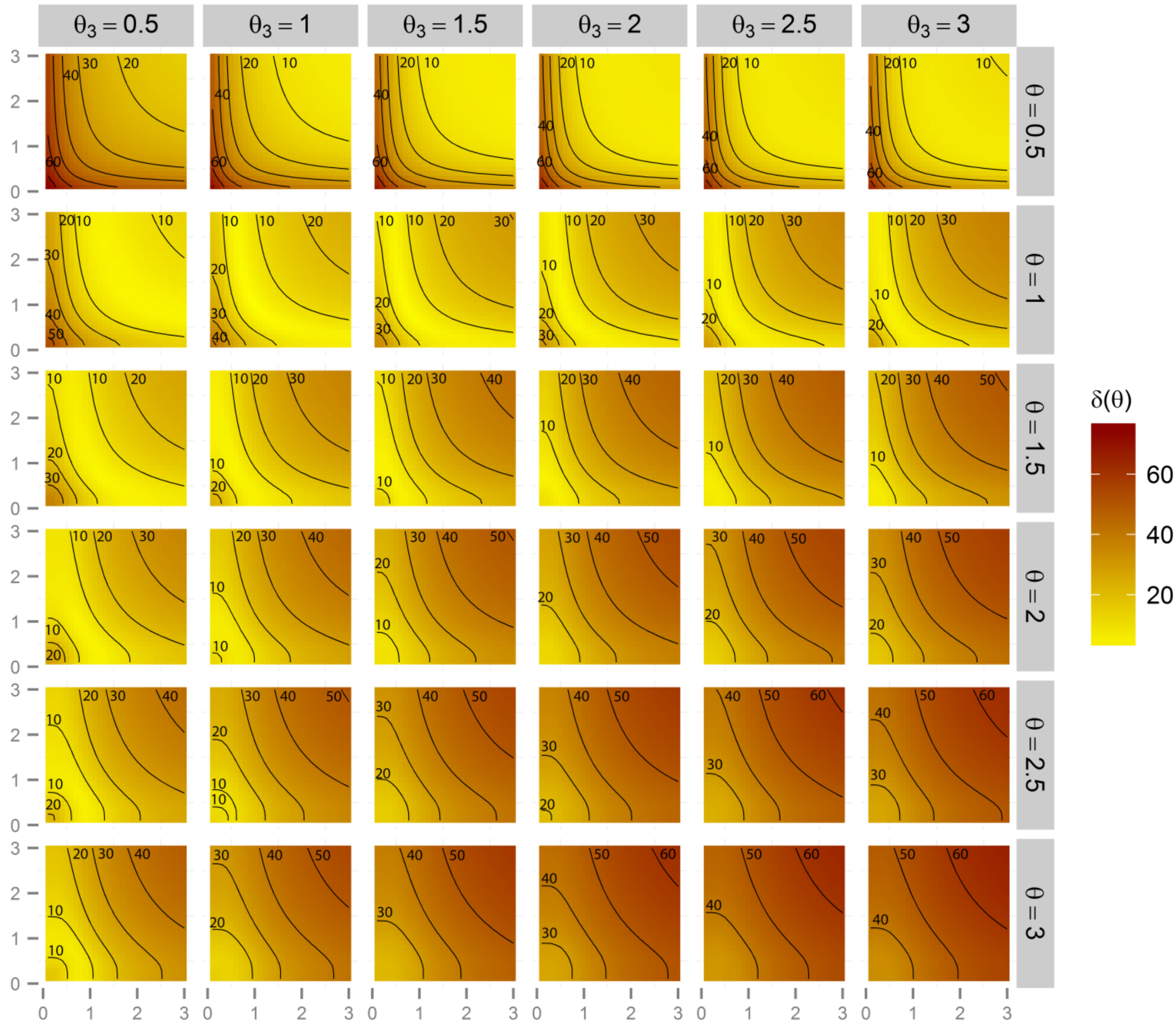

Figure 4. Values and level contours of the distance function $\delta(\boldsymbol{\theta})$ when $\boldsymbol{\theta}=(\theta, \theta_1, \theta_2, \theta_3)\in(0,3]^4$. In each square, the horizontal and vertical axes show the values of θ_1 and θ_2 respectively. Source: Author's calculations using DHS data.

Tables

Group	(A) PP	(B) RR	(C) RP	(D) $H(P_4^1)$	(E) $H(P_4^*)$	(F) $M_0(P_4^1)$	(G) $M_0(P_4^*)$
Hispanic	17.5%	62.4%	20.1%	0.345	0.143	0.202	0.097
Non-his White	3.4%	91.2%	5.3%	0.084	0.031	0.047	0.02
Non-his Black	10.3%	79.2%	10.4%	0.201	0.093	0.117	0.06
Others	4.6%	88.9%	6.6%	0.107	0.044	0.058	0.027
Total	7.1%	83.9%	9.0%	0.128	0.052	0.073	0.033

Table 1. Comparison of P_4^1-poverty and P_4^*-poverty for different ethnic/racial groups in the US. Source: Author's calculations using the 2004 US National Health Interview Survey.

Variables	Hispanic	Non-his White	Non-his Black	Others	Total
Income	26.10%	46.96%	38.01%	50.00%	35.97%
Educ	48.85%	57.40%	36.96%	50.39%	49.77%
Health_Insurance	57.84%	55.47%	53.90%	60.87%	56.76%
Self-assessed Health	39.33%	56.90%	47.03%	55.32%	49.44%

Table 2. Percentage of 'mistargeted individuals' for the different variables and the different population subgroups. Source: Author's calculations using the 2004 US National Health Interview Survey.

$x \in P_4^*$	$100n_x/N$	$f(M_x;z)$	C_x
1100	1.68%	0.42	19.44%
0011	0.87%	0.46	11.11%
1110	1.79%	0.6	22.22%
1101	1.34%	0.5	25%
1011	0.41%	0.49	5.56%
0111	0.61%	0.49	8.33%
1111	0.43%	0.7	8.33%

Table 3. Profile decomposability of P_4^*-poverty. Source: Author's calculations using the 2004 US National Health Interview Survey.

Dimensions of poverty	Indicator	Deprived if…	Weight
Education	Years of Schooling	No household member has completed five years of schooling.	1/6
	Child School Attendance	Any school aged child is not attending school up to class 8.	1/6
Health	Child Mortality	Any child has died in the family.	1/6
	Nutrition	Any adult for whom there is nutritional information is malnourished.	1/6
Living Standard	Electricity	The household has no electricity.	1/18
	Improved Sanitation	The household's sanitation facility is not improved (according to MDG guidelines), or it is improved but shared with other households.	1/18
	Improved Drinking Water	The household does not have access to improved drinking water (according to MDG guidelines) or safe drinking water is more than a 30-minute walk from home, roundtrip.	1/18
	Flooring	The household has a dirt, sand or dung floor.	1/18
	Cooking Fuel	The household cooks with dung, wood or charcoal.	1/18
	Assets ownership	The household does not own more than one radio, TV, telephone, bike, motorbike or refrigerator and does not own a car or truck.	1/18

Table 4. Dimensions, indicators, deprivation cutoffs and weights on the MPI. Source: Own elaboration.

Country	$M_0(AF)$	$M_0(P_{10}^*)$	$M_0(Q_{10}^*)$	$\Pi_{\theta^*}(AF)$	$m_i(P_{10}^*)$	$m_i(Q_{10}^*)$	$\delta(\boldsymbol{\theta})_i$
Albania	0.010	0.002	0.003	0.015	1.96	1.19	56.74
Armenia	0.007	0	0	0.013	2.07	0.72	78.21
Azerbaijan	0.031	0.008	0.012	0.045	6.05	3.25	45.15
Bangladesh	0.354	0.135	0.291	0.447	49.55	15.08	26.52
Benin	0.384	0.202	0.363	0.478	41.49	14.21	24.63
Bolivia	0.113	0.036	0.099	0.157	18.50	11.96	38.48
Burkina Faso	0.46	0.296	0.413	0.558	37.56	8.13	21.26
Burundi	0.433	0.191	0.422	0.539	53.89	14.33	24.47
Cambodia	0.368	0.18	0.365	0.485	41.79	18.12	31.89
Cameroon	0.195	0.09	0.196	0.254	24.25	17.01	30.48
Colombia	0.039	0.011	0.02	0.058	7.42	3.97	48.09
Congo	0.247	0.087	0.21	0.338	37.08	19.58	36.86
Cote Ivoire	0.293	0.143	0.236	0.380	36.26	12.46	29.62
Dominic. Rep	0.034	0.007	0.019	0.049	7.18	3.65	43.25
Egypt	0.031	0.01	0.013	0.045	5.37	3.28	44.61
Ethiopia	0.513	0.36	0.501	0.629	32.53	9.11	22.53
Gabon	0.085	0.017	0.048	0.126	17.17	9.39	47.79
Guinea	0.435	0.266	0.391	0.526	37.09	9.94	21.08
Guyana	0.035	0.004	0.027	0.050	7.79	6.08	43.03
Haiti	0.335	0.171	0.308	0.439	38.47	17.92	30.74
Honduras	0.164	0.043	0.141	0.221	30.28	17.66	35.26
India	0.294	0.137	0.258	0.373	35.81	10.07	26.71
Indonesia	0.141	0.019	0.092	0.195	31.01	11.65	38.35
Jordan	0.039	0.004	0.004	0.070	10.06	3.84	78.80
Kenya	0.234	0.095	0.255	0.314	32.03	25.41	34.02
Lesotho	0.141	0.049	0.17	0.199	22.34	26.70	40.77
Liberia	0.441	0.253	0.45	0.560	40.48	16.39	27.16
Madagascar	0.377	0.268	0.393	0.498	24.36	17.85	32.20
Malawi	0.269	0.088	0.298	0.354	41.74	29.11	31.56
Maldives	0.064	0.022	0.024	0.099	10.79	8.50	56.08
Mali	0.495	0.321	0.457	0.599	37.65	9.33	21.05
Moldova	0.097	0.037	0.044	0.154	16.52	7.44	58.81
Mozambique	0.488	0.296	0.456	0.609	39.87	12.58	24.89
Namibia	0.197	0.075	0.209	0.267	27.95	19.89	35.10
Nepal	0.277	0.123	0.221	0.358	36.89	11.41	29.15
Niger	0.524	0.373	0.516	0.640	31.45	8.05	22.13
Pakistan	0.24	0.098	0.159	0.319	34.14	13.80	33.27
Peru	0.054	0.009	0.047	0.075	11.28	7.25	40.25
Philippines	0.116	0.023	0.087	0.161	23.13	10.29	39.02
Rwanda	0.376	0.145	0.373	0.479	52.52	16.91	27.31
Sao Tome & Pr	0.186	0.04	0.183	0.250	35.88	15.85	33.89
Senegal	0.272	0.157	0.232	0.347	26.97	10.80	27.54
Tanzania	0.376	0.129	0.342	0.488	54.70	18.39	29.90
Timor-Leste	0.341	0.184	0.33	0.449	35.34	19.71	31.82
Uganda	0.376	0.147	0.348	0.484	50.77	19.75	28.78
Ukraine	0.002	0	0	0.003	0.54	0.17	54.66
Zambia	0.325	0.158	0.334	0.434	36.89	22.96	33.54
Zimbabwe	0.283	0.076	0.224	0.386	47.30	18.05	36.43

Table 5. Poverty measures $M_0(AF), M_0(P_{10}^*)$, $M_0(Q_{10}^*)$, $\Pi_{\theta^*}(AF)$ ($\boldsymbol{\theta}^*$=(2,3,3,0.5)), percentage of misclassified households using alternative identification criteria and values of $\delta(\boldsymbol{\theta}^*)_i$ for 48 countries. Source: Author's calculations using DHS data.